MATHEMATICS
JEE-MAIN (MARCH-Attempt) 17 MARCH
(Shift-1) Paper

SECTION – A

1. Which of the following is true for y(x) that satisfies the differential equation

$\frac{dy}{dx}$ = xy-1+x-y;y(0) = 0 :

(1) y(1) = 1

(2) y(1) = $e^{\frac{1}{2}}-1$

(3) y(1) = $e^{\frac{1}{2}}-e^{-\frac{1}{2}}$

(4) y(1) = $e^{-\frac{1}{2}}-1$

Ans. **(4)**

Sol. $\frac{dy}{dx}=(x-1)y+(x-1)$

$\frac{dy}{dx}=(x-1)(y+1)$

$\frac{dy}{y+1}=(x-1)dx$

$\ell n(y+1)=\frac{x^2}{2}-x+c$

X = 0, y = 0

$\Rightarrow c=0$

$\therefore \ell n(y+1)=\frac{x^2}{2}-x$

putting x = 1, ℓn(y + 1) = $\frac{1}{2}-1=-\frac{1}{2}$

$y+1=e^{-\frac{1}{2}}$

$y=e^{-\frac{1}{2}}-1$

$\therefore y(1)=e^{-\frac{1}{2}}-1$

2. The system of equations kx + y + z=1, x + ky + z=k and x + y + zk =k^2 has no solution if k is equal to:

(1) -2

(2)-1

(3)1

(4) 0

Ans. (1)

Sol. $D = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$

$\Rightarrow k(k^2 - 1) - (k - 1) + (1 - k) = 0$
$\Rightarrow (k - 1)(k^2 + k - 1 - 1) = 0$
$\Rightarrow (k - 1)(k^2 + k - 2) = 0$
$\Rightarrow (k - 1)(k - 1)(k + 2) = 0$
$\Rightarrow k = 1, k = 2$
for k = 1 equation identical so k = –2 for no solution.

3. The value of $4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{4 + \ldots\ldots\infty}}}}$ is:

(1) $2 + \frac{4}{\sqrt{5}}\sqrt{30}$

(2) $4 + \frac{4}{\sqrt{5}}\sqrt{30}$

(3) $2 + \frac{2}{5}\sqrt{30}$

(4) $5 + \frac{2}{5}\sqrt{30}$

Ans. (3)

Sol. $y = 4 + \cfrac{1}{5 + \cfrac{1}{y}}$

$\Rightarrow y = 4 + \frac{y}{5y + 1}$

$\Rightarrow 5y^2-20y-4 = 0$

$\Rightarrow y = \dfrac{20 \pm \sqrt{400+80}}{10}$

$\Rightarrow y = \dfrac{20 \pm 4\sqrt{30}}{10}, y > 0$

$y = \dfrac{10 + 2\sqrt{30}}{5}$

4. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim P))$ is a tautology, then the Boolean expression p * (~q) is equivalent to:

(1) $p \Rightarrow \sim q$

(2) $p \Rightarrow q$

(3) $q \Rightarrow p$

(4) $\sim q \Rightarrow p$

Ans. **(3)**

Sol. $(p \rightarrow q) \Leftrightarrow (q * \sim P)$

P	q	$p \rightarrow q$	$q * \sim P$	$\sim q$	$\sim q \wedge p$	$\sim(\sim q \wedge p)$
T	T	T	T	F	F	T
T	F	F	F	T	T	F
F	T	T	T	F	F	T
F	F	T	T	T	F	T

$\therefore \sim(\sim q \wedge p) = q \vee \sim p$

$= \sim p \vee q$

$\therefore$ * is equivalent to $\vee$

$\therefore p * \sim q = p \vee \sim q$

$= \sim q \vee p$

$= q \rightarrow p$

5. Choose the incorrect statement about the two circles whose equations are given below:

$X^2+y^2-10x-10y+41=0$ and

$X^2+y^2-16x-10y+80=0$

(1) Distance between two centres is the average of radii of both the circles.

