# MATHEMATICS
# JEE-MAIN (February-Attempt) 25 February (Shift-1) Paper

## SECTION - A

1. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

(1) $\frac{1}{54}$

(2) $\frac{1}{72}$

(3) $\frac{1}{36}$

(4) $\frac{5}{216}$

Ans. (4)

Sol. $ax^2 + bx + c = 0$

$a, b, c \in \{1,2,3,4,5,6\}$

$n(s) = 6 \times 6 \times 6 = 216$

$D = 0 \Rightarrow b^2 = 4ac$

$ac = \frac{b^2}{4}$ If $b = 2, ac = 1 \Rightarrow a = 1, c = 1$

If $b = 4, ac = 4 \Rightarrow a = 1, c = 4$

$a = 4, c = 1$

$a = 2, c = 2$

If $b = 6, ac = 9 \Rightarrow a = 3, c = 3$

$\therefore \text{probability} = \frac{5}{216}$

2. Let $\alpha$ be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is :

(1) $\frac{3}{4}$

(2) $\frac{1}{2}$

(3) $\frac{5}{8}$

(4) $\frac{3}{8}$

Ans. (3)

Sol. $l^2 + m^2 + n^2 = 1$

$\therefore 2n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore l^2 + m^2 = \frac{1}{2} \;\&\; l + m = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{2} - 2lm = \frac{1}{2}$

$\Rightarrow$ l m = 0 or m = 0

$\therefore$ l = 0, m = $\frac{1}{\sqrt{2}}$ or l = $\frac{1}{\sqrt{2}}$

$< 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ or $< \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$

$\therefore \cos\alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2}$

$\therefore \sin^4\alpha + \cos^4\alpha = 1 - \frac{1}{2}\sin^2(2\alpha) = 1 - \frac{1}{2}, \frac{3}{4} = \frac{5}{8}$

3. The value of the integral

$$\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta \text{ is}$$

(where c is a constant of integration)

(1) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(2) $\frac{1}{18}\left[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta\right]^{\frac{3}{2}} + c$

(3) $\frac{1}{18}\left[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta\right]^{\frac{3}{2}} + c$

(4) $\frac{1}{18}\left[9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta\right]^{\frac{3}{2}} + c$

Ans. (3)

Sol. $$\int \frac{2\sin^2\theta\cos\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{2\sin^2\theta} d\theta$$

Let $\sin\theta$ = t, $\cos\theta\, d\theta$ = dt

$= \int (t^6 + t^4 + t^2)\sqrt{2t^4 + 3t^2 + 6}\, dt = \int (t^5 + t^3 + t)\sqrt{2t^6 + 3t^4 + 6t^2}\, dt$

Let $2t^6 + 3t^4 + 6t^2 = z$

$12(t^5 + t^3 + t)\, dt = dz$

$= \frac{1}{12}\int \sqrt{z}\, dz = \frac{1}{18}z^{3/2} + c$

$= \frac{1}{18}[(2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta)^{3/2} + C$

$= \frac{1}{18}\left[\left(1 - \cos^2\theta\right)\left(2(1 - \cos^2\theta)^2 + 3 - 3\cos^2\theta + 6\right)\right]^{3/2} + C$

$= \frac{1}{18}[(1 - \cos^2\theta)(2\cos^4\theta - 7\cos^2\theta + 11)]^{3/2} + C$

$= \frac{1}{18}[-2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11]^{3/2} + C$

4. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

(1) $10(\sqrt{3}-1)$

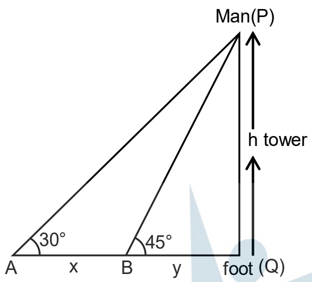
(2) $10\sqrt{3}$

(3) 10

(4) $10(\sqrt{3}+1)$

Ans. (4)

Sol.




