MATHEMATICS
JEE-MAIN (September-Attempt)
5 September (Shift-1) Paper

SECTION - A

Q.1 If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ $(n \geq 0)$, is 158 cu. units, then:

(1) $\vec{a}.\vec{c} = 17$ (2) $\vec{b}.\vec{c} = 10$ (3) n=9 (4) n=7

Sol. **2**

$$\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$$

$(12 + n^2) -(6+n) +n(2n-4)=158$
$3n^2 -5n + 6 -158 = 0$
$3n^2 - 5n - 152 = 0$
$3n^2 - 24n + 19n - 152 = 0$
$(3n + 19) (n-8) = 0$
$\Rightarrow n = 8$
$\Rightarrow \vec{b}.\vec{c} = 10$

Q.2 A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:
(1) 63 (2) 54 (3) 38 (4) 36

Sol. 4

n(coffee) = $\frac{73}{100}$

n(tea) = $\frac{65}{100}$

$n(T \cap C) = \frac{x}{100}$

$n(C \cup T)$ = n(C) + n(T) − x $\leq$ 100
= 73 + 65 − x $\leq$ 100
$\Rightarrow x \geq 38$
Ans. 36

Q.3 The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2,4,10,12,14, then the absolute difference of the remaining two observations is:
(1) 1 (2) 4 (3) 3 (4) 2

Sol. **4**

$$\text{Var}(x) = \sum \frac{x_i^2}{n} - (\bar{x})^2$$

$16 = \frac{x_1^2 + x_2^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2}{7} - 64$

$80 \times 7 = x_1^2 + x_2^2 + x_3^2 + ... + x_7^2$

Now, $x_2^6 + x_7^2 = 560 - (x_1^2 + ... x_5^2)$

$x_6^2 + x_7^2 = 560 - (4 + 16 + 100 + 144 + 196)$

$x_6^2 + x_7^2 = 100$ ......(1)

Now, $\frac{x_1 + x_2 + ... + x_7}{7} = 8$

$x_6 + x_7 = 14$ ......(2)

from (1) & (2)

$(x_6 + x_7)^2 - 2x_6x_7 = 100$

$2x_6x_7 = 96 \Rightarrow x_6 x_7 = 48$ ......(3)

Now, $|x_6 - x_7| = \sqrt{(x_6 + x_7)^7 - 4x_6 x_7}$

$= \sqrt{196 - 192} = 2$

Q.4 If $2^{10}+2^9.3^1+2^8.3^2+.....+2.3^9+3^{10}=S-2^{11}$, then S is equal to:

(1) 3^{11} (2) $\frac{3^{11}}{2} + 2^{10}$ (3) 2.3^{11} (4) $3^{11} - 2^{12}$

Sol. **1**

let

$S' = 2^{10} + 2^9 3^1 + 2^8 3^2 + ------ + 2.3^9 + 3^{10}$

$\frac{3 \times S'}{2} = 2^9 \times 3^1 + 2^8 . 3^2 + ---- + 3^{10} + \frac{3^{11}}{2}$

$\frac{-S'}{2} = 2^{10} - \frac{3^{11}}{2}$

$S' = 3^{11} - 2^{11}$

Now $S' = S - 2^{11}$

$S = 3^{11}$

Q.5 If $3^{2 \sin 2\alpha - 1}$, 14 and $3^{4-2 \sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth terms of this A.P. is:

(1) 65 (2) 81 (3) 78 (4) 66

Sol. **4**

$28 = 3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha}$

$28 = \frac{9^{\sin 2\alpha}}{3} + \frac{81}{9^{\sin 2\alpha}}$

Let $9^{\sin 2\alpha} = t$

$28 = \frac{t}{3} + \frac{81}{t}$

$t^2 - 84t + 243 = 0$

$t = 81, 3$

$9^{\sin 2\alpha} = 9^2$ or 3

$\sin 2\alpha = 2$ or $\sin 2\alpha = 1/2$

(Not possible)

Now three terms in A.P. are

1, 14, 27

Next term are

40,53,66

Q.6 If the common tangent to the parabolas, $y^2=4x$ and $x^2=4y$ also touches the circle, $x^2+y^2 = c^2$, then c is equal to:

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

Sol. **3**

$y = mx + \frac{1}{m}$

$x^2 = 4\left(mx + \frac{1}{m}\right)$

$x^2 - 4mx - \frac{4}{m} = 0$

$D = 0$

$16m^2 + \frac{16}{m} = 0$

$16\left(\frac{m^3+1}{m}\right) = 0$

$m = -1$

$\Rightarrow y + x = -1$

Now, $\left|\frac{-1}{\sqrt{2}}\right| = c$

$c = \frac{1}{\sqrt{2}}$

Q.7 If the minimum and the maximum values of the function $f:\left[\frac{\pi}{4},\frac{\pi}{2}\right]\to R$, defined by

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$ are m and M respectively, then the ordered pair (m,M) is equal to :

(1) (0,4)
(2) (-4,0)
(3) (-4,4)
(4) $\left(0,2\sqrt{2}\right)$

Sol. **2**

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$

$C_1 \to C_1 - C_2$, $C_3 \to C_3 + C_2$

$$\begin{vmatrix}1 & -1-\sin^2\theta & -\sin^2\theta\\ 1 & -1-\cos^2\theta & -\cos^2\theta\\ 2 & 10 & 8\end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix}1 & -1 & -\sin^2\theta\\ 1 & -1 & -\cos^2\theta\\ 2 & 2 & 8\end{vmatrix}$$

$1(2\cos^2\theta-8)+(8+2\cos^2\theta)-4\sin^2\theta$
$f(\theta)=4\cos 2\theta$

Q.8 Let $\lambda \in R$. The system of linear equations
$2x_1-4x_2+\lambda x_3=1$
$x_1-6x_2+x_3=2$
$\lambda x_1-10x_2+4x_3=3$
is inconsistent for:
(1) exactly two values of λ
(2) exactly one negative value of λ.
(3) every value of λ.
(4) exactly one positive value of λ.

Sol. **2**

$$D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0$$

$2(-14) + 4(4 - \lambda) + \lambda(6\lambda - 10) = 0$
$-28 + 16 - 4\lambda + 6\lambda^2 - 10\lambda = 0$
$6\lambda^2 - 14\lambda - 12 = 0$
$3\lambda^2 - 7\lambda - 6 = 0$
$3\lambda^2 - 9\lambda + 2\lambda - 6 = 0$
$(3\lambda + 2)(\lambda - 3) = 0$
$\lambda = -2/3, 3$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix}$$

$\Rightarrow -14 + 4(5) + \lambda(-2)$
$\Rightarrow -2\lambda + 6$

$$D_2 = \begin{vmatrix} 2 & 1 & \lambda \\ 1 & 2 & 1 \\ \lambda & 3 & 4 \end{vmatrix}$$

$\Rightarrow 2(5) - 1(4 - \lambda) + \lambda(3 - 2\lambda)$
$\Rightarrow 10 - 4 + \lambda + 3\lambda - 2\lambda^2$
$\Rightarrow -2\lambda^2 + 4\lambda + 6$
$\Rightarrow -2(\lambda^2 - 2\lambda - 3)$
$\Rightarrow -2[\lambda^2 - 3\lambda + \lambda - 3]$
$\Rightarrow -2(\lambda - 3)(\lambda + 1)$

$$D_3 = \begin{vmatrix} 2 & -4 & 1 \\ 1 & -6 & 2 \\ \lambda & -10 & 3 \end{vmatrix} \Rightarrow 2(-18 + 20) + 4(3 - 2\lambda) + 1(-10 + 6\lambda)$$

$= 4 + 12 - 8\lambda - 10 + 6\lambda$
$= -2\lambda + 6$
$\Rightarrow \lambda = -2/3$ is answer

Q.9 If the point P on the curve, $4x^2+5y^2=20$ is farthest from the point Q(0, -4), then PQ^2 is equal to:

(1) 48 (2) 29 (3) 21 (4) 36

Sol. **4**

Let P be $\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$

Now, PQ = $\sqrt{\left(\sqrt{5}\cos\theta\right)^2 + \left(2\sin\theta + 4\right)^2}$

PQ = $\sqrt{5\cos^2\theta + \left(2\sin\theta + 4\right)^2}$

$\frac{d(PQ)}{d\theta} = 0 \Rightarrow -10\sin\theta\cos\theta + (4\sin\theta + 8)\cos\theta = 0$

$\Rightarrow -6\sin\theta\cos\theta + 8\cos\theta = 0$

$\cos\theta = 0$ or $\sin\theta = \frac{4}{3}$

Not possible

So P is either (0,2) or (0,-2)

$PQ^2 = 36$

Q.10 The product of the roots of the equation $9x^2-18|x|+5=0$ is :

