

PHYSICS
JEE-MAIN (July-Attempt)
25 July (Shift-2) Paper Solution

SECTION - A

1. In AM modulation, a signal is modulated on a carrier wave such that maximum and minimum amplitudes are found to be 6 V and 2 V respectively. The modulation index is :
 (A) 100% (B) 80% (C) 60% (D) 50%

Sol. **D**

$$m = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

$$= \frac{6-2}{6+2} = \frac{4}{8} = \frac{1}{2} \times 100\% = 50\%$$

2. The electric current in a circular coil of 2 turns produces a magnetic induction B_1 at its centre. The coil is unwound and is rewound into a circular coil of 5 turns and the same current produces a magnetic induction B_2 at its centre. The ratio of $\frac{B_2}{B_1}$ is

- (A) $\frac{5}{2}$ (B) $\frac{25}{4}$ (C) $\frac{5}{4}$ (D) $\frac{25}{2}$

Sol. **B**

$$B_1 = \frac{2\mu_0 i}{2R_1} \qquad 2 \times 2\pi R_1 = 5 \times 2\pi R_2$$

$$B_2 = \frac{5\mu_0 i R_1}{2R_2 R_2} = \frac{5}{2}$$

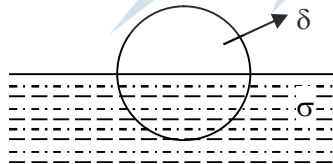
$$\frac{B_2}{B_1} = \frac{5 R_1}{2 R_2}$$

$$= \frac{5}{2} \times \frac{5}{2} = \frac{25}{4}$$

3. A drop of liquid of density ρ is floating half immersed in a liquid of density σ and surface tension $7.5 \times 10^{-4} \text{ Ncm}^{-1}$. The radius of drop in cm will be : ($g = 10 \text{ ms}^{-2}$)

- (A) $\frac{15}{\sqrt{(2\rho-\sigma)}}$ (B) $\frac{15}{\sqrt{(\rho-\sigma)}}$ (C) $\frac{3}{2\sqrt{(\rho-\sigma)}}$ (D) $\frac{3}{20\sqrt{(2\rho-\sigma)}}$

Sol. **A**



$$S \times 2\pi r = mg - F_b$$

$$= v\rho g - \frac{v\sigma}{2} g$$

$$S \times 2\pi r = v g \left(\frac{2\rho - \sigma}{2} \right)$$

$$S \times 2\pi r = \frac{4}{3} \pi r^3 g \left(\frac{2\rho - \sigma}{2} \right)$$

$$\frac{3s}{g(2\rho - \sigma)} = r^2$$

$$\frac{3(7.5 \times 10^{-4} \text{ kg m/sec}^2 \times \text{cm})}{(10 \text{ m/sec}^2)(2\rho - \sigma \times \text{kg m}^3)} = r^2 \qquad (1 \text{ m} = 100 \text{ cm})$$

$$\frac{3 \times 10^2}{2\sqrt{(2\rho - \sigma)}} = r$$

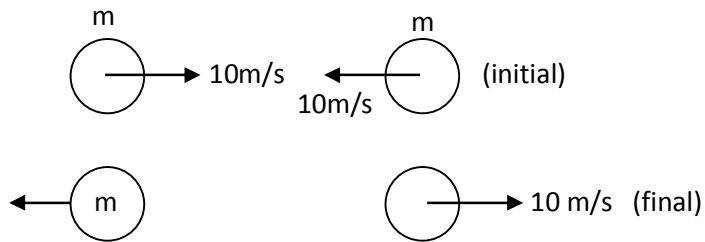
$$r = \frac{15}{\sqrt{2\rho - \sigma}}$$

4. Two billiard balls of mass 0.05 kg each moving in opposite directions with 10 ms^{-1} collide and rebound with the same speed. If the time duration of contact is $t = 0.005 \text{ s}$, then what is the force exerted on the ball due to each other?