$\frac{h}{x+y} = \tan 30°$

$x + y = \sqrt{3}h$ .......(1)

Also

$\frac{h}{y} = \tan 45°$

$h = y$ .......(2)

put in (1)

$x + y = \sqrt{3}y$

$x = (\sqrt{3}-1)y$

$\frac{x}{20} = \text{'v' speed}$

$\therefore$ time taken to reach

Foot from B

$\Rightarrow \frac{y}{V}$

$\Rightarrow \frac{x}{(\sqrt{3}-1).x} \times 20$

$\Rightarrow 10(\sqrt{3}+1)$

5. If $0 < \theta, \phi < \frac{\pi}{2}, x=\sum_{n=0}^{\infty}\cos^{2n}\theta,\ y=\sum_{n=0}^{\infty}\sin^{2n}\phi$ and

$z=\sum_{n=0}^{\infty}\cos^{2n}\theta.\sin^{2n}\phi$ then :

(1) xyz = 4
(2) xy – z = (x + y)z
(3) xy + yz + zx = z
(4) xy + z = (x +y)z

Ans. (4)

Sol. $x = 1 + \cos^2\theta + \ldots\ldots\ldots\infty$

$x = \frac{1}{1-\cos^2\theta}=\frac{1}{\sin^2\theta}$ .......(1)

$y = 1 + \sin^2\phi + \ldots\ldots\ldots\infty$

$y = \frac{1}{1-\sin^2\phi}=\frac{1}{\cos^2\phi}$ .......(2)

$z = \frac{1}{1-\cos^2\theta.\sin^2\phi}=\frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} = \frac{xy}{xy-(x-1)(y-1)}$

xz + yz – z = xy
xy + z = (x + y)z

6. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is :

(1) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$

(2) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$

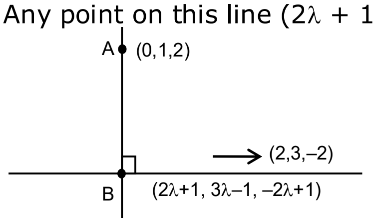
(3) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$

(4) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$

Ans. (1)

Sol. $\frac{x-1}{2} = \frac{y+1}{3}=\frac{z-1}{-2}=\lambda$

Any point on this line (2λ + 1, 3λ – 1, –2λ+1)




Direction ratio of given line (2, 3, –2)

Direction ratio of line to be found $(2\lambda + 1, 3\lambda - 2, -2\lambda - 1)$

$\therefore\ \vec{d}_1.\vec{d}_2 = 0$

$\lambda = 2/17$

Direction ratio of line $(21, -28, -21) \equiv (3, -4, -3) \equiv (-3, 4, 3)$

7. The statement $A \to (B \to A)$ is equivalent to:

(1) $A \to (A \wedge B)$

(2) $A \to (A \vee B)$

(3) $A \to (A \to B)$

(4) $A \to (A \leftrightarrow B)$

Ans. (2)

Sol. $A \to (B \to A)$

$\Rightarrow A \to (\sim B \vee A)$

$\Rightarrow \sim A \vee (\sim B \vee A)$

$\Rightarrow \sim B \vee (\sim A \vee A)$

$\Rightarrow \sim B \vee t$

= t (tantology)

From options :

(2) $A \to (A \vee B)$

$\Rightarrow \sim A \vee (A \vee B)$

$\Rightarrow (\sim A \vee A) \vee B$

$\Rightarrow t \vee B$

$\Rightarrow t$



8. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R is :

(1) 3

(2) 2

(3) 4

(4) 0

Ans. (1)

Sol. $D < 0$

$(2(3k-1))^2 - 4(8k^2 - 7) < 0$

$4(9k^2 - 6k + 1) - 4(8k^2 - 7) < 0$

$k^2 - 6k + 8 < 0$

$(k-4)(k-2) < 0$

$2 < k < 4$

then $k = 3$

9. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

(1) (0, 3)

(2) (−6, 0)

(3) (4, 5)

(4) (5, 4)

Ans. (4)

Sol. Equation of tangent : $y = mx + \frac{3}{2m}$

$m_T = \frac{1}{2}$ ($\because$ perpendicular to line $2x + y = 1$)

$\therefore$ tangent is : $y = \frac{x}{2} + 3$ $\Rightarrow x - 2y + 6 = 0$

10. Let f, g: N → N such that $f(n + 1) = f(n) + f(1)\ \forall n \in N$ and g be any arbitrary function. Which of the following statements is NOT true ?

