MATHEMATICS
JEE-MAIN (July-Attempt) 6 SEPTEMBER
(Shift-2) Paper

SECTION - A

Q.1 If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies:

(1) $e^4+2e^2-1=0$ (2) $e^2+2e-1=0$ (3) $e^4+e^2-1=0$ (4) $e^2+e-1=0$

Sol. **(3)**

Equation of normal at $\left(ae, \frac{b^2}{a}\right)$

$$\frac{a^2x}{ae} - \frac{b^2y}{\frac{b^2}{a}} = a^2 - b^2$$

It passes through (0,–b)

$ab = a^2 e^2$

$a^2 b^2 = a^4 e^4$ $\quad (b^2 = a^2(1-e^2))$

$1-e^2 = e^4$

Q.2 The set of all real values of λ for which the function $f(x) = (1-\cos^2 x).(\lambda+\sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is:

(1) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (4) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Sol. **(1)**

$f(x) = (1-\cos^2x)(\lambda + \sin x)$

$f(x) = \sin^2x(\lambda + \sin x)$

$f'(x) = 2\sin x \cos x(\lambda + \sin x) + \sin^2x(\cos x)$

$= \sin 2x \left(\lambda + \sin x + \frac{\sin x}{2}\right)$

$= \sin 2x\,(2\lambda + 3\sin x)$

$\sin 2x = 0 \Rightarrow \sin x = 0 \rightarrow$ One point

$2\lambda + 3\sin x \Rightarrow \sin x = \frac{-2\lambda}{3}$

$\sin x \in (-1,1) - \{0\}$

$-1 < \frac{-2\lambda}{3} < 1 \Rightarrow \frac{-3}{2} < \lambda < \frac{3}{2}$

$\lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right) - \{0\}$

Q.3 The probabilities of three events A, B and C are given by P(A)=0.6, P(B)=0.4 and P(C)=0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval:

(1) [0.36,0.40] (2) [0.25,0.35] (3) [0.35,0.36] (4) [0.20,0.25]

Sol. **(2)**

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$\alpha = 0.6 + 0.4 + 0.5 - P(A \cap B) - \beta - 0.3 + 0.2$

$\alpha = 1.4 - P(A \cap B) - \beta \Rightarrow \alpha + \beta = 1.4 - P(A \cap B)$ ....(1)

again

$P(A \cup B) = P(A) + P(B) - p(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.2$ ....(2)

Put the value $P(A \cap B)$ in equation (1)

$\alpha + \beta = 1.2$

$\alpha = 1.2 - \beta$

$0.85 \le \alpha \le 0.95 \Rightarrow 0.85 \le 1.2 - \beta \le 0.95$

$P \in [0.25, 0.35]$

Q.4 The common difference of the A.P. $b_1, b_2, \ldots\ldots b_m$ is 2 more than the common difference of A.P. $a_1, a_2, \ldots a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to:

(1) −127 (2) 81 (3) 127 (4) −81

Sol. **(4)**

Common diff of A.P = 2+ common difference

$(b_1, b_2, b_3 \_\_\_\_ b_m)$ A.P $(a_1, a_2, a_3 \_\_\_\_ a_n)$

$D_b = D_a + 2$

$a_{40} = -159$

$a_1 + 39 D_a = -159$ ----(1)

$a_{100} = -399$

$a_1 + 99 D_a = -399$ ----(2)

Eqn (1) − (2)

$-60 D_a = 240 \Rightarrow D_a = -4$

$D_b = -4 + 2 = -2$

$a_1 + 39(-4) = -159 \Rightarrow a_1 = -3$

$b_{100} = a_{70}$

$b_1 + 99 D_b = a_1 + 69 D_a$

$b_1 + 99(-2) = (-3) + 69(-4)$

$b_1 = -81$

Q.5 The integral $\int_1^2 e^x . x^x (2 + \log_e x) dx$ equal :

(1) $e(4e-1)$ (2) $e(4e+1)$ (3) $4e^2-1$ (4) $e(2e-1)$

Sol. **(1)**

$$\int_1^2 e^x.x^x \quad (2+\ln x)\, dx$$

$e^x.\ x^x = t$

$(e^x.x^x + e^x x^x(1+\ln x))\, dx = dt$

$e^x\ .\ x^x (2 + \ln x)\, dx\ = dt$

$$\int_e^{4.e^2} dt \;=\; [t]_e^{4.e^2} = 4.e^2 - e = e(4e-1)$$

Q.6 If the tangent to the curve, $y=f(x)=x\log_e x$, $(x>0)$ at a point $(c,f(c))$ is parallel to the line-segment joining the points (1,0) and (e,e), then c is equal to:

(1) $e^{\left(\frac{1}{1-e}\right)}$ (2) $\frac{e-1}{e}$ (3) $\frac{1}{e-1}$ (4) $e^{\left(\frac{1}{e-1}\right)}$

Sol. **(4)**

$y = f(x) = x \ln x$

$$m_1 = \frac{dy}{dx}\Big|_{(c_1, f(c))} = (\ln x+1)\Big|_{c_1, f(c))} \qquad = \ln c + 1$$

$$m_1 = \frac{e}{e-1}$$

$$m_2 = m_1 \Rightarrow \ln c + 1 = \frac{e}{e-1}$$

$$\ln c = \frac{e}{e-1} - 1 = \frac{1}{e-1}$$

$c = e^{(1/e-1)}$

Q.7 If $y = \left(\frac{2}{\pi}x - 1\right)$ cosecx is the solution of the differential equation, $\frac{dy}{dx} + p(x)y = \frac{2}{\pi}\text{cosec}x, 0 < x < \frac{\pi}{2}$, then the function p(x) is equal to:

(1) cosec x (2) cot x (3) tan x (4) sec x

Sol. **2**

$$y = \left(\frac{2}{\pi}x - 1\right) \text{cosec}x$$

Differentiate w.r.t x

$$\frac{dy}{dx} = \frac{2}{\pi}\ \text{cosec}x - \left(\frac{2x}{\pi} - 1\right)\text{cosec}x.\ \cot x$$

$$\frac{dy}{dx} + \left(\frac{2x}{\pi} - 1\right) \text{cosec}x \cot x = \frac{2}{\pi}\ \text{cosec}x$$

$$\frac{dy}{dx} + y \cot x = \frac{2}{\pi} \operatorname{cosec} x$$

Compare this differential equation with given differential equation

P(x) = cotx

Q.8 If α and β are the roots of the equation 2x(2x+1)=1, then β is equal to:

(1) $2\alpha(\alpha - 1)$ (2) $-2\alpha(\alpha + 1)$ (3) $2\alpha^2$ (4) $2\alpha(\alpha + 1)$

Sol. (2)

2x(2x+1) = 1

If α & β are the roots i.e α & β satisy this equation

$2\alpha(2\alpha + 1) = 1 \quad \Rightarrow \quad \alpha(2\alpha + 1) = \frac{1}{2}$

$4x^2 + 2x - 1 = 0$

$\alpha + \beta = \frac{-1}{2} = -\alpha(2\alpha + 1)$

$\beta = -\alpha(2\alpha + 1) - \alpha = -\alpha(2\alpha + 2) = -2\alpha(\alpha + 1)$

Q.9 For all twice differentiable functions f: R $\rightarrow$ R, with f(0)=f(1)=f′(0)=0,

(1) f″(x)=0, at every point $x \in (0,1)$ (2) f″(x) $\neq$ 0, at every point $x \in (0,1)$

(3) f″(x)=0, for some $x \in (0,1)$ (4) f″(0)=0

Sol. (3)

Applying rolle's theorem in [0,1] for function f(x)

$f'(c) = 0,\ c \in (0,1)$

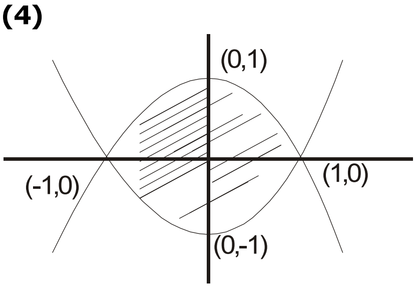
again applying rolles theorem in [0,c] for function f'(x) s

$f''(c_1) = 0,\ c_1 \in (0,c)$

Q.10 The area (in sq.units) of the region enclosed by the curves $y=x^2-1$ and $y=1-x^2$ is equal to :

(1) $\frac{4}{3}$ (2) $\frac{7}{2}$ (3) $\frac{16}{3}$ (4) $\frac{8}{3}$

Sol. (4)



Total area = $4\int_0^1 (1-x^2)dx = 4\left[x - \frac{x^3}{3}\right]_0^1$

$= 4\left[1-\frac{1}{3}\right]=\frac{8}{3} sq.unit$

Q.11 For a suitably chosen real constant a, let a function, $f: R-\{-a\} \rightarrow R$ be defined by $f(x)=\frac{a-x}{a+x}$.

Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, (fof)(x)=x.Then $f\left(-\frac{1}{2}\right)$ is equal to:

(1) –3 (2) 3 (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

Sol. **(2)**

$f(x) = \frac{a-x}{a+x}$

$f(f(x)) = \frac{a-f(x)}{a+f(x)}=x$

$\frac{a-ax}{1+x}=f(x)=\frac{a-x}{a+x}$

$a\left(\frac{1-x}{1+x}\right)=\frac{a-x}{a+x}$

$\Rightarrow$ a = 1

So $f(x) = \frac{1-x}{1+x}$

$f\left(\frac{-1}{2}\right)=3$

Q.12 Let $\theta=\frac{\pi}{5}$ and $A=\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$. If $B=A+A^4$, then det (B):

(1) is one (2) lies in (1,2) (3) lies in (2,3) (4) is zero

Sol. **(2)**

$A = \begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$

$B = A + A^4$

$A^2 = \begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

Simmilarly

$$A^4 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$B = A^4 + A = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$B = A^4 + A = \begin{bmatrix} \cos 4\theta + \cos\theta & \sin 4\theta + \sin\theta \\ -\sin 4\theta - \sin\theta & \cos 4\theta + \cos\theta \end{bmatrix}$$

$B = (\cos 4\theta + \cos\theta)^2 + (\sin 4\theta + \sin\theta)^2$
$= \cos^2 4\theta + \cos^2\theta + 2\cos 4\theta \cos\theta$
$+ \sin^2 4\theta + \sin^2\theta + 2 \sin 4\theta - \sin\theta$
$= 2 + 2 (\cos 4\theta \cos\theta + \sin 4\theta \sin\theta)$
$= 2 + 2 \cos 3\theta$

at $\theta = \frac{\pi}{5}$

$|B| = 2 + 2\cos\frac{3\pi}{5} = 2 - (1 - \sin 18)$

$|B = 2\left(1 - \frac{\sqrt{5}-1}{4}\right) = 2\left(\frac{5-\sqrt{5}}{4}\right) = \frac{5-\sqrt{5}}{2}$

Q.13 The centre of the circle passing through the point (0,1) and touching the parabola $y=x^2$ at the point (2,4) is :

(1) $\left(\frac{3}{10}, \frac{16}{5}\right)$ (2) $\left(\frac{6}{5}, \frac{53}{10}\right)$ (3) $\left(\frac{-16}{5}, \frac{53}{10}\right)$ (4) $\left(\frac{-53}{10}, \frac{16}{5}\right)$

Sol. **(3)**

Circle passing through point (0,1) and touching curve
$y = x^2$ at (2,4)
tangent at (2,4) is

$$\frac{(y+4)}{2} = x(2)$$

$\Rightarrow y + 4 = 4x \Rightarrow y - 4x - 4 = 0$
Equation of circle
$(x-2)^2 + (y-4)^2 + \lambda(4x-y-4) = 0$
Passing through (0,1)
$4 + 9 + \lambda(-5) = 0$

$\lambda = \frac{13}{5}$

Circle is

$x^2 - 4x + 4 + y^2 - 8y + 16 + \frac{13}{5}[4x - y - 4] = 0$

$x^2 + y^2 + \left(\frac{52}{5} - 4\right)x - \left(8 + \frac{13}{5}\right)y + 20 - \frac{52}{5} = 0$

$x^2 + y^2 + \frac{32}{5}x - \frac{53}{5}y + \frac{48}{5} = 0$

Centre is $\left(-\frac{16}{5}, \frac{53}{10}\right)$

Q.14 A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1,1,2). Then the equation of the line through this centroid and perpendicular to the plane P is:

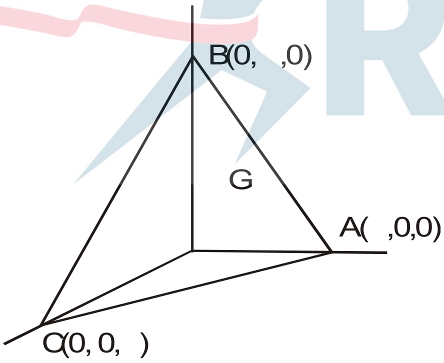
(1) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

(2) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

(3) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

(4) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

Sol. **(2)**



$G = \left(\frac{\alpha}{3}, \frac{\beta}{3}, \frac{\gamma}{3}\right) = (1, 1, 2)$

