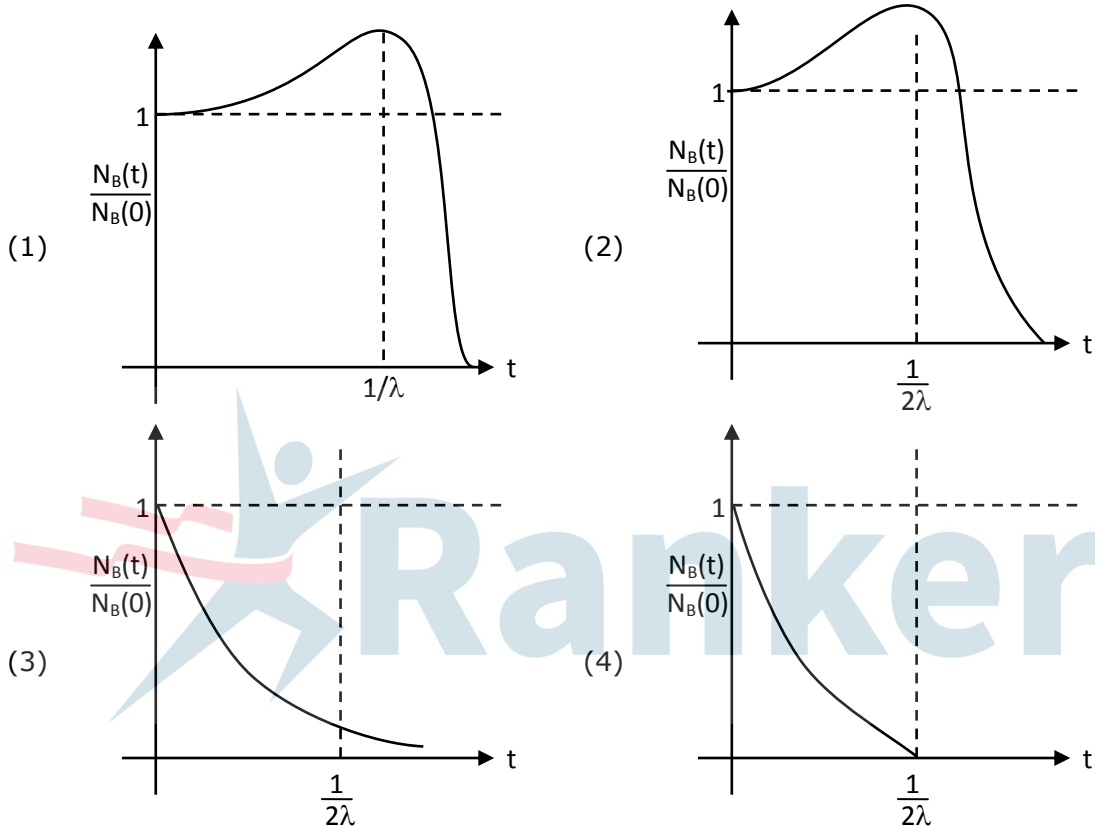


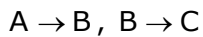
SECTION - A

1. At time $t = 0$, a material is composed of two radioactive atoms A and B, where $N_A(0) = 2N_B(0)$. The decay constant of both kind of radioactive atoms is λ . However, A disintegrates to B and B disintegrates to C. Which of the following figures represents the evolution of $N_B(t)/N_B(0)$ with respect to time t ?

$$\begin{cases} N_A(0) = \text{No. of A atoms at } t = 0 \\ N_B(0) = \text{NO. of B atoms at } t = 0 \end{cases}$$



Sol. 2



$$\frac{dN_B}{dt} = \lambda N_A - \lambda N_B, \quad \frac{dN_B}{dt} = 2\lambda N_{B_0} e^{-\lambda t} - \lambda N_B$$

$$e^{-\lambda t} \left(\frac{dN_B}{dt} + \lambda N_B \right) = 2\lambda N_{B_0} e^{-\lambda t} \times e^{\lambda t}$$

$$\frac{d}{dt} (N_B e^{\lambda t}) = 2\lambda N_{B_0}, \text{ on integrating}$$

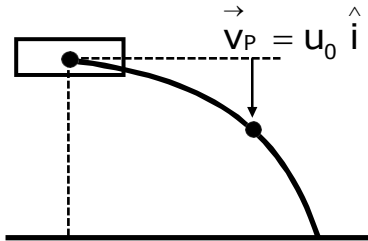
$$N_B e^{\lambda t} = 2\lambda t N_{B_0} + N_{B_0}, \quad N_B = N_{B_0} [1 + 2\lambda t] e^{-\lambda t}$$

