MATHEMATICS
JEE-MAIN (August-Attempt)
31 August (Shift-2) Paper

SECTION - A

1. The sum of the roots of the equation, $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is:

(1) $\log_2 12$ (2) $\log_2 14$ (3) $\log_2 11$ (4) $\log_2 13$

Ans. (3)

Sol. $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$

$\log_2(2^{x+1}) - \log_2(3 + 2^x)^2 + \log_2(10 - 2^{-x}) = 0$

$$\log_2\left(\frac{2^{x+1}.(10-2^{-x})}{(3+2^x)^2}\right)=0$$

$$\frac{2(10.2^x-1)}{(3+2^x)^2}=1$$

$\Rightarrow 20.2^x - 2 = 9 + 2^{2x} + 6.2^x$

$\therefore (2^x)^2 - 14(2^x) + 11 = 0$

Roots are 2^{x_1} & 2^{x_2}

$\therefore 2^{x_1}\cdot 2^{x_2} = 11$

$x_1 + x_2 = \log_2(11)$

2. The distance of the point $(-1, 2, -2)$ from the line of intersection of the planes $2x + 3y + 2z=0$ and $x-2y+z=0$ is:

(1) $\frac{\sqrt{42}}{2}$ (2) $\frac{5}{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{\sqrt{34}}{2}$

Ans. (4)

Sol. $P_1 : 2x + 3y + 2z = 0$

$\Rightarrow \vec{n}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$P_2 : x - 2y + z = 0$

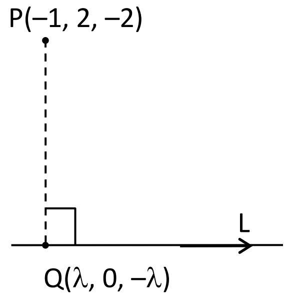
$\Rightarrow \vec{n}_2 = \hat{i} - 2\hat{j} + \hat{k}$

Direction vector of line L which is line of intersection of P_1 & P_2

$\vec{r} = \vec{n}_1 \times \vec{n}_2 = 7\hat{i} - 7\hat{k}$

DR's of L are $(1, 0, -1)$

$\Rightarrow$ Equation of L : $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1} = \lambda$



DR's of $\overrightarrow{PQ} = (\lambda + 1, -2, 2 - \lambda)$

$\because \overrightarrow{PQ} \perp \vec{r}$

$\Rightarrow (\lambda + 1)(1) + (-2)(0) + (2 - \lambda)(-1) = 0$

$\Rightarrow \lambda = \frac{1}{2} \Rightarrow Q\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$

$\Rightarrow PQ = \frac{\sqrt{34}}{2}$

3. If $\alpha = \lim_{x\to\pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ and $\beta = \lim_{x\to 0}(\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is:

(1) (−1, −3) (2) (−1, 3) (3) (1, 3) (4) (1, −3)

Ans. **(3)**

Sol. $\alpha = \lim_{x\to\frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$; $\frac{0}{0}$ form

Using L Hospital rule

$\alpha = \lim_{x\to\frac{\pi}{4}} \frac{3\tan^2 x \sec^2 x - \sec^2 x}{-\sin\left(x + \frac{\pi}{4}\right)}$

$\Rightarrow \alpha = -4$

$\beta = \lim_{x\to 0}(\cos x)^{\cot x} = e^{\lim_{x\to 0}\frac{(\cos x - 1)}{\tan x}}$

$\beta = e^{\lim_{x\to 0}\frac{-(1-\cos x)}{x^2}\cdot\frac{x^2}{\left(\frac{\tan x}{x}\right)^x}}$

$\beta = e^{\lim_{x\to 0}\left(\frac{-1}{2}\right)\cdot\frac{x}{1}} = e^0 \Rightarrow \beta = 1$

$\alpha = -4;\ \beta = 1$

If $ax^2 + bx - 4 = 0$ are the roots then

$16a - 4b - 4 = 0$ & $a + b - 4 = 0$

$\Rightarrow a = 1$ & $b = 3$

4. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If f(0) =0, f(1) = 1 and f(2) = 2, then:

(1) $f''(x) > 0$ for all $x \in (0, 2)$
(2) $f'(x) = 0$ for some $x \in [0, 2]$
(3) $f''(x) = 0$ for all $x \in (0, 2)$
(4) $f''(x) = 0$ for some $x \in (0, 2)$

Ans. (4)

Sol. f(0) = 0, f(1) = 1 and f(2) = 2

Let h(x) = f(x) – x has three roots

By Rolle's theorem h′(x) = f′(x) – 1 has at least two roots

h″(x) = f″(x) = 0 has at least one roots.

5. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is:

(1) $p \wedge q \wedge r$
(2) $\sim p \wedge q \wedge r$
(3) $p \wedge \sim q \wedge \sim r$
(4) $\sim P \wedge q \wedge \sim r$

Ans. (3)

Sol. $\because \sim(A \Rightarrow B) = A \wedge \sim B$

$\therefore \sim((p \vee r) \Rightarrow (q \vee r))$

$= (p \vee r) \wedge (\sim q \wedge \sim r)$

$= ((p \vee r) \wedge (\sim r)) \wedge (\sim q)$

$= p \wedge (\sim r) \wedge (\sim q)$

6. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)\, y + (\cos\beta)z = 0$

$(\cos\gamma)\, x + y + (\cos\alpha)z = 0$

$(\cos\beta)\, x + (\cos\alpha)y + z = 0$

has:

(1) exactly two solutions
(2) a unique solution
(3) no solution
(4) infinitely many solutions

Ans. (4)

Sol. $\alpha + \beta + \gamma = 2\pi$

$$\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$= 1 + 2\cos\alpha.\cos\beta.\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + (\cos(\alpha+\beta) + \cos(\alpha-\beta))\cos\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + \cos^2\gamma + \cos(\alpha-\beta)\cos\gamma$

$= \sin^2\alpha - \cos^2\beta + \cos(\alpha-\beta)\cos(\alpha+\beta)$

$= \sin^2\alpha - \cos^2\beta + \cos^2\alpha - \sin^2\beta = 0$

7. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\frac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots + a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to:

(1) $\frac{121}{100}$ (2) $\frac{100}{121}$ (3) $\frac{19}{21}$ (4) $\frac{21}{19}$

Ans. (4)

Sol. $$\frac{\frac{10}{2}(2a_1 + 9d)}{\frac{p}{2}(2a_1 + (p-1)d)} = \frac{100}{p^2}$$

$(2a_1 + 9d)p = 10(2a_1 + (p-1)d)$

$9dp = 20a_1 - 2pa_1 + 10d(p-1)$

$9p = (20 - 2p)\frac{a_1}{d} + 10(p-1)$

$\frac{a_1}{d} = \frac{(10-p)}{2(10-p)} = \frac{1}{2}$

$$\therefore \frac{a_{11}}{a_{10}} = \frac{a_1 + 10d}{a_1 + 9d} = \frac{\frac{1}{2} + 10}{\frac{1}{2} + 9} = \frac{21}{19}$$

8. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies

$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$, then $\vec{r}$ is equal to:

(1) $\frac{1}{2}(\vec{a} + \vec{b} + 2\vec{c})$ (2) $\frac{1}{3}(2\vec{a} + \vec{b} - \vec{c})$ (3) $\frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ (4) $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$

Ans. **(4)**

Sol. Suppose, $\vec{r} = x\vec{a} + y\vec{b} + 2\vec{c}$

and $|\vec{a}| = |\vec{b}| = |\vec{c}| = k$

$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$

$\Rightarrow k^2(\vec{r} - \vec{b}) - k^2x\vec{a} + k^2(\vec{r} - \vec{c}) - k^2y\vec{b} + k^2(\vec{r} - \vec{a}) - k^2z\vec{c} = \vec{0}$

$\Rightarrow 3\vec{r} - (\vec{a} + \vec{b} + \vec{c}) - \vec{r} = \vec{0}$

$\Rightarrow \vec{r} = \dfrac{\vec{a} + \vec{b} + \vec{c}}{2}$

9. The locus of mid-points of the line segments joining (–3, –5) and the points on the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ is:

(1) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$ (2) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

(3) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$ (4) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

Ans. **(2)**

Sol. General point on $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$, is A $(2\cos\theta, 3\sin\theta)$

Given, B (–3, –5)