(1) f is one-one

(2) If fog is one-one, then g is one-one

(3) If g is onto, then fog is one-one

(4) If f is onto, then $f(n) = n\ \forall n \in N$

Ans. (3)

Sol. $f(n + 1) = f(n) + 1$

$f(2) = 2f(1)$

$f(3) = 3f(1)$

$f(4) = 4f(1)$

….

$f(n) = nf(1)$

f(x) is one-one

11. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is :

(1) $\frac{3}{\sqrt{2}}$

(2) $3\sqrt{2}$

(3) $\frac{3}{2\sqrt{2}}$

(4) $\frac{1}{2\sqrt{2}}$

Ans. (3)

Sol. $(2-i)z=(2+i)\bar{z}$

$\Rightarrow (2-i)(x+iy)=(2+i)(x-iy)$

$\Rightarrow 2x - ix + 2iy + y = 2x + ix - 2\text{-}iy + y$

$\Rightarrow 2ix - 4iy = 0$

$L_1 : x - 2y = 0$

$\Rightarrow (2+i)z + (i-2)\bar{z} - 4i = 0.$

$\Rightarrow (2+i)(x+iy) + (i-2)(x-iy) - 4i = 0.$

$\Rightarrow 2x + ix + 2iy - y + ix - 2x + y + 2iy - 4i = 0$

$\Rightarrow 2ix + 4iy - 4i = 0$

$L_2 : x + 2y - 2 = 0$

Solve $L_1$ and $L_2$ $4y=2$, $y=\frac{1}{2}$

$\therefore x=1$

Centre $\left(1,\frac{1}{2}\right)$

$L_3 : iz + \bar{z} + 1 + i = 0$

$\Rightarrow i(x+iy) + x - iy + 1 + i = 0$

$\Rightarrow ix - y + x - iy + 1 + i = 0$

$\Rightarrow (x - y + 1) + i(x - y + 1) = 0$

Radius = distance from $\left(1,\frac{1}{2}\right)$ to $x - y + 1 = 0$

$r = \dfrac{1-\frac{1}{2}+1}{\sqrt{2}}$

$r = \dfrac{3}{2\sqrt{2}}$

12. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in:

(1) $\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$

(2) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

(3) $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

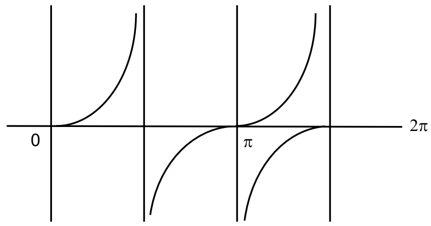
(4) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{11\pi}{6}\right)$

y

1

Ans. (2)

Sol.




$\tan 2\theta\,(1 + \cos 2\theta) > 0$

$2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

13. The image of the point (3,5) in the line $x - y + 1 = 0$, lies on :

(1) $(x - 2)^2 + (y - 4)^2 = 4$

(2) $(x - 4)^2 + (y + 2)^2 = 16$

(3) $(x - 4)^2 + (y - 4)^2 = 8$

(4) $(x - 2)^2 + (y - 2)^2 = 12$

Ans. (1)

Sol. Image of P(3, 5) on the line $x - y + 1 = 0$ is

$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2} = 1$

$x = 4,\ y = 4$

$\therefore$ Image is (4, 4)

Which lies on

$(x - 2)^2 + (y - 4)^2 = 4$



14. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

(1) (–5, 8)

(2) (5, 8)

(3) (5, –8)

(4) (–5, –8)

Ans. (2)

Sol. $f(1) = f(2)$

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

$3a - b = 7 \quad \ldots(1)$

$f'(x) = 3x^2 - 2ax + b$

$\Rightarrow f'\left(\frac{4}{3}\right) = 0 \Rightarrow 3 \times \frac{16}{9} - \frac{8}{3}a + b = 0$

$\Rightarrow -8a + 3b = -16 \quad \ldots(2)$

$a = 5,\ b = 8$

15. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90°, then which of the following relations is true ?

