MATHEMATICS
JEE-MAIN (July-Attempt)
26 July (Shift-2) Paper Solution

SECTION - A

1. The minimum value of the sum of the squares of the roots of $x^2 + (3-a)x + 1 = 2a$ is :

(A) 4 (B) 5 (C) 6 (D) 8

Sol. **C**

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2ab$

$= (3 - a)^2 - 2(1 - 2a)$

$= a^2 - 6a + 9 - 2 + 4a$

$= a^2 - 2a + 7 \Rightarrow (a-1)^2 + 6 \geq 6$

$\Rightarrow$ Ans. 6

2. If z = x + *i*y satisfies |z| – 2 = 0 and |z – *i*| – |z + 5*i*| = 0, then

(A) $x + 2y - 4 = 0$ (B) $x^2 + y - 4 = 0$

(C) $x + 2y + 4 = 0$ (D) $x^2 - y + 3 = 0$

Sol. **C**

$|z| = 2 \Rightarrow x^2 + y^2 = 4$ ...(1)

$|z-i| = |z+5i| \Rightarrow x^2 + (y-1)^2 = x^2 + (y+5)^2$

$\Rightarrow 1 - 2y = 25 + 10y$

$\Rightarrow y = -2$

from (1), x = 0

$\Rightarrow$ Ans. C

3. Let $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$, then the value of A'BA is :

(A) 1224 (B) 1042 (C) 540 (D) 539

Sol. **D**

$$A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$$

$$A'BA = [1\ 1\ 1] \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$[1\ 1\ 1] \begin{bmatrix} 81 - 100 + 121 \\ 144 + 169 - 196 \\ -225 + 256 + 289 \end{bmatrix} \Rightarrow [1 \quad 1 \quad 1] \begin{bmatrix} 102 \\ 117 \\ 320 \end{bmatrix}$$

$\Rightarrow 102 + 117 + 320 \Rightarrow 539$

$\Rightarrow$ Ans. D

4. $\sum_{i,j=0}^{n} {}^nC_i\, {}^nC_j$ is equal to

(A) $2^{2n} - {}^{2n}C_n$ (B) $2^{2n-1} - {}^{2n-1}C_{n-1}$

(C) $2^{2n} - \frac{1}{2}{}^{2n}C_n$ (D) $2^{n-1} + {}^{2n-1}C_n$

Sol. **A**

$\sum_{i,j=0}^{n} {}^nC_i\, {}^nC_j \Rightarrow \sum_{i\neq j} {}^nC_i \cdot {}^nC_j - \sum_{i=j} {}^nC_i \cdot {}^nC_j$

$\Rightarrow \sum_{i\neq j} {}^nC_i \cdot {}^nC_j = \sum_{i,j=0}^{n} {}^nC_i\, {}^nC_j - \sum_{i=j} {}^nC_i\, {}^nC_j$

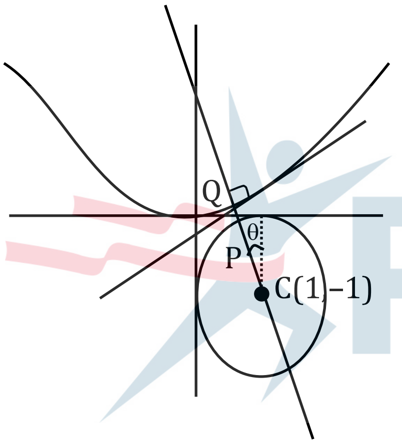
$= (2^n)^2 - (C_0^2 + C_1^2 + \ldots + C_n^2)$

$= 2^{2n} - {}^{2n}C_n$

5. Let P and Q be any points on the curves $(x-1)^2 + (y+1)^2 = 1$ and $y = x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval

(A) $\left(0, \frac{1}{4}\right)$ (B) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (C) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{4}, 1\right)$

