

MATHEMATICS
JEE-MAIN (February-Attempt) 26
February (Shift-1) Paper

SECTION - A

1. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

- (1) 77
- (2) 42
- (3) 35
- (4) 82

Ans. (1)

Sol. CASE-I: 1, 1, 1, 1, 1, 2, 3

$$\text{WAYS} = \frac{7!}{5!} = 42$$

CASE-II: 1, 1, 1, 1, 2, 2, 2

$$\text{WAYS} = \frac{7!}{4!3!} = 35$$

$$\text{TOTAL WAYS} = 42 + 35 = 77$$

2. The maximum value of the term independent of 't' in

the expansion of $\left(tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t} \right)^{10}$ where $x \in (0,1)$ is:

- (1) $\frac{10!}{\sqrt{3}(5!)^2}$
- (2) $\frac{2 \cdot 10!}{3(5!)^2}$
- (3) $\frac{10!}{3(5!)^2}$
- (4) $\frac{2 \cdot 10!}{3\sqrt{3}(5!)^2}$

Ans. (4)

Sol. $T_{r+1} = {}^{10}C_r (tx^{1/5})^{10-r} \left[\frac{(1-x)^{1/10}}{t} \right]^r$
 $= {}^{10}C_r t^{(10-2r)} \times x^{\frac{10-r}{5}} \times (1-x)^{\frac{r}{10}}$

$$\Rightarrow 10 - 2r = 0 \Rightarrow r = 5$$

$$T_6 = {}^{10}C_5 x \sqrt{1-x}$$

$$\frac{dT_6}{dx} = {}^{10}C_5 \left[\sqrt{1-x} - \frac{x}{2\sqrt{1-x}} \right] = 0$$

$$= 1 - x = x/2 \Rightarrow 3x = 2$$
$$\Rightarrow x = 2/3$$

$$T_6|_{\max} = \frac{10!}{5!5!} \times \frac{2}{3\sqrt{3}}$$

3. The value of $\sum_{n=1}^{100} \int_{n-1}^n e^{x-[x]} dx$, where $[x]$ is the greatest integer $\leq x$, is:

(1) $100(e-1)$

(2) $100e$

(3) $100(1-e)$

(4) $100(1+e)$

Ans. (1)

Sol. $\sum_{n=1}^{100} \int_{n-1}^n e^{x-[x]} dx$
 $= \int_0^1 e^{(x)} dx + \int_1^2 e^{(x)} dx + \int_2^3 e^{(x)} dx + \dots + \int_{99}^{100} e^{(x)} dx \quad (\because \{x\} = x - [x])$
 $= e^x \Big|_0^1 + e^{(x-1)} \Big|_1^2 + e^{(x-2)} \Big|_2^3 + \dots + e^{(x-99)} \Big|_{99}^{100}$
 $= (e-1) + (e-1) + (e-1) + \dots + (e-1)$
 $= 100(e-1)$

4. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e \left(\frac{6}{5} \right)}$ hours,

then $\left(\frac{k}{\log_e 2} \right)^2$ is equal to

(1) 4

(2) 2

(3) 16

(4) 8

Ans. (1)

Sol. $\frac{dx}{dt} \propto x$

$$\frac{dx}{dt} = \lambda x$$

$$\int_{1000}^x \frac{dx}{x} = \int_0^t \lambda dt$$

$$\ln x - \ln 1000 = \lambda t$$

$$\ln\left(\frac{x}{1000}\right) = \lambda t$$

Put $t = 2, x = 1200$

$$\ln\left(\frac{12}{10}\right) = 2\lambda \Rightarrow \lambda = \frac{1}{2} \ln \frac{6}{5}$$

$$\text{Now } \ln\left(\frac{x}{1000}\right) = \frac{t}{2} \ln\left(\frac{6}{5}\right)$$

$$x = 1000e^{\frac{t}{2} \ln\left(\frac{6}{5}\right)}$$

$$x = 2000 \text{ at } t = \frac{k}{\ln\left(\frac{6}{5}\right)}$$

$$\Rightarrow 2000 = 1000 e^{\frac{k}{2 \ln(6/5)} \times \ln(6/5)}$$

$$\Rightarrow 2 = e^{k/2}$$

$$\Rightarrow \ln 2 = \frac{k}{2}$$

$$\Rightarrow \frac{k}{\ln 2} = 2$$

$$\Rightarrow \left(\frac{k}{\ln 2}\right)^2 = 4$$

5. If \vec{a} & \vec{b} are perpendicular vectors, then

$\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))$ is equal to

(1) $\frac{1}{2} |\vec{a}|^4 \vec{b}$

(2) $\vec{a} \times \vec{b}$

(3) $|\vec{a}|^4 \vec{b}$

(4) $\vec{0}$

Ans. (3)

