PHYSICS JEE-MAIN (July-Attempt) 27 July (Shift-1) Paper Solution

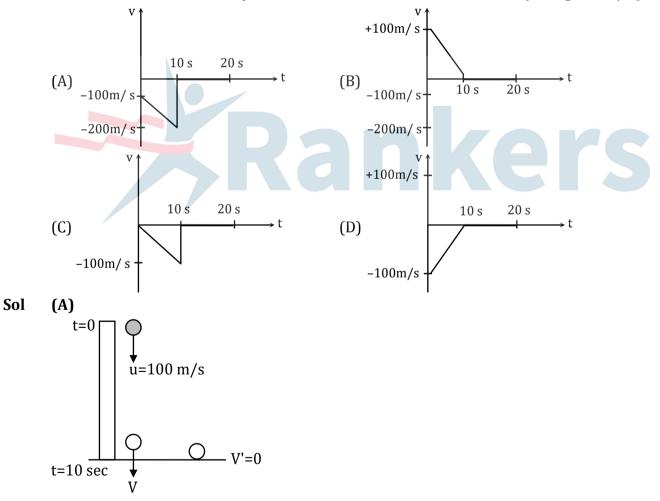
SECTION - A

- A torque meter is calibrated to reference standards of mass, length and time each with 5% accuracy. After calibration, the measured torque with this torque meter will have net accuracy of :

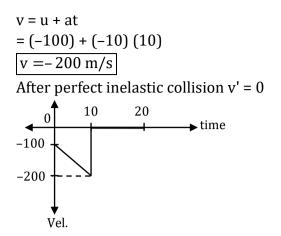
 (A) 15 %
 (B) 25 %
 (C) 75 %
 (D) 5%
- Sol. (B)

Dimension of torque $\rightarrow [ML^2 T^{-2}]$ % error in mass, length and time = 5% then $100 \times \frac{\Delta \tau}{\tau} = (\frac{\Delta m}{M} + \frac{2\Delta L}{L} + \frac{2\Delta T}{T}) \times 100$ = 5 + 2(5) + 2(5) $\Delta \tau \% = 25\%$

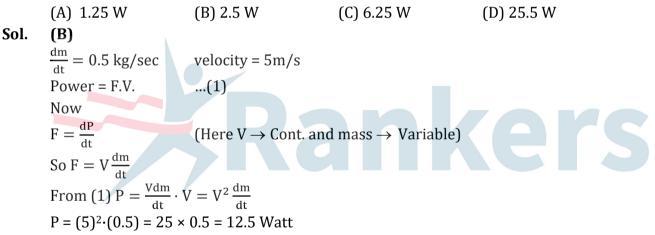
2. A bullet is shot vertically downwards with an initial velocity of 100 m/s from a certain height. Within 10s, the bullet reaches the ground and instantaneously comes to rest due to the perfectly inelastic collision. The velocity-time curve for total time t = 20s will be : (Take g = $10m/s^2$).



Velocity of bullet test after 10 sec.



3. Sand is being dropped from a stationary dropper at a rate of 0.5 kgs⁻¹ on a conveyor belt moving with a velocity of 5 ms⁻¹. The power needed to keep the belt moving with the same velocity will be :



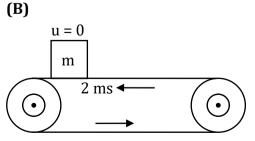
4. A bag is gently dropped on a conveyor belt moving at a speed of 2 m/s. The coefficient of friction between the conveyor belt and bag is 0.4. Initially, the bag slips on the belt before it stops due to friction. The distance travelled by the bag on the belt during slipping motion, is : $[Take g = 10 \text{ m/s}^{-2}]$

(C) 3.2 m

(D) 0.8 ms

Sol.

