MATHEMATICS
JEE-MAIN (July-Attempt)
26 July (Shift-1) Paper Solution

SECTION - A

1. Let f : R → R be a continuous function such that f(3x)-f(x) = x. If f(8) = 7, then f(14) is equal to :

(A) 4 (B) 10 (C) 11 (D) 16

Sol. **B**

$$f(3x) - f(x) = x$$

$x \to \frac{x}{3}$ $\quad f(x) - f(\frac{x}{3}) = \frac{x}{3}$

$x \to \frac{x}{3}$ $\quad f(\frac{x}{3}) - f(\frac{x}{3^2}) = \frac{x}{3^2}$

....

$\quad f(\frac{x}{3^{n-1}}) - f(\frac{x}{3^n}) = \frac{x}{3^n}$

$f(x) - f(\frac{x}{3^n}) = \frac{x}{3}\left[1 + \frac{1}{3} + ... + \frac{1}{3^{n-1}}\right]$

$= \frac{x}{3}\left[\frac{1-\frac{1}{3^n}}{1-\frac{1}{3}}\right]$

$= f(x) - f(\frac{x}{3^n}) = \frac{x}{3}\left[1 + \frac{1}{3} + ... + \frac{1}{3^{n-1}}\right]$

$= \frac{x}{3}\left[\frac{1-\frac{1}{3^n}}{1-\frac{1}{3}}\right]$

$= f(x) - f\left(\frac{x}{3^n}\right) = \frac{x}{2}\left[1 - \frac{1}{3^n}\right]$

Apply $\lim_{n\to\infty}$

$f(x) - f(0) = \frac{x}{2}$

Put x = 8 ⇒ f(0) = 3

Put x = 14

f(14) = 3 + 7 = 10

2. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $Re(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true ?

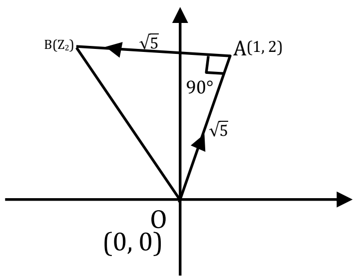
(A) $\arg z_2 = \pi - \tan^{-1}3$

(B) $\arg(z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$

(C) $|z_2| = \sqrt{10}$

(D) $|2z_1 - z_2| = 5$

Sol. **C**



$$\frac{z_2-(1+2i)}{(1+2i)-0}=e^{i\frac{\pi}{2}}$$

$z_2 - (1 + 2\, i) = i - 2$

$z_2 = 3\, i - 1 \Rightarrow z_2 = -1 + 3\, i$

$|z_2| = \sqrt{10}$

$|2z_1 - z_2| = |(2 + 4\, i) - (-1 + 3\, i)|$

$= |3 + i| = \sqrt{10}$

3. If the system of linear equations.

$8x + y + 4z = -2$

$x + y + z = 0$

$\lambda x - 3y = \mu$

has infinitely many solutions, then the distance of the point $(\lambda, \mu, -\frac{1}{2})$ from the plane $8x + y + 4z + 2 = 0$ is :

(A) $3\sqrt{5}$ (B) 4 (C) $\frac{26}{9}$ (D) $\frac{10}{3}$

Sol. **D**

$8x + y + 4z = -2$

$x + y + z = 0$

$\lambda x - 3y + 0.\, z = \mu$

$$\Delta_x=\begin{vmatrix}-2 & 1 & 4\\ 0 & 1 & 1\\ \mu & -3 & 0\end{vmatrix}=0$$

$= (-2)(3) - 1(-\mu) + 4(-\mu) = 0$

$= -6 - 3\mu = 0 \Rightarrow \mu = -2$

$\Delta = 0$

$$\begin{vmatrix}8 & 1 & 4\\ 1 & 1 & 1\\ \lambda & -3 & 0\end{vmatrix}=0$$

$= 8(3) - 1(0 - \lambda) + 4(-3 - \lambda) = 0$

$\Rightarrow 12 - 3\lambda = 0 \Rightarrow \lambda = 4$

$\left(4, -2, -\frac{1}{2}\right)$

$$Distance=\left|\frac{32-2-2+2}{\sqrt{81}}\right|=\frac{30}{9}=\frac{10}{3}$$

4. Let A be a 2 × 2 matrix with det (A) = -1 and det ((A + I) (Adj (A) + I)) = 4. Then the sum of the diagonal elements of A can be :