(1) $\frac{25}{81}$

(2) $\frac{5}{9}$

(3) $\frac{5}{27}$

(4) $\frac{25}{9}$

Sol. **1**

$9t^2 - 18t + 5 = 0$

$9t^2 - 15t - 3t + 5 = 0$

$(3t - 5)(3t - 1) = 0$

$|x| = \frac{5}{3}, \frac{1}{3}$

$\Rightarrow \quad x = \frac{5}{3}, \frac{-5}{3}, \frac{1}{3}, \frac{-1}{3}$

$\Rightarrow \quad P = \frac{25}{81}$

Q.11 If y=y(x) is the solution of the differential equation $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx} + e^x = 0$ satisfying y(0)=1, then a value of $y(\log_e 13)$ is:

(1) 1

(2) 0

(3) 2

(4) -1

Sol. **4**

$\frac{dy}{dx} + \left(e^x \times \frac{y+2}{e^x+5}\right) = 0$

$$\frac{dy}{dx}+\left(\frac{e^x}{e^x+5}\right)y=\frac{-2e^x}{e^x+5}$$

I.F. $= e^{\int \frac{e^x}{e^x+5}dx}$

$= e^{\int\left(1-\frac{5}{e^x+5}\right)dx}$

$= e^{\int\left(1-\frac{5e^{-x}}{1+5e^{-x}}\right)dx}$

$= e^{x + \ln(1+5e-x)}$

$= e^x.(1+5e^{-x}) \Rightarrow (e^x + 5)$

$y(e^x + 5) = -\int 2e^x dx$

$y(e^x + 5) = -2e^x + C$

$\Downarrow x=0$

$(6) = -2 + C \Rightarrow C = 8$

$y(\ln 13) = \frac{8-2\times 13}{13+5} = \frac{-18}{18} = -1$

Q.12 If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\tan^{-1}\left(\frac{1}{21}\right)+\ldots..,$ then tan(S) is equal to :

(1) $\frac{5}{11}$

(2) $\frac{5}{6}$

(3) $-\frac{6}{5}$

(4) $\frac{10}{11}$

Sol. **2**

$S = \tan^{-1}\left(\frac{1}{1+1\times 2}\right)+\tan^{-1}\left(\frac{1}{1+2\times 3}\right)+\ldots.$

$T_r = \tan^{-1}\left(\frac{1}{1+r(r+1)}\right)$

$T_r = \tan^{-1}(r + 1) - \tan^{-1}r$

$T_1 = \tan^{-1}2 - \tan^{-1}1$

$T_2 = \tan^{-1}3 - \tan^{-1}2$

$T_3 = \tan^{-1}4 - \tan^{-1}3$

$T_{10} = \tan^{-1}11 - \tan^{-1}10$

$\Rightarrow S = \tan^{-1}11 - \tan^{-1}1$

$\Rightarrow \tan S = \frac{10}{12} = \frac{5}{6}$

Q.13 The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$ is:

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) π (4) $\frac{3\pi}{2}$

Sol. **1**

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$$

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{e^{\sin x}}{1+e^{\sin x}} dx \Rightarrow 2I = \pi$$

$$I = \frac{\pi}{2}$$

Q.14 If (a, b, c) is the image of the point (1,2,-3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a+b+c is

(1) 2 (2) 3 (3) -1 (4) 1

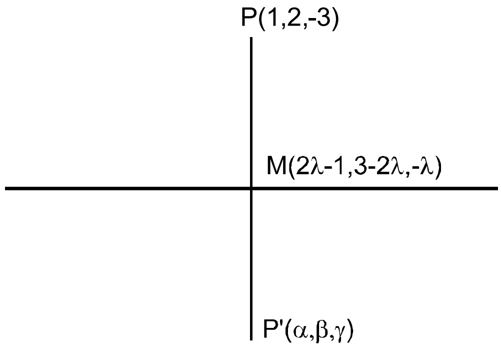
Sol. **1**

$\overrightarrow{PM} \perp (2\hat{i} - 2\hat{j} - \hat{k})$

$\Rightarrow (2\lambda-2).2 + (1-2\lambda)(-2) + (3-\lambda)(-1) = 0$

$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$

$\Rightarrow 9\lambda = 9 \Rightarrow \lambda = 1$



$\Rightarrow m(1,1,-1)$

Now, $p' = 2M - P$

$= 2(1,1,-1) - (1, 2, -3)$

$= (1, 0, 1)$

$a + b + c = 2$

Q.15 If the function $f(x)=\begin{cases}k_1(x-\pi)^2-1, & x\le\pi\\ k_2\cos x, & x>\pi\end{cases}$ is twice differentiable, then the ordered pair (k_1,k_2) is equal to:

(1) (1,1) (2) (1,0) (3) $\left(\frac{1}{2},-1\right)$ (4) $\left(\frac{1}{2},1\right)$

Sol. **4**

$f(x)=\begin{cases}2k_1(x-\pi); & x\le\pi\\ -k_2\sin x & ;x>\pi\end{cases}$

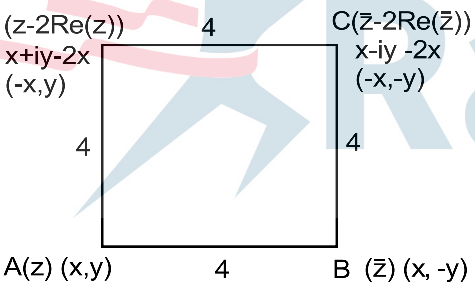
$f''(x)=\begin{cases}2k_1 & ;x\le\pi\\ -k_2\cos x; & x>\pi\end{cases}$

$2k_1 = k_2$

Q.16 If the four complex numbers $z, \bar{z}$, $\bar{z}-2\text{Re}(\bar{z})$ and $z-2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then |z| is equal to:

(1) 2 (2) 4 (3) $4\sqrt{2}$ (4) $2\sqrt{2}$

Sol. **4**



Let z = x + iy

$CA^2 = AB^2 + BC^2$

$2^2x^2 + 2^2y^2 = 32$

$x^2 + y^2 = 8$

$\sqrt{x^2+y^2} = 2\sqrt{2}$

Q.17 If $\int\left(e^{2x}+2e^x-e^{-x}-1\right)e^{\left(e^x+e^{-x}\right)}dx = g(x)e^{\left(e^x+e^{-x}\right)}+c$, where c is a constant of integration, then g(0) is equal to :

(1) 2 (2) e
(3) 1 (4) e^2

Sol. **1**

$$\int\left(e^{2x}+2e^{x}-e^{-x}-1\right)e^{\left(e^{x}+e^{-x}\right)}dx$$

$$\int\left(e^{2x}+e^{x}-1\right)e^{\left(e^{x}+e^{-x}\right)}dx+\int\left(e^{x}-e^{-x}\right)e^{\left(e^{x}+e^{-x}\right)}dx$$

$$\int\left(e^{x}+1-e^{-x}\right)e^{\left(e^{x}+e^{-x}+x\right)}dx+\int\left(e^{x}-e^{-x}\right)e^{\left(e^{x}+e^{-x}\right)}dx$$

$$e^{\left(e^{x}+e^{-x}+x\right)}+e^{e^{x}+e^{-x}}+C$$

$$\left(e^{e^{x}+e^{-x}}\right)\left[e^{x}+1\right]+C$$

$$\Downarrow$$

$$g(x)$$

$\Rightarrow$ g(0) = 2

Q.18 The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :

(1) $(x \wedge y) \wedge (\sim x \vee \sim y)$

(2) $(x \wedge y) \vee (\sim x \wedge \sim y)$

(3) $(x \wedge \sim y) \vee (\sim x \wedge y)$

(4) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

Sol. **2**

As we know

$\sim (p \leftrightarrow q) = (p \wedge \sim q) \vee (\sim p \wedge q)$

$\Rightarrow$ so, $\sim (x \leftrightarrow \sim y) = (x \wedge y) \vee (\sim x \wedge \sim y)$

Q.19 If α is positive root of the equation, p(x) =x²-x-2=0, then $\lim\limits_{x\to\alpha^+}\dfrac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to :

(1) $\dfrac{1}{2}$ (2) $\dfrac{3}{\sqrt{2}}$ (3) $\dfrac{3}{2}$ (4) $\dfrac{1}{\sqrt{2}}$

Sol. **2**

f(x) = x² –x –2 $\left\langle {}^{2}_{-1} = \alpha\right.$

$$\lim_{x\to 2^+}\frac{\sqrt{1-\cos(x-2)(x+1)}}{x+\alpha-4}$$

$$\lim_{x\to 2+}\frac{\sqrt{1-\cos(x-2)(x+1)}}{(x-2)}$$

$$\lim_{h\to 0}\frac{\sqrt{1-\cos(\text{h}\times(\text{h}+3))}}{h}$$

$$\lim_{h\to 0}\sqrt{\frac{1-\cos(\text{h }(\text{h}+3))}{h^2\times(h+3)^2}\times(h+3)^2} \Rightarrow \sqrt{\frac{1}{2}\times 9}=\frac{3}{\sqrt{2}}$$