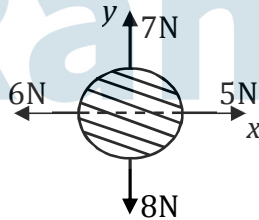
(A) 100 N (B) 200 N (C) 300 N (D) 400 N

Sol. **B**

$$\begin{aligned} \Delta \vec{p} &= \vec{p}_f - \vec{p}_i \\ &= (-) 10 \text{ m} - 10 \text{ m} \\ &= -20 \text{ m} \\ &= -20 \times 0.05 \\ &= -1 \text{ kg m/s} \\ \text{Force} &= \frac{\Delta p}{\Delta t} \\ &= \frac{1}{0.005} = \frac{1000}{5} \\ &= 200 \text{ N} \end{aligned}$$

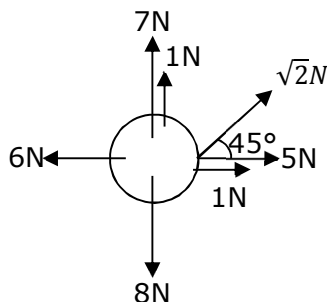


5. For a free body diagram shown in the figure, the four forces are applied in the 'x' and 'y' directions. What additional force must be applied and at what angle with positive x-axis so that the net acceleration of body is zero?



(A) $\sqrt{2} \text{ N}, 45^\circ$ (B) $\sqrt{2} \text{ N}, 135^\circ$ (C) $\frac{2}{\sqrt{3}} \text{ N}, 30^\circ$ (D) $2 \text{ N}, 45^\circ$

Sol. **A**



6. Capacitance of an isolated conducting sphere of radius R_1 becomes n times when it is enclosed by a concentric conducting sphere of radius R_2 connected to earth. The ratio of their radii $\left(\frac{R_2}{R_1}\right)$ is :

(A) $\frac{n}{n-1}$ (B) $\frac{2n}{2n+1}$ (C) $\frac{n+1}{n}$ (D) $\frac{2n+1}{n}$

Sol. A

$$C = 4\pi\epsilon_0 R_1$$

$$C' = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$$

$$C' = nC$$

$$\frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1} = n \cdot 4\pi\epsilon_0 R_1$$

$$\frac{R_2}{R_2 - R_1} = n$$

$$\frac{R_2/R_1}{R_2/R_1 - 1} = n$$

$$\frac{x}{x-1} = n$$

$$x = nx - n$$

$$n = nx - x$$

$$n = (n-1)x$$

$$x = \frac{n}{n-1}$$

7. The ratio of wavelengths of proton and deuteron accelerated by potential V_p and V_d is $1:\sqrt{2}$. Then, the ratio of V_p to V_d will be:

(A) 1 : 1 (B) $\sqrt{2} : 1$ (C) 2 : 1 (D) 4 : 1

Sol. D

$$eV_p = \frac{p^2}{2m} = \frac{h^2}{2m_p\lambda_p^2} \quad \dots(1)$$

$$eV_d = \frac{h^2}{2m_p\lambda_d^2} \quad \dots(2)$$

$$\frac{V_p}{V_d} = \left(\frac{\lambda_d}{\lambda_p}\right)^2 \times \frac{m_d}{m_p}$$

$$= \left(\frac{\sqrt{2}}{1}\right)^2 \times \frac{2m}{m} = 4$$

8. For an object placed at a distance 2.4 m from a lens, a sharp focused image is observed on a screen placed at a distance 12 cm from the lens. A glass plate of refractive index 1.5 and thickness 1 cm is introduced between lens and screen such that the glass plate plane faces parallel to the screen. By what distance should the object be shifted so that a sharp focused image is observed again on the screen?

(A) 0.8 m (B) 3.2 m (C) 1.2 m (D) 5.6 m

Sol. B

$$u = 240 \text{ cm } v = 12 \text{ cm } f = \frac{240}{21} = \frac{80}{7}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{12} + \frac{1}{240}$$

$$\text{shift} = t \left(1 - \frac{1}{\mu} \right)$$

$$\frac{1}{f} = -\frac{20+1}{240} = 1 \left(1 - \frac{1}{3/2} \right)$$

$$= \frac{3-2}{3} = \frac{1}{3}$$

$$v' = 12 - \frac{1}{3}$$

$$= \frac{35}{3} \text{ cm}$$

$$\Rightarrow \frac{1}{u'} = \frac{1}{v'} - \frac{1}{f}$$

$$= \frac{3}{35} - \frac{7}{80}$$

$$= \frac{240-245}{35 \times 80}$$

$$\frac{1}{u'} = \frac{5}{36 \times 80}$$

$$\Rightarrow u' = 560 \text{ cm}$$

$$u' = 5.6 \text{ m}$$

$$\text{shifted} = 5.6 - 2.4 = 3.2 \text{ m}$$

9. Light wave traveling in air along x-direction is given by

$E_y = 540 \sin \pi \times 10^4 (x - ct) \text{ Vm}^{-1}$. Then, the peak value of magnetic field of wave will be
(Given $c = 3 \times 10^8 \text{ ms}^{-1}$)

