MATHEMATICS
JEE-MAIN (July-Attempt) 25 July (Shift-2) Paper

SECTION - A

1. Let y = y(x) be the solution of the differential equation $xdy = (y + x^3\cos x)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $\frac{\pi^2}{2} - \frac{\pi}{4}$ (2) $\frac{\pi^2}{4} + \frac{\pi}{2}$ (3) $\frac{\pi^2}{4} - \frac{\pi}{2}$ (4) $\frac{\pi^2}{2} + \frac{\pi}{4}$

Sol. **(2)**

$xdy = (y+x^3\cos x)dx$

$xdy = ydx+x^3\cos x dx$

$\frac{xdy - ydx}{x^2} = \frac{x^3 \cos x\, dx}{x^2}$

$\int \frac{d}{dx}\left(\frac{y}{x}\right) dx = \int x \cos x\, dx$

$\Rightarrow \frac{y}{x} = x \sin x - \int 1.\sin x\, dx$

$\frac{y}{x} = x \sin x + \cos x + C$

$\Rightarrow 0 = -1 + C \Rightarrow C = 1$, When $x = \pi, y = 0$

so $\frac{y}{x} = x \sin x + \cos x + 1$

$y = x^2\sin x + x\cos x + x$

$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

2. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is :

(1) 1 (2) 2 (3) 3 (4) 4

Sol. **(1)**

$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, \frac{\pi}{4} \le x \le \frac{\pi}{4}$

Apply : $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2(\sin x + 2\cos x) = 0$

$\sin x = \cos x$ or $\sin x = -2\cos x$

$\tan x = 1$ or $\tan x = -2$

$\therefore x = \frac{\pi}{4}$ (Not valid)

3. Consider function $f : A \to B$ and $g : B \to C$ ($A, B, C \subseteq R$) such that $(gof)^{-1}$ exists, then:

(1) f and g both are one-one (2) f is onto and g is one-one

(3) f is one-one and g is onto (4) f and g both are onto

Sol. **(3)**

$\therefore (gof)^{-1}$ exist $\Rightarrow$ gof is bijective $\Rightarrow$ g f(x) must be bijective.

$\Rightarrow$ 'f' must be one-one and 'g' must be ONTO

4. The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is:

(1) 5 (2) 8 (3) 4 (4) 6

Sol. **(3)**

$n_1 = 100$ $n = 250$

$\therefore n_2 = 250 - 100 \Rightarrow n_2 = 150$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$15.6 = \frac{100(15) + (150)(\bar{x}_2)}{250}$$

$$\Rightarrow \boxed{\bar{x}_2 = 16}$$

$\bar{X}_1 = 15$ $\Rightarrow$

$\sigma_1^2 = V_1(x) = 9$ $\sigma^2 = Var(x) = 13.44$

$$\boxed{\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)}\left(\bar{x}_1 - \bar{x}_2\right)^2}$$

$n_2 = 150, \bar{x}_2 = 16, V_2(x) = \sigma_2$

$$13.44 = \frac{100 \text{ x } 9 + 150 \text{ x } \sigma_2^2}{250} + \frac{100 \text{ x } 150}{(250)^2} \text{ x } 1$$

$$\Rightarrow \sigma_2^2 = 16 \Rightarrow \boxed{\sigma_2 = 4}$$

5. The value of the integral $\int_{-1}^{1} \log\left(x + \sqrt{x^2 + 1}\right) dx$ is:

(1) 1 (2) 0 (3) −1 (4) 2

Sol. **(2)**

$$\text{Let } I = \int_{-1}^{1} \log\left(x + \sqrt{x^2 + 1}\right) dx$$

$\because \log\left(x + \sqrt{x^2 + 1}\right)$ is an odd function

$$\therefore \int_{-1}^{1} \ln(x + \sqrt{x^2 + 1})\, dx = 0$$

6. If P = $\begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$, then P^{50} is:

(1) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

Sol. **(2)**

$P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

$P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$P^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$

$P^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$\vdots$

$\therefore P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

7. Let a, b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to:

(1) $\sqrt{ab}$ (2) $\frac{a+b}{2}$ (3) $\frac{1}{a} + \frac{1}{b}$ (4) $\frac{2}{\frac{1}{a} + \frac{1}{b}}$

