MATHEMATICS
JEE-MAIN (August-Attempt)
31 August (Shift-1) Paper

SECTION - A

1. Which of the following is not correct for relation R on the set of real numbers?

(1) $(x, y) \in R \Leftrightarrow |x-y| \leq 1$ is reflexive and symmetric.

(2) $(x, y) \in R \Leftrightarrow 0 < |x-y| \leq 1$ is symmetric and transitive.

(3) $(x, y) \in R \Leftrightarrow |x|-|y| \leq 1$ is reflexive but not symmetric.

(4) $(x, y) \in R \Leftrightarrow 0 < |x|-|y| \leq 1$ is neither transitive nor symmetric.

Ans. (2)

Sol. Note that (1, 2) and (2, 3) satisfy $0 < |x-y| \leq 1$

but (1,3) does not satisfy it so

$0 \leq |x-y| \leq 1$ is symmetric but not transitive

So, (2) is correct.

2. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to:

(1) $7 + 3\sqrt{3}$ (2) $7 - \sqrt{3}$ (3) $7 - 7\sqrt{3}$ (4) $7 + \sqrt{3}$

Ans. (2)

Sol. Let numbers be $\frac{a}{r}$, a, ar $\rightarrow$ G.P

$\frac{a}{r}$, 2a, ar $\rightarrow$ A.P $\Rightarrow 4a = \frac{a}{r} + ar \Rightarrow r + \frac{1}{r} = 4$

$r = 2 \pm \sqrt{3}$

4^{th} form of G.P $= 3r^2 \Rightarrow ar^2 = 3r^2 \Rightarrow a = 3$

$r = 2 + \sqrt{3}$, a = 3, $d = 2a - \frac{a}{r} = 3\sqrt{3}$

$r^2 - d = \left(2+\sqrt{3}\right)^2 - 3\sqrt{3}$

$= 7 + 4\sqrt{3} - 3\sqrt{3}$

$= 7 + \sqrt{3}$

3. The sum of 10 terms of the series

$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots$ is:

(1) $\frac{120}{121}$ (2) 1 (3) $\frac{143}{144}$ (4) $\frac{99}{100}$

Ans. (1)

Sol. $S = \frac{2^2 - 1^2}{1^2 \times 2^2} + \frac{3^2 - 2^2}{2^2 \times 3^2} + \frac{4^2 - 3^2}{3^2 \times 4^2} + \ldots$

$= \left[\frac{1}{1^2} - \frac{1}{2^2}\right] + \left[\frac{1}{2^2} - \frac{1}{3^2}\right] + \left[\frac{1}{3^2} - \frac{1}{4^2}\right] + \ldots + \left[\frac{1}{10^2} - \frac{1}{11^2}\right]$

$= 1 - \frac{1}{121}$

$= \frac{120}{121}$

4. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is:

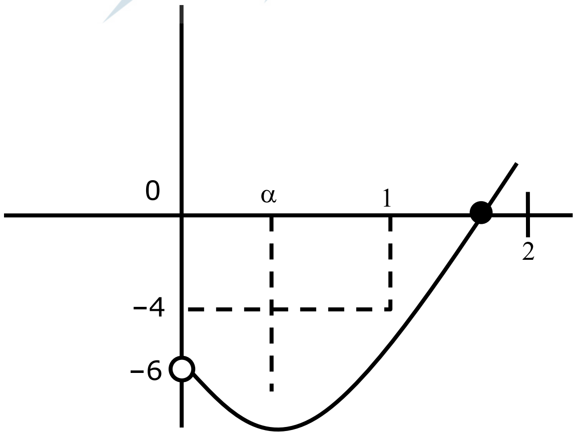
(1) 0 (2) 1 (3) 4 (4) 2

Ans. (2)

Sol. Let $e^x = t > 0$

$f(t) = t^4 + 2t^3 - t - 6 = 0$

$f'(t) = 4t^3 + 6t^2 - 1$



$f''(t) = 12t^2 + 12t > 0$

$f(0) = -6, f(1) = -4, f(2) = 24$

$\Rightarrow$ Number of real roots = 1

5. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3 (x+2)^5}} dx$ is equal to:

(where C is a constant of integration)

(1) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}} + c$ (2) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}} + c$ (3) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + c$ (4) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}} + c$

Ans. (3)

Sol. $\int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}$

$= \int \frac{dx}{\left(\frac{x+2}{x-1}\right)^{5/4} .(x-1)^2}$

put $\frac{x+2}{x-1} = t$

$= -\frac{1}{3}\int \frac{dt}{t^{5/4}}$

$= \frac{4}{3}.\frac{1}{t^{1/4}} + C$

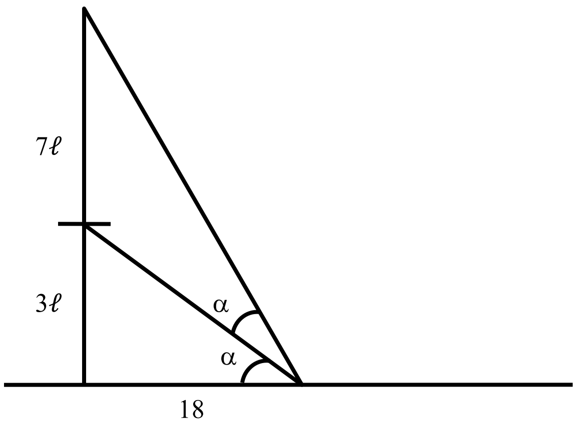
$= \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$

6. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is:

(1) $12\sqrt{15}$ (2) $8\sqrt{10}$ (3) $6\sqrt{10}$ (4) $12\sqrt{10}$

Ans. (4)

Sol.



Let height of pole = 10ℓ

$\tan\alpha = \frac{3\ell}{18} = \frac{\ell}{6}$

$\tan 2\alpha = \frac{10\ell}{18}$

$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{10\ell}{18}$

Use $\tan\alpha = \frac{\ell}{6} \Rightarrow \ell = \sqrt{\frac{72}{5}}$

height of pole = $= 10\ell = 12\sqrt{10}$

7. Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt, 0 \le x \le 1$ and f(0) = 0, then $\lim_{x\to 0}\frac{1}{x^2}\int_0^x f(t)\,dt$:

(1) equals 1 (2) does not exist (3) equals $\frac{1}{2}$ (4) equals 0

Ans. (3)

Sol. $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt \quad 0 \le x \le 1$

differentiating both the sides

$\sqrt{1-(f'(x))^2} = f(x)$

$\Rightarrow 1 - (f'(x))^2 = f^2(x)$

$$\frac{f'(x)}{\sqrt{1-f^2(x)}}=1$$

$\sin^{-1} f(x) = x + c$

$\because f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$

$$\text{Now } \lim_{x\to 0}\frac{\int_0^x \sin t\,dt}{x^2}\left(\frac{0}{0}\right) = \frac{1}{2}$$

8. If the function $f(x) = \begin{cases} \dfrac{1}{x}\log_e\left(\dfrac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , x < 0 \\ k, & , x = 0 \\ \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} & , x > 0 \end{cases}$

is continuous at $x = 0$, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to :

(1) –4 (2) –5 (3) 4 (4) 5

Ans. (2)

Sol. If $f(x)$ is continuous at $x = 0$, RHL = LHL = f(0)

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1}\cdot\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1} \text{ (Rationalisation)}$$

$$= \lim_{x\to 0^+} -\frac{2\sin^2 x}{x^2}\cdot\left(\sqrt{x^2+1}+1\right) = -4$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{1}{x}\ell n\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)$$

$$\lim_{x\to 0^-}\frac{\ell n\left(1+\frac{x}{a}\right)}{\left(\frac{x}{a}\right).a}+\frac{\ell n\left(1-\frac{x}{b}\right)}{\left(-\frac{x}{b}\right).b}$$

$$=\frac{1}{a}+\frac{1}{b}$$

So $\frac{1}{a}+\frac{1}{b}=-4=k$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{4}{k}=-4-1=-5$