(A) $18 \times 10^{-7} \text{ T}$

(B) $54 \times 10^{-7} \text{ T}$

(C) $54 \times 10^{-8} \text{ T}$

(D) $18 \times 10^{-8} \text{ T}$

Sol. A

$$\frac{E_0}{B_0} = c$$

$$B_0 = \frac{E_0}{c} = \frac{540}{3 \times 10^8}$$

$$= 18 \times 10^{-7}$$

10. When you walk through a metal detector carrying a metal object in your pocket, it raises an alarm. This phenomenon works on:

(A) Electromagnetic induction

(B) Resonance in ac circuits

(C) Mutual induction in ac circuits

(D) Interference of electromagnetic waves

Sol. B

By theory

11. An electron with energy 0.1 keV moves at right angle to the earth's magnetic field of $1 \times 10^{-4} \text{ Wbm}^{-2}$. The frequency of revolution of the electron will be (Takes mass of electron = $9.0 \times 10^{-31} \text{ kg}$)
 (A) $1.6 \times 10^5 \text{ Hz}$ (B) $5.6 \times 10^5 \text{ Hz}$ (C) $2.8 \times 10^6 \text{ Hz}$ (D) $1.8 \times 10^6 \text{ Hz}$

Sol. C

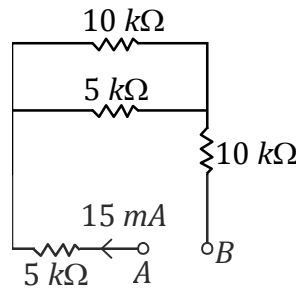
$$f = \frac{qB}{2\pi m} = \frac{1.6 \times 10^{-19} \times 1 \times 10^{-4}}{2 \times 3.14 \times 9 \times 10^{-31}}$$

$$= 0.028 \times 10^{-23+31}$$

$$= 0.028 \times 10^8$$

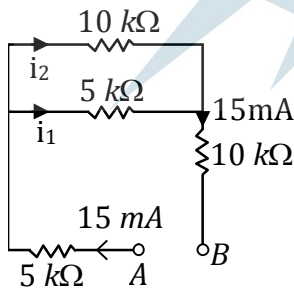
$$= 2.8 \times 10^6 \text{ Hz}$$

12. A current of 15 mA flows in the circuit as shown in figure. The value of potential difference between the points A and B will be :



- (A) 50 V (B) 75 V (C) 150 V (D) 275 V

Sol. D



$$\frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{10}{5} = \frac{2}{1}$$

$$i_1 = \frac{2}{3} \times 15 \text{ mA} = 10 \text{ mA}$$

$$V_A - 15 \times 5 - 10 \times 5 - 15 \times 10 = V_B$$

$$V_A - V_B = 275 \text{ V}$$

13. The length of a second pendulum at a height $h = 2R$ from earth surface will be : (Given $R = \text{Radius of earth}$ and acceleration due to gravity at the surface of earth, $g = \pi^2 \text{ ms}^{-2}$)

- (A) $\frac{2}{9} \text{ m}$ (B) $\frac{4}{9} \text{ m}$ (C) $\frac{8}{9} \text{ m}$ (D) $\frac{1}{9} \text{ m}$

Sol. D

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T^2 = \frac{4\pi^2 l}{g'} \quad \dots(1)$$

$$g' = g \left(\frac{R}{R+2R} \right)^2$$

$$g' = \frac{g}{9} \quad \dots(2)$$

$$T^2 = \frac{4\pi^2 l}{g/9}$$

$$\frac{2^2 \times g}{9 \times 4\pi^2} = 1$$

$$l = \frac{1}{9} m$$

14. Sound travels in a mixture of two moles of helium and n moles of hydrogen. If rms speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound, then the value of n will be :