Midpoint $C\left(\dfrac{2\cos\theta - 3}{2}, \dfrac{3\sin\theta - 5}{2}\right)$

$h = \dfrac{2\cos\theta - 3}{2}; \; k = \dfrac{3\sin\theta - 5}{2}$

$\Rightarrow \left(\dfrac{2h+3}{2}\right)^2 + \left(\dfrac{2k+5}{3}\right)^2 = 1$

$\Rightarrow 36x^2 + 16y^2 + 108x + 80y + 145 = 0$

10. If $\dfrac{dy}{dx} = \dfrac{2^x y + 2^y \cdot 2^x}{2^x + 2^{x+y}\log_e 2}, y(0) = 0$, then for y = 1, the value of x lies in the interval:

(1) $\left(\dfrac{1}{2}, 1\right]$ (2) $\left(0, \dfrac{1}{2}\right]$ (3) (1, 2) (4) (2, 3)

Ans. **(3)**

Sol. $\frac{dy}{dx} = \frac{2^x(y+2^y)}{2^x(1+2^y \ell n 2)}$

$\Rightarrow \int \frac{(1+2^y)\ell n 2}{(y+2^y)} dy = \int dx$

$\Rightarrow \ell n|y+2^y| = x + c$

$x = 0;\ y = 0 \Rightarrow c = 0$

$\Rightarrow x = \ell n|y+2^y|$

$\Rightarrow \text{at } y = 1,\ x = \ell n 3$

$\because 3 \in (e, e^2) \Rightarrow x \in (1, 2)$

11. If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is

(1) $3\sqrt{2}$ (2) $6\sqrt{2}$ (3) $2\sqrt{2}$ (4) $2\sqrt{2}-1$

Ans. **(3)**

Sol. $\frac{z-i}{z-1}$ is purely Imaginary number

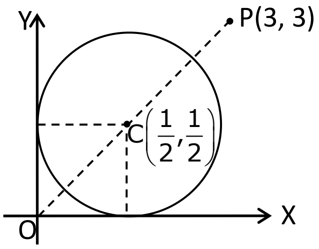
Let z = x + iy

$\therefore \frac{x+i(y-1)}{(x-1)+i(y)} \times \frac{(x-1)-iy}{(x-1)-iy}$

$\Rightarrow \frac{x(x-1)+y(y-1)+i(-y-x+1)}{(x-1)^2+y^2}$ is purely Imaginary number

$\Rightarrow x(x - 1) + y(y - 1) = 0$

$\Rightarrow \left(x-\frac{1}{2}\right)^2 + \left(y-\frac{1}{2}\right)^2 = \frac{1}{2}$



$\therefore |z-(3+3i)|_{min} = |PC| - \frac{1}{\sqrt{2}}$

$= \frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\sqrt{2}$

12. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is:

(1) $\frac{92}{5}$ (2) $\frac{134}{5}$ (3) $\frac{536}{25}$ (4) $\frac{112}{5}$

Ans. (3)

Sol. Let, 8, 16, x_1, x_2, x_3, x_4, x_5 be the observations.

Now, $\frac{x_1 + x_2 + ... + x_5 + 14}{7} = 8$

$\Rightarrow \sum_{i=1}^{5} x_i = 42$ ...(1)

Also, $\frac{x_1^2 + x_2^2 + ... + x_5^2 + 8^2 + 6^2}{7} - 64 = 16$

$\Rightarrow \sum_{i=1}^{5} x_i^2 = 560 - 100 = 460$ ...(2)

So, variance of x_1, x_2,...., x_5

$= \frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{2300 - 1764}{25} = \frac{536}{25}$

13. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81,\ 0 \le x \le \frac{\pi}{4}$ is:

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (2)

Sol. $(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$

In interval $\left[0, \frac{\pi}{4}\right]$ only one solution.