9. The function $f(x) = \left|x^2-2x-3\right| . e^{\left|9x^2-12x+4\right|}$ is not differentiable at exactly :

(1) two points (2) one point (3) four points (4) three points

Ans. (1)

Sol. $f(x) = \left|(x-3)(x+1)\right| . e^{(3x-2)^2}$

$$f(x) = \begin{cases} (x-3)(x+1).e^{(3x-2)^2} & ; x \in (3,\infty) \\ -(x-3)(x+1).e^{(3x-2)^2} & ; x \in [-1,3] \\ (x-3).(x+1).e^{(3x-2)^2} & ; x \in (-\infty,-1) \end{cases}$$

Clearly, non-differentiable at x = –1 & x = 3.

10. If $a_r = \cos\frac{2r\pi}{9}+i\sin\frac{2r\pi}{9}$, r = 1, 2, 3, ..., $i=\sqrt{-1}$, then the determinant

$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

(1) $a_2a_6 - a_4a_8$ (2) $a_1a_9 - a_3a_7$ (3) a_5 (4) a_9

Ans. (2)

Sol. $a_r = e^{\frac{i2\pi r}{9}}$, r = 1, 2, 3, ... $a_1, a_2, a_3, \ldots$ are in G.P.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_n & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_2^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix} = a_1.a_1^4.a_1^7 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix} = 0$$

Now $a_1a_9 - a_3a_7 = a_1^{10} - a_1^{10} = 0$

11. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes 3x – 2y + 4z – 7 = 0 and x + 5y – 2z + 9 = 0, be $\alpha x + \beta y + \gamma z + 3 = 0$ then $\alpha+\beta+\gamma$ is equal to:

(1) –15 (2) 15 (3) –23 (4) 23

Ans. (3)

Sol. Equation of plane is

$3x - 2y + 4z - 7 + \lambda(x + 5y - 2z + 9) = 0$

$(3 + \lambda)x + (5\lambda - 2)y + (4 - 2\lambda)z + 9\lambda - 7 = 0$

passing through (1, 4, –3)

$\Rightarrow 3 + \lambda + 20\lambda - 8 - 12 + 6\lambda + 9\lambda - 7 = 0$

$\Rightarrow \lambda = \frac{2}{3}$

$\Rightarrow$ equation of plane is

$-11x - 4y - 8z + 3 = 0$

$\Rightarrow \alpha + \beta + \gamma = -23$

12. The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves?

(1) $25x^2 + 12y^2 = 3600$ (2) $144x^2 + 25y^2 = 3600$

(3) $x^2 + y^2 = 169$ (4) $x^2 + y^2 = 60$

Ans. (2)

Sol. $12x\cos\theta + 5y\sin\theta = 60$

$\frac{x\cos\theta}{5} + \frac{y\sin\theta}{12} = 1$

is tangent to $\frac{x^2}{25} + \frac{y^2}{144} = 1$

$144x^2 + 25y^2 = 3600$

13. If p and q are the lengths of the perpendiculars from the origin on the lines, $x\operatorname{cosec}\alpha - y\sec\alpha = k\cot 2\alpha$ and $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$ respectively, then k^2 is equal to:

(1) $p^2 + 2q^2$ (2) $2p^2 + q^2$ (3) $p^2 + 4q^2$ (4) $4p^2 + q^2$

Ans. (1)

Sol. First line is $\frac{x}{\sin\alpha} - \frac{y}{\cos\alpha} = \frac{k\cos 2\alpha}{\sin 2\alpha}$

$\Rightarrow \quad x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow \quad p = \left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2P = |k\cos 2\alpha| \quad ....(1)$

second line is $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$

$\Rightarrow \quad q = |k\sin 2\alpha| \qquad \ldots(ii)$

Hence $4p^2 + q^2 = k^2$ (from (i) & (ii))

14. $\lim_{x\to 0} \dfrac{\sin^2(\pi \cos^4 x)}{x^4}$ is equal to :