(A) 1

(B) 2

(C) 3

(D) 4

Sol. B

$$V_{\text{rms}} = \sqrt{\frac{3RT}{m}}$$

$$V_{\text{sound}} = \sqrt{\frac{\gamma RT}{m}}$$

$$V_{\text{rms}} = \sqrt{2} V_{\text{sound}} \text{ (given)}$$

$$\sqrt{\frac{3RT}{m}} = \sqrt{2} \cdot \sqrt{\frac{\gamma RT}{m}}$$

$$\sqrt{3} = \sqrt{2} \sqrt{\gamma}$$

$$3 = 2\gamma$$

$$\gamma_{\text{max}} = \frac{3}{2}$$

$$\frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}} = \frac{3}{2}$$

$$\frac{2 \times \frac{5}{2}R + n \times \frac{7}{2}R}{2 \times \frac{3}{2}R + n \times \frac{5}{2}R} = \frac{3}{2}$$

$$\frac{10 + 7n}{6 + 5n} = \frac{3}{2}$$

$$20 + 14n = 18 + 15n$$

$$2 = n$$

15. Let η_1 is the efficiency of an engine at $T_1 = 447^\circ\text{C}$ and $T_2 = 147^\circ\text{C}$ while η_2 is the efficiency at $T_1 = 947^\circ\text{C}$ and $T_2 = 47^\circ\text{C}$. The ratio $\frac{\eta_1}{\eta_2}$ will be:

(A) 0.41 (B) 0.56 (C) 0.73 (D) 0.70

Sol. B

$$\begin{aligned} \eta_1 &= 1 - \frac{T_2}{T_1} & \eta_2 &= 1 - \frac{T'_2}{T'_1} \\ &= 1 - \frac{147+273}{447+273} & &= 1 - \frac{47+273}{947+273} \\ &= 1 - \frac{420}{720} & &= 1 - \frac{320}{1220} \\ &= 1 - \frac{7}{12} & &= 1 - \frac{16}{61} \\ &= \frac{5}{12} & &= \frac{45}{61} \\ \therefore \frac{\eta_1}{\eta_2} &= \frac{5 \times 61}{12 \times 45} = 0.56 \end{aligned}$$

16. An object is taken to a height above the surface of earth at a distance $\frac{5}{4}R$ from the centre of the earth. Where radius of earth, $R = 6400$ km. The percentage decrease in the weight of the object will be:

(A) 36% (B) 50% (C) 64% (D) 25%

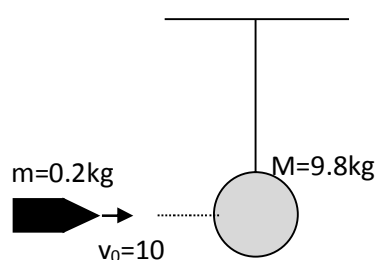
Sol. A

$$\begin{aligned} \frac{g'}{g} &= \left(\frac{R}{r}\right)^2 \\ g' &= g \left(\frac{R}{\frac{5}{4}R}\right)^2 \\ &= g \left(\frac{4R}{5R}\right)^2 \\ g' &= \frac{16}{25}g \\ \Delta g &= g - g' \\ &= g - \frac{16}{25}g \\ &= \frac{9}{25}g \\ \frac{\Delta g}{g} \times 100\% &= \frac{9}{25} \times 100 = 36\% \end{aligned}$$

17. A bag of sand of mass 9.8 kg is suspended by a rope. A bullet of 200 g travelling with speed 10 ms^{-1} gets embedded in it, then loss of kinetic energy will be :

(A) 4.9 J (B) 9.8 J (C) 14.7 J (D) 19.6 J

Sol. B



$$\begin{aligned}
p_i &= p_f \\
mv_0 &= (M + m) v \\
0.2 \times 10 &= (9.8 + 0.2) v \\
0.2 &= v \\
\text{Loss in KE} &= KE_i + KE_f \\
&= \frac{1}{2} \times 0.2 \times 10 \times 10 - \frac{1}{2} \times 10 \times 0.2 \times 0.2 \\
&= 10 - 0.2 = 9.8 \text{ J}
\end{aligned}$$

18. A ball is projected from the ground with a speed 15 ms^{-1} at an angle θ with horizontal so that its range and maximum height are equal, then 'tan θ ' will be equal to :

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 2 (D) 4

Sol. D

$$\begin{aligned}
\frac{R}{H} &= \left(\frac{2U^2 \sin\theta \cos\theta}{g} \right) \left(\frac{2g}{U^2 \sin^2\theta} \right) \\
1 &= \frac{4}{\tan\theta} \\
\tan\theta &= 4
\end{aligned}$$

19. The maximum error in the measurement of resistance, current and time for which current flows in an electrical circuit are 1%, 2% and 3% respectively. The maximum percentage error in the detection of the dissipated heat will be :