(A) 2 m

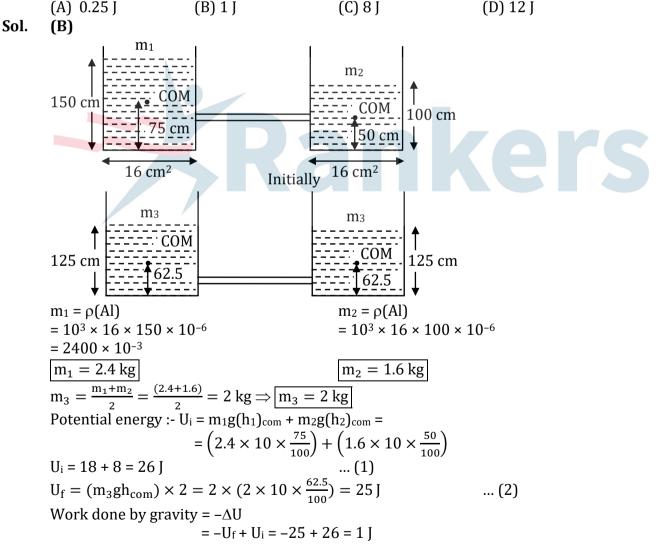


(B) 0.5 m

 $\begin{array}{l} \mu=0.4\\ \mbox{Velocity of conveyor belt}=2\ m/s\\ \mbox{Initially when bag is dropped on conveyor belt it starts slipping so kinetic friction acts on its due to which it finally stop after some time.\\ \mbox{Motion w.r.t. belt} \rightarrow \\ u_{rel}=0-(-2)=2\ m/s\\ a_{rel}=\frac{\mu mg}{m}=\mu g=0.4\times 10=4\ m/s^2\\ V_{rel}^2=u_{rel}^2+2a_{rel}\cdot S_{rel}\\ 0=(2)^2-2(4)\ (S_{rel}) \end{array}$

5. Two cylindrical vessels of equal cross-sectional area 16 cm^2 contain water upto heights 100 cm and 150 cm respectively. The vessels are interconnected so that the water levels in them become equal. The work done by the force of gravity during the process, is [Take, density of water = 10^3 kg/m^3 and g = 10 ms^{-2}]:

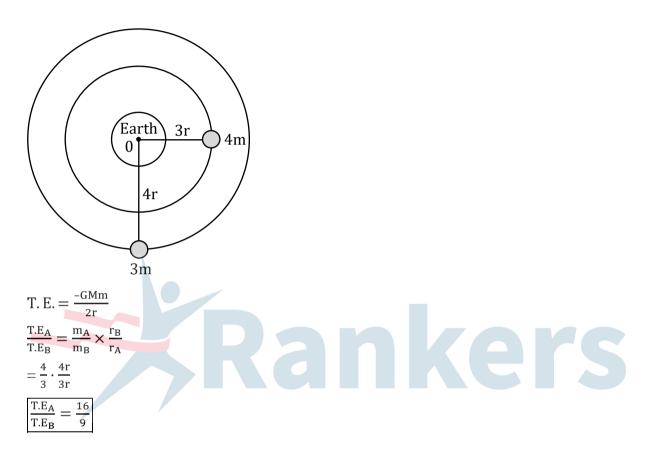
 $S_{rel} = \frac{4}{8} = \frac{1}{2} = 0.5 \text{ m}$



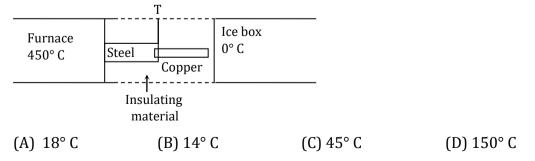
6. Two satellites A and B, having masses in the ratio 4:3, are revolving in circular orbits of radii 3r and 4r respectively around the earth. The ratio of total mechanical energy of A to B is :

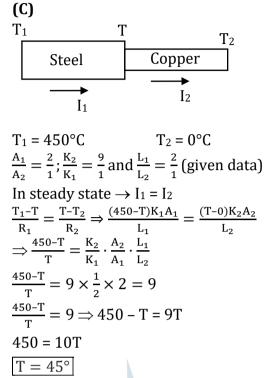
(A) 9:16 (B) 16:9 (C) 1:1 (D) 4:3

Sol. (B)



7. If K₁ and K₂ are the thermal conductivities, L₁ and L₂ are the lengths and A₁ and A₂ are the cross sectional areas of steel and copper rods respectively such that $\frac{K_2}{K_1} = 9$, $\frac{A_1}{A_2} = 2$, $\frac{L_1}{L_2} = 2$. Then, for the arrangement as shown in the figures, the value of temperature T of the steel - cooper junction in the steady state will be :





Read the following statements : 8.

> A. When small temperature difference between a liquid and its surrounding is doubled, the rate of loss of heat of the liquid becomes twice.

> B. Two bodies P and Q having equal surface areas are maintained at temperature 10° C and 20°

- C. The thermal radiation emitted in a given time by P and Q are in the ratio 1: 1.15.
- C. A Carnot Engine working between 100K and 400K has an efficiency of 75%

D. When small temperature difference between a liquid and its surrounding is quadrupled, the rate of loss of heat of the liquid becomes twice.