Sol. **C**



$Q = (t, t^2)$

$m_{CQ} = m_{normal}$

$\frac{t^2+1}{t-1} = -\frac{1}{2t}$

Let $f(t) = 2t^3 + 3t - 1$

$f\left(\frac{1}{4}\right) f\left(\frac{1}{3}\right) < 0 \Rightarrow t \in \left(\frac{1}{4}, \frac{1}{3}\right)$

$P \equiv (1 + \cos(90 + \theta), -1 + \sin(90 + \theta))$

$P = (1 - \sin\theta, -1 + \cos\theta)$

$m_{normal} = m_{CP} \Rightarrow -\frac{1}{2t} = \frac{\cos\theta}{-\sin\theta} \Rightarrow \tan\theta = 2t$

$x = 1 - \sin\theta = 1 - \frac{2t}{\sqrt{1+4t^2}} = g(t)$ (let)

$\Rightarrow g'(t) < 0$

$g(t) \downarrow$ function

$t \in \left(\frac{1}{4}, \frac{1}{3}\right)$

$\Rightarrow g(t) \in (0.44, 0.485) \in \left(\frac{1}{4}, \frac{1}{2}\right)$

6. If the maximum value of a, for which of the function $f_a(x) = \tan^{-1}2x - 3ax + 7$ is non − decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to

(A) $8 - \frac{9\pi}{4(9+\pi^2)}$ (B) $8 - \frac{4\pi}{9(4+\pi^2)}$

(C) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$ (D) $8 - \frac{\pi}{4}$

Sol. **Bonus**

$f_a(x) = \tan^{-1}2x - 3ax + 7$

$f'_a(x) = \frac{2}{1+4x^2} - 3a \geq 0$

$a \leq \left(\frac{2}{3(1+4x^2)}\right)_{\text{min.}}$ at $x = \pm\frac{\pi}{6}$

$a_{\max} = \bar{a} = \frac{6}{9+\pi^2}$

$f_{\bar{a}}\left(\frac{\pi}{8}\right) = \tan^{-1}\frac{\pi}{4} - 3\frac{6}{9+\pi^2}\frac{\pi}{8} + 7 = \tan^{-1}\frac{\pi}{4} - \frac{9\pi}{4(\pi^2+9)} + 7$

7. Let $= \lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x\,(e^{3x}-1)}$ for some $\alpha \in R$. Then the value of $\alpha + \beta$ is :

(A) $\frac{14}{5}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Sol. **C**

$\beta = \lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x\frac{(e^{3x}-1)}{3x}3x} \Rightarrow \lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{3\alpha x^2}$

$\Rightarrow \lim_{x\to 0}\frac{\alpha x+1-\left\{1+3x+\frac{(3x)^2}{2}\right\}}{3\alpha x^2}$

$\Rightarrow \lim_{x\to 0}\frac{x(\alpha-3)-\frac{9x^2}{2}}{3\alpha x^2}$

$\Rightarrow \alpha - 3 = 0 \Rightarrow \alpha = 3$

& $\beta = \lim_{x\to 0}\frac{-9x^2}{2(9x^2)} = -\frac{1}{2}$

$\Rightarrow \alpha + \beta = 3 - \frac{1}{2} = \frac{5}{2}$

8. The value of $\log_e 2\frac{d}{dx}(\log_{\cos x}\operatorname{cosec} x)$ at $x = \frac{\pi}{4}$ is

(A) $-2\sqrt{2}$ (B) $2\sqrt{2}$ (C) −4 (D) 4

Sol. **D**

$\ell n2\frac{d}{dx}\left(\frac{\ell n\operatorname{cosec}x}{\ell n\cos x}\right)$

$\ell n2\left\{\frac{\ell n(\cos x)\{-\cot x\}-\ell n(\operatorname{cosec}x)\{-\tan x\}}{(\ell n\cos x)^2}\right\}$

$x = \frac{\pi}{4}$

$\frac{\ell n2\{-\ell n\left(\frac{1}{\sqrt{2}}\right)+\ell n(\sqrt{2})\}}{(\ell n\left(\frac{1}{\sqrt{2}}\right))^2}$

$\ell n2\frac{\{\ell n\sqrt{2}+\ell n(\sqrt{2})\}}{\left(-\frac{1}{2}\ell n2\right)^2} = 4$ Ans.

