MATHEMATICS
JEE-MAIN (July-Attempt)
28 July (Shift-2) Paper Solution

SECTION - A

1. Let $S = \left\{x \in [-6,3] - \{-2,2\}: \frac{|x+3|-1}{|x|-2} \geq 0\right\}$ and $T = \{x \in Z: x^2 - 7|x| + 9 \leq 0\}$ then the number of elements of s ∩ T is :

(A) 7 (B) 5 (C) 4 (D) 3

Sol. **Official Ans. by NTA (D)**

Motion Ans. (D)

$S \cap T = \{-5, -4, 3\}$

2. let α, β be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2} + 1, \frac{1}{\beta^2} + 1$ be the roots of the equation $x^2 + ax + b = 0$. then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are :

(A) non – real complex numbers (B) real and both negative

(C) real and both positive (D) real and exactly one of them is positive

Sol. **Official Ans. by NTA (B)**

Motion Ans. (B)

$a = \frac{-1}{\alpha^2} - \frac{1}{\beta^2} - 2$

$b = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + 1 + \frac{1}{\alpha^2\beta^2}$

$a + b = \frac{1}{(\alpha\beta)^2} - 1 = \frac{1}{6} - 1 = \frac{-5}{6}$

$x^2 - \left(-\frac{5}{6} - 2\right)x + \left(2 - \frac{5}{6}\right) = 0$

$6x^2 + 17x + 7 = 0$

$x = \frac{-7}{3}, x = -\frac{1}{2}$ are the roots

Both roots are real negative.

3. Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then which of the following is NOT true ?

(A) $A^4 - B^4$ is a symmetric matrix (B) $AB - BA$ is a symmetric matrix

(C) $B^5 - A^5$ is a skew – symmetric matrix (D) $AB + BA$ is a skew – symmetric matrix

Sol. **Official Ans. by NTA (C)**

Motion Ans. (C)

Given that $A^T = A$, $B^T = -B$

(A) $C = A^4 - B^4$

$C^T = (A^4 - B^4) = (A^4)^T - (B^4)^T = A^4 - B^4 = C$

(B) $C = AB - BA$

$C^T = (AB - BA)^T = (AB)^T - (BA)^T$

$= B^TA^T - A^TB^T = -BA + AB = C$

(C) $C = B^5 - A^5$

$C^T = (B^5 - A^5)^T = (B^5)^T - (A^5)^T = -B^5 - A^5$

(D) $C = AB + BA$

$C^T = (AB + BA)^T = (AB)^T + (BA)^T$

$= -BA - AB = -C$

$\therefore$ Option C is not true.

4. Let $f(x) = ax^2 + bx + c$ be such that $f(1) = 3$, $f(-2) = \lambda$ and $f(3) = 4$. If $f(0) + f(1) + f(-2) + f(3) = 14$. then λ is equal to :

(A) -4 (B) $\frac{13}{2}$ (C) $\frac{23}{2}$ (D) 4

Sol. Official Ans. by NTA (D)

Motion Ans. (D)

$f(0) + 3 + \lambda + 4 = 14$

$\therefore f(0) = 7 - \lambda = c$

$f(1) = a + b + c = 3$ ....(i)

$f(3) = 9a + 3b + c = 4$ ....(ii)

$f(-2) = 4a - 2b + c = \lambda$ ....(iii)

(ii) – (iii)

$a + b = \frac{4-\lambda}{5}$ put in equation (i)

$\frac{4-\lambda}{5} + 7 - \lambda = 3$

$6\lambda = 24;\ \lambda = 4$

5. The function $f : R \to R$ defined by $f(x) = \lim_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$ is continuous for all x in :

(A) $R - \{-1\}$ (B) $R - \{-1, 1\}$ (C) $R - \{1\}$ (D) $R - \{0\}$

Sol. Official Ans. by NTA (B)

Motion Ans. (B)

Note : n should be given as a natural number.

$$f(= \begin{cases} -\frac{\sin(x-1)}{x-1} & x < -1 \\ -(sin2 + 1) & x = -1 \\ \cos 2\pi x & -1 < x < 1 \\ 1 & x = 1 \\ \frac{-\sin(x-1)}{x-1} & x > 1 \end{cases}$$

f(x) is discontinuous at $x = -1$ and $x = 1$

6. The function $f(x) = xe^{x(1-x)}, x \in R$ is

(A) increasing in $\left(-\frac{1}{2}, 1\right)$ (B)decreasing in $\left(\frac{1}{2}, 2\right)$