(A) 2 (B) 4 (C) 6 (D) 8

Sol. D

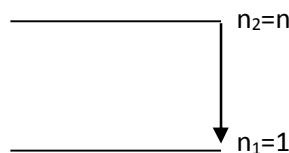
$$\begin{aligned}
H &= i^2 R t \\
\frac{\Delta H}{H} &= \frac{2\Delta i}{i} + \frac{\Delta R}{R} + \frac{\Delta t}{t} \\
&= 2 \times 2\% + 1\% + 3\% \\
&= 8\%
\end{aligned}$$

20. Hydrogen atom from excited state comes to the ground state by emitting a photon of wavelength λ . The value of principal quantum number 'n' of the excited state will be : (R : Rydberg constant)

(A) $\sqrt{\frac{\lambda R}{\lambda - 1}}$ (B) $\sqrt{\frac{\lambda R}{\lambda R - 1}}$ (C) $\sqrt{\frac{\lambda}{\lambda R - 1}}$ (D) $\sqrt{\frac{\lambda R^2}{\lambda R - 1}}$

Sol. B

$$\begin{aligned}
n_2 &= n, n_1 = 1 \\
\frac{1}{\lambda} &= RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \\
\frac{1}{\lambda} &= R(1) \left(\frac{1}{1} - \frac{1}{n^2} \right) \\
\frac{1}{\lambda} &= R \left[\frac{n^2 - 1}{n^2} \right] \\
\Rightarrow n^2 &= \lambda R n^2 - \lambda R \\
\lambda R &= n^2 (\lambda R - 1) \\
n &= \sqrt{\frac{\lambda R}{\lambda R - 1}}
\end{aligned}$$



SECTION - B

21. A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is _____ ms^{-2} at a point where its velocity is 20 ms^{-1} .

Sol. 100

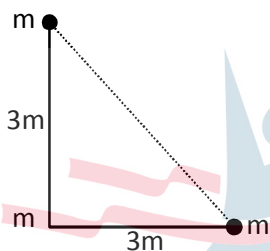
$$\frac{dv}{dx} = 5$$

$$a = v \frac{dv}{dx}$$

$$a = (20)(5) = 100 \text{ m/sec}$$

22. Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of the system will be $\sqrt{x} \text{ m}$. the value of x is _____.

Sol. 2



$$\vec{R}_{CM} = \frac{m(3\hat{i}) + m(3\hat{j})}{3m}$$

$$\vec{R}_{CM} = \hat{i} + \hat{j}$$

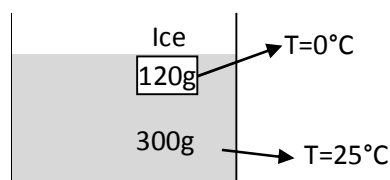
$$|\vec{R}| = \sqrt{2}$$

$$x = 2$$

23. A block of ice of mass 120 g at temperature 0°C is put in 300 g of water at 25°C . The $x \text{ g}$ of ice melts as the temperature of the water reaches 0°C . The value of x is _____.

[Use specific heat capacity of water = $4200 \text{ Jkg}^{-1}\text{K}^{-1}$, Latent heat of ice = $3.5 \times 10^5 \text{ Jkg}^{-1}$]

Sol. 90



L = Latent heat of ice

Heat absorbed = Heat transmitted

$$m \times L = m C D T$$

m → mass of ice which has been melted
 L → latent heat of fusion
 m → mass of water
 C → specific heat capacity
 D → change in temperature
 T → charge into temperature

$$(m)_{\text{melted}} \times (3.5 \times 10^5) = 300 \times 4200 \times (25)$$

$$(m_{\text{ice}})_{\text{melted}} = 90$$

24. $\frac{x}{x+4}$ is the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its

- (i) Third permitted energy level to the second level and
 (ii) The highest permitted energy level to the second permitted level.

The value of x will be _____.

Sol. 5

$$E = -13.6 \times \frac{z^2}{n^2}$$

$$\text{_____ } h=3 \quad = -1.51 \text{ eV}$$

$$\text{_____ } h=2 \quad = -3.4 \text{ eV}$$

$$\text{_____ } h=1 \quad = -13.6 \text{ eV}$$

$$E_3 - E_2 = 1.51 + 3.4 = 1.89 \text{ eV}$$

$$E_\infty - E_2 = 0 + 3.4 = 3.4 \text{ eV}$$

$$\frac{E_3 - E_2}{E_\infty - E_2} = \frac{1.89}{3.4} = \frac{x}{x+4}$$

$$\Rightarrow 1.89x + 7.56 = 3.4x$$

$$7.56 = 1.51x$$

$$x = 5$$

25. In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 36 cm length of wire. This cell is now replaced by another cell of emf 1.80 V. The difference in balancing length of potentiometer wire in above conditions will be _____ cm.