(1) $2\pi^2$ (2) 4π (3) π^2 (4) $4\pi^2$

Ans. (4)

Sol. $\lim_{x\to 0} \dfrac{\sin^2(\pi\cos^4 x)}{x^4}$

$$= \lim_{x\to 0} \frac{1-\cos(2\pi\cos^4 x)}{2x^4}$$

$$= \lim_{x\to 0} \frac{1-\cos(2\pi - 2\pi\cos^4 x)}{\left[2\pi(1-\cos^4 x)\right]^2} 4\pi^2 . \frac{\sin^4 x}{2x^4}(1+\cos^2 x)^2$$

$$= \frac{1}{4}.4\pi^2 \frac{1}{2}(2)^2 = 4\pi^2$$

15. cosec18° is a root of the equation:

(1) $x^2 - 2x - 4 = 0$ (2) $x^2 - 2x + 4 = 0$ (3) $4x^2 + 2x - 1 = 0$ (4) $x^2 + 2x - 4 = 0$

Ans. (-)

Sol. $\operatorname{cosec}18^0 = \dfrac{1}{\sin 18^0} = \dfrac{4}{\sqrt{5}-1} = \sqrt{5}+1$

Let $\operatorname{cosec}18^0 = x = \sqrt{5}+1$

$\Rightarrow \quad x - 1 = \sqrt{5}$

Squaring both sides, we get

$x^2 - 2x + 1 = 5$

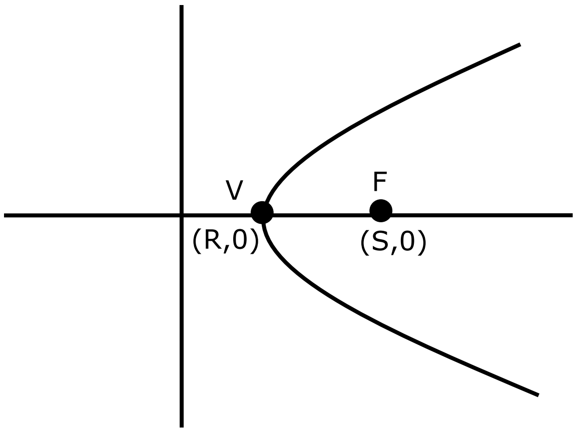
$\Rightarrow \quad x^2 - 2x - 4 = 0$

16. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x-axis at a distance R and S (>R) respectively from the origin, is:

(1) 4(S + R) (2) 2(S + R) (3) 2(S – R) (4) 4(S – R)

Ans. (4)

Sol.



$V \to$ Vertex

$F \to$ focus

$VF = S - R$

So latus rectum $= 4(S - R)$

17. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

(1) $a \neq \frac{1}{3}, b = \frac{7}{3}$ (2) $a \neq -\frac{1}{3}, b = \frac{7}{3}$ (3) $a = \frac{1}{3}, b \neq \frac{7}{3}$ (4) $a = -\frac{1}{3}, b \neq \frac{7}{3}$

Ans. (3)

Sol. Here D= $\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 2(-a-1) - 1(a-1) + 1 + 1 = 1 - 3a$

$D_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} = 2(-b-3) - 1(b-3) + 5(1+1) = 7 - 3b$

for $a = \frac{1}{3}, b \neq \frac{7}{3}$, system has no solutions

18. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a}+3\vec{b}| = |3\vec{a}+\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to:

(1) 5 (2) 8 (3) 4 (4) 6

Ans. (1)

Sol. $|3\vec{a}+\vec{b}|^2 = |2\vec{a}+3\vec{b}|^2$

$(3\vec{a}+\vec{b}).(3\vec{a}+\vec{b}) = (2\vec{a}+3\vec{b}).(2\vec{a}+3\vec{b})$

$9\vec{a}.\vec{a}+6\vec{a}.\vec{b}+\vec{b}.\vec{b} = 4\vec{a}.\vec{a}+12\vec{a}.\vec{b}+9.\vec{b}.\vec{b}$

