

MATHEMATICS
JEE-MAIN (September-Attempt)
3 September (Shift-1) Paper

SECTION - A

Q.1 The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \dots$ up to 51^{th} term) $+ (1! - 2! + 3! - \dots$ up to 51^{th} term) is equal to:

- (1) $1 - 51(51)!$ (2) $1 + (52)!$ (3) 1 (4) $1 + (51)!$

Sol. 2

2. ${}^1P_0 = \underline{2}$

3. ${}^2P_1 = \underline{3}$

4. ${}^3P_2 = \underline{4}$

$(\underline{2} - \underline{3} + \underline{4} - \underline{5} + \dots \dots \dots \underline{52}) + (\underline{1} - \underline{2} + \underline{3} - \underline{4} \dots \dots + \underline{51})$

$= \underline{52} + 1$

Q.2 Let P be a point on the parabola, $y^2=12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis

which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then:

- (1) $PN=4$ (2) $MQ=\frac{1}{3}$ (3) $PN=3$ (4) $MQ=\frac{1}{4}$

Sol. 4



Q (h, 3t) lie on
 Parabola
 $9t^2 = 12h$

$h = \frac{3t^2}{4}$

$Q = \left(\frac{3t^2}{4}, 3t \right)$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3t^2}{4} - 3t^2\right)} (x - 3t^2)$$

$$y = \frac{-4t}{3t^2} (x - 3t^2)$$

put $x = 0$

$$y = \frac{-4}{3t} (-3t^2) = 4t$$

$$4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$$

$$PN = 6t = 6 \cdot \frac{1}{3} = 2$$

$$M = \left[\frac{1}{3}, 1\right], Q = \left[\frac{1}{12}, 1\right]$$

$$MQ = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$$

Q.3

If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$, then $B+C$ is equal to:

(1) 1

(2) -1

(3) -3

(4) 9

Sol.

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 3R_1$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 1 & -x+1 & x-5 \end{vmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 0 & -1 & 3x-8 \end{vmatrix} = Ax^3+Bx^2+Cx+D$$

$$\Rightarrow -1[(3-2x)(x-2) - (3x-4)] + (3x-8)[(x-2)(-x+2) - (2x-3)] = Ax^3+Bx^2+Cx+D$$

$$\Rightarrow 3x - 2x^2 - 6 + 4x - 3x + 4 + (3x-8)[-x^2 + 4x - 4 - 2x + 3] = Ax^3+Bx^2+Cx+D$$

$$A = -3, B = 12, C = -15$$

$$B + C = -3$$

Q.4 The foot of the perpendicular drawn from the point (4,2,3) to the line joining the points (1,-2,3) and (1,1,0) lies on the plane:

(1) $x-y-2z=1$

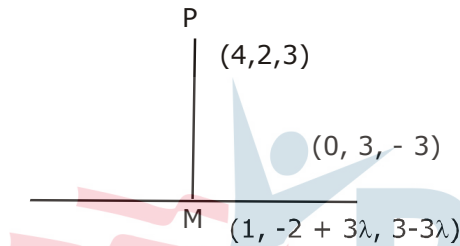
(2) $x-2y+z=1$

(3) $2x+y-z=1$

(4) $x+2y-z=1$

Sol. 3

$$\vec{r} = (1, -2, 3) + \lambda(0, 3, -3)$$



$$\vec{pm} \perp \vec{b}$$

$$\vec{pm} \cdot \vec{b} = 0$$

$$(-3, 3\lambda - 4, -3\lambda) \cdot (0, 3, -3) = 0$$

$$\Rightarrow 0 + 9\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{12}{18} = \frac{2}{3}$$

$$m = (1, 0, 1) \text{ are on } 2x + y - z = 1$$

Q.5 If $y^2 + \log_e(\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then

(1) $|y'(0)| + |y''(0)| = 1$

(2) $y''(0) = 0$

(3) $|y'(0)| + |y''(0)| = 3$

(4) $|y''(0)| = 2$

Sol. 4

$$2yy' + 2(-\tan x) = y' \quad \dots(1)$$

diff. w.r.t.x

$$2yy'' + 2(y')^2 - 2 \sec^2 x = y'' \quad \dots(2)$$

Put $x = 0$ in given equation we get $y = 0, 1$

from (1) $x = 0, y = 0 \Rightarrow y'(0) = 0$

$x = 0, y = 1, \Rightarrow y'(0) = 0$

from (2) $x = 0, y = 0, y'(0) = 0 \Rightarrow y''(0) = -2$
 $x = 0, y = 1, y'(0) = 0 \Rightarrow y''(0) = 2$
 $|y''(0)| = 2$