Sol. **(1)**

Hence $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

$\Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$

8. Let X be a random variable such that the probability function of a distribution is given by P(X = 0) = $\frac{1}{2}$, P(X = j) = $\frac{1}{3^j}$ (j = 1, 2, 3,...., ∞). Then the mean of the distribution and P(X is positive and even) respectively are:

(1) $\frac{3}{4}$ and $\frac{1}{9}$ (2) $\frac{3}{4}$ and $\frac{1}{16}$ (3) $\frac{3}{8}$ and $\frac{1}{8}$ (4) $\frac{3}{4}$ and $\frac{1}{8}$

Sol. **(4)**

mean = $\sum x_i p_i = \sum_{r=0}^{\infty} r.\frac{1}{3^r} = \frac{3}{4}$

p(x is even) = $\frac{1}{3^2} + \frac{1}{3^4} + \ldots\infty$

$= \frac{\frac{1}{9}}{1 - \frac{1}{9}} = \frac{1}{8}$

9. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following:

(1) The match will not be played and weather is not good and ground is wet.

(2) If the match will not be played, then either weather is not good or ground is wet.

(3) The match will not be played or weather is good and ground is not wet.

(4) The match will be played and weather is not good or ground is wet.

Sol. (4)

p : weather is good

q : ground is not wet

$\sim (p \wedge q) \equiv \sim p \vee \sim q$

$\equiv$weather is not good or ground is wet

10. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to:

(1) 3 (2) 1 (3) 4 (4) 2

Sol. (4)

$${}^np_r = {}^np_{r+1} \Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$$

$\Rightarrow (n-r) = 1$ ...(1)]

${}^nC_r = {}^nC_{r-1}$

$$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)!}$$

$$\Rightarrow \frac{1}{r(n-r)!} = \frac{1}{(n-r+1)(n-r)!}$$

$\Rightarrow n-r+1=r$

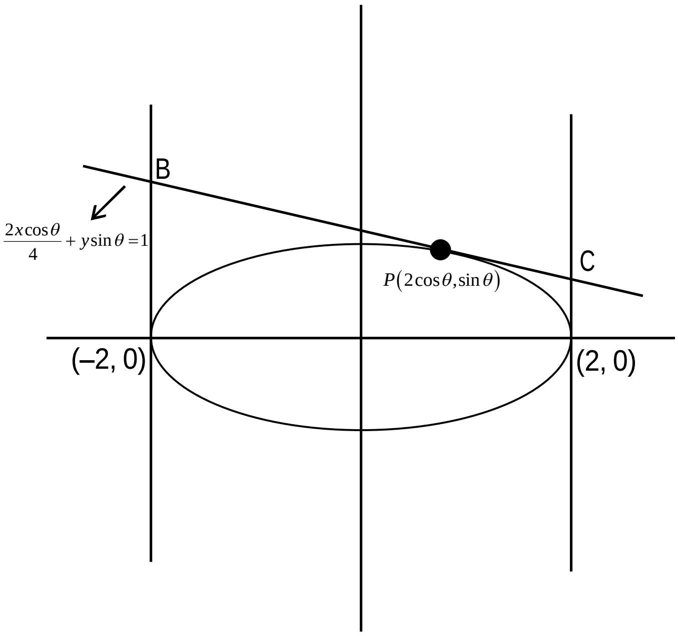
$\Rightarrow n+1=2r$ ...(2)

(1) $\Rightarrow 2r-1-r=1 \Rightarrow r=2$

11. If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point:

(1) (—1, 1) (2) (1, 1) (3) $(\sqrt{3}, 0)$ (4) $(\sqrt{2}, 0)$

Sol. (3)



$\frac{x^2}{4}+\frac{y^2}{1}=1 \Rightarrow$ a = 2 & b = 1, Let P(2cosθ, sinθ)

Equation of tangent at P is (cosθ)x+2sinθy=2

$B\left(-2, \frac{1+\cos\theta}{\sin\theta}\right)$, $C\left(2, \frac{1-\cos\theta}{\sin\theta}\right)$