$5|\vec{a}|^2 - 6\vec{a}.\vec{b} = 8|\vec{b}|^2$

$5(8)^2 - 6.8.\ |\vec{b}|\cos 60° = 8\ |\vec{b}|^2$ $\left(\begin{array}{l}\because \frac{1}{8}|\vec{a}| = 1\\ \Rightarrow |\vec{a}| = 8\end{array}\right)$

$40 - 3|\vec{b}| = |\vec{b}|^2$

$\Rightarrow |\vec{b}|^2 + 3|\vec{b}| - 40 = 0$

$|\vec{b}| = -8,\quad |\vec{b}| = 5$

(rejected)

19. If $\frac{dy}{dx} = \frac{2^{x+y}-2^x}{2^y}$, y(0) = 1, then y(1) is equal to :

(1) $\log_2(2+e)$ (2) $\log_2(1+e)$ (3) $\log_2(2e)$ (4) $\log_2(1+e^2)$

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{2^x 2^y - 2^x}{2^y}$

$2^y\frac{dy}{dx} = 2^x(2^y-1)$

$\int \frac{2^y}{2^y-1}dy = \int 2^x dx$

$\frac{\ell n(2^y-1)}{\ell n 2} = \frac{2^x}{\ell n 2} + C$

$\Rightarrow \log_2(2^y-1) = 2^x\log_2 e + C$

$\because y(0) = 1 \Rightarrow 0 = \log_2 e + C$

$C = -\log_2 e$

$\Rightarrow \ \log_2(2^y - 1) = (2^x - 1)\log_2 e$

Put $x = 1, \log_2(2^y - 1) = \log_2 e$

$2^y = e + 1$

$y = \log_2(e + 1)$

20. Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \ \square \ q)$ is a tautology. Then:

(1) $* = \vee, \square = \wedge$ (2) $* = \wedge, \square = \vee$ (3) $* = \wedge, \square = \wedge$ (4) $* = \vee, \square = \vee$

Ans. (2)

Sol. $(p \wedge \sim q) \rightarrow (p \vee q)$ is tautology

p	q	~q	$p \wedge \sim q$	$p \vee q$	$(p \wedge \sim q) \rightarrow (p \vee q)$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

SECTION-B

1. If $x\,\phi(x) = \int_5^x \left(3t^2 - 2\phi'(t)\right) dt$, $x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is ____________.

Ans. (4)

Sol. $x\phi(x) = \int\limits_5^x 3t^2 - 2\phi'(t)dt$

$x\phi(x) = x^3 - 125 - 2\left[\phi(x) - \phi(5)\right]$

$x\phi(x) = x^3 - 125 - 2\phi(x) - 2\phi(5)$

$\phi(0) = 4 \Rightarrow \phi(5) = -\dfrac{133}{2}$

$\phi(x) = \dfrac{x^3 + 8}{x + 2}$

$\phi(2) = 4$

2. The square of the distance of the point of intersection of the line $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+1}{6}$ and the plane $2x - y + z = 6$ from the point $(-1, -1, 2)$ is ______________.

Ans. (61)

Sol. $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+1}{6} = \lambda$

$x = 2\lambda + 1,\ \ y = 3\lambda + 2,\ \ z = 6\lambda - 1$

for point of intersection of line & plane

$2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$

$7\lambda = 7 \Rightarrow \lambda = 1$

point : (3,5, 5)

$(\text{distance})^2 = (3+1)^2 + (5+1)^2 + (5-2)^2$

$= 16 + 36 + 9 = 61$

3. If $\left(\dfrac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\dfrac{x}{4} - \dfrac{12}{x^2}\right)^{12}$, then k is equal to ______.

Ans. (55)

Sol. $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$

$$T_{r+1}=(-1)^r\cdot{}^{12}C_r\left(\frac{x}{4}\right)^{12-r}\left(\frac{12}{x^2}\right)^r$$

$$T_{r+1}=(-1)^r\cdot{}^{12}C_r\left(\frac{1}{4}\right)^{12-r}(12)^r.(x)^{12-3r}$$

Term independent of $x \Rightarrow 12-3r=0 \Rightarrow r=4$

$$T_5=(-1)^4\cdot{}^{12}C_4\left(\frac{1}{4}\right)^8(12)^4=\frac{3^6}{4^4}.k$$

$\Rightarrow$ k = 55

4. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to ___________.