9. $\int_0^{20\pi}(|sinx| + |cosx|)^2\ dx$ is equal to

(A) $10(\pi + 4)$ (B) $10(\pi + 2)$ (C) $20(\pi - 2)$ (D) $20(\pi + 2)$

Sol. **D**

$\int_0^{20\pi}(|\sin x| + |\cos x|)^2 dx$

$\int_0^{20\pi}(1 + |\sin 2x|)dx$

$20\pi + \int_0^{20\pi}(|\sin 2x|)dx$

$20\pi + 40\int_0^{\frac{\pi}{2}}|\sin 2x|dx$

$$20\pi + 40\int_0^{\frac{\pi}{2}}\left(-\frac{\cos 2x}{2}\right)_0^{\frac{\pi}{2}}$$

$20\pi - 20\{\cos\pi - \cos 0\}$

$20\pi + 40 \Longrightarrow 20(\pi + 2) \Rightarrow$ D Ans.

10. Let the solution curve y = f (x) of the differential equation

$\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}, x \in (-1,1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)\ dx$ is equal to

(A) $\frac{\pi}{3} - \frac{1}{4}$ (B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$ (C) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

Sol. **B**

$I.F. = e^{\int\frac{xdx}{x^2-1}}$

$\Rightarrow e^{-\frac{1}{2}\int\frac{2x}{1-x^2}} = e^{\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$

Solution is,

$y\sqrt{1-x^2} = \int\frac{x^4+2x}{\sqrt{1-x^2}}\sqrt{1-x^2}dx$

$y\sqrt{1-x^2} = \frac{x^5}{5} + x^2 + C$

$\Downarrow$ (0,0)

C = 0

$\Rightarrow y = \frac{x^5+5x^2}{5\sqrt{1-x^2}}$

$\Rightarrow y = \frac{x^5}{5\sqrt{1-x^2}} + \frac{x^2}{\sqrt{1-x^2}}$

$\int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx = 2\int_0^{\frac{\sqrt{3}}{2}}\frac{x^2}{\sqrt{1-x^2}}dx$ x = sinθ

dx = cosθdθ

$$= 2\int_0^{\frac{\pi}{3}}\frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta$$

$= \int_0^{\frac{\pi}{3}}(1 - \cos 2\theta)d\theta$

$\Rightarrow \left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\frac{\pi}{3}}$

$\Rightarrow \frac{\pi}{3} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

11. The acute angle between the pair of tangents drawn to the ellipse $2x^2 + 3y^2 = 5$ from the point (1, 3) is

(A) $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$ (B) $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

(C) $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$ (D) $\tan^{-1}\left(\frac{3+8\sqrt{5}}{35}\right)$

Sol. **B**

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$(y - mx)^2 = \frac{5}{2}m^2 + \frac{5}{3}$

$\Downarrow (1,3)$

$(3 - m)^2 = \frac{5}{6}(3m^2 + 2)$

$6(9 + m^2 - 6m) = 15m^2 + 10$

$9m^2 + 36m - 44 = 0$ ⟨ m_1, m_2

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| \Rightarrow \left|\frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{1 + m_1m_2}\right|$

$\Rightarrow \left|\frac{\sqrt{16 + 4 \times \frac{44}{9}}}{1 - \frac{44}{9}}\right|$

$\Rightarrow \left(\frac{9 \times 4\sqrt{1 + \frac{11}{9}}}{35}\right)$

$\Rightarrow \left(\frac{12\sqrt{20}}{35}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{24\sqrt{5}}{35}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{24}{7\sqrt{5}}\right) \Rightarrow B$ Ans.

12. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x - 2)^2$ is

(A) y = 4 (x – 2) (B) y = 4 (x – 1)

(C) y = 4 (x + 1) (D) y = 4 (x + 2)

Ans. **B**

$y = x^2 \Rightarrow y = mx - am^2$

$y = mx - \frac{m^2}{4}$ ...(1)

put in $y = -(x-2)^2$

$$mx - \frac{m^2}{4} = -(x-2)^2$$

$4mx - m^2 = -4(x^2 - 4x + 4)$

$4x^2 + 4x(m - 4) + (16 - m^2) = 0$

$D = 0$

$16(m - 4)^2 - 16(16 - m^2) = 0$

$m^2 - 8m + 16 - 16 + m^2 = 0$

$2m^2 = 8m \Rightarrow m = 0, 4$

put m = 4 in (1)

$y = 4x - 4 \Rightarrow$ B Ans.

13. Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of $(a+b-c)$ is

(A) 12 (B) 13 (C) 14 (D) 16

Sol. **A**

$x^2 - 4x - 6 = 0$ ⟨ x_1, x_2 $\qquad y^2 + 2y - 7 = 0$ ⟨ y_1, y_2

equation of circle:

$x^2 + y^2 - 4x + 2y - 13 = 0$

$a = -2, b = 1, c = -13$

$\Rightarrow a + b - c = -2 + 1 + 13 = 12$ Ans.

14. If the line $x - 1 = 0$ is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point

(A) $(-2\sqrt{5}, 6)$ (B) $(-\sqrt{5}, 3)$ (C) $(\sqrt{5}, -2)$ (D) $(2\sqrt{5}, 3\sqrt{6})$

Sol. **C**

$$\frac{x^2}{\frac{6}{k}} - \frac{y^2}{6} = 1$$

$$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{6\times k}{6}$$

$$e^2 = \sqrt{1+k}$$

equation of directrix is $x = \pm\frac{a}{e} = 1$

$a^2 = e^2$

$\frac{6}{k} = k + 1$

$k^2 + k - 6 = 0 \Rightarrow k = 2$

$\Rightarrow$ equation is $2x^2 - y^2 = 6 \Rightarrow$ [C] Ans.

15. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{\imath}$, $\hat{\imath}+\hat{\jmath}$ and the plane determined by the vectors $\hat{\imath}-\hat{\jmath}, \hat{\imath}+\hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b}=\hat{\imath}-2\hat{\jmath}+2\hat{k}$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{2\pi}{3}$ (C) $\frac{4\pi}{5}$ (D) $\frac{5\pi}{6}$

Sol. **A**

$\overrightarrow{n_1}=\hat{\imath}\times(\hat{\imath}+\hat{\jmath})=\hat{k}$

$\overrightarrow{n_2}=(\hat{\imath}+\hat{k})(\hat{\imath}-\hat{\jmath})=\hat{\imath}+\hat{\jmath}-\hat{k}$

Since line is parallel to both planes, then line is parallel to $\overrightarrow{n_1}\times\overrightarrow{n_2}$

$\overrightarrow{n_1}\times\overrightarrow{n_2} = \hat{k}\times(\hat{\imath}+\hat{\jmath}-\hat{k})=-\hat{\imath}+\hat{\jmath}$

D.R. of $\vec{a}=-\hat{\imath}+\hat{\jmath}$

D.R. of $\vec{b}=\hat{\imath}-2\hat{\jmath}+2\hat{k}$

$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$=\frac{-3}{\sqrt{2}\times 3}$

$\theta=\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)=\frac{3\pi}{4}$

16. If $0<x<\frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1}x}{\alpha}=\frac{\cos^{-1}x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is