$B\left(-2, \cot\frac{\theta}{2}\right)$ $C\left(2, \tan\frac{\theta}{2}\right)$

Equation of circle is

$$(x+2)(x-2)+\left(y-\cot\frac{\theta}{2}\right)\left(y-\tan\frac{\theta}{2}\right)=0$$

$$x^2-4+y^2-\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)y+1=0 \ \ldots (1)$$

so, $\left(\sqrt{3}, 0\right)$ satisfying equation (1)

12. The number of real solutions of the equation, $x^2-|x|-12=0$ is:

(1) 3 (2) 1 (3) 2 (4) 4

Sol. (3)

$|x|^2-|x|-12=0$

$(|x|+3)(|x|-4)=0$

$|x|=4 \Rightarrow x=\pm 2 \quad (\because |x| \neq -3)$

13. The sum of all those terms which are rational numbers in the expansion of $\left(2^{1/3}+3^{1/4}\right)^{12}$ is:

(1) 27 (2) 89 (3) 35 (4) 43

Sol. (4)

$T_{r+1}={}^{12}C_r\left(2^{1/3}\right)^r.\left(3^{1/4}\right)^{12-r}$

T_{r+1} will be rational number

Where r = 0, 3, 6, 9, 12

& r = 0, 4, 8, 12

$\Rightarrow$ r = 0, 12

$T_1+T_{13}=1 \times 3^3+1 \times 2^4 \times 1$

$\Rightarrow$ 27 + 16 = 43

14. If $|\vec{a}|=2$, $|\vec{b}|=5$ and $|\vec{a}\times\vec{b}|=8$, then $|\vec{a}.\vec{b}|$ is equal to :

(1) 5 (2) 4 (3) 6 (4) 3

Sol. (3)

$|\vec{a}|=2, |\vec{b}|=5$

$|\vec{a} \times \vec{b}|=|\vec{a}||\vec{b}|\sin\theta=\pm 8$

$\sin\theta = \pm\frac{4}{5}$

$\therefore \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= 10.\left(\pm\frac{3}{5}\right) = \pm 6$

$|\vec{a}.\vec{b}| = 6$

15. If the greatest value of the term independent of 'x' in the expansion of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to:

(1) 2 (2) –1 (3) 1 (4) –2

Sol. **(1)**

$T_{r+1} = {}^{10}C_r (x\sin\alpha)^{10-r}\left(\frac{a\cos\alpha}{x}\right)^r$

r = 0, 1, 2, ..., 10

T_{r+1} will be independent of x

When 10 – 2r = 0 ⇒ $\boxed{r = 5}$

$T_6 = {}^{10}C_5 (x\sin\alpha)^5 \times \left(\frac{a\cos\alpha}{x}\right)^5$

$= {}^{10}C_5 \times a^5 \times \frac{1}{2^5}(\sin 2\alpha)^5$

will be greatest when sin2α = 1

$\Rightarrow {}^{10}C_5 \frac{a^5}{2^5} = {}^{10}C_5 \Rightarrow \boxed{a = 2}$

16. The value of $\cot\frac{\pi}{24}$ is :

(1) $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$ (2) $3\sqrt{2} - \sqrt{3} - \sqrt{6}$

(3) $\sqrt{2} - \sqrt{3} + 2 - \sqrt{6}$ (4) $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

Sol. **(4)**

$\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta} = \left\{\begin{matrix} \therefore 1 + \cos 2\theta = 2\cos^2\theta \\ \& \sin 2\theta = 2\sin\theta\cos\theta \end{matrix}\right\}$

put, $\theta = \frac{\pi}{24}$

$\left\{\therefore \cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} \ \& \ \sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}\right\}$

$$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \frac{\left(2\sqrt{2}+\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$$

