

MATHEMATICS
JEE-MAIN (September-Attempt)
2 September (Shift-1) Paper

SECTION - A

Q.1 A line parallel to the straight line $2x-y=0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1) . Then $x_1^2 + 5y_1^2$ is equal to :

- (1) 6 (2) 10 (3) 8 (4) 5

Sol. 1

$$T : \frac{xx_1}{4} - \frac{yy_1}{2} = 1 \quad \dots(1)$$

$t : 2x - y = 0$ is parallel to T

$$\Rightarrow T : 2x - y = \lambda \quad \dots\dots(2)$$

Now compare (1) & (2)

$$\frac{x_1}{4} = \frac{y_1}{1} = \frac{1}{\lambda}$$

$$x_1 = 4/\lambda \text{ \& } y_1 = 1/\lambda$$

$$(x_1, y_1) \text{ lies on hyperbola } \Rightarrow \frac{64}{4\lambda^2} - \frac{4}{\lambda^2} = 1$$

$$\Rightarrow 14 = \lambda^2$$

$$\text{Now } x_1^2 + 5y_1^2$$

$$= \frac{64}{\lambda^2} + 5 \frac{4}{\lambda^2}$$

$$= \frac{84}{14}$$

$$= 6 \text{ Ans.}$$



Q.2 The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$. Then a is equal to :

- (1) $\frac{\sqrt{17}-1}{2}$ (2) $\frac{\sqrt{17}}{2}$ (3) $\frac{1+\sqrt{17}}{2}$ (4) $\frac{\sqrt{17}}{2} + 1$

Sol. 3

$$-1 \leq \frac{|x|+5}{x^2+1} \leq 1$$

$$-x^2-1 \leq |x|+5 \leq x^2+1$$

case - I

$$-x^2-1 \leq |x|+5$$

this inequality is always right $\forall x \in R$

case - II

$$|x|+5 \leq x^2+1$$

$$x^2 - |x| \geq 4$$

$$|x|^2 - |x| - 4 \geq 0$$

$$\left(|x| - \left(\frac{1 + \sqrt{17}}{2} \right) \right) \left(|x| - \left(\frac{1 - \sqrt{17}}{2} \right) \right) \geq 0$$

$$|x| \leq \frac{1 - \sqrt{17}}{2} \cup |x| \geq \frac{1 + \sqrt{17}}{2}$$

not possible

$$x \in \left(-\infty, \frac{-1 - \sqrt{17}}{2} \right] \cup \left[\frac{1 + \sqrt{17}}{2}, \infty \right)$$

$$a = \frac{1 + \sqrt{17}}{2}$$

Q.3 If a function $f(x)$ defined by $f(x) = \begin{cases} ae^x + be^{-x}, & -1 \leq x < 1 \\ cx^2, & 1 \leq x \leq 3 \\ ax^2 + 2cx, & 3 < x \leq 4 \end{cases}$ be continuous for some $a, b, c \in \mathbb{R}$ and

$f'(0) + f'(2) = e$, then the value of a is :

(1) $\frac{1}{e^2 - 3e + 13}$ (2) $\frac{e}{e^2 - 3e - 13}$ (3) $\frac{e}{e^2 + 3e + 13}$ (4) $\frac{e}{e^2 - 3e + 13}$

Sol.

4 $f(x)$ is continuous

at $x=1 \Rightarrow \boxed{ae + \frac{b}{e} = c}$

at $x=3 \Rightarrow 9c = 9a + 6c \Rightarrow c = 3a$

Now $f'(0) + f'(2) = e$

$\Rightarrow a - b + 4c = e$

$\Rightarrow a - e(3a - ae) + 4 \cdot 3a = e$

$\Rightarrow a - 3ae + ae^2 + 12a = e$

$\Rightarrow 13a - 3ae + ae^2 = e$

$\Rightarrow \boxed{a = \frac{e}{13 - 3e + e^2}}$

Q.4 The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

(1) $(-\infty, -9] \cup [3, \infty)$ (2) $[-3, \infty)$ (3) $(-\infty, 9]$ (4) $(-\infty, -3] \cup [9, \infty)$

Sol.