(C) increasing in $\left(-1, -\frac{1}{2}\right)$ (D) decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Sol. Official Ans. by NTA (A)

Motion Ans. (A)

$f(x) = x\, e^{x(1-x)}$

$f'(x) = -e^{x(1-x)}\,(2x + 1)\,(x - 1)$

f(x) is increasing in $\left(-\frac{1}{2}, 1\right)$

7. The sum of the absolute maximum and absolute minimum values of the function f(x) = $\tan^{-1}$ (sinx – cosx) in the interval [0, π] is :

(A) 0 (B) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{4}$ (C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$ (D) $-\frac{\pi}{12}$

Sol. Official Ans. by NTA (C)

Motion Ans. (C)

$f(x) = \tan^{-1}(\sin x - \cos x)$

$f'(x) = \frac{\cos x + \sin x}{(\sin x - \cos x)^2 + 1} = 0$

$\therefore x = \frac{3\pi}{4}$

x	0	$\frac{3\pi}{4}$	π
f(x)	$-\frac{\pi}{4}$	$\tan^{-1}\sqrt{2}$	$\frac{\pi}{4}$

$f_{\max} = \tan^{-1}\sqrt{2}$

$f_{\min} = -\frac{\pi}{4}$]

sum = $\tan^{-1}\sqrt{2} - \frac{\pi}{4}$

= $\cos^{-1}\frac{1}{\sqrt{3}} - \frac{\pi}{4}$

8. let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}, t \in \left(0, \frac{\pi}{2}\right)$ then $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to

(A) $-\frac{2\sqrt{2}}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{-2}{3}$

Sol. Official Ans. by NTA (D)

Motion Ans. (D)

$x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$

$\frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}}$

$y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$

$\frac{dy}{dt} = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$
$\frac{dy}{dx} = \tan 3t$
$\frac{dy}{dx} = 1$ at $t = \frac{\pi}{4}$
$\frac{d^2y}{dx^2} = \frac{3}{2\sqrt{2}}\sec 3t \;.\sqrt{\sin 2t} = -3$ at $t = \frac{\pi}{4}$
$\therefore \frac{1+(\frac{dy}{dx})^2}{\frac{d^2y}{dx^2}} = \frac{1+1}{-3} = -\frac{2}{3}$

9. Let $I_n(x) = \int_0^x \frac{1}{(t^2+5)^n} dt$, n = 1, 2, 3,...Then :

(A) $50I_6 - 9I_5 = xI_5'$
(B) $50I_6 - 11I_5 = xI_5'$
(C) $50I_6 - 9I_5 = I_5'$
(D) $50I_6 - 11I_5 = I_5'$

Sol. **Official Ans. by NTA (A)**
Motion Ans. (A)

$I_n(x) = \int_0^x \frac{dt}{(t^2+5)^n}$

Applying integral by parts

$I_n(x) = \left[\frac{t}{(t^2+5)^n}\right]_0^x - \int_0^x n(t^2+5)^{-n-1}.\, 2t^2$

$I_n(x) = \frac{x}{(x^2+5)^n} + 2n\int_0^x \frac{t^2}{(t^2+5)^{n+1}} dt$

$I_n(x) = \frac{x}{(x^2+5)^n} + 2n\int_0^x \frac{(t^2+5)-5}{(t^2+5)^{n+1}} dt$

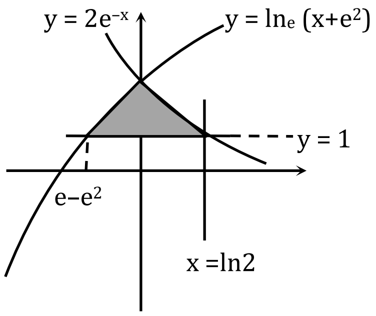
$I_n(x) = \frac{x}{(x^2+5)^n} + 2nI_n(x) - 10n\, I_{n+1}(x)$

$10_n\, I_{n+1}(x) + (1-2)I_n(x) = \frac{x}{(x^2+5)^n}$ Put n = 5

10. The area enclosed by the curves $y = \log_e(x+e^2), x = \log_e\left(\frac{2}{y}\right)$ and x = $\log_e 2$, above the line y = 1 is :