(A) -1 (B) 2 (C) 1 (D) $-\sqrt{2}$

Sol. **B**

$|A| = -1$

$|(A + I)\, (adj(A) + I)| = 4$

$\Rightarrow$ |A adj A + A + adj A + I| = 4

$\because$ A adjA = |A| I $\Rightarrow$ |- I + A + adj A + I| = 4

$\left|\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right| = 4$

$\left|\begin{bmatrix} a+d & 0 \\ 0 & a+d \end{bmatrix}\right| = 4$

(a + d) = ± 2

a + d = 2

5. The odd natural number a, such that the area of the region bounded by y=1, y = 3, x=0, x=y^a is $\frac{364}{3}$, is equal to :

(A) 3 (B) 5 (C) 7 (D) 9

Sol. **B**

$\int_1^3 x dy = \frac{364}{3}$

$\int_1^3 y^a dy = \frac{364}{3}$

$[\frac{y^{a+1}}{a+1}]_1^3 = \frac{364}{3}$

$\frac{3^{a+1}-1}{a+1} = \frac{364}{3}$

a = 5

6. Consider two G.Ps. 2, 2^2, 2^3,.... and 4, 4^2, 4^3,... of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :

(A) 560 (B) 1540 (C) 1330 (D) 2600

Sol. **C**

$\ln N = ([2.2^2 \ldots . 2^{60}][4.4^2 \ldots . 4^n])^{\frac{1}{60+n}}$

$= \left[2^{(1+\cdots..60)}.4^{(1+2+\cdots+n)}\right]^{\frac{1}{60+n}}$

$= \left[2^{(1830)}.4^{\frac{n(n+1)}{2}}\right]^{\frac{1}{60+n}}$

$= 2^{\frac{1830+n(n+1)}{60+n}} = 2^{\left(\frac{225}{8}\right)}$

$= \frac{1830+n^2+n}{60+n} = \frac{225}{8}$

$\Rightarrow 8n^2 - 217 n + 1140 = 0$

n = 20, $\frac{57}{8}$

$\sum_{k=1}^{20}(nk - k^2)$

$(20)\left[\frac{(20)(21)}{2}\right] - \frac{(20)(21)(41)}{6}$

$\frac{(20)(21)}{2}\left[20 - \frac{41}{3}\right]$

$\frac{(20)(21)(19)}{6} = (10)(7)(19) = 1330$

7. If the function f(x)= $\begin{cases} \frac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x \in \left(\frac{-\pi}{2},\frac{\pi}{2}\right)-\{0\} \\ K & x=0 \end{cases}$

is continuous at x=0, then k is equal to :

(A) 1 (B) -1 (C) e (D) 0

Sol. **A**

$f_0 = k$

$f_0 = f_{0^+}$

$$\lim_{x\to 0}\frac{\left[\frac{\log(x^4+x^2+1)}{(x^2+x^4)}\right](x^2+x^4)}{\left(\frac{1-\cos^2 x}{\cos x}\right)}$$

$$= \lim_{x\to 0}\frac{\left[\frac{\log(1+(x^2+x^4))}{(x^2+x^4)}.x^2(1+x^2)\right]}{\left(\frac{\sin^2 x}{x^2}\right)(x^2)} \times cosx$$

$$k = \frac{(1)}{(1)}(1) = 1$$

8. If f(x) = $\begin{cases} x+a, & x \le 0 \\ |x-4|, & x > 0 \end{cases}$ and g(x) = $\begin{cases} x+1, & x < 0 \\ (x-4)^2+b, & x \ge 0 \end{cases}$

are continuous on R, then (gof) (2) + (fog) (-2) is equal to :