Choose the correct answer from the options given below :

(C) A, C only (A) A, B, C only (B) A, B only (D) B, C, D only

Sol. **(C)**

 $\frac{dQ}{dt} \propto K(T - T_0) \text{ here } \rightarrow T - T_0 = \Delta T$ $\frac{dQ}{dt} \propto K\Delta T \text{ If } \Delta T \text{ is twice then } \frac{dQ}{dt} \text{ will be 2 times}$ (A)

(B)
$$I \propto T^4$$

 $\frac{I_P}{I_Q} = \frac{(273+10)^4}{(273+20)^4} = (\frac{283}{293})^4 \simeq 0.92$

(C) $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{400} = \frac{3}{4} \Rightarrow \eta\% = \frac{3}{4} \times 100 = 75\%$

(D)
$$\frac{dQ}{dt} \propto \Delta T$$

Statements \rightarrow A and C are correct.

9. Same gas is filled in two vessels of the same volume at the same temperature. If the ratio of the number of molecules is 1:4, then

A. The r.m.s. velocity of gas molecules in two vessels will be the same.

B. The ratio of pressure in these vessels will be 1 : 4.

C. The ratio of pressure will be 1 : 1.

D. The r.m.s. velocity of gas molecules in two vessels will be in the ratio of 1 : 4.

Choose the correct answer from the options given below :

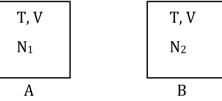
(A) A and C only (B) B and D only (C) A and B only (D) C and D only **(C)**

Sol.

PV = nRT

(C)

Same gas, same volume and same temperature.



 $V_{\rm rms} = \sqrt{\frac{3RT}{M}}$ (T and M (molar mass) are same so $V_{\text{rms}} \rightarrow \text{same})$ (A)

(B)
$$P \propto n$$

(C) $\frac{P_1}{P_2} = \frac{n_1}{n_2} = \frac{N_1/N_A}{N_2/N_A} = \frac{N_1}{N_2} = \frac{1}{4}$

(b)
$$P_2 = 1$$

(D) $\frac{(V_{rms})_1}{(V_{rms})} = \frac{1}{1}$

 $(V_{rms})_2$ Option \rightarrow A and B are correct.

10. Two identical positive charges Q each are fixed at a distance of '2a' apart from each other. Another point charge q₀ with mass 'm' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q₀ executes SHM. The time period of oscillation of charge q₀ will be :

(A)
$$\sqrt{\frac{4\pi^{3}\varepsilon_{0}ma^{3}}{q_{0}Q}}$$
 (B) $\sqrt{\frac{q_{0}Q}{4\pi^{3}\varepsilon_{0}ma^{3}}}$ (C) $\sqrt{\frac{2\pi^{2}\varepsilon_{0}ma^{3}}{q_{0}Q}}$ (D) $\sqrt{\frac{8\pi^{2}\varepsilon_{0}ma^{3}}{q_{0}Q}}$
Sol. (A)

 $4\pi^{3}\varepsilon_{0}ma^{3}$ (C) $\sqrt{\frac{2\pi^{2}\varepsilon_{0}ma^{3}}{q_{0}Q}}$ (D) $\sqrt{\frac{8\pi^{2}\varepsilon_{0}ma^{3}}{q_{0}Q}}$
 f_{2} (D) $\sqrt{\frac{8\pi^{2}\varepsilon_{0}ma^{3}}{q_{0}Q}}$
 f_{1} (D) $\sqrt{\frac{8\pi^{2}\varepsilon_{0}ma^{3}}{q_{0}Q}}$
 f_{2} (D) $\sqrt{\frac{8\pi^{2}\varepsilon_{0}ma^{3}}{q_{0}Q}}$
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 f_{2} (D) $\sqrt{\frac{8\pi^{2}\varepsilon_{0}ma^{3}}{$

$$F_{\text{net}} = \frac{\text{kqQ}[-4ax]}{(a^2 - x^2)^2} a \gg x$$

$$F_{\text{net}} = \frac{-\text{kqQ4ax}}{a^4} = \frac{-4qQx}{a^3} \cdot \frac{1}{4\pi\epsilon_0} = -\left(\frac{qQ}{\pi\epsilon_0 \text{ma}^3}\right) x$$

$$\text{macc.} = \frac{-aQx}{(\pi\epsilon_0 a^3)} \Rightarrow \text{acc.} = -\left(\frac{qQ}{\pi\epsilon_0 \text{ma}^3}\right) x$$

$$T = 2\pi \sqrt{\frac{\pi\epsilon_0 \text{ma}^3}{qQ}} = \sqrt{\frac{4\pi^3 \text{ma}^3\epsilon_0}{qQ}}$$