$$= \sqrt{6}+\sqrt{2}+\sqrt{3}+2$$

17. If f(x) = $\begin{cases} \int_0^x \left(5+|1-t|\right)dt, & x>2 \\ 5x+1, & x \le 2 \end{cases}$, then

(1) f(x) is not differentiable at x = 1

(2) f(x) is continuous but not differentiable at x = 2

(3) f(x) is not continuous at x = 2

(4) f(x) is everywhere differentiable

Sol. **(2)**

$$f(x) = \int_0^1 \left(5+(1-t)\right)dt + \int_1^x \left(5+(t-1)\right)dt$$

$$= 6 - \frac{1}{2} + \left(4t + \frac{t^2}{2}\right)\Bigg|_1^x$$

$$= \frac{11}{2} + 4x + \frac{x^2}{2} - 4 - \frac{1}{2}$$

$$= \frac{x^2}{2} + 4x + 1$$

$$f\left(2^+\right) = 2+8+1 = 11$$

$\Rightarrow$continuous at x = 2

Clearly differentiable at x = 1

Lf'(2) = 5

Rf'(2) = 6

$\Rightarrow$not differentiable at x = 2

18. The lowest integer which is greater than $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$ is _________.

(1) 3 (2) 4 (3) 2 (4) 1

Sol. **(1)**

Let $P = \left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$,

Let $x = 10^{100}$

$\Rightarrow P = \left(1 + \frac{1}{x}\right)^{x}$

$\Rightarrow P = 1 + (x)\left(\frac{1}{x}\right) + \frac{(x)(x-1)}{\lfloor 2} \cdot \frac{1}{x^2} + \frac{(x)(x-1)(x-2)}{\lfloor 3} \cdot \frac{1}{x^3} + \ldots$

(upto 10^{100} + 1 terms)

$\Rightarrow P = 1 + 1 + \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 2 x^2}\right) + \left(\frac{1}{\lfloor 3} - \ldots\right) + \ldots$ so on

$\Rightarrow P = 2 + \left(\text{Positive value less than } \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} + \ldots\right)$

$\Rightarrow P \in (2, 3)$

Also $e = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} + \ldots$

$\Rightarrow \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} + \ldots = e - 2$

$\Rightarrow P = 2 + (\text{Positive value less than } e - 2)$

$\Rightarrow P \in (2, 3)$

$\Rightarrow$ least integer value of P is 3

19. If [x] be the greatest integer less than or equal to x, then $\sum_{n=8}^{100} \left[\frac{(-1)^n n}{2}\right]$ is equal to:

(1) –2 (2) 4 (3) 2 (4) 0

Sol. **(2)**

$\sum_{n=8}^{100} \left[\frac{(-1)^n . n}{2}\right] = \left[\frac{8}{2}\right] + \left[\frac{-9}{2}\right] + \left[\frac{10}{2}\right] + \left[\frac{-11}{2}\right] + \ldots + \ldots \left[\frac{-99}{2}\right] + \left[\frac{100}{2}\right]$

$= 4 - 5 + 5 - 6 + 6 + \ldots - 50 + 50 = 4$

20. Let the equation of the pair of lines, y = px and y = qx, can be written as (y–px)(y–qx) = 0. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is:

(1) $x^2 - 3xy - y^2 = 0$ (2) $x^2 + 3xy - y^2 = 0$

(3) $x^2 - 3xy + y^2 = 0$ (4) $x^2 + 4xy - y^2 = 0$

Sol. **(2)**

$\left\{\text{using formula for angle bisector of } ax^2 + 2hxy + by^2 = 0 \text{ as } \boxed{\frac{x^2 - y^2}{a - b} = \frac{xy}{h}}\right\}$

$\frac{x^2 - y^2}{1 - (-5)} = \frac{xy}{-2}$

$\frac{x^2 - y^2}{6} = \frac{xy}{-2}$

$\Rightarrow x^2 - y^2 = -3xy$

$\Rightarrow x^2 + 3xy - y^2 = 0$

SECTION - B

1. If a + b + c = 1, ab + bc + ca = 2 and abc = 3, then the value of $a^4 + b^4 + c^4$ is equal to __________.

Sol. (13)

$a^2 + b^2 + c^2 = (a+b+c)^2 - 2\sum ab = -3$

$(ab + bc + ca)^2 = \sum(ab)^2 + 2abc\sum a$

$\Rightarrow \sum(ab)^2 = -2$

$a^4 + b^4 + c^4 = (a^2 + b^2 + c^2)^2 - 2\sum(ab)^2$

$= 9 - 2(-2) = 13$

2. If the coefficients of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal, then the value of n is equal to ________.