Q.6 $2\pi - \left(\sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{25} \right)$ is equal to:

- (1) $\frac{5\pi}{4}$ (2) $\frac{3\pi}{2}$ (3) $\frac{7\pi}{4}$ (4) $\frac{\pi}{2}$

Sol. 2

$$2\pi - \left[\tan^{-1} \left(\frac{4}{3} \right) + \tan^{-1} \left(\frac{5}{12} \right) + \tan^{-1} \frac{16}{63} \right]$$

$$2\pi - \tan^{-1} \left(\frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} \right) - \tan^{-1} \left(\frac{16}{63} \right)$$

$$\Rightarrow 2\pi - \tan^{-1} \left(\frac{48 + 15}{36 - 20} \right) - \tan^{-1} \left(\frac{16}{63} \right)$$

$$\Rightarrow 2\pi - \left[\tan^{-1} \left(\frac{63}{16} \right) + \cot^{-1} \left(\frac{63}{16} \right) \right]$$

$$\Rightarrow 2\pi - \frac{\pi}{2}$$

$$= \frac{3\pi}{2}$$

Q.7 A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points ?

- (1) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}} \right)$ (2) $\left(1, -\frac{1}{\sqrt{2}} \right)$ (3) $\left(\frac{1}{\sqrt{2}}, 0 \right)$ (4) $\left(-\sqrt{\frac{3}{2}}, 1 \right)$

Sol. 1

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$b_1^2 = a_1^2 (1 - e_1^2)$$

$$3 = 4(1 - e_1^2)$$

$$e_1 = \frac{1}{2}$$

$$\text{focus} = (\pm a_1 e_1, 0) \\ = (\pm 1, 0)$$

$$\text{Length of transverse axis } 2a_2 = \sqrt{2} \rightarrow a_2 = \frac{1}{\sqrt{2}}$$

$$a_2 e_2 = 1$$

$$= e_2 = \sqrt{2}$$

$$b_2^2 = a_2^2 (e_2^2 - 1)$$

$$b_2^2 = \frac{1}{2} (2 - 1) = \frac{1}{2}$$

equation of Hyperbola

$$x^2 - y^2 = \frac{1}{2}$$

Q.8 For the frequency distribution:

Variate(x):	x_1	x_2	$x_3 \dots x_{15}$
Frequency(f):	f_1	f_2	$f_3 \dots f_{15}$

where $0 < x_1 < x_2 < x_3 < \dots < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be:

(1) 1

(2) 4

(3) 6

(4) 2

Sol.

$$\sigma^2 \leq \frac{1}{4} (M - m)^2$$

(M = upper bound of value of any random variable,
m = Lower bound of value of any random variable)

$$\sigma^2 \leq \frac{1}{4} (10 - 0)^2$$

$$\sigma^2 < 25$$

$$-5 < \sigma < 5$$

$$\sigma \neq 6$$

Q.9 A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is:

(1) $\frac{1}{3}$

(2) $\frac{1}{4}$

(3) $\frac{1}{8}$

(4) $\frac{1}{9}$

Sol. 4

Total Possibilities = (1, 3), (3, 1), (2, 2),
(2, 6), (6, 2) (4, 4)
(3, 5), (5, 3) (6, 6)
fav. = 1 = (4, 4)

$$\text{prob.} = \frac{1}{9}$$

Q.10 If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$ is exactly 33, then the least value

of n is:

(1) 128

(2) 248

(3) 256

(4) 264

Sol. 3

$$T_{r+1} = {}^nC_r \left(3^{\frac{1}{2}}\right)^{n-r} \left(5^{\frac{1}{8}}\right)^r$$

$$\left. \begin{array}{l} \frac{n-r}{2} \rightarrow n-r = 0, 2, 4, 6, 8, \dots \\ \frac{r}{8} \rightarrow r = 0, 8, 16, 24, \dots \end{array} \right\}$$

$$\text{common } r = 0, 8, 16, 24, \dots$$

no. of integral term = 33.

$$L = 0 + (33 - 1) \times 8 \rightarrow L = 32 \times 8 = 256$$

Q.11 $\int_{-\pi}^{\pi} |\pi - |x|| dx$ is equal to:

(1) π^2

(2) $\frac{\pi^2}{2}$

(3) $\sqrt{2}\pi^2$

(4) $2\pi^2$

Sol. 1

$$\int_{-\pi}^{\pi} |\pi - |x|| dx$$

even function

$$2 \int_0^{\pi} |\pi - x| dx$$

$$= 2 \int_0^{\pi} (\pi - x) dx \Rightarrow 2 \left[\pi x - \frac{x^2}{2} \right]_0^{\pi}$$

$$= 2 \left[\frac{\pi^2}{2} \right] = \pi^2$$

Q.12 Consider the two sets:

$A = \{m \in \mathbb{R} : \text{both the roots of } x^2 - (m+1)x + m + 4 = 0 \text{ are real}\}$ and $B = [-3, 5)$.