(A) -10 (B) 10 (C) 8 (D) -8

Sol. **D**

g(f(2)) + f(1-2)

g[2] + f(-1)

(4 + b) + (a - 1)

a + b + 3 ... (1)

Now f(x) is continuous $f(0^-) = f(o^+)$

a = 4

g(x) is continuous

$g(0^-) = g(0^+)$

⇒ 1 = b + 16

b = – 15

∴ Ans : a + b + 3

4 – 15 + 3

= - 8

9. Let f(x) = $\begin{cases} x^3 - x^2 + 10x - 7, & x \le 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$

Then the set of all values of b, for which f(x) has maximum value at x = 1, is :

(A) (-6, -2) (B) (2, 6) (C) [-6, -2) ∩ (2,6) (D) $[-\sqrt{6}, -2) \cup (2, \sqrt{6}]$

Sol. **D**

$f(1) \ge f(1^+)$

$1 - 1 + 10 - 7 \geqslant -2 + \log_2(b^2 - 4)$

$5 \ge \log_2(b^2 - 4)$

$36 \ge b^2$

$b \in [-6, 6]$

$\because b^2 - 4 > 0 \quad \Rightarrow b \in (-\infty, -2) \cup (2, \infty)$

$\therefore b \in [-6, -2] \cup [2,6]$

10. If a = $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2n}{n^2+k^2}$ and f(x) = $\sqrt{\frac{1-\cos x}{1+\cos x}}, x \in (0,1)$, then :

(A) $2\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$ (B) $f\left(\frac{a}{2}\right) f'\left(\frac{a}{2}\right) = \sqrt{2}$

(C) $\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$ (D) $f\left(\frac{a}{2}\right) = \sqrt{2} f'\left(\frac{a}{2}\right)$

Sol. **D**

$a = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n[1+(\frac{k}{n})^2]}$

$a = \int_0^1 \frac{2dx}{1+x^2}$

$= (2\tan^{-1}x)_0^1$

$a = \frac{\pi}{2}$

$f(x) = \left|\tan\frac{x}{2}\right|$

$f(x) = \tan\left(\frac{x}{2}\right) \quad x \in (0,1)$

$f\left(\frac{a}{2}\right) = \tan\left(\frac{a}{2}\right) \Rightarrow \quad \frac{x}{2} \in \left(0, \frac{1}{2}\right)$

$f'(x) = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

$f'\left(\frac{a}{2}\right) = \frac{1}{2}\sec^2\left(\frac{a}{4}\right)$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{2}\sec^2\left(\frac{\pi}{8}\right)$

$= \frac{1}{2}\left[1 + \tan^2\frac{\pi}{8}\right]$

$=\frac{1}{2}\left[1 + \left(\sqrt{2} - 1\right)^2\right]$

$= \frac{1}{2}\left[4 - 2\sqrt{2}\right]$

$= 2 - \sqrt{2}$

$f'\left(\frac{\pi}{4}\right) = \sqrt{2}\, f\left(\frac{\pi}{4}\right)$

$f'\left(\frac{a}{2}\right) = \sqrt{2}\, f\left(\frac{a}{2}\right)$

11. If $\frac{dy}{dx} + 2y\tan x = \sin x$, $0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of y(x) is :

(A) $\frac{1}{8}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $\frac{3}{8}$

Sol. **A**

$I.F = e^{\int 2\tan x\, dx}$

$= e^{2(\ln \sec x)}$

$= \sec^2 x$

$y(\sec^2 x) = \int (\sin x)(\sec^2)dx$

$y(\sec^2 x) = \sec x + C$ ... (1)