11. Two sources of equal emfs are connected in series. This combination is connected to an external resistance R. The internal resistances of the two sources are r_1 and r_2 ($r_1 > r_2$). If the potential difference across the source of internal resistance r_1 is zero, then the value of R will be :

12. Two bar magnets oscillate in a horizontal plane in earth's magnetic field with time periods of 3s and 4s respectively. If their moments of inertia are in the ratio of 3 : 2, then the ratio of their magnetic moments will be :

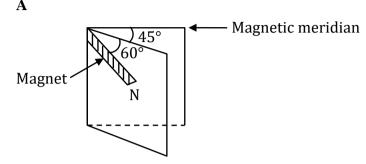
(A) 2:1 (B) 8:3 (C) 1:3 (D) 27:16(B)

$$T = 2\pi \sqrt{\frac{I}{MB}} B \rightarrow \text{Earth horizontal magnetic field}$$
$$T \propto \sqrt{\frac{I}{M}} \Rightarrow M \propto \frac{I}{T^2}$$
$$\text{So,} \frac{M_1}{M_2} = \frac{I_1}{I_2} \times \left(\frac{T_2}{T_1}\right)^2$$
$$\frac{M_1}{M_2} = \frac{3}{2} \times \left(\frac{4}{3}\right)^2 = \frac{3}{2} \times \frac{16}{9} = \frac{8}{3}$$

Sol.

13. A magnet hung at 45° with magnetic meridian makes an angle of 60° with the horizontal. The actual value of the angle of dip is -

(A)
$$\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$$
 (B) $\tan^{-1}\left(\sqrt{6}\right)$ (C) $\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$ (D) $\tan^{-1}\left(\sqrt{\frac{1}{2}}\right)$



Sol.

Sol.

Angle between real magnetic meridian and apparent magnetic meridian

$$\alpha = 45^{\circ} \qquad \dots (1)$$
Apparent angle of dip $\delta_{A} = 60^{\circ}$

$$\tan \delta_{A} = \frac{\tan \delta}{\cos \alpha}$$

$$\tan 60^{\circ} = \frac{\tan \delta}{\cos (45^{\circ})}$$

$$\sqrt{3} = \frac{\tan \delta}{\frac{1}{\sqrt{2}}}$$

$$\tan \delta = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \delta = \tan^{-1} \left(\sqrt{\frac{3}{2}}\right)$$

14. A direct current of 4 A and an alternating current of peak value 4 A flow through resistance of 3 Ω and 2 Ω respectively. The ratio of heat produced in the two resistances in same interval of time will be :

(A) 3:2 (B) 3:1 (C) 3:4 (D) 4:3

(B) For D.C. current $R_1 = 3\Omega I_1 = 4A$ And for A.C. current $\rightarrow I_2 = 4A$ (Peak value) $(I_2)_{rms} = \frac{4A}{\sqrt{2}} = \sqrt{2}A$ $\frac{H_1}{H_2} = \frac{I_1^2 R_1 t}{(I_2)_{rms}^2 R_2 t} = \frac{(4)^2 (3)}{(2\sqrt{2})^2 \cdot 2} = \frac{16 \times 3}{8 \times 2} = 3:1$

15. A beam of light travelling along X-axis is described by the electric field $E_y = 900 \sin \omega(t - x/c)$. The ratio of electric force to magnetic force on a charge q moving along Y-axis with a speed of $3 \times 10^7 \text{ ms}^{-1}$ will be : (Given speed of light = $3 \times 10^8 \text{ ms}^{-1}$) (A) 1 : 1 (B) 1 : 10 (C) 10 : 1 (D) 1 : 2 Sol. **(C)**

$$E_{y} = 900 \sin(wt - \frac{wx}{c})$$

$$C = \frac{E_{0}}{B_{0}} \Rightarrow B_{0} = \frac{E_{0}}{C} = \frac{900}{3 \times 10^{8}} = 300 \times 10^{-8} = 3 \times 10^{-6} \text{T}$$

$$\frac{F_{e}}{F_{m}} = \frac{qE}{qVB} = \frac{900}{3 \times 10^{7} \times 3 \times 10^{-6}} = \frac{900}{90} = \frac{10}{1}$$

- 16. A microscope was initially placed in air (refractive index 1). It is then immersed in oil (refractive index 2). For a light whose wavelength in air λ , calculate the change of microscope's resolving power due to oil and choose the correct option.
 - (A) Resolving power will be $\frac{1}{4}$ in the oil than it was in the air.
 - (B) Resolving power will be twice in the oil than it was in the air.
 - (C) Resolving power will be four times in the oil than it was in the air.
 - (D) Resolving power will be $\frac{1}{2}$ in the oil than it was in the air.