Sol. (55)

$${}^nC_7 2^{n-7} \frac{1}{3^7} = {}^nC_8 2^{n-8} \frac{1}{3^8}$$

$$\Rightarrow \frac{n!}{(n-7)!7!} 2^{n-7} \frac{1}{3^7} = \frac{n!}{(n-8)!8!} 2^{n-8} \frac{1}{3^8} \Rightarrow \frac{1}{(n-7)} = \frac{1}{8}\cdot\frac{1}{2}\cdot\frac{1}{3}$$

$\Rightarrow n - 7 = 48 \Rightarrow n = 55$

3. Consider the function $\begin{cases} \dfrac{P(x)}{\sin(x-2)} & x \neq 2 \\ 7 & x = 2 \end{cases}$

where P(x) is a polynomial such that P" (x) is always a constant and P(3) = 9. If f(x) is continuous at x = 2, then P(5) is equal to ______.

Sol. (39)

$$f(x) = \begin{cases} \dfrac{P(x)}{\sin(x-2)}, & x \neq 2 \\ 7, & x = 2 \end{cases}$$

P"(x) = const. $\Rightarrow$P(x) is a 2 degree polynomial

f(x) is cont. at x = 2

$f(2^+) = f(2^-)$

$$\lim_{x\to 2^+} \frac{P(x)}{\sin(x-2)} = 7$$

$$\lim_{x\to 2^+} \frac{(x-2)(ax+b)}{\sin(x-2)} = 7 \Rightarrow \boxed{2a+b=7}$$

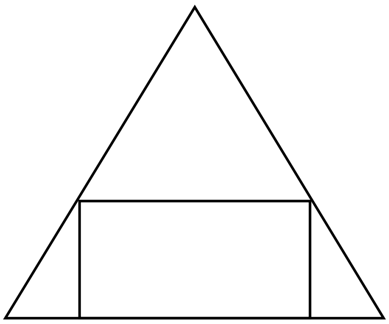
$P(x) = (x-2)(ax+b)$

$P(3) = (3-2)(3a+b) = 9 \Rightarrow \boxed{3a+b=9}$

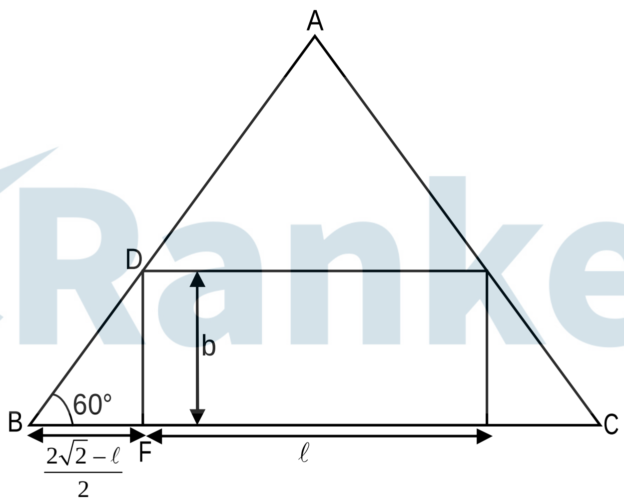
$\boxed{a=2,\ b=3}$

$P(5) = (5-2)(2.5+3) = 3.13 = 39$

4. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is _________.



Sol. **(3)**



In ΔDBF

$$\tan 60^\circ = \frac{2b}{2\sqrt{2}-\ell} \Rightarrow \boxed{b = \frac{\sqrt{3}\left(2\sqrt{2}-\ell\right)}{2}}$$

A = Area of rectangle = ℓ x b

$$A = \ell \times \frac{\sqrt{3}}{2}\left(2\sqrt{2}-\ell\right)$$

$$\frac{dA}{d\ell} = \frac{\sqrt{3}}{2}\left(2\sqrt{2}-\ell\right) - \ell\sqrt{3} = 0$$

$$\boxed{\ell = \sqrt{2}}$$

$$A = \ell \times b = \sqrt{2} \times \frac{\sqrt{3}}{2}\left(\sqrt{2}\right) = \sqrt{3}$$

$$\Rightarrow \boxed{A^2 = 3}$$

5. Let a curve y = f(x) pass through the point (2, $(\log_e 2)^2$) and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x. Then the value of f(e) is equal to ________.