$put\ x = \frac{\pi}{3}, y = 0$

$c = -2$

$y(\sec^2 x) = \sec x - 2$

$y = \cos x - 2\cos^2 x$

$y = -2\left[\left(\cos x - \frac{1}{4}\right)^2 - \frac{1}{16}\right]$

$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$= y_{max.} = \frac{1}{8}$

12. A point P moves so that the sum of squares of its distances from the points (1,2) and (-2,1) is 14. Let f(x,y) = 0 be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral ACBD is equal to :

(A) $\frac{9}{2}$ (B) $\frac{3\sqrt{17}}{2}$ (C) $\frac{3\sqrt{17}}{4}$ (D) 9

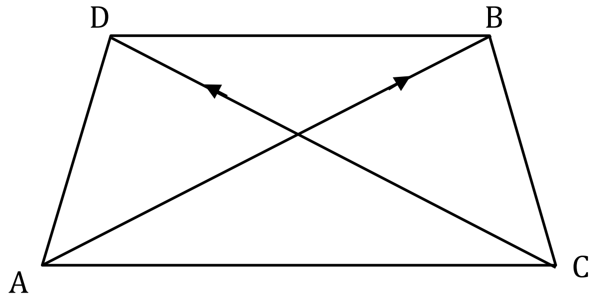
Sol. **B**

Let P(h, k)

$[(h - 1)^2 + (k - 2)^2] + [(h + 2)^2 + (k - 1)^2 = 14$

$h^2 + k^2 + h - 3k = 2$

$x^2 + y^2 + x - 3y - 2 = 0$
If y = 0
$x^2 + x - 2 = 0 \Rightarrow (x + 2)(x - 1) = 0$
x = - 2, 1
A(-2, 0), B(1, 0)
$y^2 - 3y - 2 = 0$
$y = \frac{3\pm\sqrt{17}}{2}$
$\Rightarrow C = \left(0, \frac{3-\sqrt{17}}{2}\right)$
$D = \left(0, \frac{3+\sqrt{17}}{2}\right)$



Area of quadrilateral = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{CD}| = \frac{3\sqrt{17}}{2}$

13. Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α,β) is perpendicular to the line 2x+2y=5. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha+4, \beta+4)$ does NOT pass through the point :

(A) (25, 10) (B) (20, 12) (C) (30, 8) (D) (15, 13)

Sol. **D**

$\beta^2 = 24\alpha$ .... (1)
$\frac{dy}{dx} = \frac{12}{y}$
$\left(\frac{dy}{dx}\right)_{\alpha,\beta} = \frac{12}{\beta}$
$\left(\frac{12}{\beta}\right)(-1) = -1$
$\beta = 12$
$\alpha = 6$
Now point = (10, 16)
equation of hyperbola $\frac{x^2}{36} - \frac{y^2}{144} = 1$
equation of Normal 2x + 5y = 100
which not passes through (15, 13)

14. The length of the perpendicular from the point (1, – 2, 5) on the line passing through (1,2,4) and parallel to the line x + y – z = 0 = x – 2y + 3z – 5 is :

(A) $\sqrt{\frac{21}{2}}$ (B) $\sqrt{\frac{9}{2}}$ (C) $\sqrt{\frac{73}{2}}$ (D) 1

Sol. **A**

$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix}$

$\left(\hat{\imath} - \hat{\jmath}(4) + \hat{k}(-3)\right)$

= $\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

Equation of line

$\frac{x-1}{1} = \frac{y-2}{-4} = \frac{z-4}{-3} = \lambda$

$P\vec{M}.(1, -4, -3) = 0$

$\Rightarrow (\lambda, -4\lambda + 4, -3\lambda - 1).(1, -4, -3) = 0$

$\Rightarrow \lambda + 16\lambda - 16 + 9\lambda + 3 = 0$

$\Rightarrow 26\lambda = 13$

$\lambda = \frac{1}{2}$

$PM = \sqrt{\frac{1}{4} + 4 + \frac{25}{4}} = \sqrt{\frac{13}{2} + 4} = \sqrt{\frac{21}{2}}$

15. Let $\vec{a} = \alpha\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - \alpha\hat{k}$, α > 0. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ is 30, then α is equal to :