Sol. $\vec{a} \times (\vec{a} \times ((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2 \vec{b}))$

$$\vec{a} \times (-|\vec{a}|^2 (\vec{a} \times \vec{b})) = -|\vec{a}|^2 ((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2 \vec{b})$$

$$= -(\vec{a} \cdot \vec{b})\vec{a} |\vec{a}|^2 + |\vec{a}|^4 \vec{b}$$

$$= |\vec{a}|^4 \vec{b} \quad (\because \vec{a} \cdot \vec{b} = 0)$$

6. In an increasing, geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4th, 6th and 8th terms is equal to :

(1) 35

(2) 30

(3) 26

(4) 32

Ans (1)

Sol. $ar + ar^5 = \frac{25}{2}$

$$ar^2 \times ar^4 = 25$$

$$a^2 r^6 = 25$$

$$ar^3 = 5$$

$$\boxed{a = \frac{5}{r^3}} \quad \dots(1)$$

$$\frac{5r}{r^3} + \frac{5r^5}{r^3} = \frac{25}{2}$$

$$\frac{1}{r^2} + r^2 = \frac{5}{2}$$

Put $r^2 = t$

$$\frac{t^2 + 1}{t} = \frac{5}{2}$$

$$2t^2 - 5t + 2 = 0$$

$$2t^2 - 4t - t + 2 = 0$$

$$(2t - 1)(t - 2) = 0$$

$$t = \frac{1}{2}, 2 \Rightarrow \boxed{r^2 = \frac{1}{2}, 2}$$

$$r = \sqrt{2}$$

$$\begin{aligned} &= ar^3 + ar^5 + ar^7 \\ &= ar^3 (1 + r^2 + r^4) \\ &= 5[1 + 2 + 4] = 35 \end{aligned}$$

7. Consider the three planes

$$P_1 : 3x + 15y + 21z = 9,$$

$$P_2 : x - 3y - z = 5, \text{ and}$$

$$P_3 : 2x + 10y + 14z = 5$$

Then, which one of the following is true?

(1) P_1 and P_3 are parallel.

(2) P_2 and P_3 are parallel.

(3) P_1 and P_2 are parallel.

(4) P_1 , P_2 and P_3 all are parallel.

Ans. (1)

Sol. $P_1 = x + 5y + 7z = 3$

$$P_2 = x - 3y - z = 5$$

$$P_3 = x + 5y + 7z = 5/2$$

$$\Rightarrow P_1 \parallel P_3$$

8. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots$ is equal to

(1) $\frac{9}{4}$

(2) $\frac{15}{4}$

(3) $\frac{13}{4}$

(4) $\frac{11}{4}$

Ans. (3)

Sol. $s = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots$

$$\frac{s}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \dots \infty$$

$$\frac{2s}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \dots \infty$$

$$\frac{2s}{3} = \frac{4}{3} + \frac{5}{3} \left\{ \frac{1/3}{1 - \frac{1}{3}} \right\} = \frac{5}{6} + \frac{4}{3} = \frac{13}{6}$$

$$\boxed{s = \frac{13}{4}}$$

9. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) -2

(2) (a+1)(a+2)(a+3)

(3) 0

(4) (a+2)(a+3)(a+4)

Ans. (1)

Sol. $C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3$

$$= \begin{vmatrix} (a+2)a & a+1 & 1 \\ (a+3)(a+1) & a+2 & 1 \\ (a+4)(a+2) & a+3 & 1 \end{vmatrix}$$

$R_2 \rightarrow R_2 - R_1$ & $R_3 \rightarrow R_3 - R_1$

$$= \begin{vmatrix} a^2 + 2a & a+1 & 1 \\ 2a+3 & 1 & 0 \\ 4a+8 & 2 & 0 \end{vmatrix}$$

$$= 6 - 8 = -2$$

10. If $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$; $0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is:

(1) $\frac{1-y^2}{2y}$

(2) $\frac{1-y^2}{1+y^2}$

(3) $1-y^2$

(4) $\frac{1-y^2}{y\sqrt{y}}$

Ans. (2)

Sol. $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$

$$\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\sin^{-1} x + \cos^{-1} x}{a+b} = \frac{\pi}{2(a+b)}$$

Now, $\frac{\tan^{-1} y}{c} = \frac{\pi}{2(a+b)}$

$$2 \tan^{-1} y = \frac{\pi c}{a+b}$$

$$\Rightarrow \cos\left(\frac{\pi c}{a+b}\right) = \cos(2 \tan^{-1} y) = \frac{1-y^2}{1+y^2}$$

11. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is:

(1) 6

(2) 1

(3) 4

(4) 12

Ans. (3)

Sol. Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & ab + bc \\ ab + bc & c^2 + b^2 \end{bmatrix}$$

$$= a^2 + 2b^2 + c^2 = 1$$

$$a = 1, b = 0, c = 0$$

$$a = 0, b = 0, c = 1$$

$$a = -1, b = 0, c = 0$$

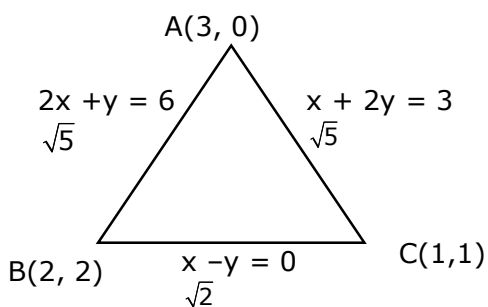
$$c = -1, b = 0, a = 0$$

12. The intersection of three lines $x - y = 0$, $x +$

$2y = 3$ and $2x + y = 6$ is a:

- (1) Equilateral triangle
- (2) None of the above
- (3) Isosceles triangle
- (4) Right angled triangle

Ans. (3)
Sol.



13. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the

point:

(1) (2, 9)

(2) (2, 2)

(3) $\left(3, \frac{21}{2}\right)$

(4) (0, 0)

Ans. (2)

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Let $f(x) = 2x^3 - 15x^2 + 36x - 19$

$f'(x) = 6x^2 - 30x + 36 = 0$

$x^2 - 5x + 6 = 0$

$x = 2, 3$

$f''(x) = 12x - 30$

$f''(x) < 0$ for $x = 2$

At $x = 2$

$y = 8 - 40 + 72 - 38$

$y = 72 - 70 = 2$

$\Rightarrow (2, 2)$

14. Let f be any function defined on \mathbb{R} and let it satisfy the condition :

$$|f(x) - f(y)| \leq |x - y|^2, \forall (x, y) \in \mathbb{R}$$

If $f(0) = 1$, then:

(1) $f(x) < 0, \forall x \in \mathbb{R}$

(2) $f(x)$ can take any value in \mathbb{R}

(3) $f(x) = 0, \forall x \in \mathbb{R}$

(4) $f(x) > 0, \forall x \in \mathbb{R}$

Ans. (4)

Sol. $|f(x) - f(y)| \leq |x - y|^2, \forall (x, y) \in \mathbb{R}$

$$\left| \frac{f(x) - f(y)}{x - y} \right| \leq |x - y|$$

$$\lim_{x \rightarrow y} \left| \frac{f(x) - f(y)}{x - y} \right| \leq 0$$

$$|f'(y)| \leq 0 \Rightarrow f'(y) = 0$$

$$f(y) = C$$

$$\Rightarrow \boxed{C = 1} \quad \because f(0) = 1$$

$$\Rightarrow f(x) = 1$$

15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1 + 3^x} dx$ is:

(1) 2π

(2) 4π

(3) $\frac{\pi}{2}$

(4) $\frac{\pi}{4}$

Ans. (4)

Sol. Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$

$$I = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \cos^2 x dx$$

$$I = \int_0^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$$

16. The value of $\lim_{h \rightarrow 0} \left\{ \frac{\sqrt{3} \sin\left(\frac{\pi}{6} + h\right) - \cos\left(\frac{\pi}{6} + h\right)}{\sqrt{3}h(\sqrt{3} \cosh - \sinh)} \right\}$ is:

(1) $\frac{3}{4}$

(2) $\frac{2}{\sqrt{3}}$

(3) $\frac{4}{3}$

(4) $\frac{2}{3}$

Ans. (3)

Sol. $\lim_{h \rightarrow 0} 2 \times 2 \left\{ \frac{\sin\left(\frac{\pi}{6} + h - \frac{\pi}{6}\right)}{2\sqrt{3}h \left(\cos\left(h + \frac{\pi}{6}\right)\right)} \right\}$

$$= \frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = \frac{4}{3}$$

17. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

(1) $\frac{15}{2^{12}}$

(2) $\frac{15}{2^{13}}$

(3) $\frac{15}{2^{14}}$

(4) $\frac{15}{2^8}$

Ans. (2)

Sol. $p(x = 9) = p(x = 7)$

$${}^n C_9 \left(\frac{1}{2}\right)^{n-9} \times \left(\frac{1}{2}\right)^9 = {}^n C_7 \left(\frac{1}{2}\right)^{n-7} \times \left(\frac{1}{2}\right)^7$$

$${}^n C_9 \times \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2 \times {}^n C_7$$

$$x + y = n \Rightarrow n = 16$$

$$p(x = 2) = {}^{16} C_2 \times \left(\frac{1}{2}\right)^{14} \times \left(\frac{1}{2}\right)^2$$

$$= {}^{16} C_2 \times \left(\frac{1}{2}\right)^{16} = \frac{15}{2^{13}}$$

18. If $(1, 5, 35)$, $(7, 5, 5)$, $(1, \lambda, 7)$ and $(2\lambda, 1, 2)$ are coplanar, then the sum of all possible values of λ is:

(1) $-\frac{44}{5}$

(2) $\frac{39}{5}$

(3) $-\frac{39}{5}$

(4) $\frac{44}{5}$

Ans. (4)

Sol. Let $P(1,5,35)$, $Q(7,5,5)$, $R(1,\lambda,7)$, $S(2\lambda,1,2)$

$$\begin{bmatrix} \vec{PQ} & \vec{PR} & \vec{PS} \end{bmatrix} = 0$$

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda - 5 & -28 \\ 2\lambda - 1 & -4 & -33 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & -5 \\ 0 & \lambda - 5 & -28 \\ 2\lambda - 1 & -4 & -33 \end{vmatrix} = 0$$

$$\{-33\lambda + 165 - 112\} + 5(\lambda - 5)(2\lambda - 1) = 0$$

$$53 - 33\lambda + 5\{2\lambda^2 - 11\lambda + 5\} = 0$$

$$16\lambda^2 - 88\lambda + 78 = 0$$

$$5\lambda^2 - 44\lambda + 39 = 0 \begin{matrix} \lambda_1 \\ \lambda_2 \end{matrix}$$

$$\Rightarrow \lambda_1 + \lambda_2 = 44 / 5$$

19. Let $R = \{P,Q\} | P \text{ and } Q \text{ are at the same distance from the origin}$ be a relation, then the equivalence class of $(1,-1)$ is the set:

(1) $S = \{(x,y) | x^2 + y^2 = 1\}$

(2) $S = \{(x,y) | x^2 + y^2 = 4\}$

(3) $S = \{(x,y) | x^2 + y^2 = \sqrt{2}\}$

(4) $S = \{(x,y) | x^2 + y^2 = 2\}$

Ans. (4)

Sol. $P(a, b)$, $Q(c, d)$, $PO = QO$

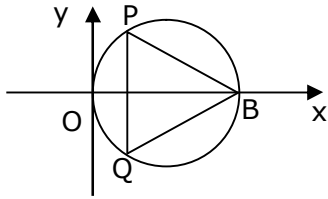
$$\Rightarrow a^2 + b^2 = c^2 + d^2$$

$$R(x,y) \quad s = (1, -1) \Rightarrow RO = SO$$

(\because equivalence class)