(2) Circles have two intersection points.

(3) Both circles' centres lie inside region of one another.

(4) Both circles pass through the centre of each other.

Ans. **(3)**

Sol. C_1 (5, 5), C_2 (8, 5)

position of $C_1(5, 5)$ in $S_2 = 0$

$= 25 + 25 - 80 - 50 + 80$

$= 0$

position of $C_2(8, 5)$ in $S_1 = 0$

$\equiv 64 + 25 - 80 - 50 + 41$

$= 0$

6. The sum of possible values of x for $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$ is:

(1) $-\frac{33}{4}$

(2) $-\frac{32}{4}$

(3) $-\frac{31}{4}$

(4) $-\frac{30}{4}$

Ans. **(2)**

Sol. Taking tan both sides

$$\frac{(1+x)+(x-1)}{1-(1+x)(x-1)}=\frac{8}{31}$$

$$\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31}$$

$\Rightarrow 4x^2+31x-8=0$

$$\Rightarrow x=-8,\frac{1}{4}$$

but at $x=\frac{1}{4}$

$\text{LHS}>\frac{\pi}{2}$ and $\text{RHS}<\frac{\pi}{2}$

So, only solution is $x=-8$

7. Lt $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b}=7\hat{i}+\hat{j}-6\hat{k}$

If $\vec{r}\times\vec{a}=\vec{r}\times\vec{b}, \vec{r}\cdot\left(\hat{i}+2\hat{j}+\hat{k}\right)=-3$, then $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+\hat{k}\right)$ is equal to :

(1) 10
(2) 13
(3) 12
(4) 8

Ans. **(3)**

Sol. $\vec{a}=(2,-3,4)$, $\vec{b}=(7,1,-6)$

$\vec{r}\times\vec{a}-\vec{r}\times\vec{b}=0$

$\bar{r}\times\left(\vec{a}-\vec{b}\right)=0$

$\vec{r}=\lambda\left(\vec{a}-\vec{b}\right)$

$\vec{r}=\lambda\left(-5\hat{i}-4\hat{j}+10\hat{k}\right)$

$\vec{r}\cdot\left(2,-3,1\right)=?$

if $\vec{r}\cdot\left(1,\,2,\,1\right)=-3$

$\lambda\left(-5-8+10\right)=-3 \quad\Rightarrow\quad \lambda = 1$

$\therefore$ (-5, -4, 10) . (2, -3, 1)

$\Rightarrow -10 + 12 + 10 = 12$

8. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3)is:

(1) $3x + z = 6$

(2) $3x - z = 0$

(3) $x + 3z = 10$

(4) $x + 3z = 0$

Ans. **(2)**

Sol. Let the equation of the plane is $a(x - 1) + b(y - 2) + c(z - 3) = 0$

Y−axis lies on it D.R.'s of y-axis are 0, 1, 0

$\therefore 0.a + 1.b + 0.c = 0 \Rightarrow b = 0$

$\therefore$ Equation of plane is $a(x - 1) + c(z - 3) = 0$

$x = 0, z = 0$ also satisfy it $-a - 3c = 0 \Rightarrow a = -3c$

$-3c(x - 1) + c(z - 3) = 0$

$-3x + 3 + z - 3 = 0$

$3x - z = 0$

9. If $A=\begin{pmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{pmatrix}$ and $\det\left(A^2-\frac{1}{2}I\right)=0$, then a possible value of α is:

(1) $\frac{\pi}{6}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{4}$

Ans. **(4)**

Sol. $A^2 = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}\begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix} = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix}$

$A^2 - \frac{1}{2}I = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{bmatrix}$

$\Rightarrow \left|A^2 - \frac{1}{2}I\right| = 0$

$\Rightarrow \begin{vmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{vmatrix} = 0$

$\Rightarrow \left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0 \Rightarrow \sin^2\alpha = \frac{1}{2} \qquad \Rightarrow \sin\alpha = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