(A) $\frac{15}{2}$ (B) 8 (C) $\frac{13}{2}$ (D) 7

Sol. **D**

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \alpha & 1 & -1 \\ 2 & 1 & -\alpha \end{vmatrix} (1-\alpha)\hat{\imath} + (\alpha^2 - 2)\hat{\jmath} + (\alpha - 2)\hat{k}$

$\therefore$ projection $(\vec{a} \times \vec{b}).\hat{c} = 30$

$\frac{-(1-\alpha)+2(\alpha^2-2)-2(\alpha-2)}{3} = 30$

$\Rightarrow 2\alpha^2 - \alpha = 91$

If $\alpha = 7$ then $2\alpha^2 - \alpha = 91$

16. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If P(x=1) = $\frac{4}{243}$, then P(x = 4 or 5) is equal to :

(A) $\frac{5}{9}$ (B) $\frac{64}{81}$ (C) $\frac{16}{27}$ (D) $\frac{145}{243}$

Sol. **C**

$np = \alpha$ ... (1)

$npq = \frac{\alpha}{3}$ ... (2)

(2) ÷ (1)

q = 1/3

p = 2/3

n = 3α/2

$= {}^nC_1(\frac{2}{3})'(\frac{1}{3})^{n-1} = \frac{4}{243}$

$n\left(\frac{1}{3}\right)^{n-1} = \frac{2}{81}$

n = 6

P(x = 4 or x= 5) = ${}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{2}{3}\right)^2$

$= \frac{15\times16}{36} + \frac{(6)(32)}{36}$

$= \frac{240+192}{36}$

$= \frac{432}{36} = \frac{144}{3^5} = \frac{48}{3^4} = \frac{16}{27}$

17. Let E_1, E_2, E_3 be three mutually exclusive events such that P(E_1) = $\frac{2+3p}{6}, P(E_2) = \frac{2-p}{8}$ and P(E_3) = $\frac{1-p}{2}$. If the maximum and minimum values of p are P_1 and P_2, then (p_1 + P_2) is equal to :

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$ (C) $\frac{5}{4}$ (D) 1

Sol. **B**

$0 \le P(E) \le 1$

$0 \le \frac{2+3p}{6} + \frac{2-p}{8} + \frac{1-p}{2} \le 1$

$\frac{2}{3} \le p \le 1$

$p_1 = 1, p_2 = \frac{2}{3}$

$p_1 + p_2 = \frac{5}{3}$

18. Let S = {θ∈[0, 2π] : $8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16$}. Then n(S) + $\sum_{\theta\in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\text{cos}ec\left(\frac{\pi}{4}+2\theta\right)\right)$ is equal to :

(A) 0 (B) -2 (C) -4 (D) 12

Sol. **C**

$8^{2\sin^2\theta} + \frac{8^2}{8^{2\sin^2\theta}} = 16$

$t + \frac{6^4}{t} = 16$

t = 8

$\Rightarrow 8^{2\sin^2\theta} = 8$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

n(5) = 4

$$\sum_{\theta\in S}\frac{2}{2\sin(\frac{\pi}{4}+2\theta)\cos(\frac{\pi}{4}+2\theta)}$$

$$\sum_{\theta \in S} \frac{2}{\sin(\frac{\pi}{2} + 4\theta)}$$

$\sum_{\theta \in S} 2\sec 4\theta$

$= 2[\sec\pi + \sec 3\pi + \sec 5\pi + sec 7\pi]$

= - 8

$nn(5) + \sum_{\theta \in S} \sec(\frac{\pi}{4} + 2\theta) cosec(\frac{\pi}{4} + 2\theta)$ = 4 – 8 = - 4

19. $\tan(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8})$ is equal to :

(A) 1 (B) 2 (C) $\frac{1}{4}$ (D) $\frac{5}{4}$

Sol. **B**

$\tan(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8})$

$= \tan\left(2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left(2\tan^{-1}\left(\frac{13}{39}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$