14. Let S = {1, 2, 3, 4, 5, 6}. Then the probability that a randomly chosen onto function g from S to S satisfies g(3) = 2g (1) is:

(1) $\frac{1}{5}$ (2) $\frac{1}{30}$ (3) $\frac{1}{15}$ (4) $\frac{1}{10}$

Ans. (4)

Sol. $g(3) = 2g(1)$ can be defined in 3 ways

number of onto functions in this condition = $3 \times 4!$

Total number of onto functions = $6!$

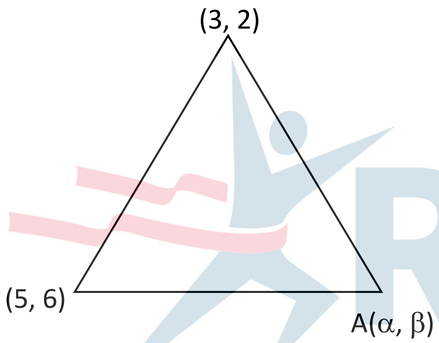
Required probability $= \frac{3 \times 4!}{6!} = \frac{1}{10}$

15. Let A be the set of all points (α, β) such that the area of triangle formed by the points (5,6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is:

(1) $\frac{16}{\sqrt{5}}$ (2) $\frac{12}{\sqrt{5}}$ (3) $\frac{8}{\sqrt{5}}$ (4) $\frac{4}{\sqrt{5}}$

Ans. **(3)**

Sol.



$$\left|\frac{1}{2}\begin{vmatrix} 5 & 6 & 1 \\ 3 & 2 & 1 \\ \alpha & \beta & 1 \end{vmatrix}\right| = 12$$

$4\alpha - 2\beta = \pm 24 + 8$

$\Rightarrow 4\alpha - 2\beta = +24 + 8 \Rightarrow 2\alpha - \beta = 16$

$2x - y - 16 = 0$ ....(1)

$\Rightarrow 4\alpha - 2\beta = -24 + 8 \Rightarrow 2\alpha - \beta = -8$

$2x - y + 8 = 0$ ....(2)

Perpendicular distance of (1) from (0, 0)

$$\left|\frac{0 - 0 - 16}{\sqrt{5}}\right| = \frac{16}{\sqrt{5}}$$

Perpendicular distance of (2) from (0, 0) is $\left|\frac{0 - 0 + 8}{\sqrt{5}}\right| = \frac{8}{\sqrt{5}}$

16. An angle of intersection of the curves, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2 + y^2 = ab$, $a > b$, is:

(1) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$ (2) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (3) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$ (4) $\tan^{-1}\left(2\sqrt{ab}\right)$

Ans. (3)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, x^2+y^2=ab$

$\frac{2x_1}{a^2}+\frac{2y_1y'}{b^2}=0$

$\Rightarrow y_1'=\frac{-x_1}{a^2}\frac{b^2}{y_1}$ ...(1)

$\therefore 2x_1+2y_1y'=0$

$\Rightarrow y_2'=\frac{-x_1}{y_1}$ ...(2)

Here (x_1, y_1) is point of intersection of both curves

$\therefore x_1^2=\frac{a^2b}{a+b}, y_1^2=\frac{ab^2}{a+b}$

$\therefore \tan\theta=\left|\frac{y_1'-y_2'}{1+y_1'y_2'}\right|=\left|\frac{\frac{-x_1b^2}{a^2y_1}+\frac{x_1}{y_1}}{1+\frac{x_1^2b^2}{a^2y_1^2}}\right|$

$\tan\theta=\left|\frac{-b^2x_1y_1+a^2x_1y_1}{a^2y_1^2+b^2x_1^2}\right|$

$\tan\theta=\left|\frac{a-b}{\sqrt{ab}}\right|$

17. If $[x]$ is the greatest integer $\le x$, then $\pi^2\int_0^2\left(\sin\frac{\pi x}{2}\right)(x-[x])^{[x]}\,dx$ is equal to:

(1) $4(\pi-1)$ (2) $2(\pi-1)$ (3) $2(\pi+1)$ (4) $4(\pi+1)$

Ans. (1)

Sol. $\pi^2\left[\int_0^1\sin\frac{\pi x}{2}dx+\int_1^2\sin\frac{\pi x}{2}(x-1)\,dx\right]$

$$= \pi^2 \left[-\frac{2}{\pi}\left(\cos\frac{\pi x}{2}\right) + \left((x-1)\left(-\frac{2}{\pi}\cos\frac{\pi x}{2}\right)\right)_1^2 - \int_1^2 -\frac{2}{\pi}\cos\frac{\pi x}{2}\,dx \right]$$

$$= \pi^2 \left[0 + \frac{2}{\pi} + \frac{2}{\pi} + \frac{2}{\pi}\cdot\frac{2}{\pi}\left(\sin\frac{\pi x}{2}\right)_1^2 \right]$$