(A) $4\sqrt{(1-x^2)}\,(1-2x^2)$ (B) $4x\sqrt{(1-x^2)}\,(1-2x^2)$

(C) $2x\sqrt{(1-x^2)}\,(1-4x^2)$ (D) $4\sqrt{(1-x^2)}\,(1-4x^2)$

Sol. **B**

$\frac{\sin^{-1}x}{x}=\frac{\cos^{-1}x}{\beta}=\lambda$

$\sin^{-1}x=\lambda\propto$

$\cos^{-1}x=\lambda\beta$

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

$\Rightarrow\lambda=\frac{\pi}{2(\alpha+\beta)}$

$\sin^{-}\left(\frac{2\pi\propto}{\alpha+\beta}\right)=\sin(4\sin^{-1}x)$

$=2\sin(2\sin^{-1}x)\cos(2\sin^{-1}x)$

$=4x\sqrt{1-x^2}(1-2x^2)$

17. Negation of the Boolean expression $p\Leftrightarrow(q\Rightarrow p)$ is

(A) $(\sim p)$ (B) p (C) $(\sim p)\vee(\sim q)$ (D) $(\sim p)$

Sol. **D**

$\sim p\leftrightarrow(q\rightarrow p)$

$(p\wedge\sim Q)\vee(Q\wedge\sim p)$

$(p\wedge\sim(q\rightarrow p))\vee((q\rightarrow p)\wedge\sim p)$

$(p\wedge(q\wedge\sim p))\vee((\sim q\vee p)\wedge\sim p)$

$((p \wedge \sim p) \wedge q) \vee ((\sim p \wedge \sim q) \vee (\sim p \wedge p))$

$F \vee \big((\sim p \wedge \sim q) \vee F\big)$

$F \vee (\sim p \wedge \sim q) \Rightarrow (\sim p \vee \sim q)$

18. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then, $54P(x \leq 2)$ is equal to

(A) $\frac{73}{27}$ (B) $\frac{146}{27}$ (C) $\frac{146}{81}$ (D) $\frac{126}{81}$

Sol. **B**

np = 4

npq = 4/3

n = 6, p = 2/3, q = 1/3

54(P(X = 2) + P (X = 1) + P(X = 0))

$54\left({}^6C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4 + {}^6C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^5 + {}^6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6 \right)$

$= \frac{146}{27}$

19. $\int \frac{(1-\frac{1}{\sqrt{3}})(cosx-sinx)}{(1+\frac{2}{\sqrt{3}}\sin 2x)} dx$ is equal to

(A) $\frac{1}{2}\log_e \left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right| + C$ (B) $\frac{1}{2}\log_e \left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right| + C$

(C) $\log_e \left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right| + C$ (D) $\frac{1}{2}\log_e \left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right| + C$

Sol. **A**

$I = \int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)} dx$

$\frac{\sqrt{3}}{2}\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(\frac{\sqrt{3}}{2}+\sin 2x\right)} dx$

$\int \frac{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)(\cos x-\sin x)}{\sin 60^o+\sin 2x} dx$

$\int \frac{\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\sin x\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)} dx$

$\int \frac{\left(\cos\left(x-\frac{\pi}{6}\right)-\sin\left(x+\frac{\pi}{6}\right)\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)} dx$

$\frac{1}{2}\left(\int \frac{dx}{\sin\left(x+\frac{\pi}{6}\right)} - \int \frac{dx}{\cos\left(x-\frac{\pi}{6}\right)}\right)$

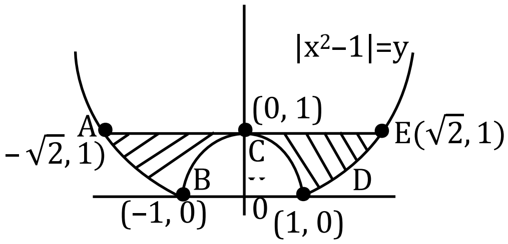
$\frac{1}{2}\ln\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|$

20. The area bounded by the curves $y = |x^2 - 1|$ and $y = 1$ is

(A) $\frac{2}{3}(\sqrt{2}+1)$ (B) $\frac{4}{3}(\sqrt{2}-1)$ (C) $2(\sqrt{2}-1)$ (D) $\frac{8}{3}(\sqrt{2}-1)$