$\alpha = \frac{\pi}{4}$

10 The line $2x-y+1=0$ is a tangent to the circle at the point $(2,5)$ and the centre of the circle lies on $x-2y=4$. Then, the radius of the circle is:

(1) $4\sqrt{5}$

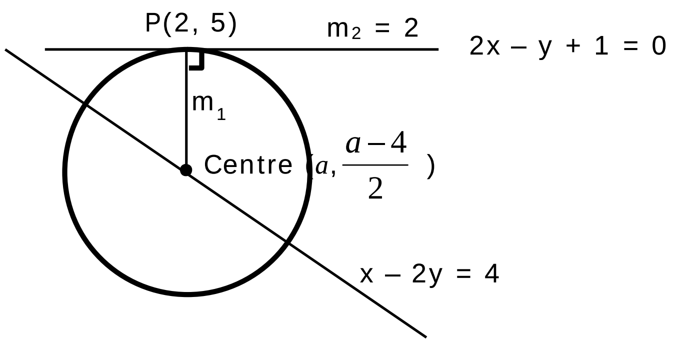
(2) $5\sqrt{3}$

(3) $3\sqrt{5}$

(4) $5\sqrt{4}$

Ans. (3)

Sol.



$m_1 \times m_2 = -1$

$\dfrac{\frac{a-4}{2} - 5}{a-2} \times 2 = -1$

$$\frac{a-14}{a-2} = -1$$

a – 14 = 2 – a

2a = 16

$\boxed{a = 8}$

∴ Centre (8, 2)

Radius = $\sqrt{36+9}$

= $\sqrt{45}$

= $3\sqrt{5}$

11. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to:

(1) 5
(2) 6
(3) 2
(4) 4

Ans. **(4)**

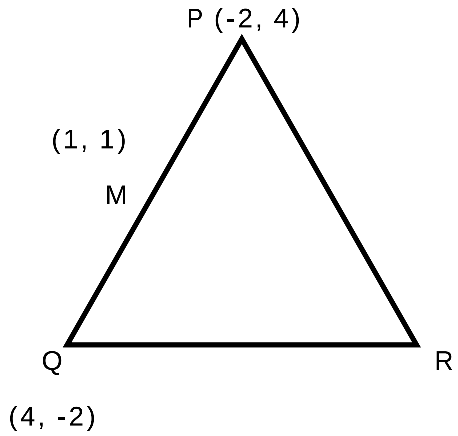
Sol.

7 × 4 + 6 × n = 52
6n = 24
⇒ n = 4

12. In a triangle PQR, the co-ordinates of the points P and Q are (-2, 4) and (4, -2) respectively. If the equation of the perpendicular bisector of PR is 2x – y + 2 = 0, then the centre of the circumcircle of the ΔPQR is:

(1) (-2, -2)
(2) (0, 2)
(3) (-1, 0)
(4) (1, 4)

Ans. **(1)**



Sol.

Perpendicular bisector of PR : $2x - y + 2 = 0$ .....(1)
Mid-point of PQ $\rightarrow$ M(1,1)
Equation of perpendicular bisector of PQ : $x - y = 0$ ......(2)
$\therefore$ POI of equation (1) & (2) is circumcentre
So, circumcentre $(-2,-2)$

13. If $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \ldots\ldots$ upto 100 terms, then α is:
(1) 1.03
(2) 1.00
(3) 1.01
(4) 1.02

Ans. **(3)**

Sol. RHS $= \sum_{n=1}^{100} \cot^{-1} 2n^2 = \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$= \tan^{-1} 201 - \tan^{-1} 1$

$= \tan^{-1}\left(\frac{200}{202}\right)$

$\Rightarrow \cot^{-1}\alpha = \cot^{-1}\left(\frac{101}{100}\right)$

$\Rightarrow \alpha = 1.01$

14. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha \in R$ such that

$$g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^{\alpha} x}{\cos^{\alpha} x + \sin^{\alpha} x}\, dx$$

(1) g(α) is a strictly decreasing function

(2) g(α) has an inflection point at $\alpha = -\frac{1}{2}$

(3) g(α) is an even function

(4) g(α) is a strictly increasing function.