$$\frac{dN_B}{dt} = 0 \text{ at } -\lambda [1 + 2\lambda t] e^{-\lambda t} + 2\lambda e^{-\lambda t} = 0$$

$$N_{B_{\max}} \text{ at } t = \frac{1}{2\lambda}$$

2. A bomb is dropped by a fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a:
- (1) Parabola in a direction opposite to the motion of plane
 - (2) Straight line vertically down the plane
 - (3) hyperbola
 - (4) parabola in the direction of motion of plane

Sol. 2



$$\vec{v}_B = u_0 \hat{i} - gt \hat{j}$$

$$\vec{v}_{B/P} = \vec{v}_B - \vec{v}_P$$

$$\vec{v}_{B/P} = -gt \hat{j}$$

Straight line vertically down

3. The temperature of equal masses of three different liquids x, y and z are 10°C , 20°C and 30°C respectively. The temperature of mixture when x is mixed with y is 16°C and that when y is mixed with z is 26°C . The temperature of mixture when x and z are mixed will be:

- (1) 28.32°C (2) 23.84°C (3) 20.28°C (4) 25.62°C

Sol. 2

X

$$m_1 = m$$

$$T_1 = 10^\circ\text{C}$$

$$S_1$$

Y

$$m_2 = m$$

$$T_2 = 20^\circ\text{C}$$

$$S_2$$

Z

$$m_3 = m$$

$$T_3 = 30^\circ\text{C}$$

$$S_3$$

When x & y are mixed, $T_f = 16^\circ\text{C}$

$$m_1 s_1 T_1 + m_2 s_2 T_2 = (m_1 s_1 + m_2 s_2) T_f$$

$$s_1 \times 10 + s_2 \times 20 = (s_1 + s_2) \times 16$$

$$s_1 = \frac{2}{3} s_2 \quad \dots(1)$$

When y & z are mixed, $T_f = 26^\circ\text{C}$

$$m_2 s_2 T_2 + m_3 s_3 T_3 = (m_2 s_2 + m_3 s_3) T_f$$

$$s_2 \times 20 + s_3 \times 30 = (s_2 + s_3) \times 26$$

$$s_3 = \frac{3}{2} s_2 \quad \dots(2)$$

When x & z are mixed

$$m_1 s_1 T_1 + m_3 s_3 T_3 = (m_1 s_1 + m_3 s_3) T_f$$

$$\frac{2}{3} s_2 \times 10 + \frac{3}{2} s_2 \times 30 = \left(\frac{2}{3} s_2 + \frac{3}{2} s_2 \right) T_f$$

$$T_f = 23.84^\circ\text{C}$$

4. Match List-I with List-II

List-I

- (a) Magnetic Induction
- (b) Magnetic Flux
- (c) Magnetic Permeability
- (d) Magnetization

List-II

- (i) $ML^2T^{-2}A^{-1}$
- (ii) $M^0L^{-1}A$
- (iii) $MT^{-2}A^{-1}$
- (iv) $MLT^{-2}A^{-2}$

Choose the most appropriate answer from the options given below

- (1) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)
- (2) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
- (3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
- (4) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Sol. 3

- (a) Magnetic Induction = $MT^{-2}A^{-1}$
- (b) Magnetic Flux = $ML^2T^{-2}A^{-1}$
- (c) Magnetic Permeability = $MLT^{-2}A^{-2}$
- (d) Magnetization = $M^0L^{-1}A$

5. The de-Broglie wavelength of a particle having kinetic energy E is λ . How much extra energy must be given to this particle so that the de-Broglie wavelength reduces to 75% of the initial value ?