4

$\frac{a}{r} \cdot aar = 27 \Rightarrow a = 3$

$$\frac{a}{r} + a + ar = S$$

$$\frac{1}{r} + 1 + r = \frac{S}{3}$$

$$r + \frac{1}{r} = \frac{S}{3} - 1$$

$$r + \frac{1}{r} \geq 2 \text{ or } r + \frac{1}{r} \leq -2$$

$$\frac{S}{3} \geq 3 \text{ or } \frac{S}{3} \leq -1$$

$$S \geq 9 \text{ or } S \leq -3$$

$$S \in (-\infty, -3] \cup [9, \infty)$$

Q.5 If $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z , then the domain of R^{-1} is :

(1) $\{-1, 0, 1\}$

(2) $\{-2, -1, 1, 2\}$

(3) $\{0, 1\}$

(4) $\{-2, -1, 0, 1, 2\}$

Sol.

1

$$3y^2 \leq 8 - x^2$$

$$R : \{(0, 1), (0, -1), (1, 0), (-1, 0), (1, 1), (1, -1)$$

$$(-1, 1), (-1, -1), (2, 0), (-2, 0), (-2, 0), (2, 1), (2, -1), (-2, 1), (-2, -1)\}$$

$$\Rightarrow R : \{-2, -1, 0, 1, 2\} \rightarrow \{-1, 0, -1\}$$

$$\text{Hence } R^{-1} : \{-1, 0, 1\} \rightarrow \{-2, -1, 0, 1, 2\}$$

Q.6

The value of $\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}} \right)^3$ is :

(1) $-\frac{1}{2}(1 - i\sqrt{3})$

(2) $\frac{1}{2}(1 - i\sqrt{3})$

(3) $-\frac{1}{2}(\sqrt{3} - i)$

(4) $\frac{1}{2}(\sqrt{3} - i)$

Sol.

3

$$\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}} \right)^3$$

$$= \left(\frac{1 + \cos \left(\frac{\pi}{2} - \frac{2\pi}{9} \right) + i \sin \left(\frac{\pi}{2} - \frac{2\pi}{9} \right)}{1 + \cos \left(\frac{\pi}{2} - \frac{2\pi}{9} \right) - i \sin \left(\frac{\pi}{2} - \frac{2\pi}{9} \right)} \right)^3$$

$$\begin{aligned}
&= \left(\frac{1 + \cos \frac{5\pi}{18} + i \sin \frac{5\pi}{18}}{1 + \cos \frac{5\pi}{18} - i \sin \frac{5\pi}{18}} \right)^3 \\
&= \left(\frac{2 \cos \frac{5\pi}{36} \left\{ \cos \frac{5\pi}{36} + i \sin \frac{5\pi}{36} \right\}}{2 \cos \frac{5\pi}{36} \left\{ \cos \frac{5\pi}{36} - i \sin \frac{5\pi}{36} \right\}} \right)^3 \\
&= \left(\frac{\operatorname{cis} \left(\frac{5\pi}{36} \right)}{\operatorname{cis} \left(\frac{-5\pi}{36} \right)} \right)^3 \\
&= \operatorname{cis} \left(\frac{5\pi}{36} \times 3 + \frac{5\pi}{36} \times 3 \right) \\
&= \operatorname{cis} \left(\frac{10\pi}{12} \right) \\
&= \operatorname{cis} \left(\frac{5\pi}{6} \right) = -\frac{\sqrt{3}}{2} + \frac{i}{2}
\end{aligned}$$