Ans. (1 OR 2 OR 3/Bonus)

Sol. g(x) = $\frac{\pi}{12}$ by applying prop. x $\to$ a + b – x.

15. If the fourth term in the expansion of $\left(x + x^{\log_2 x}\right)^7$ is 4480, then the value of x where $x \in N$ is equal to:

(1) 4

(2) 3

(3) 2

(4) 1

Ans. (3)

Sol. ${}^7C_3 x^4 \left(x^{\log_2 x}\right)^3 = 4480$

$\Rightarrow 35x^4 \left(x^{\log_2 x}\right)^3 = 4480$

$\Rightarrow x^4 \left(x^{\log_2 x}\right)^3 = 128$

take log w.r.t base 2 we get $4\log_2 x + 3\log_2 \left(x^{\log_2 x}\right) = \log_2 128$

Let $\log_2 x = y$

$4y + 3y^2 = 7$

$\Rightarrow y = 1, \frac{-7}{3}$

$\Rightarrow \log_2 x = 1, \frac{-7}{3}$

$x = 2, x = 2^{-7/3}$

16. Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is:

(1) $\frac{17}{36}$

(2) $\frac{4}{9}$

(3) $\frac{5}{12}$

(4) $\frac{1}{2}$

Ans. (1)

Sol. n(S) = 36

possible ordered pair ; (1, 1), (1, 2), (1, 3), (1, 5), (1, 7), (2, 1), (2, 2), (2, 3), (2, 5), (3, 1), (3, 2), (3, 3), (3, 5), (5, 1), (5, 2), (5, 3), (7, 1)

Number of ordered pair = 17

Probability = $\frac{17}{36}$

17. The inverse of $y = 5^{\log x}$ is:

(1) $x = 5^{\log y}$

(2) $x = y^{\log 5}$

(3) $x = y^{\frac{1}{\log 5}}$

(4) $x = 5^{\frac{1}{\log y}}$

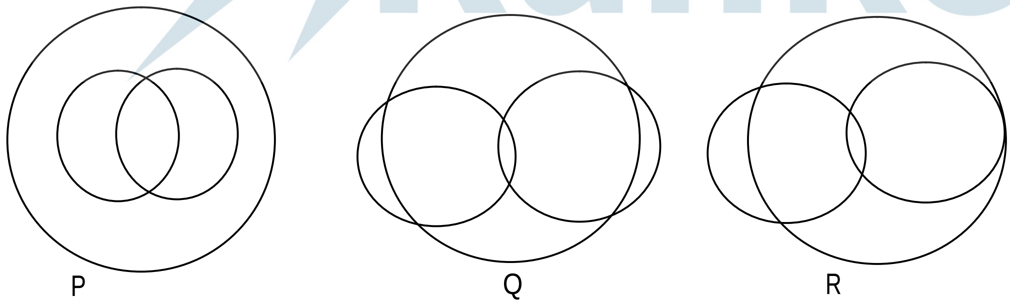
Ans. **(3)**

Sol. $y = 5^{\log_e x}$

$\log_5 y = \log_e x$

$x = e^{\log_5 y}$

18. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statements.



(1) P and R

(2) P and Q

(3) None of these

(4) Q and R

Ans. **(3)**

Sol. In are the (A), (B), (C) there are some students which play all the three games hence no venn diagram is correct.