- (1) $\frac{1}{9}E$
- (2) E
- (3) $\frac{7}{9}E$
- (4) $\frac{16}{9}E$

Sol. 3

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}, \quad mv = \sqrt{2mE}$$

$$\lambda \propto \frac{1}{\sqrt{E}}$$

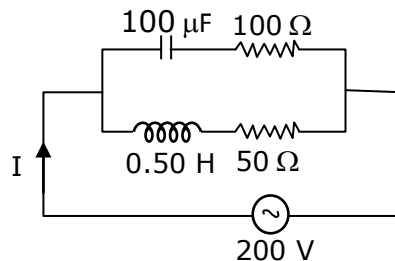
$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{E_1}{E_2}} = \frac{3}{4}, \quad \lambda_2 = 0.75\lambda_1$$

$$\frac{E_1}{E_2} = \left(\frac{3}{4}\right)^2$$

$$E_2 = \frac{16}{9}E_1 = \frac{16}{9}E \quad (E_1 = E)$$

$$\text{Extra energy given} = \frac{16}{9}E - E = \frac{7}{9}E$$

6. In the given circuit the AC source has $\omega = 100 \text{ rad s}^{-1}$. Considering the inductor and capacitor to be ideal, what will be the current I flowing through the circuit?



- (1) 5.9 A
- (2) 0.94 A
- (3) 4.24 A
- (4) 6 A

Sol. NTA Ans. 4.24
Motion Ans. 3

$$Z_C = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$$

$$= \sqrt{\left(\frac{1}{100 \times 100 \times 10^{-6}}\right)^2 + 100^2}$$

$$Z_C = \sqrt{(100)^2 + (100)^2}$$

$$= 100\sqrt{2}$$

$$Z_L = \sqrt{(\omega L)^2 + R^2}$$

$$= \sqrt{(100 \times 0.5)^2 + 50^2}$$

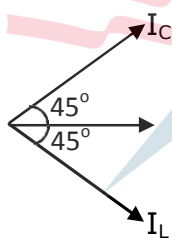
$$= 50\sqrt{2}$$

$$i_C = \frac{200}{Z_C} = \frac{200}{100\sqrt{2}} = \sqrt{2}$$

$$i_L = \frac{200}{Z_L} = \frac{200}{50\sqrt{2}} = 2\sqrt{2}$$

$$\cos \phi_1 = \frac{100}{10\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_1 = 45^\circ$$

$$\cos \phi_2 = \frac{50}{50\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_2 = 45^\circ$$



$$I = \sqrt{I_C^2 + I_L^2} = \sqrt{2 + 8} = \sqrt{10}$$

$$I = 3.16 \text{ A}$$

7. If the length of the pendulum in pendulum clock increases by 0.1%, then the error in time per day is ;

(1) 8.64 s

(2) 86.4 s

(3) 4.32 s

(4) 43.2 s

Sol. 4

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\frac{\Delta T}{T} = \frac{1}{2} \frac{\Delta \ell}{\ell}$$

$$\Delta T = \frac{1}{2} \times \frac{0.1}{100} \times 24 \times 3600$$

$$\Delta T = 43.2$$

8. The two thin coaxial rings, each of radius 'a' and having charges +Q and -Q respectively are separated by a distance of 's'. The potential difference between the centre of the two rings is:

(1) $\frac{Q}{4\pi\epsilon_0} \left[\frac{1}{a} + \frac{1}{\sqrt{s^2 + a^2}} \right]$ (2) $\frac{Q}{2\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{\sqrt{s^2 + a^2}} \right]$

(3) $\frac{Q}{4\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{\sqrt{s^2 + a^2}} \right]$ (4) $\frac{Q}{2\pi\epsilon_0} \left[\frac{1}{a} + \frac{1}{\sqrt{s^2 + a^2}} \right]$

Sol. 2

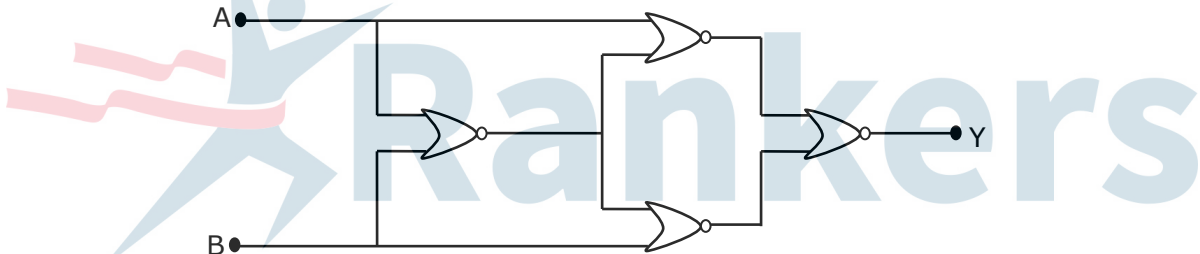
$$V_A = \frac{KQ}{a} - \frac{KQ}{\sqrt{a^2 + s^2}}$$

$$V_B = \frac{-KQ}{a} + \frac{KQ}{\sqrt{a^2 + s^2}}$$

$$V_A - V_B = \frac{2KQ}{a} - \frac{2KQ}{\sqrt{a^2 + s^2}}$$

$$= \frac{Q}{2\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{s^2 + a^2} \right)$$