(A) $2 + e - \log_e 2$ (B) $1 + e - \log_e 2$ (C) $e - \log_e 2$ (D) $1 + \log_e 2$

Sol. **Official Ans. by NTA (B)**
Motion Ans. (B)



Required area is

$= \int_{e-e^2}^{0} \ell n(x+e^2) - 1dx + \int_0^{\ell n2} 2e^{-x} - 1dx = 1 + e - \ell n2$

11. let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{1/2}.x>1$ passing through the point $\left(2,\sqrt{\frac{1}{3}}\right)$, then $\left(2,\sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}y(8)$ is equal to

(A) $11 + 6\log_e 3$ (B) 19 (C) $12 - 2\log_e 3$ (D) $19 - 6\log_e 3$

Sol. **Official Ans. by NTA (D)**

Motion Ans. (D)

$\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$,

$\frac{dy}{dx}+Py=Q$

$I.F.=e^{\int Pdx}=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$

$y\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}=\int\left(\frac{x-1}{x+1}\right)^{1}dx$

$= x - 2\log_e |x + 1| + C$

Curve passes through $\left(2,\frac{1}{\sqrt{3}}\right)$

$\Rightarrow C = 2\log_e 3 - \frac{5}{3}$

at x = 8,

$\sqrt{7}y(8) = 19 - 6\log_e 3$

12. The differential equation of the family of circles passing through the points (0, 2) and (0, –2) is :

(A) $2xy\frac{dy}{dx}+(x^2-y^2+4)=0$ (B) $2xy\frac{dy}{dx}+(x^2+y^2-4)=0$

(C) $2xy\frac{dy}{dx}+(y^2-x^2+4)=0$ (D) $2xy\frac{dy}{dx}-(x^2-y^2+4)=0$

Sol. **Official Ans. by NTA (A)**

Motion Ans. (A)

Equation of circle passing through (0, –2) and (0, 2) is

$x^2 + (y^2 - 4) + \lambda x = 0, (\lambda \in R)$

Divide by x we get

$\frac{x^2+(y^2-4)}{x}+\lambda=0$

Differentiating with respect to x

$\frac{x\left[2x+2y.\frac{dy}{dx}\right]-[x^2+y^2-4].1}{x^2}=0$

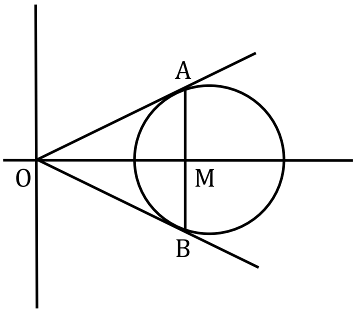
$\Rightarrow 2xy.\frac{dy}{dx} + (x^2 - y^2 + 4) = 0$

13. Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at orgin O(0, 0) then the area of the triangle OAB is :

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{4}$ (C) $\frac{3}{2\sqrt{3}}$ (D) $\frac{3}{4\sqrt{3}}$

Sol. **Official Ans. by NTA (B)**

Motion Ans. (B)



C : $(x - 2)^2 + y^2 = 1$

Equation of chord AB : $2x = 3$

OA = OB = $\sqrt{3}$

$AM = \frac{\sqrt{3}}{2}$

Area of triangle OAB = $\frac{1}{2}$ (2AM) (OM)

$= \frac{3\sqrt{3}}{4} sq. units$

14. Let the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ pass through the point $(2\sqrt{2}, -2\sqrt{2})$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola ?

(A) $(2\sqrt{3}, 3\sqrt{2})$ (B) $(3\sqrt{3}, -6\sqrt{2})$ (C) $(\sqrt{3}, -\sqrt{6})$ (D) $(3\sqrt{6}, 6\sqrt{2})$

Sol. **Official Ans. by NTA (B)**

Motion Ans. (B)

H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Foci : S (ae, 0), S' (–ae, 0)

Foot of directrix of parabola is (–ae, 0)

Focus of parabola is (ae, 0)

Now, semi latus rectum of parabola = SS' = |2ae|

Given, 4ae = $e\left(\frac{2b^2}{a}\right)$

$\Rightarrow b^2 = 2a^2$ ..... (1)

Given, $\left(2\sqrt{2}, -2\sqrt{2}\right)$ lies on H

$\Rightarrow \frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{8}$ .....(2)

From (1) and (2)

$a^2 = 4, b^2 = 8$

$\because b^2 = a^2(e^2 - 1)$

$\therefore e = \sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2 = 8\sqrt{3x}$

15. Let the lines $\frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. then which of the following points does NOT lie on P ?