Q.20 If the co-ordinates of two points A and B are $\left(\sqrt{7},0\right)$ and $\left(-\sqrt{7},0\right)$ respectively and P is any point on the conic, $9x^2+16y^2=144$, then PA+PB is equal to :

(1) 6 (2) 16

(3) 9 (4) 8

Sol. **4**

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$$e=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$$

$$F_1\left(\sqrt{7},0\right),F_2\left(-\sqrt{7},0\right)$$

$PF_1 + PF_2 = 2a$

$PA + PB = 2 \times 4 = 8$

Q.21 The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m+\frac{1}{x^2}\right)^{22}$ is 1540, is ....

Sol. 13

$$T_{r+1}={}^{22}C_r\left(x^m\right)^{22-r}\left(\frac{1}{x^2}\right)^r$$

$$= {}^{22}C_r\left(x\right)^{22m-mr-2r}$$

Given ${}^{22}C_r = 1540={}^{22}C_{19} \Rightarrow r=19$

$\because 22m - rm - 2r = 1$

$$\Rightarrow m=\frac{2r+1}{22-r}$$

m = 13(At r=19)

Q.22 Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ....

Sol. **11**

(atteat 2 or 3) = $^4C_2\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right)^1 + {}^4C_4\left(\frac{2}{6}\right)^4$

= $6 \times \frac{1}{9} \times \frac{4}{9} + 4 \times \frac{1}{27} \times \frac{2}{3} + \frac{1}{81}$

$= \frac{33}{81} = \frac{11}{27} \Rightarrow nP \qquad \Rightarrow 11$

Q.23 Let $f(x) = x.\left[\frac{x}{2}\right]$, for-10<x<10, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to...........

Sol. **8**

f(x) = $x\left[\frac{x}{2}\right]$, $-10 < x < 10$

$-5 < \frac{x}{2} < 5$

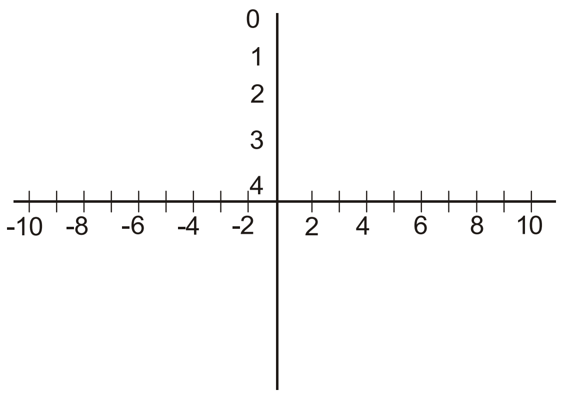
$-5x \quad -5 < \frac{x}{2} < -4$

$-4x \quad -4 < \frac{x}{2} < 3$

$-3x \quad -3 < x/2 < -2$

$-2x \quad -2 < x/2 < -1$

$-x \quad -1 < x/2 < 0$



Number of point of discontinuity = 8

Q.24 The number of words, with or without meaning, that can be formed by taking 4 lettersat a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is

Sol. **240**

SS, Y, LL, A, B, U

| S | S | | |

$\Rightarrow {}^5C_2 \times \frac{4!}{2!} \times {}^2C_1$

$\Rightarrow 120 \times 2$

$= 240$

Q.25 If the line, 2x-y+3=0 is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x-2y+\alpha=0$ and $6x-3y+\beta=0$, respectively, then the sum of all possible values of α and β is

Sol. **30**

$L_1 : 2x - y + 3 = 0$

$L_2 : 4x - 2y + \alpha = 0$

$L_3 : 6x - 3y + \beta = 0$

$\frac{\left|\frac{\alpha}{2} - 3\right|}{\sqrt{5}} = \frac{1}{\sqrt{5}}$ $\Rightarrow \frac{\alpha}{2} - 3 = 1, -1$

$\Rightarrow \alpha = 8, 4$

$\frac{\left|\frac{\beta}{3} - 3\right|}{\sqrt{5}} = \frac{2}{\sqrt{5}}$ $\Rightarrow \frac{\beta}{3} - 3 = 2, -2$

$\Rightarrow \beta = 15, 3$