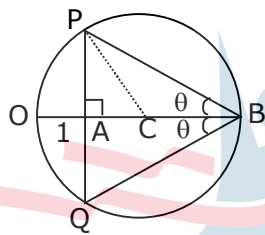
$$x^2 + y^2 = 2$$

20. In the circle given below, let $OA = 1$ unit, $OB = 13$ unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is:



- (1) $26\sqrt{3}$
 (2) $24\sqrt{2}$
 (3) $24\sqrt{3}$
 (4) $26\sqrt{2}$

Ans. (3)
Sol.



$$OC = \frac{13}{2} = 6.5$$

$$AC = CO - AO$$

$$= 6.5 - 1$$

$$= 5.5$$

In ΔPAC

$$PA = \sqrt{6.5^2 - 5.5^2}$$

$$PA = \sqrt{12}$$

$$\Rightarrow PQ = 2PA = 2\sqrt{12}$$

$$\text{Now, area of } \Delta PQB = \frac{1}{2} \times PQ \times AB$$

$$= \frac{1}{2} \times 2\sqrt{12} \times 12$$

$$= 12\sqrt{12}$$

$$= 24\sqrt{3}$$

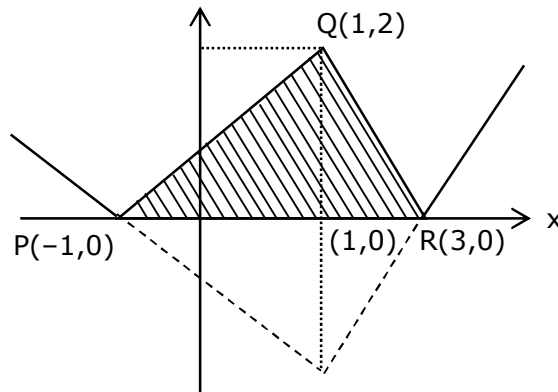
Section-B

1. The area bounded by the lines $y = ||x-1|-2|$ is.....

Ans. Bonus

NTA Ans. (8)

Sol.



2. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in \mathbb{R}$ is _____.

Ans. (11)

Sol. $3\sin x + 4\cos x = k+1$

$$-5 \leq k + 1 \leq 5$$

$$-6 \leq k \leq 4$$

$$\boxed{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4} \Rightarrow 11 \text{ integral values}$$

3. Let $m, n \in \mathbb{N}$ and $\gcd(2, n) = 1$. If $30 \binom{30}{0} + 29 \binom{30}{1} + \dots + 2 \binom{30}{28} + 1 \binom{30}{29} = n \cdot 2^m$,

then $n + m$ is equal to_____.

Ans. (45)

Sol. Let $S = \sum_{r=0}^{30} (30-r) {}^{30}C_r$

$$= 30 \sum_{r=0}^{30} {}^{30}C_r - \sum_{r=0}^{30} r {}^{30}C_r$$

$$\begin{aligned}
&= 20 \times 2^{30} - \sum_{r=1}^{30} r \cdot \frac{30}{4} \cdot {}^{29}C_{r-1} \\
&= 30 \times 2^{30} - 30 \cdot 2^{29} \\
&= (30 \times 2 - 30) \cdot 2^{29} = 30 \cdot 2^{29} \Rightarrow 15 \cdot 2^{30} \\
&= n = 15, m = 30 \\
&n + m = 45
\end{aligned}$$

4. If $y = y(x)$ is the solution of the equation $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x, y(0) = 0$; then

$1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2} y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}} y\left(\frac{\pi}{4}\right)$ is equal to _____.

Ans. (1)

Sol. $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$

Put $e^{\sin y} = t$

$$e^{\sin y} \times \cos y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dt}{dx} + t \cos x = \cos x$$

$$\text{I.F.} = e^{\int \cos x dx} = e^{\sin x}$$

Solution of diff equation:

$$t \cdot e^{\sin x} = \int e^{\sin x} \cdot \cos x dx$$

$$e^{\sin y} \cdot e^{\sin x} = e^{\sin x} + c$$

$$\text{at } x = 0, y = 0$$

$$1 = 1 + c \Rightarrow c = 0$$

$$e^{\sin x + \sin y} = e^{\sin x}$$

$$\sin x + \sin y = \sin x$$

$$y = 0$$

$$\Rightarrow y\left(\frac{\pi}{6}\right) = 0, \quad y\left(\frac{\pi}{3}\right) = 0, \quad y\left(\frac{\pi}{4}\right) = 0$$

$$\Rightarrow 1 + 0 + 0 + 0 = 1$$

5. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is _____.