(1) a + b = c + d

(2) a − b = c − d

(3) ab = $\frac{c+d}{a+b}$

(4) a − c = b + d

Ans. (2)

Sol. $\frac{x^2}{a}+\frac{y^2}{b}=1$ .........(1)

diff : $\frac{2x}{a}+\frac{2y}{b}\frac{dy}{dx}=0 \Rightarrow \frac{y}{b}\frac{dy}{dx}=\frac{-x}{a}$

$\frac{dy}{dx}=\frac{-bx}{ay}$ ......(2)

$\frac{x^2}{c}+\frac{y^2}{d}=1$ .......(3)

Diff : $\frac{dy}{dx}=\frac{-dx}{cy}$ .......(4)

$m_1m_2 = -1 \Rightarrow \frac{-bx}{ay}\times\frac{-dx}{cy} = -1$

$\Rightarrow bdx^2 = -acy^2$ .......(5)

$(1)-(3) \Rightarrow \left(\frac{1}{a}-\frac{1}{c}\right)x^2 + \left(\frac{1}{b}-\frac{1}{d}\right)y^2 = 0$

$\Rightarrow \frac{c-a}{ac}x^2+\frac{d-b}{bd}\times\left(\frac{-bd}{ac}\right)x^2=0$ (using 5)

$\Rightarrow (c-a)-(d-b)=0$

$\Rightarrow c-a = d-b$

$\Rightarrow c-d = a-b$

16. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+\ldots\ldots+\frac{1}{n}}{n^2}\right)^n$ is equal to :

(1) $\frac{1}{2}$

(2) $\frac{1}{e}$

(3) 1

(4) 0

Ans. (3)

Sol. It is $1^{\infty}$ form

$$L = e^{\lim_{n\to\infty}\left(\frac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n}\right)}$$

$$S = 1+\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)+\left(\frac{1}{8}+........+\frac{1}{15}\right)$$

$$S < 1+\left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right).........+\underbrace{\left(\frac{1}{2^P}+........+\frac{1}{2^P}\right)}_{2^P \text{ times}}$$

$S < 1+ 1 + 1 + 1 +........ +1$

$S < P + 1$

$\therefore \quad L = e^{\lim_{n\to\infty}\frac{(P+1)}{2^P}}$

$\Rightarrow L = e^{o} = 1$

17. The total number of positive integral solutions (x, y, z) such that xyz = 24 is

(1) 36

(2) 45

(3) 24

(4) 30

Ans. (4)

Sol. x.y.z = 24

$x.y.z=2^3.3^1$

Now using beggars method.

3 things to be distributed among 3 persons

Each may receive none, one or more

$\therefore$ ${}^5C_2$ ways

Similarly for '1' $\quad \therefore$ ${}^3C_2$ ways

Total ways = ${}^5C_2 \,.\, {}^3C_2$ = 30 ways

18. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$ ,then this curve also passes through the point :

(1) (4, 5)

(2) (5, 4)

(3) (4, 4)

(4) (5, 5)

Ans. (4)

Sol. $\frac{dy}{dx} = \frac{(x-2)^2+y+4}{(x-2)} = (x-2)+\frac{y+4}{(x-2)}$

Let $x - 2 = t \Rightarrow dx = dt$

and $y + 4 = u \Rightarrow dy = du$

$\frac{dy}{dx}=\frac{du}{dt}$

$\frac{du}{dt}$ =t + $\frac{u}{t}\Rightarrow\frac{du}{dt}-\frac{u}{t}=t$

I.F = $e^{\int\frac{-1}{t}dt}=e^{-\ln t}=\frac{1}{t}$

u. $\frac{1}{t}=\int t.\frac{1}{t}dt\Rightarrow\frac{u}{t}=t+c$

$\frac{y+4}{x-2}$ = (x – 2) + c

Passing through (0, 0)

c = 0

$\Rightarrow$ (y + 4) = $(x - 2)^2$

19. The value of $\int_{-1}^{1}x^2e^{[x^3]}dx$, where [t] denotes the greatest integer $\leq$ t, is :

(1) $\frac{e+1}{3}$

(2) $\frac{e-1}{3e}$

(3) $\frac{e+1}{3e}$

(4) $\frac{1}{3e}$

Ans. (3)

Sol. I = $\int_{-1}^{0}x^2.e^{-1}dx+\int_{0}^{1}x^2dx$

$\therefore$ I = $\left.\frac{x^3}{3e}\right|_{-1}^{0}+\left.\frac{x^3}{3}\right|_{0}^{1}$