Which of the following is not true ?

(1) $A - B = (-\infty, -3) \cup (5, \infty)$

(2) $A \cap B = \{-3\}$

(3) $B - A = (-3, 5)$

(4) $A \cup B = \mathbb{R}$

Sol. 1

$$D \geq 0$$

$$(m + 1)^2 - 4(m + 4) \geq 0$$

$$\Rightarrow m^2 - 2m - 15 \geq 0$$

$$(m - 5)(m + 3) \geq 0$$

$$m \in (-\infty, -3] \cup [5, \infty)$$

$$A = (-\infty, -3] \cup [5, \infty)$$

$$B = [-3, 5)$$

$$A - B = (-\infty, -3) \cup [5, \infty)$$

$$A \cup B = \mathbb{R}$$

Q.13 The proposition $p \rightarrow \sim(p \wedge \sim q)$ is equivalent to :

(1) $(\sim p) \vee (\sim q)$

(2) $(\sim p) \wedge q$

(3) q

(4) $(\sim p) \vee q$

Sol. 4

$$\sim(p \wedge \sim q) \rightarrow \sim p \vee q$$

$$p \rightarrow (\sim p \vee q)$$

$$\Rightarrow \sim p \vee (\sim p \vee q)$$

$$\Rightarrow \sim p \vee q$$

Q.14 The function, $f(x) = (3x-7)x^{2/3}$, $x \in \mathbb{R}$ is increasing for all x lying in:

(1) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

(2) $\left(-\infty, \frac{14}{15}\right)$

(3) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(4) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

Sol. 3

$$f(x) = (3x - 7) \cdot \frac{2}{3x^{\frac{1}{3}}} + x^{\frac{2}{3}} \cdot 3$$

$$= \frac{6x - 14 + 9x}{3x^{\frac{1}{3}}}$$

$$= \frac{15x - 14}{3x^{\frac{1}{3}}}$$

$$f(x) > 0 \uparrow \Rightarrow x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$$

Q.15 If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:

- (1) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{7}$

Sol. 1
a = 3

$$\frac{25}{2}[2a + 24d] = \frac{15}{2}[2 \times (a + 25d) + 14d]$$

$$\Rightarrow 50a + 600d = 15[2a + 50d + 14d]$$

$$\Rightarrow 20a + 600d = 960d$$

$$\Rightarrow 60 = 360d$$

$$d = \frac{1}{6}$$

Q.16 The solution curve of the differential equation, $(1 + e^{-x})(1 + y^2) \frac{dy}{dx} = y^2$, which passes through the point (0,1), is:

$$(1) y^2 = 1 + y \log_e \left(\frac{1 + e^{-x}}{2} \right)$$

$$(2) y^2 + 1 = y \left(\log_e \left(\frac{1 + e^{-x}}{2} \right) + 2 \right)$$

$$(3) y^2 + 1 = y \left(\log_e \left(\frac{1 + e^x}{2} \right) + 2 \right)$$

$$(4) y^2 = 1 + y \left(\log_e \left(\frac{1 + e^x}{2} \right) \right)$$

Sol. 4

$$\int \left(\frac{1 + y^2}{y^2} \right) dy = \int \left(\frac{1}{1 + e^{-x}} \right) dx$$

$$\int \frac{1}{y^2} dy + \int dy = \int \left(\frac{e^x}{e^x + 1} \right) dx$$

$$\Rightarrow \frac{-1}{y} + y = \ln|e^x + 1| + c$$

$$x = 0, y = 1$$

$$\Rightarrow -1 + 1 = \ln 2 + c \Rightarrow c = -\ln 2$$

$$\Rightarrow \frac{-1}{y} + y = \ln|e^x + 1| - \ln 2$$

$$\Rightarrow y^2 = 1 + y \left[\ln \left(\frac{e^x + 1}{2} \right) \right]$$

Q.17 The area (in sq. units) of the region $\left\{ (x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2 \right\}$ is