19. The area of the triangle with vertices A(z), B(iz) and C(z+iz) is:

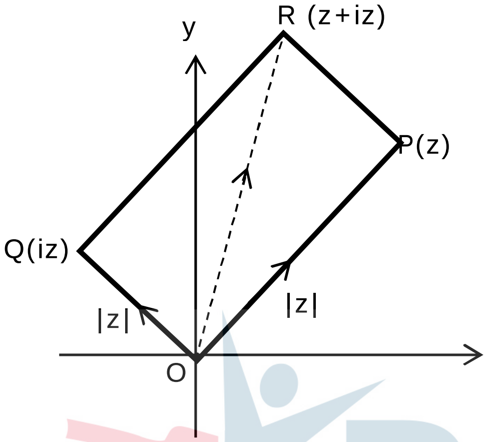
(1) $\frac{1}{2}|z+iz|^2$

(2) 1

(3) $\frac{1}{2}$

(4) $\frac{1}{2}|z|^2$

Ans. **(4)**



Sol.

Area of $\Delta = \frac{1}{2}$(area of square)

$= \frac{1}{2}|z|^2$

20. The value of $\lim_{x \to 0^+} \frac{\cos^{-1}\left(x-[x]^2\right).\sin^{-1}\left(x-[x]^2\right)}{x-x^3}$, where [x] denotes the greatest integer ≤ x is :

(1) 0

(2) $\frac{\pi}{4}$

(3) $\frac{\pi}{2}$

(4) π

Ans. **(3)**

Sol. $\lim_{x \to 0^+} \frac{\cos^{-1}\left(x-[x]^2\right)\sin^{-1}\left(x-[x]^2\right)}{x\left(1-x^2\right)}$

$\Rightarrow \lim_{x \to 0^+} \frac{\cos^{-1} x \sin^{-1} x}{x} = \frac{\pi}{2}$

SECTION - B

1. Let there by three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1)

Then, $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to_____________.

Ans. **(6)**

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1 - y)(1 - z) = \alpha \Rightarrow y(1 - x)(1 - z) = \beta \Rightarrow z(1 - x)(1 - y) = \gamma \Rightarrow (1 - x)$
$(1 - y)(1 - z) = P$

Putting in the given relation we get $x = 2y$ and $y = 3z \Rightarrow x = 6z \Rightarrow \frac{x}{z} = 6$

2. If the equation of the plane passing through the line of intersection of the planes 2x-7y+4z-3=0, 3x-5y+4z+11 = 0 and the point (-2, 1, 3) is ax+by+cz-7=0, then the value of 2a+b+c-7 is___________.

Ans. **(4)**

Sol.

Equation of plane can be written using family of planes: $P_1 + \lambda P_2 = 0$

$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0$

It passes through (-2, 1, 3)

$\therefore (-4 + 7 + 12 - 3) + \lambda(-6 - 5 + 12 + 11) = 0$

$-2 + \lambda(12) = 0$

$\lambda = \frac{1}{6}.$

$\therefore 12x - 42y + 24z - 18 + 3x - 5y + 4z + 11 = 0$

$15x - 47y + 28z - 7 = 0$

$a = 15, b = -47, c = 28$

$\therefore 2a + b + c - 7 = 30 - 47 + 28 - 7$

$= 4$

$\boxed{\text{Ans : 4}}$

3. If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det(A^{10} - \text{Adj}(2A))^{10})$ is equal to_________.

Ans. **(16)**

Sol. $|A| = -2 \Rightarrow |A|^4 = 16$

$$A^{10} = \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1024 & 1023 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$$

$$\text{adj}(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$$

$$\text{adj}(2A) = -2\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$$

$$\left(\text{adj}(2A)\right)^{10} = 2^{10}\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}^{10}$$

$$= 2^{10}\begin{bmatrix} 1 & -\left(2^{10}-1\right) \\ 0 & 2^{10} \end{bmatrix}$$

$$= 2^{10}\begin{bmatrix} 1 & -1023 \\ 0 & 1024 \end{bmatrix}$$

$$A^{10} - \left(\text{adj}(2A)\right)^{10} = \begin{bmatrix} 0 & 2^{11}\times 1023 \\ 0 & 1-(1024)^2 \end{bmatrix}$$

$$\left|A^{10} - \text{adj}(2A)^{10}\right| = 0$$

4. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 one the other circle for the given circles' equations