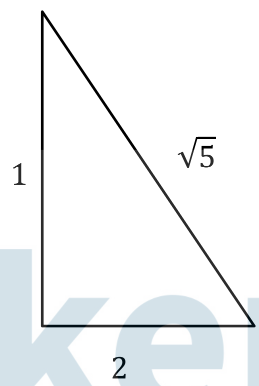
$= \tan\left(\tan^{-1}\left(\frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{3}}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{2\times 3}{8}\right) + \tan^{-1}\frac{1}{2}\right)$

$= \tan\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{4}\cdot\frac{1}{2}}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{5}{4} + \frac{8}{5}\right)\right)$

= 2



20. The statement $(\sim (p \Leftrightarrow \ \sim q)) \wedge q$ is :

(A) a tautology

(B) contradiction

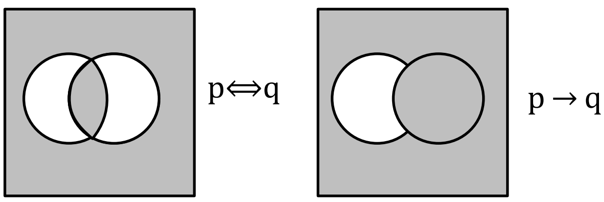
(C) equivalent to $(p \Rightarrow q) \wedge q$

(D) equivalent to $(p \Rightarrow q) \wedge p$

Sol. **D**

$(\sim(p \Leftrightarrow \sim q)) \wedge q \equiv (p \Leftrightarrow q) \wedge q$

$(p \Leftrightarrow q) \wedge q \equiv p \wedge q$



21. If for some p, q, r ε R, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2$-$2q(p+r)x+q^2(p+r)x+q^2+r^2=0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2+r^2}{p^2}$ is equal to ______.

Sol. **272**

$(px - q)^2 + (qx - r)^2 = 0$

$px - q = 0$, $2x - r = 0$

$x = \frac{q}{p} = \frac{r}{2}$

Now root's of equation $x^2 + 2x - 8 = 0$

$x = -4, 2$

∴ q and p one not of same sign

$\therefore \frac{q}{p} = \frac{r}{q} = -4$

$\frac{q^2+r^2}{p^2} = 272$

22. The number of 5-digit natural numbers, such that the product of their digits is 36, is ________.

Sol. **180**

$(2,2,3,3,1) \rightarrow \frac{5!}{2!2!}$

$(1,4,3,3,1) \rightarrow \frac{5!}{2!2!}$

$(2,2,1,9,1) \rightarrow \frac{5!}{2!2!}$

$(2,1,6,3,1) \rightarrow \frac{5!}{2!}$

$(4,9,1,1,1) \rightarrow \frac{5!}{3!}$

$(6,6,1,1,1) \rightarrow \frac{5!}{2!3!}$

Add All

$\Rightarrow 30 + 30 + 30 + 60 + 20 + 10 = 180$

23. The series of positive multiples of 3 is divided into sets : {3}, {6, 9, 12}, {15, 18, 21, 24, 27}, . . Then the sum of the elements in the 11th set is equal to _____.

Sol **6993**

3

6 9 12

15 18 21 24 27

In 11th set total no. of elements = $2 \times 11 - 1 = 21$

Total no. of element till 10th group

$= 3[1 + 3 + 5 + ... + 19]$

$= 300$

First element of 11th group = 303

Sum of element of 11th group= $\frac{21}{2}[2 \times 303 + (10) \times 3]$

= 21(303 + 30]

= 6993

24. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is ________.

Sol. **3**

$x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$

= $(x-1)^2(x+1)(x^5 + 3x - 1) = 0$

= $f(x) = x^5 + 3x - 1$

= $f'(x) = 5x^4 + 3$

= f'(x) > 0 f(x) ↑

Total = 3 distinct real root's

25. If the coefficients of x and x^2 in the expansion of $(1 + x)^p (1 - x)^q$, p, q ≤ 15, are -3 and -5 respectively, then the coefficient of x^3 is equal to _______.