$= 4\pi - 4 = 4(\pi - 1)$

18. Let $f : N \to N$ be a function such that $f(m+n) = f(m) + f(n)$ for every $m, n \in N$. If $f(6) = 18$, then $f(2) \bullet f(3)$ is equal to:

(1) 36 (2) 6 (3) 18 (4) 54

Ans. (4)

Sol. $f(m + n) = f(m) + f(n)$

Put $m = 1, n = 1$

$f(2) = 2f(1)$

Put $m = 2, n = 1$

$f(3) = f(2) + f(1) = 3f(1)$

Put $m = 3, n = 3$

$f(6) = 2f(3) \Rightarrow f(3) = 9$

$\Rightarrow f(1) = 3, f(2) = 6$

$f(2).f(3) = 6 \times 9 = 54$

19. The domain of the function

$$f(x) = \sin^{-1}\left(\frac{3x^2 + x - 1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right) \text{ is:}$$

(1) $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$ (2) $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$ (3) $\left[0, \frac{1}{4}\right]$ (4) $\left[0, \frac{1}{2}\right]$

Ans. (1)

Sol. $$f(x) = \sin^{-1}\left(\frac{3x^2 + x - 1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$$

$$-1 \le \frac{x-1}{x+1} \le 1 \Rightarrow 0 \le x < \infty \quad ...(1)$$

$$-1 \le \frac{3x^2 + x - 1}{(x-1)^2} \le 1 \Rightarrow x \in \left[\frac{-1}{4}, \frac{1}{2}\right] \cup \{0\} \quad ...(2)$$

Equation (1) & (2)

$$\Rightarrow \text{Domin} = \left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$$

20. If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, x > 0, ϕ > 0 and y(1) = −1, then $\phi\left(\frac{y^2}{4}\right)$ is equal to:

(1) $\phi(1)$ (2) $4\,\phi(2)$ (3) $2\,\phi(1)$ (4) $4\,\phi(1)$

Ans. **(4)**

Sol. Let, y = tx

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$\therefore\ tx\left(t + x\frac{dt}{dx}\right) = x\left(t^2 + \frac{\phi(t^2)}{\phi'(t^2)}\right)$$

$$t^2 + xt\frac{dt}{dx} = t^2 + \frac{\phi(t^2)}{\phi'(t^2)}$$

$$\int \frac{t\phi'(t^2)}{\phi(t^2)}dt = \int \frac{dx}{x}$$

Let $\phi(t^2) = p$

$\therefore\ \phi'(t^2)2t\ dt = dp$

$$\Rightarrow \int \frac{dy}{2p} = \int \frac{dx}{x}$$

$$\frac{1}{2}\ell n\phi(t^2) = \ell nx + \ell nc$$

$\Phi(t^2) = x^2k$

$$\phi\left(\frac{y^2}{x^2}\right) = kx^2,\ \phi(1) = k$$

$$\phi\left(\frac{y^2}{4}\right) = 4\phi(1)$$

SECTION-B

1. Suppose the line $\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ lies on the plane x + 3y − 2z + β = 0, then (α+β) is equal to___

Ans. **(7)**

Sol. Point (2, 2, −2) also lies on given plane

So, $2 + 3 \times 2 - 2(-2) + \beta = 0$

$\Rightarrow 2 + 6 + 4 + \beta = 0 \Rightarrow \beta = -12$

Also, $\alpha \times 1 - 5 \times 3 + 2 \times -2 = 0$

$\Rightarrow \alpha - 15 - 4 = 0 \Rightarrow \alpha = 19$

$\therefore \alpha + \beta = 19 - 12 = 7$

2. If $\int \frac{\sin x}{\sin^3 x+\cos^3 x}dx = \alpha\log_e|1+\tan x| + \beta\log_e|1-\tan x+\tan^2 x| + \gamma\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$,

When C is constant of integration, then the value of 18 (α + β + γ²) is ________

Ans. **(3)**

Sol. $\int \frac{\sin x}{\sin^3 x+\cos^3 x}dx = \int \frac{\frac{\sin x}{\cos^3 x}}{1+\tan^3 x}dx = \int \frac{\tan x\cdot\sec^2 x}{(\tan x+1)(1+\tan^2 x-\tan x)}dx$