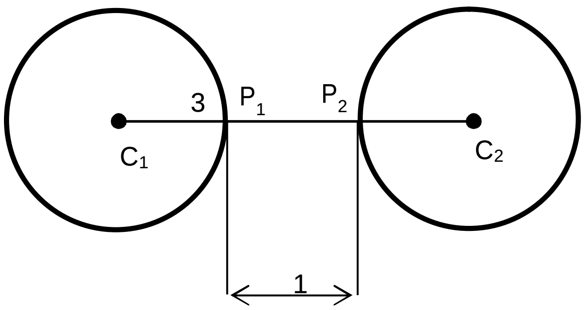
$X^2+y^2-10x-10y+41=0$

$X^2+y^2-24x-10y+160=0$___________.

Ans. **(1)**

Sol. $S_1 : (x-5)^2 + (y-5)^2 = 9$ centre $(5, 5)$, $r_1 = 3$

$S_2 : (x-12)^2 + (y-5)^2 = 9$ centre $(12, 5)$, $r_2 = 3$



So $(P_1P_2)_{min} = 1$

5. If $(2021)^{3762}$ is divided by 17, then the remainder is___________.

Ans. **(4)**

Sol. $(2021)^{3762} = (2023 - 2)^{3762}$ = multiple of 17 + 2^{3762}

$= 17\lambda + 2^2\,(2^4)^{940}$

$= 17\,\lambda + 4\,(17 - 1)^{940}$

$= 17\,\lambda + 4\,(17\mu + 1)$

$17k + 4;\ (k \in I)$

Remainder = 4

6. If [·] represents the greatest integer function, then the value of $\left|\int_0^{\sqrt{\frac{\pi}{2}}}\left[\left[x^2\right] - \cos x\right]dx\right|$ is__________.

Ans. **(1)**

Sol.

$$\int_0^{\sqrt{\frac{\pi}{2}}}\left[\left[x^2\right] - \cos x\right]dx$$

$$= \int_0^1\left[-\cos x\right]dx + \int_1^{\sqrt{\frac{\pi}{2}}}\left[1 - \cos x\right]dx$$

$$= \int_0^1 -1 - \left[\cos x\right]dx + \int_1^{\sqrt{\frac{\pi}{2}}} dx + \int_1^{\sqrt{\frac{\pi}{2}}} -1 - \left[\cos x\right]dx$$

$$= -\int_0^1 dx + \int_1^{\sqrt{\frac{\pi}{2}}} dx - \int_1^{\sqrt{\frac{\pi}{2}}} dx$$

$$= -(x)_0^1 = -1$$

$$\therefore \boxed{\text{Ans : }1}$$

7. If $f(x) = \sin\left(\cos^{-1}\left(\frac{1 - 2^{2x}}{1 + 2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $\left|a^2 - b^2\right|$ is__________.

Ans. **(481)**

Sol.

$\cos^{-1}\left(\frac{1 - 4^x}{1 + 4^x}\right)$ Let $2^x = t > 0$

$\cos^{-1}\left(\frac{1-t^2}{1+t^2}\right)$ $t > 0$

$t = \tan\theta$

$\cos^{-1}(\cos 2\theta)$ $\theta \in \left(0, \frac{\pi}{2}\right)$

$2\theta \in (0, \pi)$

$\Rightarrow 2\theta$

$\therefore \sin\left\{\cos^{-1}\left(\frac{1-4^x}{1+4^x}\right)\right\}$

$= \sin 2\theta$

$y = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2.t}{1+t^2} = \frac{2.2^x}{1+4^x}$

$\therefore \frac{dy}{dx} = \frac{(1+4^x).2.2^x \ell n 2 - 2.2^x(4^x).\ell n 4}{(1+4^x)^2}$

$\frac{dy}{dx} = \frac{5.2.2.\ell n 2 - 2.2.4.2\ell n 2}{25}$

$= \frac{20\ell n 2 - 32\ell n 2}{25} = \frac{-12\ell n 2}{25}$

a=25, b= 12

$\therefore \left|a^2 - b^2\right|_{min} = \left|25^2 - 12^2\right|$

= (481)