$\alpha = 3, \beta = 3, \gamma = 6$

Equation of plane is

$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

$\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$2x + 2y + z = 6$

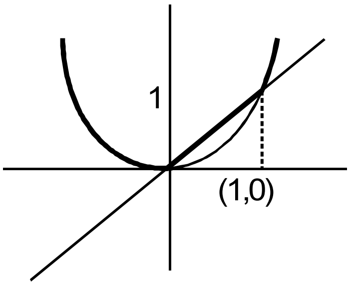
Require line is $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

Q.15 Let $f:R \to R$ be a function defined by f(x)=max $\{x,x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then

(1) {0,1} (2) ϕ (an empty set)

(3) {1} (4) {0}

Sol. **(1)**



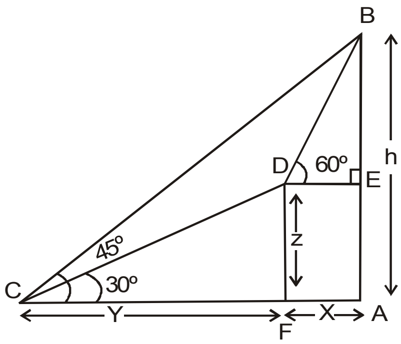
Function is not differentiable at two point

$\{0,1\}$

Q.16 The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is:

(1) $\frac{1}{\sqrt{3}+1}$ (2) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (4) $\frac{1}{\sqrt{3}-1}$

Sol. **(4)**



If Δ CDF

Sin30º = $\frac{z}{1} \Rightarrow Z = \frac{1}{2}$ km

$\cos 30^\circ = \frac{y}{1} \Rightarrow y = \frac{\sqrt{3}}{2}$ km

Now in ΔABC

$\tan 45 = \frac{h}{x+y} \Rightarrow h = x + y$

$X = h - \frac{\sqrt{3}}{2}$

Now in ΔBDE

$\tan 60^\circ = \frac{h-z}{x}$

$\sqrt{3}x = h - \frac{1}{2}$

$\sqrt{3}\left(h - \frac{\sqrt{3}}{2}\right) = h - \frac{1}{2} \Rightarrow h = \frac{1}{\sqrt{3}-1}$ km

Q.17 If the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then |k| equals:

(1) 1 (2) 9 (3) 2 (4) 3

Sol. **(4)**

$^{10}C_r \left(\frac{-k}{x^2}\right)^r \left(\sqrt{x}\right)^{10-r}$

$^{10}C_r (-k)^r (x)^{5-\frac{5r}{2}}$

For constant term

$5 - \frac{5r}{2} = 0 \Rightarrow r = 2$

$T_3 = {}^{10}C_2 k^2 = 405$

$k^2 = \frac{405}{45} = \frac{81}{9} = 9$

$|k| = 3$

Q.18 Let z=x+iy be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the

(1) line, y=x (2) real axis (3) imaginary axis (4) line, y=−x

Sol. **(1)**

$Z = x + iy$

$Z^2 = i\,|Z|^2$

$x^2 - y^2 + 2ixy = i(x^2 + y^2)$

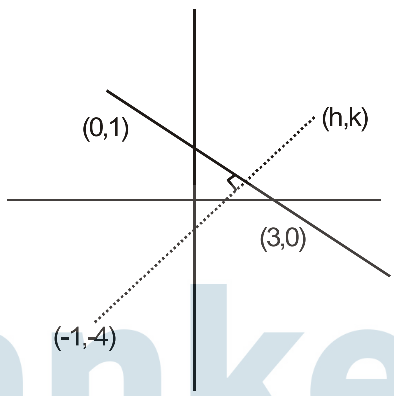
$x^2 - y^2 = 0 \Rightarrow x^2 = y^2$
$2xy = x^2 + y^2$
$(x - y)^2 = 0 \Rightarrow x = y$

Q.19 Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point (–1, –4) in this line is:

(1) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (2) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (3) $\left(\frac{29}{5}, \frac{11}{5}\right)$ (4) $\left(\frac{29}{5}, \frac{8}{5}\right)$

Sol. **(1)**

$\frac{x}{3} + \frac{y}{1} = 1$
$x + 3y = 3$
$L_2: 3x - y + \lambda = 0$
$-3 + 4 + \lambda = 0$
$\lambda = -1$
$3x - y = 1$
(h,k) satisfy the equation of line L_2
$3h - k = 1$ ______(1)