Sol. **(1)**

$y' = \frac{2y}{x\ell nx} \quad \Rightarrow \frac{dy}{dx} = \frac{2y}{x \ln x}$

$\Rightarrow \frac{dy}{y} = \frac{2dx}{x\ell nx}$

$\Rightarrow \ell n|y| = 2\ell n|\ell nx| + C$

Put $x = 2, y = (\ell n2)^2$

$\Rightarrow \ln|(\ln 2)^2| = \ln|(\ln 2)^2| + c$

$\Rightarrow c = 0$

$\Rightarrow y = (\ell nx)^2$

$\Rightarrow f(e) = 1$

6. A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is ________.

Sol. **(4)**

$P(\text{Head}) = \frac{1}{2}$

$1 - \left(\frac{1}{2}\right)^n \geq 0.9$

$\Rightarrow \left(\frac{1}{2}\right)^n \leq \frac{1}{10}$

$\Rightarrow n_{min} = 4$

7. The equation of a circle is $Re(z^2) + 2(Im(z))^2 + 2Re(z) = 0$, where z = x + iy. A line which passes through the center of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to ______.

Sol. **(1)**

Equation of circle is $(x^2 - y^2) + 2y^2 + 2x = 0$

$X^2 + y^2 + 2x = 0$

Centre : (–1, 0)

Parabola : $x^2 - 6x - y + 13 = 0$

$(x - 3)^2 = y - 4$

Vertex : (3, 4)

$\equiv y - 0 = \frac{4-0}{3+1}(x+1)$

$y = x + 1$

y = x + 1

y-intercept = 1

8. Let $n \in N$ and [x] denote the greatest integer less than or equal to x. If the sum of (n + 1) terms ${}^nC_0$, $3.{}^nC_1$, $5.{}^nC_2$, $7.{}^nC_3$, ... is equal to 2^{100}. 101, then $2\left[\frac{n-1}{2}\right]$ is equal to ________.

Sol. **(98)**

$$1.{}^nC_0 + 3.{}^nC_1 + 5.{}^nC_2 + \ldots + (2n+1).{}^nC_n$$

$$T_r = (2r+1)^n C_r$$

$S = \sum T_r$

$S = \sum(2r+1)\ {}^nC_r = \sum 2r\,{}^nC_r + \sum {}^nC_r$

$$S = 2\left(n.2^{n-1}\right) + 2^n = 2^n(n+1)$$

$$2^n(n+1) = 2^{100}.101 \Rightarrow n = 100$$

$$2\left[\frac{n-1}{2}\right] = 2\left[\frac{99}{2}\right] = 98$$

9. If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar, then the value of k is ________.

Sol. **(1)**

$$\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

{$\therefore$ Shortest distance between then is zero}

(k + 1)[2 – 6] –4[1 – 9] + 6[2 – 6] = 0

K = 1

10. If $(\vec{a} + 3\vec{b})$ is perpendicular to $(7\vec{a} - 5\vec{b})$ and $(\vec{a} - 4\vec{b})$ is perpendicular to $(7\vec{a} - 2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is _________.

Sol. **(60)**

$$(\vec{a} + 3\vec{b}) \perp (7\vec{a} - 5\vec{b})$$

$$(\vec{a} + 3\vec{b}).(7\vec{a} - 5\vec{b}) = 0$$

$$7|\vec{a}|^2 - 15|\vec{b}|^2 + 16\vec{a}.\vec{b} = 0 \ \ldots(1)$$

$$(\vec{a} - 4\vec{b}).(7\vec{a} - 2\vec{b}) = 0$$

$$7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\vec{a}.\vec{b} = 0 \ \ldots(2)$$

From (1) & (2)

$$\boxed{|\vec{a}| = |\vec{b}|}$$

as $|\vec{a}| = |\vec{b}|$

$$\therefore 7|\vec{a}|^2 - 15|\vec{a}|^2 + 16\vec{a}.\vec{b} = 0$$

$$\Rightarrow \vec{a}.\vec{b} = \frac{|\vec{a}|^2}{2}$$

$$\therefore \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{|\vec{a}|^2}{2|\vec{a}||\vec{a}|}$$

$$\therefore \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60^o$$