Sol. **D**

$Y = |x^2-1|$



Area = ABCDEA

$= 2\left(\int_0^1 (1-(1-x^2))dx + \int_1^{\sqrt{2}} (1-(x^2-1))dx\right)$

$= \frac{8}{3}\left(\sqrt{2}-1\right)$

21. Let A = {1,2,3,4,5,6,7} and B = {3,6,7,9}. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \phi\}$ is ___________

Sol. **112**

Total subsets of A

$= 2^7 = 128$

number of subsets of A when $C \cap B = \phi$

$= 2^4 = 16$ (C is subset of {1,2,4,5})

required answer $= 128 - 16$

$= 112$

22. The largest value of a, for which the perpendicular distance of the plane containing the lines $\vec{r} -= (\hat{\imath}+\hat{\jmath}) + \lambda(\hat{\imath}+a\hat{\jmath}-\hat{k})$ and $\vec{r} = (\hat{\imath}+\hat{\jmath}) + \mu(-\hat{\imath}+\hat{\jmath}-a\hat{k})$ from the point (2, 1, 4) is $\sqrt{3}$, is ___________.

Sol. **2**

$\vec{r} = (\hat{\imath}+\hat{\jmath}) + \lambda(\hat{\imath}+a\hat{\jmath}-\hat{k})$

$\vec{r} = (\hat{\imath}+\hat{\jmath}) + M(-\hat{\imath}+\hat{\jmath}-a\hat{k})$

$\vec{n} -= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix}$

$= \hat{\imath}(-a^2 + 1) - \hat{\jmath}(-a - 1) + \hat{k}(1 + a)$

$\langle -a^2 + 1, a + 1, a + 1 \rangle$

$(-a^2 + 1)(\alpha - 1) + (a + 1)(y - 1) + (\alpha + 1)Z = 0$

$\left|\frac{(-a^2+1)+4(a+1)}{\sqrt{(-a^2+1)^2+(a+1)^2+(a+1)^2}}\right| = \sqrt{3}$

$\Rightarrow (-a^2 + 4a + 5)^2 = 3((-a^2 + 1)^2 + (a + 1)^2 + (a + 1)^2)$

$\Rightarrow a^4 + 16a^2 + 25 - 8a^3 + 409 - 109^2$

$\Rightarrow 3(a^4 + 1 - 2a^2 + 2a^2 + 2 + 49)$

$\Rightarrow a^4 + 16a^2 + 25 - 8a^3 + 409 - 109^2$

$\Rightarrow 3a^4 + 3 + 6 + 12a$

$\Rightarrow 2a^4 + 8a^3 - 6a^2 - 28a - 16 = 0$

$\Rightarrow a^4 + 4a^3 - 3a^2 - 14a - 8 = 0$

$\Rightarrow (a + 1)^2 (a - 2) (a + 4) = 0$

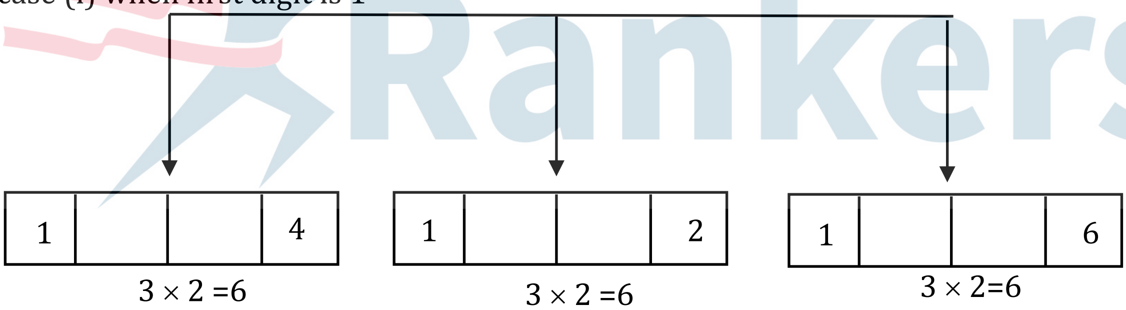
$\Rightarrow a = -1, 2, -4$

Largest value of a = 2

23. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1,2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is __________.