$\left|\frac{-1-12-3}{\sqrt{1+9}}\right| = \left|\frac{h+3k-3}{\sqrt{1+9}}\right|$
$16 = |h + 3k - 3|$
$h + 3k = 19$ ______(2)
$h + 3k = -13$ ______(3)
From equation (2) & (3) put the value of h in equation (1)
$h = 19 - 3k$, $h = -13 - 3k$
$3(19 - 3k) - k = 1$ $3(-13 - 3k) - k = 1$

$-10k = -56 = \frac{28}{5}$ $-10k = 40 \Rightarrow k = -4$

$k = \frac{28}{5}, h = 19 - 3\left(\frac{28}{5}\right) = \frac{95-84}{5} = \frac{11}{5}$

Image = $\left(\frac{11}{5}, \frac{28}{5}\right)$

Q.20 Consider the statement : "For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is:
(1) For an integer n, if n is even, then $n^3 - 1$ is even
(2) For an integer n, if n is odd, then $n^3 - 1$ is even
(3) For an integer n, if $n^3 - 1$ is not even, then n is not odd.
(4) For an integer n, if n is even, then $n^3 - 1$ is odd

Sol. **(4)**

P:n^3-1 is even, q : n is odd

Contrapositive of $p \to q$ = $\sim q \to \sim p$

$\Rightarrow$ If n is not odd then n^3-1 is not even

$\Rightarrow$ For an integer n, if n is even, then n^3-1 is odd

Q.21 The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is______

Sol. **120**

Consonants $\to$ LTTR

Vowels $\to$ EE

Total No of words = $\frac{6!}{2!2!} = 180$

Total no of words if vowels are together

$= \frac{5!}{2!} = 60$

Required = 180 – 60 = 120

Q.22 If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}| = |\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is ______

Sol. **1**

$|\overline{x}+\overline{y}|^2 = |\overline{x}|^2$

$\Rightarrow |\overline{y}|^2 + 2\overline{x}.\overline{y} = 0$ ______(1)

and $(2\overline{x}+\lambda\overline{y})\overline{y} = 0$

$\Rightarrow \lambda\left(|\overline{y}|^2\right) + 2\overline{x}.\overline{y} = 0$ _____(2)

by comparing (1) & (2)

we get λ = 1

Q.23 Consider the data on x taking the values 0, 2, 4, 8, ...,2^n with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \ldots {}^nC_n$, respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to _______

Sol. **6**

X_i(observation)	0	2	2^2	.....	2^n
f_i (frequency)	${}^nC_0$	${}^nC_1$	${}^nC_2$	.....	${}^nC_n$

$\overline{x} = \frac{\sum f_i X_i}{\sum f_i}$

$$= \frac{0 \times {}^nC_0 + 2\,{}^nc_1 + 2^2\,{}^nc_2 + ... + 2^n\,{}^nc_n}{{}^nc_0 + {}^nc_1 + ... + {}^nc_n}$$

$$= \frac{3^n - 1}{2^n} = \frac{728}{2^n}$$

$3^n = 729 = 3^6$

n = 6

Q.24 Suppose that function $f : R \to R$ satisfies f(x+y)=f(x)f(y) for all x, y $\in$ R and f(1)=3.

If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to ....

Sol. **5**

f(x+y) = f(x) f(y)

$f(x)=a^x$

$\Rightarrow$ f(1) = a = 3

So $f(x) = 3^x$

$$\sum_{i=1}^{n} f(i) = 363$$

$\Rightarrow 3 + 3^2 + 3^3 + ... + 3^n = 363$

$$\Rightarrow \frac{3\left(3^n - 1\right)}{2} = 363$$

n = 5

Q.25 The sum of distinct values of λ for which the system of equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$,

has non-zero solutions,is _______

Sol. **3**

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ 0 & \lambda-3 & -\lambda+3 \\ 3-\lambda & 0 & \lambda-3 \end{vmatrix}$$

$C_1 \rightarrow C_1 + C_3$

$$\begin{vmatrix} 3\lambda-1 & 3\lambda+1 & 2\lambda \\ -\lambda+3 & \lambda-3 & -\lambda+3 \\ 0 & 0 & \lambda-3 \end{vmatrix}$$

$(\lambda-3)\ [(3\lambda-1)\ (\lambda-3) - (3-\lambda)\ (3\lambda + 1)] = 0$
$(\lambda-3)\ [3\lambda^2 - 10\lambda + 3 - (8\lambda - 3\lambda^2 + 3)]$
$(\lambda-3)\ (6\lambda^2 - 18\lambda) = 0$
$(6\lambda)\ (\lambda-3)^2 = 0$
$\lambda = 0, 3$
sum of values of $\lambda = 0 + 3 = 3$