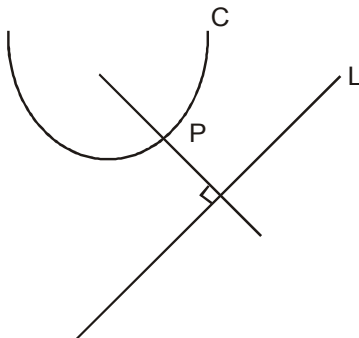
Q.7 Let P(h,k) be a point on the curve $y=x^2+7x+2$, nearest to the line, $y=3x-3$. Then the equation of the normal to the curve at P is:

- (1) $x+3y-62=0$ (2) $x-3y-11=0$ (3) $x-3y+22=0$ (4) $x+3y+26=0$

Sol.

4
 $C : y = x^2 + 7x + 2$

Let P : (h, k) lies on



Curve = $k = h^2 + 7h + 2$

Now for shortest distance

$$M_T \Big|_p^c = m_L = 2h+7 = 3$$

$$h = -2$$

$$k = -8$$

$$P : (-2, -8)$$

equation of normal to the curve is perpendicular to $L : 3x - y = 3$

$$N : x + 3y = \lambda$$

↓ Pass $(-2, -8)$

$$\lambda = -26$$

$$N : x + 3y + 26 = 0$$

Q.8 Let A be a 2×2 real matrix with entries from $\{0, 1\}$ and $|A| \neq 0$. Consider the following two statements:

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $\text{tr}(A) = 2$,

where I_2 denotes 2×2 identity matrix and $\text{tr}(A)$ denotes the sum of the diagonal entries of A . Then:

(1) Both (P) and (Q) are false

(2) (P) is true and (Q) is false

(3) Both (P) and (Q) are true

(4) (P) is false and (Q) is true

Sol. 4

$$P : A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \neq I_2 \text{ \& } |A| \neq 0 \text{ \& } |A| = 1 \text{ (false)}$$

$$Q : A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = I_2 \text{ then } \text{Tr}(A) = 2 \text{ (true)}$$

Q.9 Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is:

(1) $\frac{4}{17}$

(2) $\frac{8}{17}$

(3) $\frac{2}{5}$

(4) $\frac{2}{3}$

Sol. 2

1 to 30

box I

Prime on I

$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

31 to 50

box II

Prime on II

$\{31, 37, 41, 43, 47\}$

A : selected number on card is non - prime

$$P(A) = P(I) \cdot P(A/I) + P(II) \cdot P(A/II)$$

$$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \cdot \frac{15}{20}$$

$$\text{Now, } P(I/A) = \frac{P(II) \cdot P(A/I)}{P(A)}$$

$$= \frac{\frac{1}{2} \cdot \frac{20}{30}}{\frac{1}{2} \cdot \frac{20}{30} + \frac{1}{2} \cdot \frac{15}{20}} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{3}{4}} = \frac{8}{17}$$

Q.10 If $p(x)$ be a polynomial of degree three that has a local maximum value 8 at $x=1$ and a local minimum value 4 at $x=2$; then $p(0)$ is equal to :

- (1) 12 (2) -12 (3) -24 (4) 6

Sol. 2

$$p'(1) = 0 \text{ \& } p'(2) = 0$$

$$p'(x) = a(x-1)(x-2)$$

$$p(x) = a \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right) + b$$

$$p(1) = 8 \Rightarrow a \left(\frac{1}{3} - \frac{3}{2} + 2 \right) + b = 8 \quad \dots(i)$$

$$p(2) = 4 \Rightarrow a \left(\frac{8}{3} - \frac{3 \cdot 4}{2} + 2 \cdot 2 \right) + b = 4 \quad \dots(ii)$$

from equation (i) and (ii)

$$a = 24 \text{ \& } b = -12$$

$$p(0) = b = \boxed{-12}$$

Q.11 The contrapositive of the statement "If I reach the station in time, then I will catch the train" is:

- (1) If I will catch the train, then I reach the station in time.
 (2) If I do not reach the station in time, then I will catch the train.
 (3) If I do not reach the station in time, then I will not catch the train.
 (4) If I will not catch the train, then I do not reach the station in time.