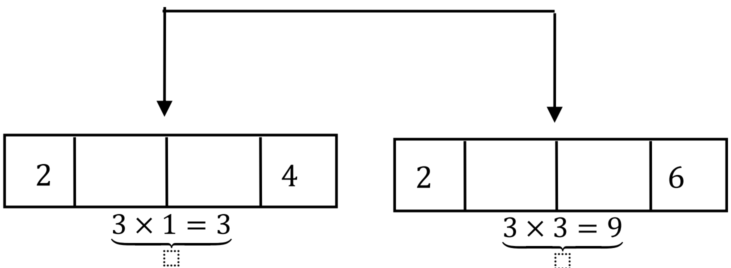
Sol. **30**

case (i) when first digit is 1



case (ii)

when first digit is 2



Total such numbers are 6 + 6 + 6 + 3 + 9 = 30

24. If $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, where m and n are co-prime, then m + n is equal to

Sol. **166**

$\sum_{k=1}^{10} \frac{k}{k^4+k^2+1}$

$= \sum_{k=1}^{10} \frac{k}{(k^2+k+1)(k^2-k+1)}$

$= \sum_{k=1}^{10} \frac{1}{2}\left(\frac{1}{k^2-k+1} - \frac{1}{k^2+k+1}\right)$

$= \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{7} \ldots\ldots \frac{1}{91} - \frac{1}{111}\right]$

$= \frac{1}{2}\left[1 - \frac{1}{111}\right]$

$= \frac{55}{111} = \frac{m}{n}$

m + n = 166

25. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to ________________,

Sol. **3**

$2\sin^2\theta - \cos 2\theta = 0$

$\Rightarrow 2\sin^2\theta - 1 + 2\sin^2\theta = 0$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin^2\theta = \pm\frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2(1 - \sin^2\theta) + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow \sin^2\theta = \frac{-1}{2}, 2$ (rejected)

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

sum of solutions

$\frac{7\pi}{6} + \frac{11\pi}{6}$

$\Rightarrow 3\pi = kp$

$\Rightarrow k = 3$

26. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, the $38\sigma^2$ is equal to _________.

Sol. **238**

$\overline{X} = 30$

$\sum x_i = 30 \times 40 = 1200$

$\overline{X}_{new} = \frac{\sum x_i - Sum\ of\ incorrect\ observations}{38}$

$= \frac{1200-22}{38} = 31$

$\sigma^2 = \frac{\sum x_i^2}{n} - (\overline{X})^2$

$\Rightarrow\ 25 = \frac{\sum x_i^2}{40} - (30)^2$

$\sum x_i^2 = (925)40$

$= 37000$

$\sum x_{i\,new}^2 = \sum x_i^2 - \frac{Sum\ of\ squares\ of\ incorrect\ observation}{38}$

$= 3700 - (10^2 + 12^2)$

$= 36756$

$\sigma_{new}^2 = \frac{\sum x_{i\,new}^2}{38} - (\overline{X}_{new})^2$

$= \frac{36756}{38} - (31)^2$

$= \frac{238}{38}$

$38\ \sigma_{new}^2 = 238$

27. The plane passing through the line L : $lx - y + 3(1 - l)z = 1$, $x + 2y - z = 2$ and perpendicular to the plane $3x + 2y + z = 6$ is $3x - 8y + 7z = 4$. If θ is the acute angle between the line L and the y – axis, then $415\cos^2\theta$ is equal to ______________.