(A) (0, -2, -2) (B) (-5, 0, -1) (C) (3, - 1, 0) (D) (0, 4, 5)

Sol. **Official Ans. by NTA (D)**

Motion Ans. (D)

Given, $L_1 : \frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$

and $L_2 : \frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$

are coplanar

$\Rightarrow \begin{bmatrix} 27 & 20 & 31 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{bmatrix} = 0$

$\Rightarrow \lambda = 3$

Now, normal of plane P, which contains L_1 L_2

$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{bmatrix}$

$= -3\hat{i} - 13\hat{j} + 11\hat{k}$

$\Rightarrow$ Equation of required plane P :

$3x + 13y - 11z + 4 = 0$

(0, 4, 5) does not lie on plane P.

16. A plane P is parallel to two lines whose direction ratios are −2, 1, −3 and −1, 2, −2 and it contains the point (2, 2, −2). Let P intersect the co – ordinate axes at the points A, B, C making the intercepts α, β, γ. If V is the volume of the tetrahedron OABC, where O is the origin, and $p = \alpha + \beta + \gamma$, then the ordered pair (V, p) is equal to:

(A) (48, −13) (B) (24, −13) (C) (48, 11) (D) (24, −5)

Sol. **Official Ans. by NTA (B)**

Motion Ans. (B)

Normal of plane P :

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -3 \\ -1 & 2 & -2 \end{bmatrix} = 4\hat{i} - \hat{j} - 3\hat{k}$$

Equation of plane P which passes through (2, 2,–2)

is $4x - y - 3z - 12 = 0$

Now, A (3, 0, 0), B (0, – 12, 0), C (0, 0, –4)

$\Rightarrow \alpha = 3, \beta = -12, \gamma = -4$

$\Rightarrow p = \alpha + \beta + \gamma = -13$

Now, volume of tetrahedron OABC

$V = \left|\frac{1}{6}\overrightarrow{OA} \times \overrightarrow{OC}\right| = 24$

(V, p) = (24, – 13)

17. let S be the set of all $a \in R$ for which the angle between the vectors.

$\vec{u} = a(\log_e b)\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$ and $\vec{v} = (\log_e b)\hat{\imath} + 2\hat{\jmath} + 2a(\log_e b)\hat{k}, (b > 1)$ is acute. Then S is equal to :

(A) $\left(-\infty, -\frac{4}{3}\right)$ (B) ϕ (C) $\left(-\frac{4}{3}, 0\right)$ (D) $\left(\frac{12}{7}, \infty\right)$

Sol. **Official Ans. by NTA (C)**

Motion Ans. (B)

For angle to be acute

$\vec{u}.\vec{v} > 0$

$\Rightarrow a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$

$\forall\ b > 1$

let $\log_e b = t \Rightarrow t > 0$ as $b > 1$

$y = at^2 + 6at - 12$ & $y > 0, \forall\ t > 0$

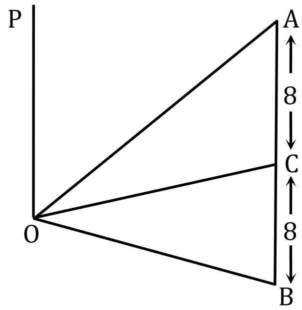
$\Rightarrow a \in \phi$

18. A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is erected at the point o such that $\angle PAO = \angle PBO = 15°$ and $\angle PCO = 45°$, where C is the midpoint of AB. then $(OP)^2$ is equal to :

(A) $\frac{32}{\sqrt{3}}(\sqrt{3} - 1)$ (B) $\frac{32}{\sqrt{3}}(2 - \sqrt{3})$ (C) $\frac{16}{\sqrt{3}}(\sqrt{3} - 1)$ (D) $\frac{16}{\sqrt{3}}(2 - \sqrt{3})$

Sol. **Official Ans. by NTA (B)**

Motion Ans. (B)