Sol. 4

Statement p and q are true

Statement, then the contra positive of the implication

$$p \rightarrow q = (\sim q) \rightarrow (\sim p)$$

hence correct Ans. is 4

Q.12 Let α and β be the roots of the equation, $5x^2+6x-2=0$. If $S_n = \alpha^n + \beta^n$, $n=1,2,3,\dots$, then:

- (1) $5S_6+6S_5+2S_4=0$ (2) $6S_6+5S_5=2S_4$
 (3) $6S_6+5S_5+2S_4=0$ (4) $5S_6+6S_5=2S_4$

Sol. 4

$$5x^2 + 6x - 2 = 0 \Rightarrow \frac{\alpha}{\beta} = 5\alpha^2 + 6\alpha = 2$$

$$6\alpha - 2 = -5\alpha^2$$

Similarly

$$6\beta - 2 = -5\beta^2$$

$$S_6 = \alpha^6 + \beta^6$$

$$S_5 = \alpha^5 + \beta^5$$

$$S_4 = \alpha^4 + \beta^4$$

$$\text{Now } 6S_5 - 2S_4$$

$$= 6\alpha^5 - 2\alpha^4 + 6\beta^5 - 2\beta^4$$

$$= \alpha^4(6\alpha - 2) + \beta^4(6\beta - 2)$$

$$= \alpha^4(-5\alpha^2) + \beta^4(-5\beta^2)$$

$$= -5(\alpha^6 + \beta^6)$$

$$= -5S_6$$

$$= 6S_5 + 5S_6 = 2S_4$$

Q.13 If the tangent to the curve $y=x+\sin y$ at a point (a,b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and

$\left(\frac{1}{2}, 2\right)$, then:

(1) $b = \frac{\pi}{2} + a$

(2) $|a+b|=1$

(3) $|b-a|=1$

(4) $b=a$

Sol. 3

$$\left. \frac{dy}{dx} \right|_{p(a,b)} = \frac{2 - \frac{3}{2}}{\frac{1}{2} - 0} = 1$$

$$1 + \cos b = 1 \quad \left| \begin{array}{l} p : (a,b) \text{ lies on curve} \\ \cos b = 0 \end{array} \right. \quad b = a + \sin b$$

$$\boxed{b = a \pm 1}$$

$$b - a = \pm 1$$

$$\boxed{|b - a| = 1}$$

Q.14 Area (in sq. units) of the region outside $\frac{|x|}{2} + \frac{|y|}{3} = 1$ and inside the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is:

(1) $3(\pi - 2)$

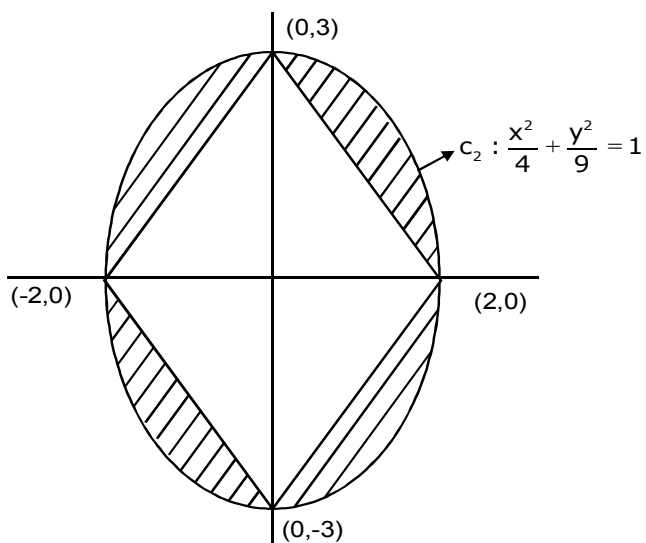
(2) $6(\pi - 2)$

(3) $6(4 - \pi)$

(4) $3(4 - \pi)$

Sol. 2

$$c_1 : \frac{|x|}{2} + \frac{|y|}{3} = 1$$



$$A = 4 \left(\frac{\pi ab}{4} - \frac{1}{2} \cdot 2 \cdot 3 \right)$$

$$A = \pi \cdot 2 \cdot 3 - 12$$

$$A = 6(\pi - 2)$$

Q.15 If $|x| < 1, |y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \dots$ is:

(1) $\frac{x+y+xy}{(1-x)(1-y)}$ (2) $\frac{x+y-xy}{(1-x)(1-y)}$ (3) $\frac{x+y+xy}{(1+x)(1+y)}$ (4) $\frac{x+y-xy}{(1+x)(1+y)}$

Sol. 2

$$(x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \dots \infty$$

$$= \frac{1}{(x-y)} \{ (x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4) + \dots \infty \}$$

$$= \frac{\frac{x^2}{1-x} - \frac{y^2}{1-y}}{x-y}$$

$$= \frac{x^2(1-y) - y^2(1-x)}{(1-x)(1-y)(x-y)}$$

$$= \frac{(x^2 - y^2) - xy(x-y)}{(1-x)(1-y)(x-y)} = \frac{((x+y) - xy)(x-y)}{(1-x)(1-y)(x-y)}$$

$$= \frac{x+y-xy}{(1-x)(1-y)}$$

Q.16 Let $\alpha > 0, \beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in

the binomial expansion of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$ is $10k$, then k is equal to:

(1) 176

(2) 336

(3) 352

(4) 84

Sol.

2
For term independent of x

$$T_{r+1} = {}^{10}C_r \left(\alpha x^{\frac{1}{9}}\right)^{10-r} \cdot \left(\beta x^{-\frac{1}{6}}\right)^r$$

$$T_{r+1} = {}^{10}C_r \alpha^{10-r} \beta^r \cdot x^{\frac{10-r}{9}} \cdot x^{-\frac{r}{6}}$$

$$\therefore \frac{10-r}{9} - \frac{r}{6} = 0 \Rightarrow r=4$$

$$T_5 = {}^{10}C_4 \alpha^6 \beta^4$$

$\therefore AM \geq GM$

$$\text{Now } \frac{\left(\frac{\alpha^3}{2} + \frac{\alpha^3}{2} + \frac{\beta^2}{2} + \frac{\beta^2}{2}\right)}{4} \geq \sqrt[4]{\frac{\alpha^6 \beta^4}{2^4}}$$

$$\left(\frac{4}{4}\right)^4 \geq \frac{\alpha^6 \beta^4}{2^4}$$

$$\alpha^6 \beta^4 \leq 2^4$$

$${}^{10}C_4 \cdot \alpha^6 \beta^4 \leq {}^{10}C_4 \cdot 2^4$$

$$T_5 \leq {}^{10}C_4 \cdot 2^4$$

$$T_5 \leq \frac{10!}{6!4!} \cdot 2^4$$

$$T_5 \leq \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 2^4}{4 \cdot 3 \cdot 2 \cdot 1}$$

maximum value of $T_5 = 10 \cdot 3 \cdot 7 \cdot 16 = 10k$

$$k = 16 \cdot 7 \cdot 3$$

$$k = 336$$

Q.17 Let S be the set of all $\lambda \in \mathbb{R}$ for which the system of linear equations

$$2x - y + 2z = 2$$

$$x - 2y + \lambda z = -4$$

$$x + \lambda y + z = 4$$

has no solution. Then the set S

(1) is an empty set.

(2) is a singleton.

(3) contains more than two elements.

(4) contains exactly two elements.