Sol. **(B)**

Resolving power $\propto \frac{\mu_{med}}{\lambda_{med}}$ $\frac{(R.P.)_2}{(R.P.)_1} = \frac{2}{1} = \left(\frac{\mu_{med}}{\lambda_{med}}\right) \times \left(\frac{\lambda_{air}}{\mu_{air}}\right) = \frac{\mu_m}{\mu_a} \times \left(\frac{\lambda_{air}}{\lambda_{air}/\mu_m}\right) = \mu_m^2 = \frac{4}{1}$

An electron (mass m) with an initial velocity $\vec{v} = v_0 \hat{i} (v_0 > 0)$ is moving in an electric field $\vec{E} =$ 17. $E_0\hat{i}$ ($E_0 > 0$) where E_0 is constant. If at t = 0 de Broglie wavelength is $\lambda_0 = \frac{h}{mv_0}$, then its de Broglie wavelength after times t is given by (A) λ_0 (B) $\lambda_0 \left(1 + \frac{eE_0t}{mv0}\right)$ (C) $\lambda_0 t$ (D) $\frac{\lambda_0}{\left(1+\frac{eE_0t}{mv_0}\right)}$

(A)
$$\lambda_0$$
 (B) λ_0 (

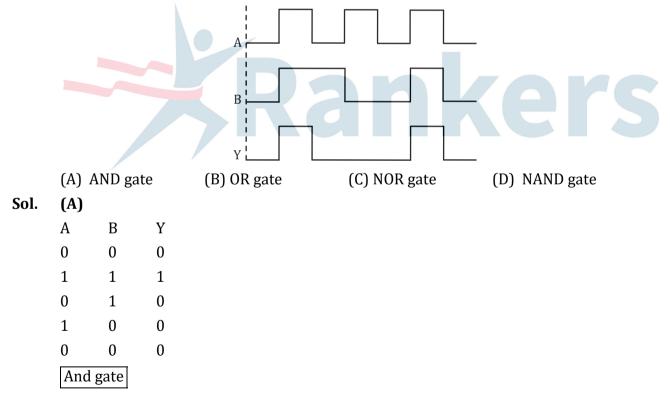
$$\lambda_0$$

Sol. **(D)**

$$\bar{V} = V_0 \hat{i} \text{ and } \vec{E} = -E_0 \hat{i} = \frac{-e\bar{E}}{m} = \frac{eE_0}{m} \hat{i}$$

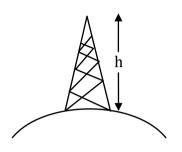
at t = 0 wavelength is $\lambda_0 = \frac{h}{mv_0}$... (1)
at time 't'
 $\bar{V} = \vec{u} + \vec{a}t$
 $\bar{V} = V_0 \hat{i} + \frac{eE_0}{m} \hat{i} t$
 $\therefore V = (V_0 + \frac{eE_0}{m} t)$
Now
 $\lambda = \frac{h}{mv} = \frac{h}{m(V_0 + \frac{eE_0}{m} t)} = \frac{h}{mV_0(1 + \frac{eE_0}{mV_0} t)}$
 $\bar{\lambda} = \frac{\lambda_0}{(1 + \frac{eE_0}{mV_0} t)}$

- **18.** What is the half-life period of a radioactive material if its activity drops to 1/16th of its initial value in 30 years ?
- (A) 9.5 years (B) 8.5 years (C) 7.5 years (D) 10.5 years Sol. (C) $A = \frac{A_0}{16}$ (given) t = 30 year (given) Now $A = \frac{A_0}{2^n}$ where $n = \frac{t}{T}$ $T \rightarrow$ Half life So $\frac{A_0}{16} = \frac{A_0}{2^n} \Rightarrow \boxed{n = 4}$ and $4 = \frac{30}{T} \Rightarrow \boxed{T = \frac{30}{4} = 7.5}$ year
- **19.** A logic gate circuit has two inputs A and B and output Y. The voltage waveforms of A, B and Y are shown below.