Let tan x = t ⇒ $\sec^2 x$. dx = dt

$= \int \frac{t}{(t+1)(t^2-t+1)}dt$

$= \int\left(\frac{A}{t+1}+\frac{B(2t-1)}{t^2-t+1}+\frac{C}{t^2-t+1}\right)dx$

$\Rightarrow A(t^2 - t + 1) + B(2t - 1)(t^2 - t + 1) + C(t + 1) = t$

$\Rightarrow t^2(A + 2B) + t(-A + B + C) + A - B + C = 1$

$\therefore A + 2B = 0$ ...(1)

$-A + B + C = 1$ ....(2)

$A - B + C = 1$ ...(3)

$\Rightarrow C=\frac{1}{2} \Rightarrow A-B=-\frac{1}{2}$ ...(4)

$A + 2B = 0$

$A - B = -\frac{1}{2}$

$\Rightarrow 3B = \frac{1}{2} \Rightarrow B = \frac{1}{6}$

$A = -\frac{1}{3}$

$I = -\frac{1}{3}\int \frac{dt}{1+t} + \frac{1}{6}\int \frac{2t-1}{t^2-t+1}dt + \frac{1}{2}\int \frac{dt}{t^2-t+1}$

$= -\frac{1}{3}\ell n|(1+\tan x)| + \frac{1}{6}\ell n|\tan^2 x - \tan x + 1| + \frac{1}{2}.\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\left(\tan x - \frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right)$

$= -\frac{1}{3}\ell n|1+\tan x| + \frac{1}{6}\ell n|\tan^2 x - \tan x + 1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$

$\alpha = -\frac{1}{3},\ \beta = \frac{1}{6},\ \gamma = \frac{1}{\sqrt{3}}$

$18(\alpha + \beta + \gamma^2) = 18\left(-\frac{1}{3} + \frac{1}{6} + \frac{1}{3}\right) = 3$

3. A tangent line L is drawn at the point (2, –4) on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to __________________.

Ans. (2)

Sol. Tangent of $y^2 = 8x$ is $y = mx + \frac{2}{m}$

$P(2, -4) \Rightarrow -4 = 2m + \frac{2}{m}$

$\Rightarrow m + \frac{1}{m} = -2 \Rightarrow m = -1$

$\therefore$ tangent is $y = -x - 2$

$\Rightarrow x + y + 2 = 0$ ...(1)

(1) is also tangent to $x^2 + y^2 = a$

So, $\frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow \sqrt{a} = \sqrt{2}$

$\Rightarrow a = 2$

4. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is_______

Ans. (5143)

Sol. A = 4 – digit numbers divisible by 3

A = 1002, 1005, …., 9999.

9999 = 1002 + (n – 1)3

$\Rightarrow$ (n – 1)3 = 8997 $\Rightarrow$ n = 3000

B = 4 – digit numbers divisible by 7

B = 1001, 1008, …., 9996

$\Rightarrow$ 9996 = 1001 + (n – 1)7

$\Rightarrow$ n = 1286

A $\cap$ B = 1008, 1029, …., 9996

9996 = 1008 + (n – 1)21

$\Rightarrow$ n = 429

So, no divisible by either 3 or 7

= 3000 + 1286 – 429 = 3857

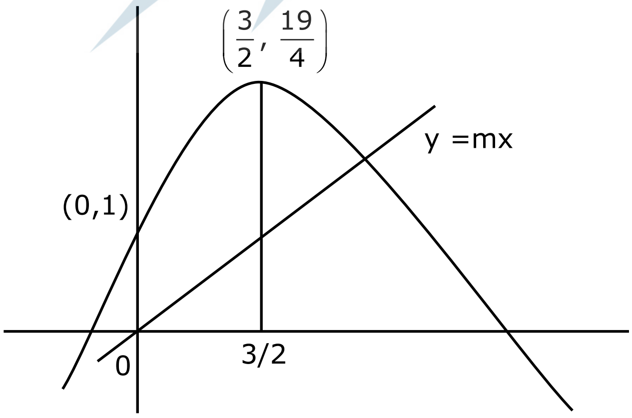
total 4-digits numbers = 9000

required numbers = 9000 – 3857 = 5143

5. If the line y = mx bisects the area enclosed by the lines x = 0, y = 0, $x=\frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, then 12 m is equal to________.