Sol. 4

For no solution

$$\Delta = 0 \text{ \& } \Delta_1 | \Delta_2 | \Delta_3 \neq 0$$

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$$2(-2 - \lambda^2) + 1(1 - \lambda) + 2(\lambda + 2) = 0$$

$$-4 - 2\lambda^2 + 1 - \lambda + 2\lambda + 4 = 0$$

$$-2\lambda^2 + \lambda + 1 = 0$$

$$2\lambda^2 - \lambda - 1 = 0 \Rightarrow \lambda = 1, -1/2$$

Equation has exactly 2 solution

Q.18 Let $X = \{x \in \mathbb{N} : 1 \leq x \leq 17\}$ and $Y = \{ax + b : x \in X \text{ and } a, b \in \mathbb{R}, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then a+b is equal to:

(1) -27

(2) 7

(3) -7

(4) 9

Sol. 3

$$X : \{1, 2, \dots, 17\}$$

$$Y : \{ax + b : x \in X \text{ \& } a, b \in \mathbb{R}, a > 0\}$$

$$\text{Given Var}(Y) = 216$$

$$\frac{\sum y_i^2}{n} - (\text{mean})^2 = 216$$

$$\frac{\sum y_i^2}{17} - 289 = 216$$

$$\sum y_i = 8585$$

$$(a+b)^2 + (2a+b)^2 + \dots + (17a+b)^2 = 8585$$

$$105a^2 + b^2 + 18ab = 505 \dots (1)$$

$$\text{Now } \sum y_i = 17 \times 17$$

$$a(17 \times 9) + 17b = 17 \times 17$$

$$9a + b = 17 \dots (2)$$

from equation (1) & (2)

$$a = 3 \text{ \& } b = -10$$

$$a+b = -7$$

Q.19 Let $y=y(x)$ be the solution of the differential equation, $\frac{2 + \sin x}{y + 1} \cdot \frac{dy}{dx} = -\cos x, y > 0, y(0) = 1$. If

$y(\pi) = a$, and $\frac{dy}{dx}$ at $x = \pi$ is b , then the ordered pair (a,b) is equal to:

- (1) $(2, \frac{3}{2})$ (2) $(1,1)$ (3) $(2,1)$ (4) $(1,-1)$

Sol. 2

$$\int \frac{dy}{y+1} = \int \frac{-\cos x \, dx}{2 + \sin x}$$

$$\ln |y+1| = -\ln |2 + \sin x| + k$$

$$\downarrow (0, 1)$$

$$k = \ln 4$$

$$\text{Now C : } (y+1)(2 + \sin x) = 4$$

$$y(\pi) = a \Rightarrow (a+1)(2+0) = 4 \Rightarrow (a=1)$$

$$\left. \frac{dy}{dx} \right|_{x=\pi} = b \Rightarrow b = -(-1) \left(\frac{2+0}{1+1} \right)$$

$$\Rightarrow b = 1$$

$$(a,b) = (1,1)$$

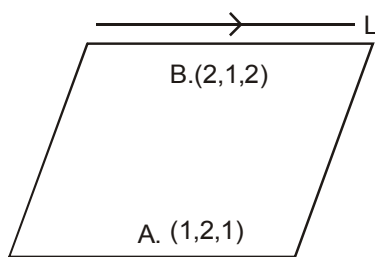
Q.20 The plane passing through the points $(1,2,1), (2,1,2)$ and parallel to the line, $2x=3y, z=1$ also passes through the point:

- (1) $(0,-6,2)$ (2) $(0,6,-2)$ (3) $(-2,0,1)$ (4) $(2,0,-1)$

Sol. 3

$$L : \begin{cases} 2x = 3y \\ z = 1 \end{cases} \begin{matrix} P : (0,0,1) \\ Q : (3,2,1) \end{matrix}$$

$$\vec{V}_L \text{ Dr of line } (3,2,0)$$



$$\vec{n}_p = \vec{AB} \times \vec{V}_L$$

$$\vec{n}_p = \langle 1, -1, 1 \rangle \times \langle 3, 2, 0 \rangle$$

$$\vec{n}_p = \langle -2, +3, 5 \rangle$$

$$\text{Plane : } -2(x-1) + 3(y-2) + 5(z-1) = 0$$

Plane : $-2x+3y+5z+2-6-5=0$
 Plane : $2x - 3y - 5z = -9$

Q.21 The number of integral values of k for which the line, $3x+4y=k$ intersects the circle, $x^2+y^2-2x-4y+4=0$ at two distinct points is.....