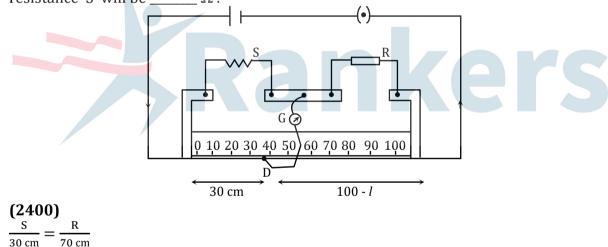
20. At a particular station, the TV transmission tower has a height of 100 m. To triple its coverage range, height of the tower should be increased to
 (A) 200 m
 (B) 300 m
 (C) 600 m
 (D) 900 m

Sol. (D) h₁ = 100 m Range becomes triple than new height is h₂ than D = $\sqrt{2Rh}$ \downarrow Range D $\propto \sqrt{h}$ $\frac{D_2}{D_1} = \sqrt{\frac{h_2}{h_1}}$ $\frac{3D_1}{D_1} = \sqrt{\frac{h_2}{h_1}}$ $9 = \frac{h_2}{h_1} \Rightarrow \boxed{h_2 = 9h_1}$ $h_2 = 9 \times 100$ $\boxed{h_2 = 900 \text{ m}}$



SECTION - B

21. In a meter bridge experiment, for measuring unknown resistance 'S', the null point is obtained at a distance 30 cm from the left side as shown at point D. If R is 5.6 k Ω , the value of unknown resistance 'S' will be _____ Ω .



 $\frac{\frac{3}{30 \text{ cm}} = \frac{R}{70 \text{ cm}}}{\frac{S}{3} = \frac{5.6 \text{ k}\Omega}{7}}$ $S = \frac{3}{7} \times 5.6 \text{ k}\Omega = 2.4 \text{ k}\Omega$

Sol.

22. The one division of main scale of Vernier calipers reads 1 mm and 10 divisions of Vernier scale is equal to the 9 division on main scale. When the two jaws of the instrument touch each other, the zero of the Vernier lies to the right of zero of the main scale and its fourth division coincides with a main scale division. When a spherical bob is tightly placed between the two jaws, the zero of the Vernier scale lies in between 4.1 cm 4.2cm and 6th Vernier division coincides with a main scale division. The diameter of the bob will be _____ × 10⁻² cm.

Sol. (412)

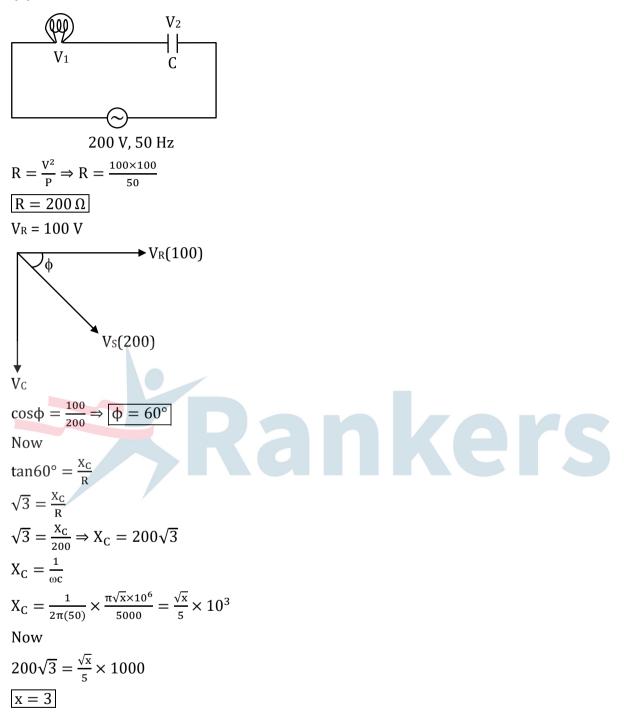
MSD = 1 mm $10 \text{ VSD} = 9 \text{ MSD} \qquad \text{L.C.} = 1 \text{ MSD} - 1 \text{ VSD}$ $1 \text{ VSD} = 0.9 \text{ MSD} \qquad = 1 \text{ mm} - 0.9 \text{ mm}$ $1 \text{ VSD} = 0.9 \text{ mm} \qquad |\text{L.C.} = 0.1 \text{ mm} = 0.01 \text{ cm}$ Zero error = 4(0.1 mm) = 0.4 mm = 0.04 cm Reading for spherical bob = Main reading + n(L.C.) - Error = (4.1) + 6(0.01) - 0.04 = 4.1 + 0.02 = 4.12 \text{ cm}} = 412 \times 10^{-2} \text{ cm}

23. Two beams of light having intensities I and 4I interfere to produce a fringe pattern on a screen. The phase difference between the two beams are $\pi/2$ and $\pi/3$ at points A and B respectively. The difference between the resultant intensities at the two points is xI. The value of x wil be _____.