Sol. **23**

$(1 + x)^p (1 - x)^q$

$\left[1 + px + \frac{P(p-1)}{2}x^2\right]\left[1 - qx + \frac{q(q-1)}{2}x^2\right]$

coefficient of x ⇒ (p – q) = - 3 ... (1)

coefficient of x_2 ⇒ $\frac{p(p-1)}{2} + \frac{q(q-1)}{2} - pq = -5$

$p^2 + q^2 - p - q - 2pq = -10$

⇒ $(p - q)^2 - (p + q) = -10$

⇒ p + q = 19 ... (2)

(1) & (2)

p = 8

q = 11

Now $(1 + x)^8 (1 - x)^{11}$

⇒ $(1 - x^2)^8(1 - x)^3$

$[1-8x^2][1 - 3x + 3x^2 - x^3]$

coefficient of x^3 = -1 + 24 = 23

26. If $n(2n + 1) \int_0^1 (1 - x^n)^{2n} dx = 1177 \int_0^1 (1 - x^n)^{2n+1} dx$, then n∈N is equal to _______.

Sol. **24**

$I_1 = \int_0^1 (1 - x^n)^{2n+1}.I\, dx$ Let $I_2 = \int_0^1 (1 - x^n)^{2n} dx$

= $[(1 - x^n)^{2n+1}.x]_0^1 - \int_0^1 (2n + 1)(1 - x^4)^{2n}(-nx^{n-1}.n)dx$

$= (1-x^n)^{2n+1}dx = n(2n+1)\int_0^1[(1-x^n)^{2n}]x^n dx$

$=-(n|2n+1|)\int_0^1[(1-x^n)^{2n}][1-x^n-1]dx$

$I_1 = -n(2n+1)(\int_0^1(1-x^n)^{2n+1}_{dx} - \int_0^1(1-x^n)^{2n}dx)$

$I_1 = -n(2n+1)I_1 + n(2n+1)I_2$

$= [1 + n(2n+1)]I_1 = n(2n+1)I_2$

$\therefore 1 + n(2n+1) = 1177 \Rightarrow 2n^2 + n + 1176 = 0$

$n = 24$

27. Let a curve y = y(x) pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and x(>3) be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $(\alpha, 6\sqrt{10})$ in the first quadrant, then α is equal to ________.

Sol. **6**

$\int_3^x f(x)dx = (y/x)^3$

differentiate

$f(x) = 3(y/x)^2 d(y/x)$

$\frac{y}{x}.x = 3\left(\frac{y}{x}\right)^2 d\left(\frac{y}{x}\right)$

$\Rightarrow \int x dx = 3\int \left(\frac{y}{x}\right) d\left(\frac{y}{x}\right)$

$\frac{x^2}{2} = 3\frac{(y/x)^2}{2} + C$

(3, 3)

$\frac{9}{2} = \frac{3}{2}(1) + C \Rightarrow C = 3$

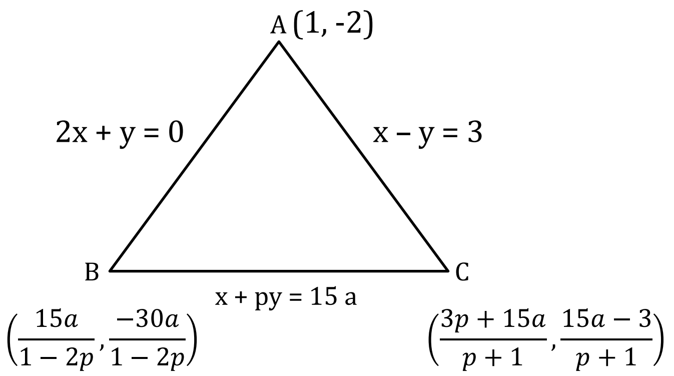
$\frac{x^2}{2} = \frac{3}{2}(y/x)^2 + 3$

$\Rightarrow$ put $y = 6\sqrt{10}$

$x = 6$

28. The equations of the sides AB, BC and CA of a triangle ABC are 2x + y = 0, x + py = 15a and x - y = 3 respectively. If its orthocentre is (2, a), $-\frac{1}{2} < a < 2$, then p is equal to _________.