9. Four NOR gates are connected as shown in figure. The truth table for the given figure is:



(1)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(2)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

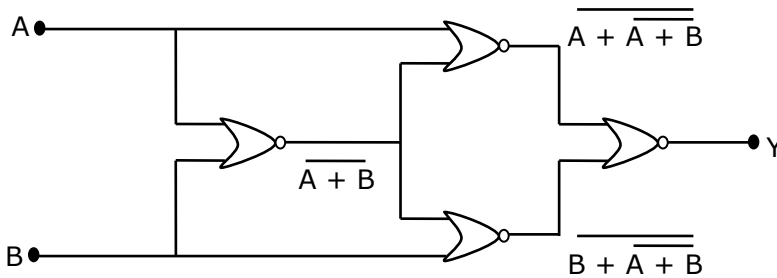
(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Sol. 4



$$y = \overline{\overline{(A + A + B)} + \overline{(B + A + B)}}$$

$$y = (A + \overline{A + B}) \cdot (B + \overline{A + B})$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

10. A cylindrical container of volume $4.0 \times 10^{-3} \text{ m}^3$ contains one mole of hydrogen and two moles of carbon dioxide. Assume the temperature of the mixture is 400 K. The pressure of the mixture of gases is : [Take gas constant as $8.3 \text{ J mol}^{-1}\text{K}^{-1}$]
- (1) $24.9 \times 10^3 \text{ Pa}$ (2) 24.9 Pa (3) $249 \times 10^1 \text{ Pa}$ (4) $24.9 \times 10^5 \text{ Pa}$

Sol. 4

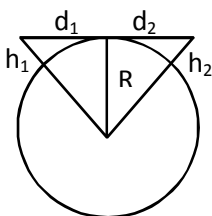
$$V = 4 \times 10^{-3} \text{ m}^3, n = 3 \text{ moles}, T = 400 \text{ K}$$

$$PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$P = \frac{3 \times 8.3 \times 400}{4 \times 10^{-3}} = 24.9 \times 10^5 \text{ Pa}$$

11. A transmitting antenna at top of a tower has height of 50 m and the height of receiving antenna is 80m. What is the range of communication for line of Sight (LoS) mode?
[use radius of earth = 6400 km]
- (1) 45.5 km (2) 57.28 km (3) 80.2 km (4) 144.1 km

Sol. 2



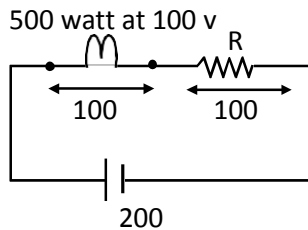
$$d_t = \sqrt{2Rh_1} + \sqrt{2Rh_2} = \sqrt{2R} (\sqrt{h_1} + \sqrt{h_2})$$

$$= (2 \times 6400 \times 10^3)^{1/2} (\sqrt{50} + \sqrt{80}) = 3578 (7.07 + 8.94) = 57.28 \text{ Km}$$

12. An electric bulb of 500 watt at 100 volt is used in a circuit having a 200 v supply. Calculate the resistance R to be connected in series with the bulb so that the power delivered by the bulb is 500 W.

- (1) 20 Ω (2) 5 Ω (3) 30 Ω (4) 10 Ω

Sol. 1



$$P = Vi$$

$$500 = Vi$$

$$I = 5 \text{ Amp}$$

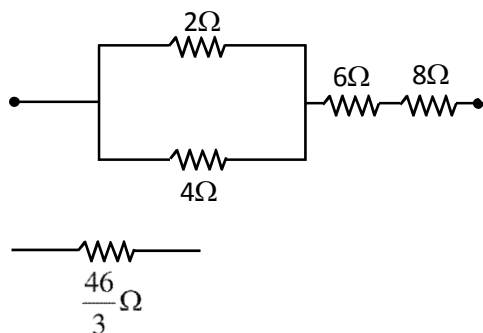
$$V = i \times R$$

$$R = 20$$

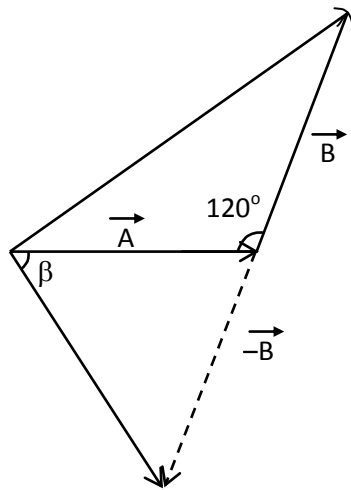
13. If you are provided a set of resistances 2 Ω , 4 Ω , 6 Ω and 8 Ω . Connect these resistances so as to obtain an equivalent resistance of $\frac{46}{3} \Omega$.