Ans. (1)

Sol. $\frac{1}{2} \log_2(x-1) = \log_2(x-3)$

$$x-1 = (x-3)^2$$

$$x^2 - 6x + 9 = x - 1$$

$$x^2 - 7x + 10 = 0$$

$$x = 2, 5$$

$x = 2$ Not possible as $\log_2(x-3)$ is not defined

\Rightarrow No. of solution = 1

6. If $\sqrt{3}(\cos^2 x) = (\sqrt{3}-1)\cos x + 1$, the number of solutions of the given equation when

$x \in \left[0, \frac{\pi}{2}\right]$ is _____.

Ans. (1)

Sol. $\sqrt{3}t^2 - (\sqrt{3}-1)t - 1 = 0$ (where $t = \cos x$)

$$\text{Now, } t = \frac{(\sqrt{3}-1) \pm \sqrt{4+2\sqrt{3}}}{2\sqrt{3}}$$

$$t = \cos x = 1 \text{ or } -\frac{1}{\sqrt{3}} \rightarrow \text{rejected as } x \in \left[0, \frac{\pi}{2}\right]$$

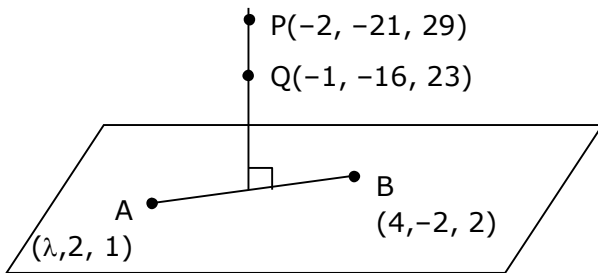
$$\Rightarrow \cos x = 1$$

\Rightarrow No. of solution = 1

7. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the point $(-2, -21, 29)$ and $(-1, -16, 23)$, then

$$\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4 \text{ is equal to } \underline{\hspace{2cm}}.$$

Ans. (8)
Sol.



$$\vec{AB} \perp \vec{PQ}$$

$$[(4 - \lambda)\hat{i} - 4\hat{j} + \hat{k}] \cdot [\hat{i} + 5\hat{j} - 6\hat{k}] = 0$$

$$4 - \lambda - 20 - 6 = 0$$

$$\lambda = -22$$

Now, $\frac{\lambda}{11} = -2$

$$\Rightarrow \left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$$

$$\Rightarrow 4 + 8 - 4 = 8$$

8. The difference between degree and order of differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$, $a > 0$ is _____.

Ans. (2)

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$

$$2yy' = a$$

$$y^2 = 2yy' \left(x + \frac{\sqrt{2yy'}}{2}\right)$$

$$y = 2y' \left(x + \frac{\sqrt{yy'}}{\sqrt{2}} \right)$$

$$y - 2xy' = \sqrt{2}y' \sqrt{yy'}$$

$$\left(y - 2x \frac{dy}{dx} \right)^2 = 2y \left(\frac{dy}{dx} \right)^3$$

$$D = 3 \quad \& \quad O = 1$$

$$D - O = 3 - 1 = 2$$

9. The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is _____.

Ans. (3)

Sol. Let roots of $x^3 - 2x^2 + 2x - 1 = 0$ are α, β, γ

$$(x - 1)(x^2 - x + 1) = 0$$

$$x = \underset{\alpha}{1}, \underset{\beta}{-\omega}, \underset{\gamma}{-\omega^2}$$

$$\text{Now } \alpha^{162} + \beta^{162} + \gamma^{162}$$

$$= 1 + \omega^{162} + (\omega^2)^{162}$$

$$= 1 + (\omega^3)^{54} + (\omega^3)^{108}$$

$$= 3$$

10. The value of the integral $\int_0^{\pi} |\sin 2x| dx$ is _____

Ans. (2)

Sol. $I = \int_0^{\pi} |\sin 2x| dx$

$$I = 2 \int_0^{\pi/2} |\sin 2x| dx = 2 \int_0^{\pi/2} \sin 2x dx$$

$$I = 2 \left[\frac{-\cos(2x)}{2} \right]_0^{\pi/2} = 2$$