Ans. (28)

Sol. I_1 = first unit is functioning

I_2 = second unit is functioning

$P(I_1)=0.9,\ P(I_2)=0.8$

$P(\bar{I}_1)=0.1,\ P(\bar{I}_2)=0.2$

$$P=\frac{0.8\times0.1}{0.1\times0.2+0.9\times0.2+0.1\times0.8}=\frac{8}{28}$$

$$98P=\frac{8}{28}\times98=28$$

5. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$, then the minimum value of $\left|z-9\sqrt{2}-2i\right|^2$ is equal to __________.

Ans. (98)

Sol. Let z = x + iy

$$\arg\left(\frac{x-2+iy}{x+2+iy}\right)=\frac{\pi}{4}$$

$$\arg(x-2+iy)-\arg(x+2+iy)=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y}{x-2}\right)-\tan^{-1}\left(\frac{y}{x+2}\right)=\frac{\pi}{4}$$

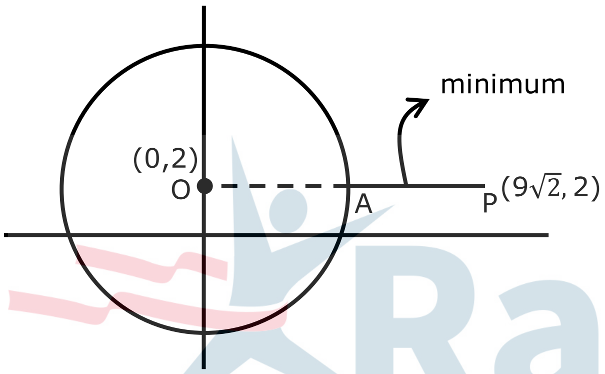
$$\frac{\frac{y}{x-2}-\frac{y}{x+2}}{1+\left(\frac{y}{x-2}\right).\left(\frac{y}{x+2}\right)}=\tan\frac{\pi}{4}=1$$

$$\frac{xy+2y-xy+2y}{x^2-4+y^2}=1$$

$$4y=x^2-4+y^2$$

$$x^2+y^2-4y-4=0$$

locus is a circle with centre (0, 2) & radius = $2\sqrt{2}$



min. value = $(AP)^2$ = $(OP-OA)^2$

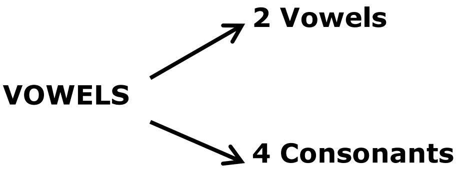
$$=\left(9\sqrt{2}-2\sqrt{2}\right)^2$$

$$=\left(7\sqrt{2}\right)^2=98$$

6. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _______.

Ans. (576)

Sol.



All Consonants should not be together

= Total – All consonants together,

= 6! – 3! 4! = 576

7. The mean of 10 numbers 7 × 8, 10 × 10, 13 × 12, 16 × 14, ... is _________.

Ans. (398)

Sol. 7 × 8, 10 × 10, 13 × 12, 16 × 14 ...

$T_n = (3n + 4)(2n + 6) = 2(3n + 4)(n + 3)$

$= 2(3n^2 + 13n + 12) = 6n^2 + 26n + 24$

$$S_{10} = \sum_{n=1}^{10} T_n = 6\sum_{n=1}^{10} n^2 + 26\sum_{n=1}^{10} n + 24\sum_{n=1}^{10} 1$$

$$= \frac{6(10\times1\times21)}{6} + 26\times\frac{10\times11}{2} + 24\times10$$

$= 10 \times 11 (21 + 13) + 240$

$= 3980$

$$\text{Mean} = \frac{S_{10}}{10} = \frac{3980}{10} = 398$$

8. If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and 'S" is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of $|R - S|$ is _________.