- (1) 2 Ω and 4 Ω are in parallel with 6 Ω and 8 Ω in series
 (2) 6 Ω and 8 Ω are in parallel with 2 Ω and 4 Ω in series
 (3) 4 Ω and 6 Ω are in parallel with 2 Ω and 8 Ω in series
 (4) 2 Ω and 6 Ω are in parallel with 4 Ω and 8 Ω in series

Sol. 1



16. The angle between vector (\vec{A}) and $(\vec{A}-\vec{B})$ is :



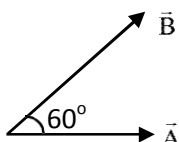
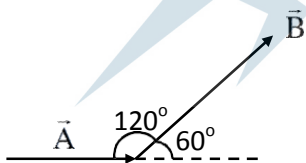
(1) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)$

(2) $\tan^{-1}\left(\frac{-\frac{B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$

(3) $\tan^{-1}\left(\frac{A}{0.7B}\right)$

(4) $\tan^{-1}\left(\frac{B \cos \theta}{A - B \sin \theta}\right)$

Sol. 1



Angle between \vec{A} and \vec{B} , $\theta = 60^\circ$

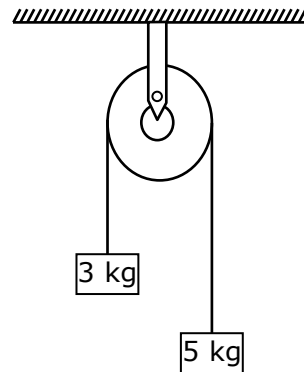
Angle between \vec{A} and $\vec{A} - \vec{B}$

$$\tan \alpha = \frac{B \sin \theta}{A - B \cos \theta}$$

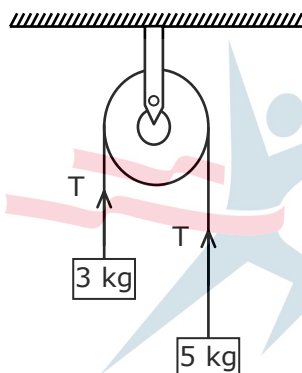
$$= \frac{B \frac{\sqrt{3}}{2}}{A - B \times \frac{1}{2}}$$

$$\tan \alpha = \frac{\sqrt{3}B}{2A - B}$$

- 17.** Two blocks of masses 3 kg and 5 kg are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $\frac{24}{\pi} \times 10^2 \text{ Nm}^{-2}$. What is the minimum radius of the wire?
(take $g = 10 \text{ ms}^{-2}$)



- Sol.** (1) 125 cm (2) 150 cm (3) 1.25 cm (4) 12.5 cm
4



$$T = \frac{2m_1m_2g}{m_1 + m_2} = \frac{2 \times 3 \times 5 \times 10}{8} = \frac{75}{2}$$

$$\text{Stress} = \frac{T}{A}$$

$$\frac{24}{\pi} \times 10^2 = \frac{75}{2 \times \pi R^2}$$

$$R^2 = \frac{75}{2 \times 24 \times 100} = \frac{3}{8 \times 24}$$

$$\Rightarrow R = 0.125 \text{ m}$$

$$R = 12.5 \text{ cm}$$

- 18.** A refrigerator consumes an average 35W power to operate between temperature -10°C to 25°C . If there is no loss of energy then how much average heat per second does it transfer?