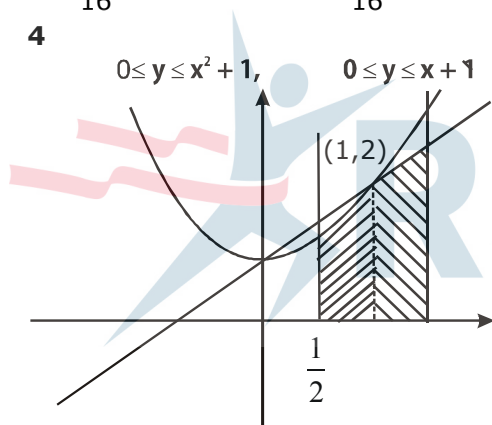
(1) $\frac{23}{16}$

(2) $\frac{79}{16}$

(3) $\frac{23}{6}$

(4) $\frac{79}{24}$

Sol. 4



$$A = \int_{\frac{1}{2}}^1 (x^2 + 1) dx + \int_1^2 (x + 1) dx$$

$$\left(\frac{x^3}{3} + x \right)_{\frac{1}{2}}^1 + \left(\frac{x^2}{2} + x \right)_1^2$$

$$= \left(\frac{1}{3} + 1 \right) - \left(\frac{1}{24} + \frac{1}{2} \right) + \left((2 + 2) - \left(\frac{3}{2} \right) \right)$$

$$= \left(\frac{4}{3} - \frac{13}{24} \right) + \left(\frac{5}{2} \right)$$

$$= \left(\frac{32 - 13}{24} \right) + \left(\frac{5}{2} \right) = \frac{19 + 60}{24} = \frac{79}{24}$$

Q.18 If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the

equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha} \right) \left(\beta - \frac{1}{\beta} \right) \left(\alpha + \frac{1}{\beta} \right) \left(\beta + \frac{1}{\alpha} \right)$ is equal to :

(1) $\frac{9}{4}(9 + p^2)$ (2) $\frac{9}{4}(9 + q^2)$ (3) $\frac{9}{4}(9 - p^2)$ (4) $\frac{9}{4}(9 - q^2)$

Sol. 3

$$\alpha + \beta = -p, \alpha\beta = 2$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = -q, \frac{1}{\alpha\beta} = \frac{1}{2}$$

$$\frac{\alpha + \beta}{\alpha\beta} = -q \Rightarrow \frac{-p}{2} = -q$$

$$\Rightarrow p = 2q$$

$$\left(\alpha + \frac{1}{\beta} \right) \left(\beta + \frac{1}{\alpha} \right) = \alpha\beta + \frac{1}{\alpha\beta} + 2$$

$$= 2 + \frac{1}{2} + 2 = \frac{9}{2}$$

$$\left(\alpha - \frac{1}{\alpha} \right) \left(\beta - \frac{1}{\beta} \right) = \alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}$$

$$= 2 + \frac{1}{2} - \left[\frac{\alpha^2 + \beta^2}{\alpha\beta} \right]$$

$$= \frac{5}{2} - \left[\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \right]$$

$$= \frac{5}{2} - \left[\frac{p^2 - 4}{2} \right]$$

$$= \frac{9 - p^2}{2}$$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \left(\frac{9 - p^2}{2}\right)\left(\frac{9}{2}\right)$$

$$= \frac{9}{4}(9 - p^2)$$

Q.19 The lines $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

(1) do not intersect for any values of l and m

(2) intersect when l=1 and m=2

(3) intersect when l=2 and m= $\frac{1}{2}$

(4) intersect for all values of l and m

Sol. 1

$$\frac{2}{1} \neq \frac{0}{1} \neq \frac{1}{-1} \rightarrow \text{lines are intersecting}$$

$$\vec{r} = (1 + 2l)\hat{i} - \hat{j} + l\hat{k} \quad \dots(1)$$

$$\vec{r} = (2 + m)\hat{i} + (m - 1)\hat{j} - m\hat{k} \quad \dots(2)$$

compare coeff. of $\hat{i}, \hat{j}, \hat{k}$

$$1 + 2l = 2 + m \quad \left| \begin{array}{l} -1 = m - 1 \\ m = 0 \end{array} \right| \quad l = 0$$

Lines do not intersect

Q.20 Let $[t]$ denote the greatest integer $\leq t$. if for some $\lambda \in \mathbb{R} - \{0, 1\}$

$$\lim_{x \rightarrow 0} \frac{1 - x + |x|}{\lambda - x + [x]} = L, \text{ then } L \text{ is equal to:}$$