Ans. (2)

Sol. $f(x) = x^2 + ax + 1$

$f'(x) = 2x + a$

when f(x) is increasing on [1, 2]

$2x + a \geq 0 \ \forall\ x \in [1, 2]$

$a \geq -2x \ \forall\ x \in [1, 2]$

$R = -4$

when f(x) is decreasing on [1, 2]

$2x + a \leq 0 \ \forall\ x \in [1, 2]$

$a \leq -2 \ \forall\ x \in [1, 2]$

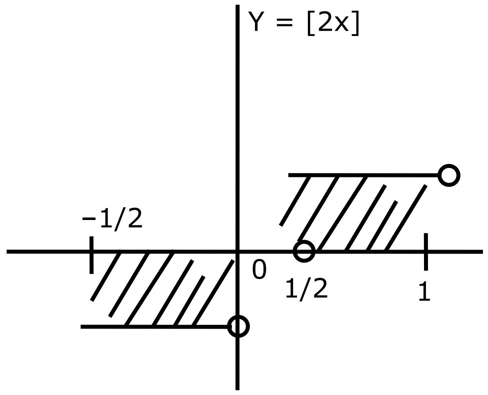
$S = -2$

$|R - S| = |-4 + 2| = 2$

9. Let [t] denote the greatest integer $\leq$ t. Then the value of 8. $\int_{-\frac{1}{2}}^{1}([2x] + |x|)\,dx$ is __________.

Ans. (5)

Sol. $I = \int_{-1/2}^{1} ([2x] + |x|)\,dx$



$= \int_{-1/2}^{1} [2x]\,dx + \int_{-1/2}^{1} |x|\,dx$

$= \left(-\frac{x^2}{2}\right)_{-1/2}^{0} + \left(\frac{x^2}{2}\right)_{0}^{1}$

$= \left(0 + \frac{1}{8}\right) + \frac{1}{2}$

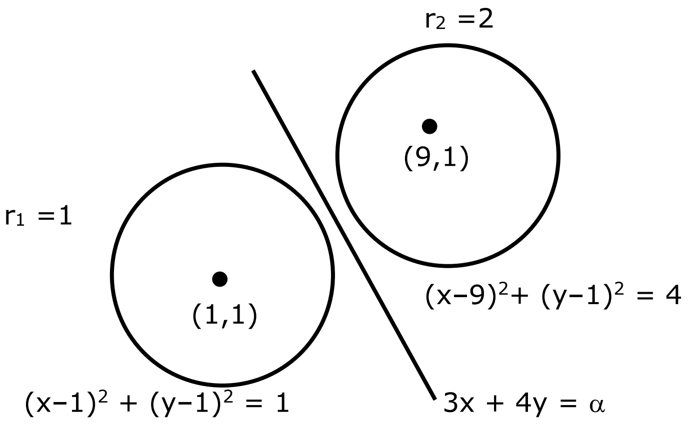
$= \frac{5}{8}$

$8I = 5$

10. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is _________.

Ans. (165)

Sol.



Both centres should lie on either side of the line as well as line can be tangent to circle.

$(3 + 4 - \alpha) . (27 + 4 - \alpha) < 0$

$(7 - \alpha) . (31 - \alpha) < 0 \Rightarrow \alpha \in (7, 31) \quad ...(1)$

d_1 = distance of (1, 1) from line

d_2 = distance of (9, 1) from line

$$d_1 \geq r_1 \Rightarrow \frac{|7-\alpha|}{5} \geq 1 \Rightarrow \alpha \in (-\infty, 2) \cup [12, \infty)$$

$$d_1 \geq r_2 \Rightarrow \frac{|31-\alpha|}{5} \geq 2 \Rightarrow \alpha \in (-\infty, 21) \cup [41, \infty) \quad ...(3)$$

$(1) \cap (2) \cap (3) \Rightarrow \alpha \in [12, 21]$

Sum of integers = 165