Sol. 9

$c : (1,2)$ & $r = 1$
 $|cp| < r$

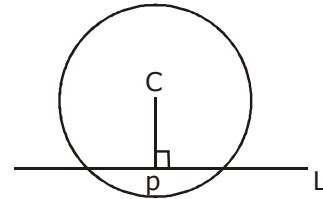
$$\left| \frac{3 \cdot 1 + 4 \cdot 2 - k}{5} \right| < 1$$

$$|11-k| < 5$$

$$-5 < k-11 < 5$$

$$6 < k < 16$$

$k = 7, 8, 9, \dots, 15 \Rightarrow$ total 9 value of k



Q.22 Let \vec{a}, \vec{b} and \vec{c} be three unit vectors such that $|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$. Then $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$ is equal to :

Sol. 2

$$|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$$

$$(\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) + (\vec{a} - \vec{c}) \cdot (\vec{a} - \vec{c}) = 8$$

$$a^2 + b^2 - 2\vec{a} \cdot \vec{b} + a^2 + c^2 - 2\vec{a} \cdot \vec{c} = 8$$

$$2a^2 + b^2 + c^2 - 2\vec{a} \cdot \vec{b} - 2\vec{a} \cdot \vec{c} = 8$$

$$\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = -2$$

$$\text{Now } |\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$$

$$= 2a^2 + 4b^2 + 4c^2 + 4\vec{a} \cdot \vec{b} + 4\vec{a} \cdot \vec{c}$$

$$= 2 + 4 + 4 + 4(-2)$$

$$= 2$$

Q.23 If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is.....

Sol. 309

E H M O R T

E - - - - - = 5!

H - - - - - = 5!

M E - - - - = 4!

M H - - - - = 4!

M O E - - - = 3!

M O H - - - = 3!

M O R - - - = 3!

M O T E - - = 2!

M O T H E R = 1

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Q.24. If $\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 + \dots + x^n - n}{x - 1} = 820, (n \in \mathbb{N})$ then the value of n is equal to :

Sol. 40

$$\lim_{x \rightarrow 1} \frac{(x-1)}{x-1} + \frac{(x^2-1)}{x-1} + \dots + \frac{(x^n-1)}{x-1} = 820$$

$$\Rightarrow 1 + 2 + 3 + \dots + n = 820$$

$$\Rightarrow \sum n = 820$$

$$\Rightarrow \frac{n(n+1)}{2} = 820$$

$$\Rightarrow n = 40$$

Q.25 The integral $\int_0^2 ||x-1| - x| dx$ is equal to :

Sol. 1.5

$$\int_0^2 ||x-1| - x| dx$$

$$= \int_0^1 |1-x-x| dx + \int_1^2 |x-1-x| dx$$

$$= \int_0^1 |2x-1| dx + \int_1^2 1 dx$$

$$= \int_0^{\frac{1}{2}} (1-2x) dx + \int_{\frac{1}{2}}^1 (2x-1) dx + \int_1^2 1 dx$$

$$= \left[\left(\frac{1}{2} - 0 \right) - \left(\frac{1}{4} - 0 \right) \right] + \left(1 - \frac{1}{4} \right) - \left(1 - \frac{1}{2} \right) + 1$$

$$= \frac{1}{2} - \frac{1}{4} + \frac{3}{4} - \frac{1}{2} + 1$$

$$= \frac{3}{2}$$