- (1) 0 (2) 2 (3) $\frac{1}{2}$ (4) 1

Sol. 2

$$\lim_{x \rightarrow 0} \frac{|1-x+|x||}{|\lambda-x+[x]|} = L$$

$$\lim_{h \rightarrow 0} \frac{|1-h+h|}{|\lambda-h+[h]|}$$

$$\lim_{h \rightarrow 0} \frac{|1|}{|\lambda-h+0|} = \frac{|1|}{|\lambda|} \quad [x] = 0$$

$$\lim_{h \rightarrow 0} \frac{|1+h+h|}{|\lambda+h+[-h]|}$$

$$= \frac{|1|}{|\lambda-1|} \quad [-h] = -1$$

$$\therefore |\lambda| = |\lambda-1|$$

$$\lambda^2 = \lambda^2 - 2\lambda + 1 \Rightarrow \lambda = \frac{1}{2}$$

$$L = 2$$

Q.21 If $\lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^{-k}$, then the value of k is

Sol. 8

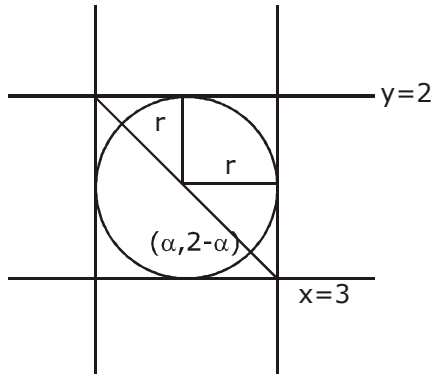
$$\lim_{x \rightarrow 0} \frac{\left(1 - \cos \frac{x^2}{2} \right) \left(1 - \cos \frac{x^2}{4} \right) \left(\frac{x^2}{2} \right) \cdot \left(\frac{x^2}{4} \right)^2}{\left(\frac{x^2}{2} \right)^2 \left(\frac{x^2}{4} \right)^2 \cdot x^8}$$

$$\lim_{x \rightarrow 0} \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{16} \Rightarrow \frac{1}{256} = 2^{-k}$$

$$2^{-8} = 2^{-k} \Rightarrow k = 8$$

Q.22 The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x=3$ and $y=2$, is

Sol. 2



$$p = r$$

for $y = 2$

$$r = \left| \frac{2 - \alpha - 2}{1} \right| = |\alpha|$$

for $x = 3$

$$r = \left| \frac{\alpha - 3}{1} \right| = |\alpha - 3|$$

$$|\alpha| = |\alpha - 3|$$

$$\Rightarrow \alpha^2 + \alpha^2 - 6\alpha + 9 \Rightarrow \alpha = \frac{3}{2}$$

$$2\alpha = 3 = 2r$$

Q.23 The value of $(0.16)^{\log_{2.5} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{to } \infty \right)}$ is equal to.....

Sol. 4

$$\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \infty = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$$

$$\log_{2.5} \left(\frac{1}{2} \right) \Rightarrow \log_{\frac{5}{2}} \frac{1}{2}$$

$$.16 = \frac{16}{100} = \frac{4}{25} = \left(\frac{2}{5} \right)^2$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2\log_{\frac{1}{2}} \frac{1}{2}} = \left(\frac{5}{2}\right)^{-2\log_{\frac{1}{2}} \frac{1}{2}}$$

$$\Rightarrow \left(\frac{5}{2}\right)^{\log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-2}}$$

$$= 4$$

Q.24 Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in \mathbb{R}$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to

Sol. 10

$$A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x^3 + x + x & x^2 + 1 \\ x^2 + 1 & x \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^3 + 2x & x^2 + 1 \\ x^2 + 1 & x \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x^4 + 2x^2 + x^2 + 1 & x^3 + 2x \\ x^3 + x + x & x^2 + 1 \end{bmatrix}$$

$$a_{11} \Rightarrow x^4 + 3x^2 + 1 = 109$$

$$x^4 + 3x^2 - 108 = 0$$

$$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$$

$$x = \pm 3$$

$$a_{11} = x^2 + 1 = 10$$

Q.25 If $\left(\frac{1+i}{1-i}\right)^m = \left(\frac{1+i}{i-1}\right)^n = 1$, ($m, n \in \mathbb{N}$) then the greatest common divisor of the least values of m and n is

Sol. 4

$$\left[\frac{(1+i)(1+i)}{(1+i)(1-i)}\right]^m = \left[\left(\frac{1+i}{-1+i}\right)\left(\frac{-1-i}{-1-i}\right)\right]^n = 1$$

$$= \left(\frac{2i}{2}\right)^m = 1 \quad \left|\quad \left(\frac{-1-i-i+1}{1+1}\right)^n = 1\right.$$

$$m = 8$$

$$\begin{aligned} (-i)^{n/3} &= 1 \\ n &= 12 \end{aligned}$$

greatest common divisor of m & n is 4