8. If the function $f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$, then k is _____________.

Ans. **(6)**

Sol. $\frac{1}{k} = \lim\limits_{x\to 0} \frac{2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x - \sin x}{2}\right)}{x^4}$

$= \lim\limits_{x\to 0} \frac{(\sin x + x)(x - \sin x)}{2x^4} = \left(\lim\limits_{x\to 0}\frac{\sin x + x}{2x}\right)\left(\lim\limits_{x\to 0}\frac{x - \sin x}{x^3}\right)$

$= 1 \times \lim\limits_{x\to 0}\frac{1-\cos x}{3x^2} = \frac{1}{6}$

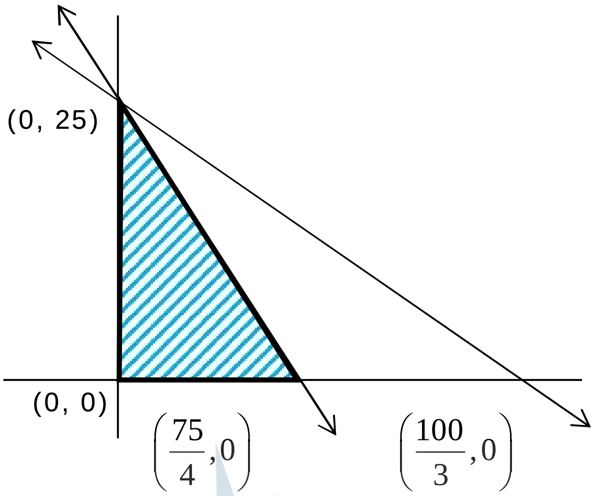
9. The maximum value of z in the following equation $z = 6xy+y^2$, where $3x+4y \le 100$ and $4x + 3y \le 75$ for $x \ge 0$ and $y \ge 0$ is ___________.

Ans. **(625)**

Sol. $x \ge 0,\ y \ge 0$

$4x + 3y \le 75$

$3x + 4y \le 100$



(x,y)	$z = 6xy + y^2$
(0,25)	z = 0 + 625 = 625
$\left(\frac{75}{4}, 0\right)$	z = 0 + 0 = 0
(0, 0)	z = 0 + 0 = 0

z_{max} = 625 at x = 0, y = 25

10. If $\vec{a} = \alpha\hat{i} + \beta\hat{j} + 3\hat{k}$,

$\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and

$\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$

such that $\vec{a} \cdot \vec{b} = 1$ and $\vec{b} \cdot \vec{c} = -3$, then $\frac{1}{3}\left(\left(\vec{a} \times \vec{b}\right).\vec{c}\right)$ is equal to__________.

Ans. **(2)**

Sol.

$\vec{a} = (\alpha, \beta, 3)$ $\qquad$ $\vec{b} = (-\beta, -\alpha, -1)$

$\vec{c} = (1, -2, -1)$

- $\vec{a} \cdot \vec{b} = 1 \Rightarrow (-\alpha\beta - \alpha\beta - 3) = 1$

$-2\alpha\beta = 4$

$\boxed{\alpha\beta = -2}$

- $\vec{b}\cdot\vec{c} = -3$

$-\beta + 2\alpha + 1 = -3$

$\boxed{2\alpha - \beta = -4}$ $\quad \therefore \begin{cases} \alpha = -1 \\ \beta = 2 \end{cases}$

$\vec{a} = (-1, 2, 3)$ $\quad \vec{b} = (-2, 1, -1)$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ -2 & 1 & -1 \end{vmatrix} = \hat{i}(-5) - \hat{j}(7) + \hat{k}(3) = (-5, -7, -3)$$

$$\therefore (\vec{a} \times \vec{b}).\vec{c} = (-5, -7, 3).(1, -2, -1)$$

$= (-5+14-3) = 6$