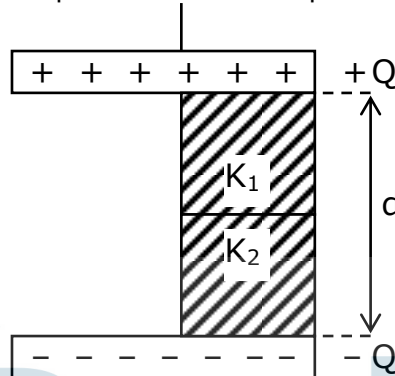
- Sol.** (1) 263 J/s (2) 298 J/s (3) 35 J/s (4) 350 J/s
1

$$\frac{T_L}{T_H - T_L} = \text{C.O.P.} = \frac{dH}{dW}$$

$$\frac{263}{35} \times 35 = \frac{dH}{dt}$$

$$\frac{dH}{dt} = 263 \text{ watts}$$

19. A parallel-plate capacitor with plate area A has separation d between the plates. Two dielectric slabs of dielectric constant K_1 and K_2 of same area $A/2$ and thickness $d/2$ are inserted in the space between the plates. The capacitance of the capacitor will be given by :



(1) $\frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{K_1 K_2}{K_1 + K_2} \right)$

(2) $\frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{2(K_1 + K_2)}{K_1 K_2} \right)$

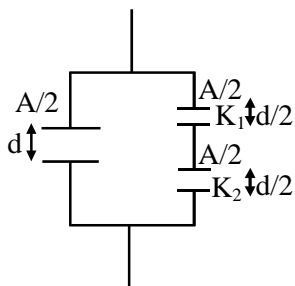
(3) $\frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{K_1 + K_2}{K_1 K_2} \right)$

(4) $\frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{K_1 K_2}{2(K_1 + K_2)} \right)$

Sol. 1

$$C_{\text{eq}} = \frac{\frac{A}{2} \epsilon_0}{d} + \frac{A \epsilon_0}{d} \frac{K_1 K_2}{K_1 + K_2}$$

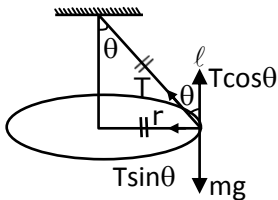
$$= \frac{A \epsilon_0}{d} \left(\frac{1}{2} + \frac{K_1 K_2}{K_1 + K_2} \right)$$



20. A particle of mass m is suspended from a ceiling through a string of length L . The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$. The speed of particle will be;

- (1) $2\sqrt{rg}$ (2) $\sqrt{2rg}$ (3) \sqrt{rg} (4) $\sqrt{\frac{rg}{2}}$

Sol. 3
Conical pendulum



$$r = \frac{l}{\sqrt{2}}$$

$$\sin \theta = \frac{r}{l} = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

$$T \sin \theta = \frac{mv^2}{r}$$

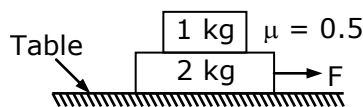
$$T \cos \theta = mg$$

$$\tan \theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$$

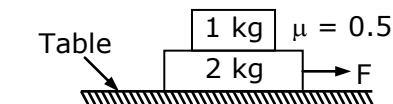
Rankers

Section B

1. The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is _____ N. (take $g = 10 \text{ ms}^{-2}$)



Sol. 15



$$F = 3a \text{ (for system) } \dots(i)$$



$$f_{s_{\max}} = 1a \text{ (for 1kg block) } \dots(ii)$$

$$\mu \times 1 \times g = a$$

$$\Rightarrow 5 = a$$

$$F = 15 \text{ N}$$

2. The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum of known mass 'm' to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as _____%.

Sol. 14

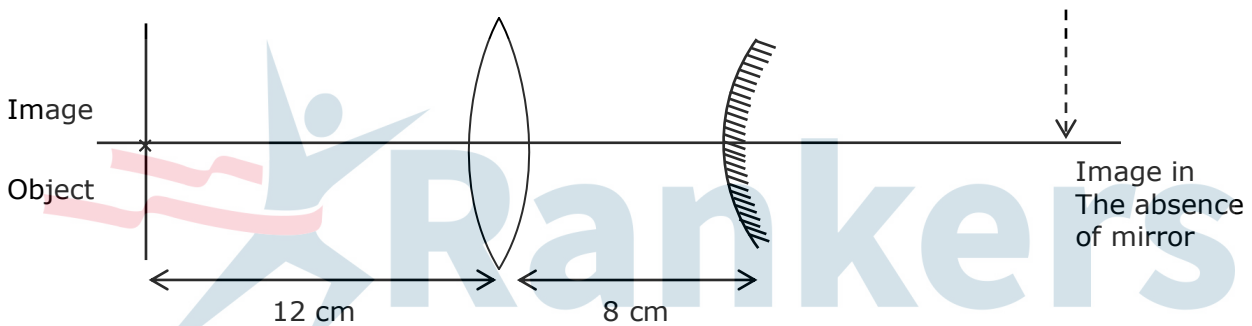
$$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \ell = \frac{T^2 g}{4\pi^2}$$