 $I_1 = I \text{ and } I_2 = 4I$ at point A $\rightarrow \phi = \frac{\pi}{2}$ at point $B \rightarrow \phi = \frac{\pi}{3}$ nkers $I_{A} = I_{1} + I_{2} + 2\sqrt{I_{1}}\sqrt{I_{2}}\cos\phi$ $I_{A} = I + 4I + 2\sqrt{I}\sqrt{4I}\cos(\frac{\pi}{2})$ = 5I + 0 $I_A = 5I$... (1) Now $I_{\rm B} = I + 4I + 2\sqrt{I}\sqrt{4I}\cos(\frac{\pi}{3})$ $5I + 4I \cdot \frac{1}{2} = 7I$ $I_B = 7I$... (2) Difference between two intensities will be = $|I_B - I_A|$ = |7I - 5I|= 2I So x = 2

24. To light, a 50 W, 100 V lamp is connected, in series with a capacitor of capacitance $\frac{50}{\pi\sqrt{x}}\mu$ F, with 200V, 50Hz AC source. The value of x will be _____.

Sol. (3)



25. A 1 m long copper wire carries a current of 1 A. If the cross section of the wire is 2.0 mm² and the resistivity of copper is $1.7 \times 10^{-8} \Omega m$, the force experienced by moving electron in the wire is _____ × 10⁻²³ N. (charge on electorn = 1.6×10^{-19} C)

Sol. (136)

$$\frac{1 \text{ m}}{1 \text{ m}} = \frac{1}{2} \text{ m}^{2}$$

$$R = \frac{\rho \ell}{A} = \frac{1.7 \times 10^{-8} \times 1}{2 \times 10^{-6}} = \frac{1.7 \times 10^{-8}}{2} \Omega$$

$$V = \text{IR} = 1 \times \frac{1.7}{2} \times 10^{-2} = \frac{1.7}{2} \times 10^{-2} \text{ Volt}$$

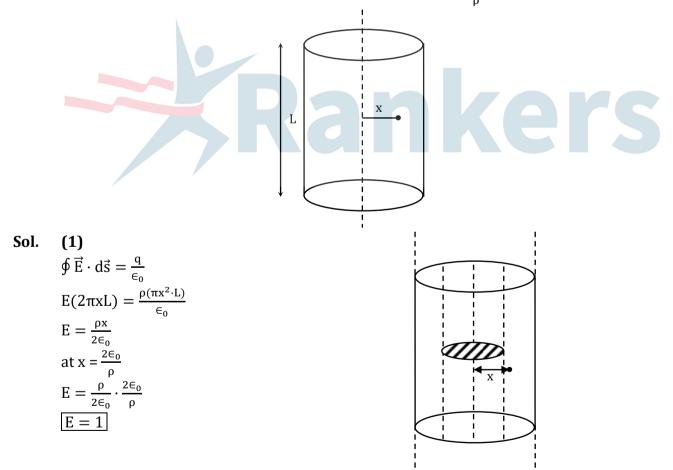
$$E = \frac{V}{\ell} = \frac{1.7}{2} \times 10^{-2} \frac{N}{C}$$

$$F = qE$$

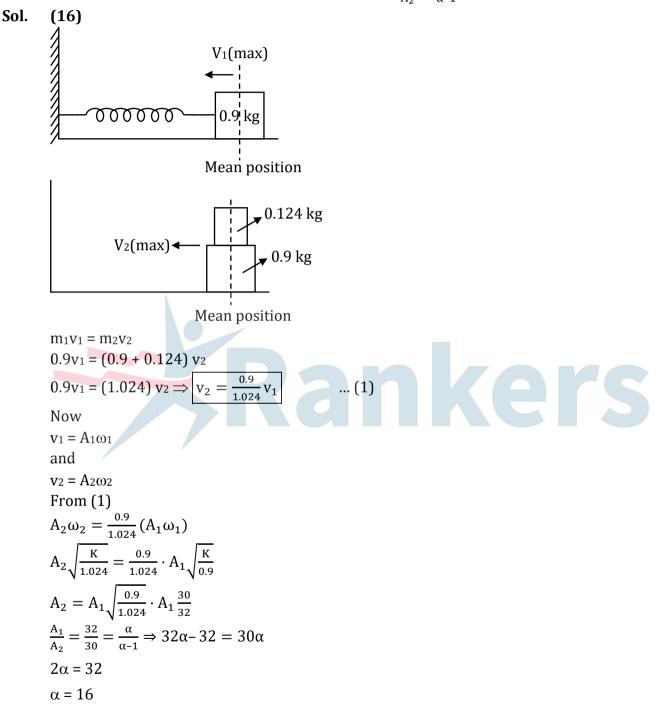
$$F = 1.6 \times 10^{-19} \times \frac{1.7}{2} \times 10^{-2} = 1.36 \times 10^{-21} = 136 \times 10^{-23}$$

$$Ans \to 136$$

26. A long cylindrical volume contains a uniformly distributed charge of density ρ Cm⁻³. The electric field inside the cylindrical volume at a distance $x = \frac{2\epsilon 0}{\rho}$ m from its axis is _____ Vm⁻¹.