$\Rightarrow$ I = $\frac{1}{3e}+\frac{1}{3}$

20. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is:

(1) $\frac{1}{8}$

(2) $\frac{1}{27}$

(3) $\frac{3}{4}$

(4) $\frac{3}{8}$

Ans. (1)

Sol. Probability of not getting intercepted = $\frac{2}{3}$

Probability of missile hitting target = $\frac{3}{4}$

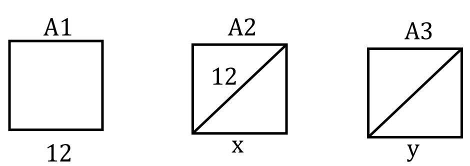
$\therefore$ Probability that all 3 hit the target = $\left(\frac{2}{3}\times\frac{3}{4}\right)^3 = \frac{1}{8}$

## Section : Mathematics Section B

1. Let $A_1, A_2, A_3, \ldots\ldots$ be squares such that for each $n \geq 1$, the length of the side of $A_n$ equals the length of diagonal of $A_{n+1}$. If the length of $A_1$ is 12 cm, then the smallest value of n for which area of $A_n$ is less than one, is _____________.

Ans. (9)

Sol.




$x = \frac{12}{\sqrt{2}}$ $\qquad$ $y = \frac{12}{(\sqrt{2})^2}$

$\therefore$ Side lengths are in G.P.

$T_n = \frac{12}{(\sqrt{2})^{n-1}}$

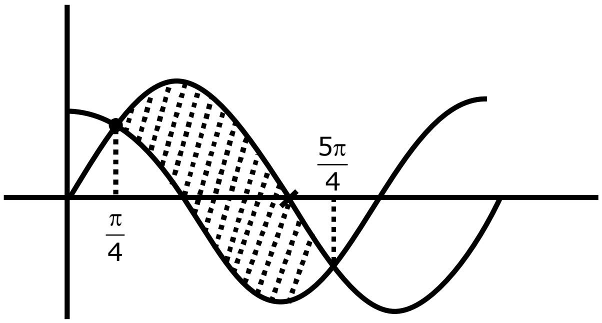
$\therefore$ Area = $\frac{144}{2^{n-1}} < 1 \qquad \Rightarrow 2^{n-1} > 144$

Smallest n = 9

2. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then $A^4$ is equal to _________

Ans. (64)

Sol.




$$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx = [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$$

$$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = \left(2\sqrt{2}\right)^4 = 64$$

3. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4(\sqrt{3})k = 0$ is a conic, whose eccentricity is _________.

Ans. (2)

Sol. $\sqrt{3}kx + ky = 4\sqrt{3}$ ......... (1)

$\sqrt{3}kx - ky = 4\sqrt{3}k^2$ .........(2)

Adding equation (1) & (2)

$2\sqrt{3}kx = 4\sqrt{3}(k^2 + 1)$

$x = 2\left(k + \frac{1}{k}\right)$ ........(3)

Substracting equation (1) & (2)

$y = 2\sqrt{3}\left(\frac{1}{k} - k\right)$ .........(4)

$\therefore \frac{x^2}{4} - \frac{y^2}{12} = 4$

$\frac{x^2}{16}-\frac{y^2}{48}=1$ Hyperbola

$\therefore e^2 = 1 + \frac{48}{16}$

e = 2

4. If A = $\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1}$

$= \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to _______.

Ans. (13)

Sol. $A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$

$\Rightarrow I + A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\Rightarrow I - A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ $\{\therefore |I - A| = \sec^2\theta/2\}$

$\Rightarrow (I - A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\Rightarrow (I+A)(I-A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$= \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1-\tan^2\frac{\theta}{2} & -2\tan\frac{\theta}{2} \\ 2\tan\frac{\theta}{2} & 1-\tan^2\frac{\theta}{2} \end{bmatrix}$

$a = \frac{1-\tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$

$b = \dfrac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$

$\therefore a^2 + b^2 = 1$

5. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of $x^6$ is unity and it has extrema at x = −1 and x = 1. If $\lim_{x\to 0}\frac{f(x)}{x^3} = 1$, then 5.f(2) is equal to __________