$$E = mg\ell \frac{\theta^2}{2} = mg^2 \frac{T^2 \theta^2}{8\pi^2}$$

$$\frac{dE}{E} = 2 \left(\frac{dg}{g} + \frac{dT}{T} \right)$$

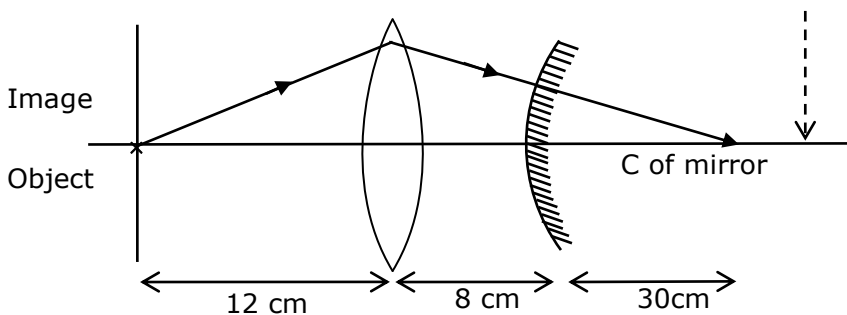
$$= (4 + 3) = 14\%$$

3. An object is placed at a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8 cm as shown in the figure. Image of object coincides with the object.



When the convex mirror is removed, a real and inverted image is formed at a position. The distance of the image from the object will be _____ (cm).

Sol. 50



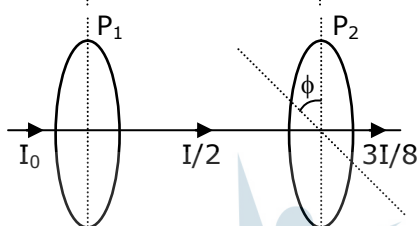
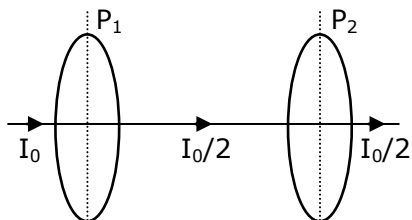
For the object to coincide with image, the light must fall perpendicularly to mirror. Which means that the light will have to converge at C of mirror. Without the mirror also, the light would converge at C.

So, the distance is : $12 + 8 + 30 = 50$ cm

4. A source of light is placed in front of a screen. Intensity of light on the screen is I . Two Polaroids P_1 and P_2 are so placed in between the source of light and screen that the intensity of light on screen is $I/2$. P_2 should be rotated by an angle of _____ (degrees) so that the intensity of light on the screen becomes $\frac{3I}{8}$.

Sol. 30

$$I = \frac{I_0}{2} \cos^2 \phi$$



$$\frac{I}{2} \cos^2 \phi = \frac{3I}{8}$$

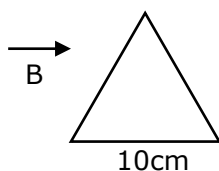
$$\cos^2 \phi = \frac{3}{4}$$

$$\cos^2 \phi = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \phi = 30$$

5. A coil in the shape of an equilateral triangle of side 10 cm lies in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 20 mT. The torque acting on the coil when a current of 0.2 A is passed through it and its plane becomes parallel to the magnetic field will be $\sqrt{x} \times 10^{-5} \text{ Nm}$. The value of x is _____.

Sol. 3



$$\vec{\tau} = \vec{M} \times \vec{B} = MB \sin 90^\circ$$

$$= MB = \frac{i\sqrt{3}l^2}{4} B$$

$$= \sqrt{3} \times 10^{-5} \text{ N-m}$$

6. If the maximum value of accelerating potential provided by a radio frequency oscillator is 12 kV. The number of revolution made by a proton in a cyclotron to achieve one sixth of the speed of light is _____.

$$[m_p = 1.67 \times 10^{-27} \text{kg}, e = 1.6 \times 10^{-19} \text{ C}, \text{ Speed of light} = 3 \times 10^8 \text{ m/s}]$$

Sol. 543

$$V = 12 \text{ kV}$$

Number of revolution = n

$$n [2 \times q_p \times V] = \frac{1}{2} m_p \times v_p^2$$

$$n [2 \times 1.6 \times 10^{-19} \times 12 \times 10^3]$$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times \left[\frac{3 \times 10^8}{6} \right]^2$$

$$n (38.4 \times 10^{-16}) = 0.2087 \times 10^{11}$$

$$n = 543.4$$

7. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s⁻¹ in a uniform horizontal magnetic field of 3.0 × 10⁻²T. The maximum emf induced the coil will be _____ × 10⁻² volt (rounded off to the nearest integer).