$\frac{OP}{OA} = \tan 15°$

$\Rightarrow$ OA = OP cot 15°

$\frac{OP}{OC}$ = tan 45° $\Rightarrow$ OP = OC

Now, OP = $\sqrt{OA^2 - 8^2}$

$\Rightarrow OP^2 = (OP)^2 \cot^2 15° - 64$

$\Rightarrow OP^2 = \frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$

19. Let A and B be two events such that $(B|A) = \frac{2}{5}$, $P(A|B) = \frac{1}{7}$ and $P(A \cap B) = \frac{1}{9}$. consider

$(S1) P(A' \cup B) = \frac{5}{6}$

$(S2) P(A' \cap B') = \frac{1}{18}$

(A) Both (S1) and (S2) are true
(B) Both (S1) and (S2) are false
(C) Only (S1) is true
(D) Only (S2) is true

Sol. **Official Ans. by NTA (A)**

Motion Ans. (A)

P(A l B) = $\frac{1}{7} \Rightarrow \frac{P(A\cap B)}{P(B)} = \frac{1}{7}$

P (B) = $\frac{7}{9}$

P(B l A) = $\frac{2}{5} \Rightarrow \frac{P(A\cap B)}{P(A)} = \frac{2}{5}$

$\Rightarrow$ P(A) = = $\frac{5}{18}$

Now, $P(A' \cup B) = 1 - P(A \cup B) + P(B)$

$= 1 - P(A) + P(A \cap B) = \frac{5}{6}$

$P(A' \cup B') = 1 - P(A \cap B)$

$= 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{18}$

$\Rightarrow$ Both (S1) and (S2) are true.

20. Let

P : Ramesh listens to music

q : Ramesh is out of his village

r : It is sunday

S : It is saturday

Then the statement "Ramesh listens to music only if he is in his village and it is sunday or saturday" can be expressed as

(A) $((\sim q) \wedge (r \vee s)) \Rightarrow p$ (B) $(q \wedge (r \vee s)) \Rightarrow p$

(C) $p \Rightarrow (q \wedge (r \vee s))$ (D) $p \Rightarrow ((\sim q) \wedge (r \vee s))$

Sol. **Official Ans. by NTA (D)**

Motion Ans. (D)

p ≡ Ramesh listens to music

$\sim$ q ≡ He is in village

$r \vee s$ ≡ Saturday or sunday

$p \Rightarrow ((\sim q) \wedge (r \vee s))$

21. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}} + \beta x\right)^4$, $(1 - 3\beta x)^2$ and $\left(1 - \frac{\beta}{2}x\right)^6$, $\beta > 0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P., then $50 - \frac{2d}{\beta^2}$ is equal to :

Sol. **Official Ans. by NTA (57)**

Motion Ans. (57)

${}^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P

$\beta^2 - \frac{5}{2}\beta^3 = -12\beta$

$\beta = \frac{12}{5}$ or $\beta = -2 \therefore \beta = \frac{12}{5}$

$d = \frac{-72}{5} - \frac{144}{25} = -\frac{504}{25}$

$\therefore 50 - \frac{2d}{\beta^2} = 57$

22. A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then b + 3g is equal to :

Sol. **Official Ans. by NTA (17)**

Motion Ans. (17)

${}^bC_3 \times {}^gC_2 = 168$

b(b – 1)(b – 2) (g)(g – 1) = 8 × 7 × 6 × 3 × 2

b = 8, g = 3

b + 3 g = 17

23. Let the tangents at the point P and Q on the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ meet at the point $R(\sqrt{2}, 2\sqrt{2}-2)$. If S is the focus of the ellipse on its negative major axis, then $SP^2 + SQ^2$ is equal to :

Sol. **Official Ans. by NTA (13)**

Motion Ans. (13)

Ellipse is

$\frac{x^2}{2}+\frac{y^2}{4}=1$; e = $\frac{1}{\sqrt{2}}$; $S \equiv (0, -\sqrt{2})$

Chord of contact is

$\frac{x}{\sqrt{2}}+\frac{(2\sqrt{2}-2)}{4}=1$

$\Rightarrow \frac{x}{\sqrt{2}} = 1-\frac{(\sqrt{2}-1)}{2}$ solving with ellipse

$\Rightarrow y = 0, \sqrt{2} \ \therefore x = \sqrt{2}, 1$

$P \equiv (1, \sqrt{2}), Q \equiv (\sqrt{2}, 0)$

$\therefore (SP)^2 + (SQ)^2 = 13$

24. If $1 + (2 + {}^{49}C_1 + {}^{49}C_2 + \ldots + {}^{49}C_{49})({}^{50}C_2 + {}^{50}C_4 + \ldots + {}^{50}C_{50})$ is equal to $2^n.m$, where m is odd , then n + m is equal to :