Ans. (144)

Sol. $f(x) = x^6 + ax^5 + bx^4 + x^3$

$\therefore f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

Roots 1 & − 1

$\therefore 6 + 5a + 4b + 3 = 0$ & $-6 + 5a - 4b + 3 = 0$ solving

$a = -\frac{3}{5}$ $\quad\quad b = -\frac{3}{2}$

$\therefore f(x) = x^6 - \frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3$

$\therefore 5.f(2) = 5\left[64 - \frac{96}{5} - 24 + 8\right] = 144$

6. The number of points, at which the function $f(x) = |2x + 1| - 3|x+2| + |x^2 + x - 2|$, $x \in R$ is not differentiable, is _________.

Ans. (2)

Sol. $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$$f(x) = \begin{cases} x^2 - 7 \ ; & x > 1 \\ -x^2 - 2x - 3 \ ; & -\frac{1}{2} < x < 1 \\ -x^2 - 6x - 5 \ ; & -2 < x < \frac{-1}{2} \\ x^2 + 2x + 3; & x < -2 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 2x \ ; & x > 1 \\ -2x - 3 \ ; & -\frac{1}{2} < x < 1 \\ -2x - 6; & -2 < x < \frac{-1}{2} \\ 2x + 2; & x < -2 \end{cases}$$

Check at 1, −2 and $\frac{-1}{2}$

Non. Differentiable at x = 1 and $\frac{-1}{2}$

7. If the system of equations

$$kx + y + 2z = 1$$
$$3x - y - 2z = 2$$
$$-2x - 2y - 4z = 3$$

has infinitely many solutions, then k is equal to __________.

Ans. (21)

Sol. D = 0

$$\Rightarrow \begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & -4 \end{vmatrix} = 0$$

$\Rightarrow$ k (4 – 4)– 1(–12 –4) +2(–6 –2)

$\Rightarrow$ 16 – 16 = 0

Also. $D_1 = D_2 = D_3 = 0$

$$\Rightarrow D_2 = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow$ k(–8+6)–1(–12–4)+2(9+4)=0

$\Rightarrow$ –2k + 16 + 26 = 0

$\Rightarrow$ 2k = 42

$\Rightarrow$ k = 21

8. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$, then $\vec{r}.\vec{a}$ is equal to _______

Ans. (12)

Sol. $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\vec{r} \times \vec{a} - \vec{c} \times \vec{a} = 0$

$(\vec{r} - \vec{c}) \times \vec{a} = 0$

$\therefore \ \vec{r} - \vec{c} = \lambda \vec{a}$

$\vec{r} = \lambda \vec{a} + \vec{c}$

$\vec{r}.\vec{b} = \lambda \vec{a}.\vec{b} + \vec{c}.\vec{b} = 0$

$\Rightarrow \lambda$ (1–2) + 2 = 0

$\Rightarrow \lambda$ = 2

$\therefore \ \vec{r} = 2\vec{a} + \vec{c}$

$\vec{r}.\vec{a} = 2|\vec{a}|^2 + \vec{a}.\vec{c}$

= 2(1 + 4 + 1) + (1 – 2 + 1)

= 12

9. Let A = $\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that x + y + z > 0 and xyz = 2.

If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is _______.

Ans. (7)

Sol. A = $\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ $\therefore$ |A| = $\left(x^3+y^3+z^3-3xyz\right)$

$A^2 = I_3$

$|A^2| = 1$

$\therefore (x^3 + y^3 + z^3 - 3xyz)^2 = 1$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 1$ only as (x+y+z>0)

$\Rightarrow x^3 + y^3 + z^3 = 6 + 1 = 7$

10. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4,5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5 is ________.

Ans. (32)

Sol. □□□ divisible by $\rightarrow$ 3

$12 \rightarrow 3, 4, 5 \rightarrow 3! = 6$

$15 \rightarrow 2, 3, 4 \rightarrow 3! = 6$

$24 \rightarrow 1, 3, 5 \rightarrow 3! = 6$

$42 \rightarrow 1, 2, 3 \rightarrow 3! = 6$

24

divisible by 5

□□5 = 12

4 × 3

Required No. = 24 + 12 − 4 = 32