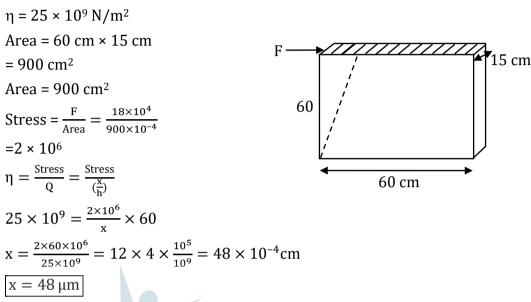


27. A mass 0.9 kg, attached to a horizontal spring, executes SHM with an amplitude A₁. When this mass passes through its mean position, then a smaller mass of 124 g is placed over it and both masses move tighter with amplitude A₂. If the ratio $\frac{A_1}{A_2}$ is $\frac{\alpha}{\alpha-1}$, then the value of α will be _____.



28. A square aluminum (shear modulus is 25×10^9 Nm⁻²) slab of side 60 cm and thickness 15 cm is subjected to a shearing force (on its narrow face) of 18.0×10^4 N. The lower edge is riveted to the floor. The displacement of the upper edge is _____ μ m.

Sol. (48)



29. A pulley of radius 1.5 m is rotated about its axis by a farce $F = (12t - 3t^2)$ N applied tangentially (while t is measured in seconds). If moment of inertia of the pulley about its axis of rotation is 4.5 kg m², the number of rotations made by the pulley before its direction of motion is reversed, will be $\frac{K}{\pi}$. The value of K is _____.

Sol. (18)

$$\tau = F.R$$

$$I\alpha = F.R$$

$$(4.5)\alpha = (12t - 3t^{2})\frac{3}{2}$$

$$\alpha = (12t - 3t^{2})\frac{3}{2} \times \frac{2}{9}$$

$$\alpha = 4t - t^{2}$$

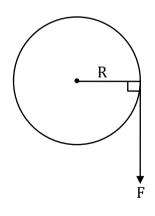
$$\frac{d\omega}{dt} = 4t - t^{2}$$

$$\int d\omega = \int (4t - t^{2}) dt$$

$$\int d\omega = \int (\frac{4t^{2}}{2} - \frac{t^{3}}{3})^{t}_{0}$$

$$(\omega - 0) = 2t^{2} - \frac{t^{3}}{3} \Rightarrow \omega = 2t^{2} - \frac{t^{3}}{3}$$
For direction change $\omega = 0$

$$2t^{2} - \frac{t^{3}}{3} = 0 \Rightarrow \boxed{t = 6 \sec}$$



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Now

$$\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$$

$$\int_0^{\theta} d\theta = \int_0^6 (2t^2 - \frac{t^3}{3}) dt$$

$$(\theta - 0) = (\frac{2t^2}{3} - \frac{t^4}{12})_0^3$$

$$\theta = \frac{2}{3}(6)^3 - \frac{1}{12} \cdot (6)^4$$

$$\theta = 144 - 108$$

$$\theta = 36$$
No. of rotation $= \frac{\theta}{2\pi} = \frac{36}{2\pi} = \frac{18}{\pi}$
So $\frac{K}{\pi} = \frac{18}{\pi} \Rightarrow \overline{K} = 18$

30. A ball of mass m is thrown vertically upward. Another ball of mass 2 m is thrown at an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the balls respectively is $\frac{1}{x}$. The value of x is _____.

Sol. (1)
For 1st ball

$$v = u + at$$

at max ht. $v = 0$
So
 $0 = v_1 - gt_1$
 $t_1 = \frac{v_1}{g}$... (1)
For 2nd ball
 $t_2 = \frac{v_2 cos\theta}{g}$... (2)
Given
 $t_1 = t_2$
From (1) and (2)
 $\frac{v_1}{g} = \frac{v_2 cos\theta}{g}$
 $v_1 = v_2 cos\theta$
 $h_1 = \frac{v_1^2}{2g}$ and $h_2 = \frac{v_1^2 cos^2\theta}{2g}$
Now
 $\frac{h_1}{h_2} = (\frac{v_1^2}{2g}) \times \frac{2g}{v_2^2 cos^2\theta} = \frac{v_2^2 cos^2\theta}{v_2^2 cos^2\theta} = 1$
 $\frac{h_1}{h_2} = 1$

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