Ans. **(26)**

Sol.



Total area = $\int_0^{3/2}(1+4x-x^2)\,dx$

$= x + 2x^2 - \frac{x^3}{3}\Big|_0^{3/2} = \frac{39}{8}$

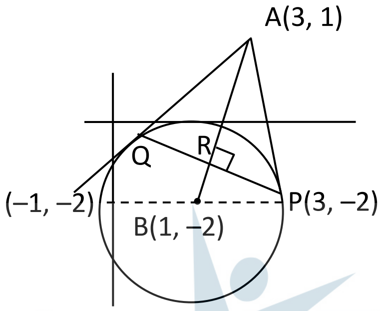
$$\& \ \frac{39}{16} = \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} m$$

$$\Rightarrow 3m = \frac{13}{2} \Rightarrow 12m = 26$$

6. Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8.\left(\frac{\text{area}\,\Delta APQ}{\text{area}\,\Delta BPQ}\right)$ is equal to______________.

Ans. **(18)**

Sol.



$$\tan\theta = \frac{3}{2}$$

$$\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ} = \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$$

$$8\left(\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ}\right) = 18$$

7. Let f(x) be a cubic polynomial with f(1) = –10, f(–1) = 6, and has a local minima at x = 1, and f′(x) has a local minima at x = –1. Then f(3) is equal to __________.

Ans. **(22)**

Sol. $F'(x) = a(x - 1)(x + 3)$

$F''(x) = 6a(x + 1)$

$F'(x) = 3a(x + 1)^2 + b$

$F'(1) = 0 \Rightarrow b = -12a$

$F(x) = a(x + 1)^3 - 12ax + c$

$= (x + 1)^3 - 12x - 6$

$F(3) = 64 - 36 - 6 = 22$

8. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + ...$, then 160 S is equal to_______.

Ans. (305)

Sol. $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + ...$

$\frac{1}{5}S = \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + ...$

On subtracting

$\frac{4}{5}S = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + ...$

$S = \frac{7}{4} + \frac{1}{10}\left(1 + \frac{1}{2} + \frac{3}{5^2} + ...\right)$

$S = \frac{7}{4} + \frac{1}{10}\left(1 - \frac{1}{5}\right)^{-2}$

$= \frac{7}{4} + \frac{1}{10} \times \frac{25}{16} = \frac{61}{32}$

$\Rightarrow 160S = 5 \times 61 = 305$

9. If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $K \cdot 2^{16}$, then K is equal to_______.

Ans. (315)

Sol. $\frac{10!}{\alpha!\beta!\gamma!} a^{\alpha}(2b)^{\beta} \cdot (4ab)^{\gamma}$

$\frac{10!}{\alpha!\beta!\gamma!} a^{\alpha+\beta} \cdot b^{\beta+\gamma} \cdot 2^{\beta} \cdot 4^{\gamma}$

$\alpha + \beta + \gamma = 10$ ...(1)

$\alpha + \gamma = 7$ ...(2)

$\beta + \gamma = 8$ ...(3)

Equation (2) + (3) – (1) $\Rightarrow \gamma = 5$

$\alpha = 2$

$\beta = 3$

So, coefficients = $\frac{10!}{2!3!5!} 2^3 \cdot 2^{10}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{2 \times 3 \times 2 \times 5!} \times 2^{13}$

$= 315 \times 2^{16} \Rightarrow k = 315$

10. The number of elements in the set

$\left\{A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}; a, b, d \in \{-1, 0, 1\} \text{ and } (I-A)^3 = I - A^3\right\}$, where I is 2×2 identity matrix, is________

Ans. **(8)**

Sol. $(I - A)^3 = I^3 - A^3 - 3A(I - A) = I - A^3$

$\Rightarrow 3A(I - A) = 0$ or $A^2 = A$

$\Rightarrow \begin{bmatrix} a^2 & ab + bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$

$\Rightarrow a^2 = a,\ b(a + d - 1) = 0,\ d^2 = d$

If $b \neq 0$, $a + d = 1 \Rightarrow 4$ ways

If $b = 0$, $a = 0, 1$ & $d = 0, 1 \Rightarrow 4$ ways

$\Rightarrow$ Total 8 matrices