Sol. **Official Ans. by NTA (99)**

Motion Ans. (99)

$1 + (1 + 2^{49})(2^{49}-1) = 2^{98}$

m = 1, n = 98

m + n = 99

25. Two tangent lines l_1 and l_2 are drawn from the point (2, 0) to the parabola $2y^2 = -x$. if the lines l_1 and l_2 are also tangent to the circle $(x-5)^2 + y^2 = r$, then 17 r is equal to :

Sol. **Official Ans. by NTA (9)**

Motion Ans. (9)

$y^2 = -\frac{x}{2}$

$y = mx - \frac{1}{8m}$

This tangent pass through (2, 0)

m = $\pm\frac{1}{4}$ i.e., one tangent is x – 4y – 2= 0

17r = 9

26. If $\frac{6}{3^{12}}+\frac{10}{3^{11}}+\frac{20}{3^{10}}+\frac{40}{3^{9}}+\ldots+\frac{10240}{3}=2^{n}\cdot m.$ where m is odd, then m.n is equal to :

Sol. **Official Ans. by NTA (12)**

Motion Ans. (12)

$\frac{6}{3^{12}}+10\left(\frac{1}{3^{11}}+\frac{2}{3^{10}}+\frac{2^2}{3^{9}}+\frac{2^3}{3^{8}}+\ldots+\frac{2^{10}}{3}\right)$

$\frac{6}{3^{12}}+\frac{10}{3^{11}}\left(\frac{6^{11}-1}{6-1}\right)$

2^{12}. 1; m.n = 12

27. Let $S=\left[-\pi,\frac{\pi}{2}\right)-\left\{-\frac{\pi}{2},-\frac{\pi}{4},-\frac{3\pi}{4},\frac{\pi}{4}\right\}$ then the number of elements in the set $A=\{\theta\in S:\tan\theta(1+\sqrt{5}\tan(2\theta))=\sqrt{5}-\tan(2\theta)\}$

Sol. **Official Ans. by NTA (5)**

Motion Ans. (5)

$\tan\theta+\sqrt{5}\ \tan 2\theta\tan\theta=\sqrt{5}\ -\tan 2\theta$

$\tan 3\theta=\sqrt{5}$

$\theta=\frac{n\pi}{3}+\frac{\alpha}{3}$; $\tan\alpha=\sqrt{5}$

Five solution

28. Let z = a + ib, b $\neq$ 0 be complex numbers satisfying $z^2=\bar{z}.2^{1-|z|}$. then the least value of n $\in$ N, such that $z^n=(z+1)^n$, is equal to :

Sol. **Official Ans. by NTA (6)**

Motion Ans. (6)

$|z^2|=|\bar{z}|.2^{1-|z|}\Rightarrow|z|=1$

$z^2=\bar{z}\Rightarrow z^3=1\ \therefore\ z=\omega$ pr ω^2

$\omega^2=(1+\omega)^n=(-\omega^2)^n$

Least natural value of n is 6.

29. A bag contains 4 white and 6 black balls, three balls are drawn at random from the bag. let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then $100\sigma^2$ is equal to : ....

Sol. **Official Ans. by NTA (56)**

Motion Ans. (56)

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$\sigma^2=\Sigma X^2P(X)-\left(\Sigma XP(X)\right)^2=\frac{56}{100}$

$100\ \sigma^2=56$

30. The value of the integral $\int_0^{\frac{\pi}{2}} 60\frac{\sin(6x)}{\sin x}\ dx$

Sol. **Official Ans. by NTA (104)**

Motion Ans. (104)

$$I = 60\int_0^{\pi/2}\left(\frac{\sin 6x - \sin 4x}{\sin x} + \frac{\sin 4x - \sin 2x}{\sin x} + \frac{\sin 2x}{\sin x}\right) dx$$

$$I = 60\int_0^{\pi/2}(2cos\,5x\ + 2\cos 3x + 2\cos x)dx$$

$$I = 60\left(\frac{2}{5}\sin 5x + \frac{2}{3}\sin 3x + 2\sin x\right)\Big|_0^{\pi/2} = 104$$