Sol. 60

Maximum emf $\varepsilon = N\omega AB$

$$N = 20, \omega = 50, B = 3 \times 10^{-2} \text{ T}$$

$$\varepsilon = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2} = 60.28 \times 10^{-2}$$

Rounded off to nearest integer = 60

8. Two waves are simultaneously passing through a string and their equations are: $y_1 = A_1 \sin k(x - vt)$, $y_2 = A_2 \sin k(x - vt + x_0)$. Given amplitudes $A_1 = 12 \text{ mm}$ and $A_2 = 5 \text{ mm}$, $x_0 = 3.5 \text{ cm}$ and wave number $k = 6.28 \text{ cm}^{-1}$. The amplitude of resulting wave will be _____ mm.

Sol. 7

$$y_1 = A_1 \sin k(x - vt)$$

$$y_2 = 12 \sin 6.28(x - vt)$$

$$y_2 = 5 \sin 6.28(x - vt + 3.5)$$

$$\Delta\phi = \frac{2\pi}{\lambda} (\Delta x)$$

$$= K (\Delta x)$$

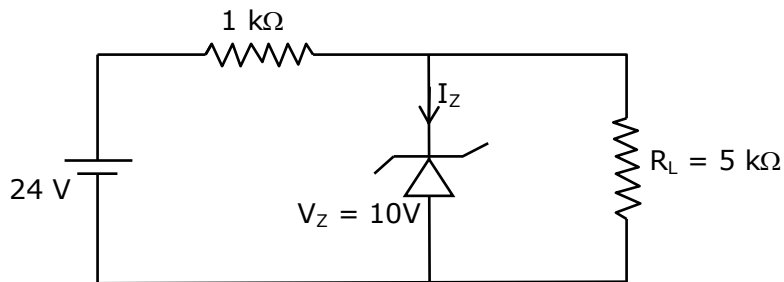
$$= 6.28 \times 3.5 = \frac{7}{2} \times 2\pi = 7\pi$$

$$A_{\text{net}} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

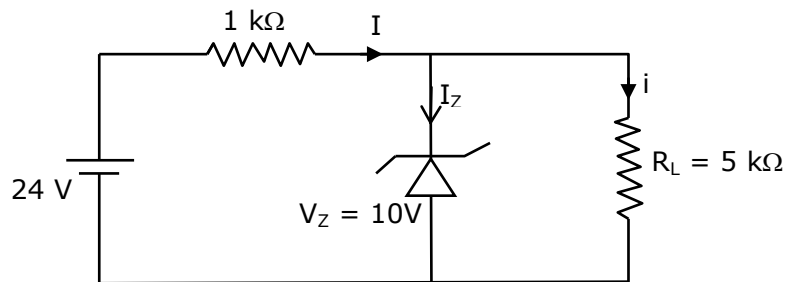
$$A_{\text{net}} = \sqrt{(12)^2 + (5)^2 + 2(12)(5) \cos(7\pi)}$$

$$= \sqrt{144 + 25 - 120} = 7$$

9. For the given circuit, the power across zener diode is _____ mW.



Sol. 120



$$i = \frac{10V}{5k\Omega} = 2mA$$

$$I = \frac{14V}{1k\Omega} = 14mA$$

$$\therefore I_z = 12mA$$

$$\therefore P = I_z V_z = 120 \text{ mW}$$

10. Two simple harmonic motions are represented by the equations $x_1 = 5 \sin\left(2\pi t + \frac{\pi}{4}\right)$ and $x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$. The amplitude of second motion is _____ times the amplitude in first motion.

Sol. 2

$$x_2 = 5\sqrt{2} \left(\frac{1}{\sqrt{2}} \sin 2\pi t + \frac{1}{\sqrt{2}} \cos 2\pi t \right) \sqrt{2}$$

$$= 10 \sin \left(2\pi t + \frac{\pi}{4} \right)$$

$$\frac{A_2}{A_1} = \frac{10}{5} = 2$$