Sol. **125**

$\vec{n}_1 = \ell\hat{\imath} - \hat{\jmath} + 3(1-\ell)\hat{k}$

$\vec{n}_2 = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

Direction ratio of line = $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \ell & -1 & 3(1-\ell) \\ 1 & 2 & -1 \end{vmatrix}$

$= (6\ell - 5)\hat{\imath} + (3 - 2\ell)\hat{\jmath} + (2\ell + 1)\hat{k}$

$3x - 8y + 7z = 4$ will contain the line

$(6\ell - 5)\hat{\imath} + (3 - 2\ell)\hat{\jmath} + (2\ell + 1)\hat{k}$

Normal of $3x - 8y + 7z = 4$ will be perpendicular to the line

$= 3(6\ell - 5) + (3 - 2\ell)(-8) + 7(2\ell + 1) = 0$

$\Rightarrow \ell = \frac{2}{3}$

$\therefore$ direction ratio of line $\left(-1, \frac{5}{3}, \frac{7}{3}\right)$

Angle with y axis

$\cos\theta = \frac{5/3}{\sqrt{1+\frac{25}{9}+\frac{49}{9}}}$

$\therefore 415\cos^2\theta = \frac{25}{83} \times 415 = 125$

28. Suppose y = y(x) be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim_{x\to\infty} y(x)$ is finite. If a and b are respectively the x – and y – intercepts of the tangent to the curve at x = 0, then the value of a – 4b is equal to ______________.

Sol. **3**

$\frac{dy}{dx} - y = 2 - e^{-x}$

is linear differential equation

I.F. = $e^{\int (-1)dx}$

= e^{-x}

$ye^{-x} = \int e^{-x}(2 - e^{-x})dx$

$= \int (2e^{-x} - e^{-2x})dx$

$= -2e^{-x} + \frac{e^{-2x}}{2} + c$

$\Rightarrow y = -2 + \frac{e^{-x}}{2} + ce^{x}$

$\lim_{x\to\infty} y(x)$ = finite

only possible when c = 0

$y = -2 + \frac{e^{-x}}{2}$

$\frac{dy}{dx} = \frac{-e^{-x}}{2}$

$\frac{dy}{dx}|_{x=0} = \frac{-1}{2}$

$y(0) = -\frac{3}{2}$

equation of tangent

$\left(y + \frac{3}{2}\right) = \frac{-1}{2}(x - 0)$

x + 2y + 3 = 0

a = –3

$b = \frac{-3}{2}$

a – 4b = –3 + 6 = 3

29. Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is ______________.

Sol. **53**

Let common difference is d and number of terms is n

199 = 100 + (n–1)d

$\Rightarrow d = \frac{99}{n-1}$

n	d
4	33

10	11
12	9

required answer = 33 + 11 + 9 = 53

30. The number of matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a, b, c, d, ∈ {–1, 0, 1, 2, 3, ……. 10}, such that A = A^{-1}, is ________________.

Sol. **50**

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given A = A^{-1}

$\therefore A^2 = A \cdot A^{-1} = I$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore a^2 + bc = 1$ ...(1)

ab + bd = 0 ...(2)

ac + cd = 0 ...(3)

$bc + d^2 = 1$ ...(4)

(1) – (4) gives

$a^2 - d^2 = 0$

⇒ (a + d) = 0 or a – d = 0

Case – I

a + d = 0 ⇒ (a,d) = (–1, 1), (0,0), (1,–1)

(i) (a,d) = (–1,1)

∴ from equation (1)

1 + bc = 1 ⇒ bc = 0

b = 0, C = 12 possibilities

c = 0, b = 12 possibilities

but (0, 0) is repeated

$\therefore 2 \times 12 = 24$

24 – 1 (repeated) = 23 pairs

(ii) (a, d) = (1, –1) ⇒ bc = 0 → 23 pairs

(iii) (a, d) = (0, 0) ⇒ bc=1

⇒ (b, c) = (1, 1) & (–1, –1), 2 pairs

Case – II

a = d

from (2) and (3)

$a \neq 0$ then b = c = 0

$a^2 = 1$

$a = \pm 1 = d$

(a, d) = (1, 1), (–1, –1) → 2 pairs

∴ Total = 23 + 23 + 2 + 2 = 50 pairs