Sol. **3**



orthocenter n = (2, a)

$m_{AH} = \frac{9+2}{1} = p$ ... (1)

$m_{BH} = -1 \Rightarrow 31a - 3ab = 15a + 4p - 2$ ... (2)

from (1) & (2)

a = 1

p = 3

29. Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in (0, a) and increasing in (a, 4). A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point (8a, 8a -1) but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then α + β is equal to ________.

Sol. **45**

$\frac{dy}{dx} = 4x - \frac{1}{x}$

$= \frac{(2x+1)(2x-1)}{x}$

↓ in (0, ½)

↑ (1/2, ∞)

$y^2 = 4ax$ $P(at^2, 2at)$

tangent $yt = x + at^2$

pass (4, 3)

$3t = 4 + \frac{1}{2}t^2$

$t^2 - 6t + 8 = 0 \Rightarrow t = 2, 4$

t = 2 $2y = x + 2$

pass (-2, 0)

$(-\frac{1}{a}, 0) = (-2, 0)$

∴ t = 2 not possible

t = 4 $4y = x + 8$ p(8, 4)

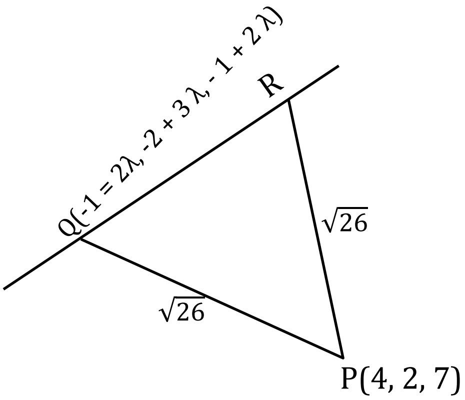
equation of normal at (8, 4) $y - 4 = -4(x - 8)$

$4x + y = 36$

$\frac{x}{9} + \frac{y}{36} - 1$ $\alpha + \beta = 45$

30. Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point P(4, 2, 7). then the square of the area of the triangle PQR is .....

Sol. **153**



$PQ^2 = 26$

$(4+1-2\lambda)^2 + (2+2-3\lambda)^2 + (7-1-2\lambda)^2 = 26$

$(5-2\lambda)^2 + (4-3\lambda)^2 + (6-2\lambda)^2 = 26$

$\Rightarrow 25 + 4\lambda^2 - 20\lambda + 16 + 9\lambda^2 - 24\lambda + 36 + 4\lambda^2 - 24\lambda = 26$

$\Rightarrow 17\lambda^2 - 68\lambda + 77 = 26$

$\Rightarrow 17\lambda^2 - 68\lambda + 51 = 0$

$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$

$\Rightarrow (\lambda - 3)(\lambda - 1) = 0$

$\Rightarrow \lambda = 1, 3$

$\therefore$ Q(1, 1, 3)

R(5, 7, 7)

P(4, 2, 3)

$\overrightarrow{PQ} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$

$\overrightarrow{PR} = \hat{\imath} + 5\hat{\jmath} + 0\hat{k}$

$A = \frac{1}{2}\left\|\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3 & 1 & 4\\ 1 & 5 & 0\end{vmatrix}\right\| = \frac{1}{2}\left|\hat{\imath}(0-20) - \hat{\jmath}(0-4) + \hat{k}(15-1)\right|$

$= \frac{1}{2}\left|-20\hat{\imath} + 4\hat{\jmath} + 14\hat{k}\right|$

$= \frac{1}{2}\sqrt{400 + 16 + 196}$

$= \frac{1}{2}\sqrt{612}$

$\therefore A^2 = \frac{1}{4}(612) = 153$