Sol. 18

$$E_1 \propto l_1$$

$$E_2 \propto l_2$$

(assumed balanced point at length of wire)

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

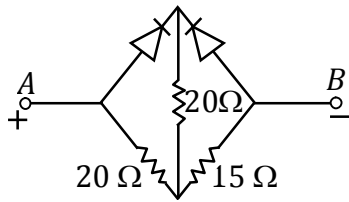
$$\frac{1.20}{1.80} = \frac{36}{l_2} \text{ cm}$$

$$\Rightarrow l_2 = 36 \times \frac{1.80}{1.20} = 54 \text{ cm}$$

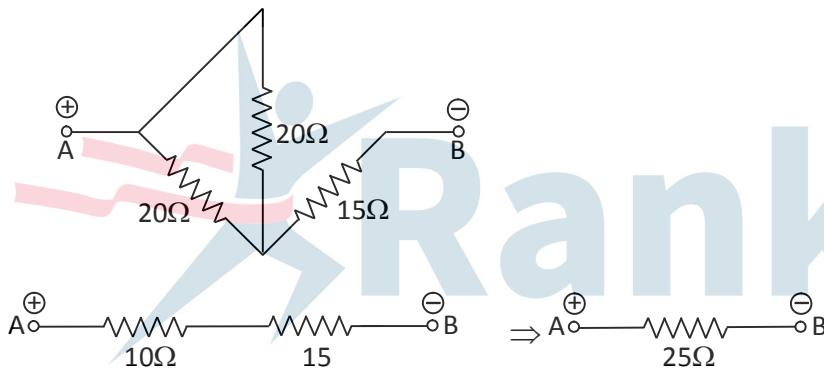
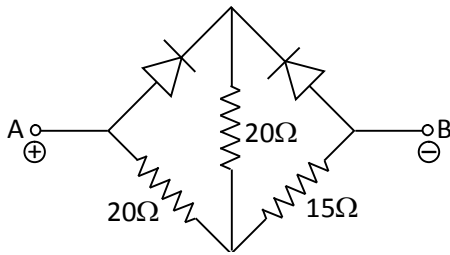
The difference in balancing length of potentiometer

$$l_2 - l_1 = 54 - 36 = 18 \text{ cm}$$

26. Two ideal diodes are connected in the network as shown in figure. The equivalent resistance between A and B is _____ Ω .



Sol. 25



27. Two waves executing simple harmonic motions travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is _____ (degree).

Sol. 60

$$R = \sqrt{A^2 + A^2 + 2A^2 \cos \Delta \phi}$$

$$R = \sqrt{3}A = \sqrt{A^2 + A^2 + 2A^2 \cos \Delta \phi}$$

$$A_1 = A_2 = A$$

$$\sqrt{3}A = \sqrt{2A^2 + 2A^2 \cos \Delta \phi}$$

$$\Rightarrow (\cos \theta = 2 \cos 2\theta - 1)$$

$$\sqrt{3}A = \sqrt{2A^2 \left(2 \cos^2 \frac{\Delta \phi}{2} \right)}$$

$$\sqrt{3}A = 2A \cos \frac{\Delta\phi}{2}$$

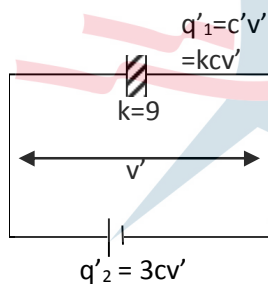
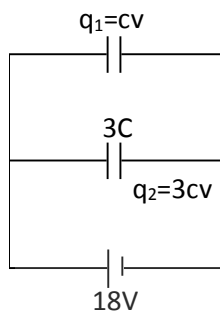
$$\cos \frac{\Delta\phi}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{\Delta\phi}{2} = 30^\circ$$

$$\Delta\phi = 60^\circ$$

28. Two parallel plate capacitors of capacity C and $3C$ are connected in parallel combination and charged to a potential difference 18 V . The battery is then disconnected and the space between the plates of the capacitor of capacity C is completely filled with a material of dielectric constant 9 . The final potential difference across the combination of capacitors will be _____ V .

Sol. 6



due to charge conservation

$$q_1 + q_2 = q'_1 + q'_2$$

$$CV + 3CV = C'v' + 3Cv'$$

$$4Cv = v'(kC + 3C), \quad k = 9$$

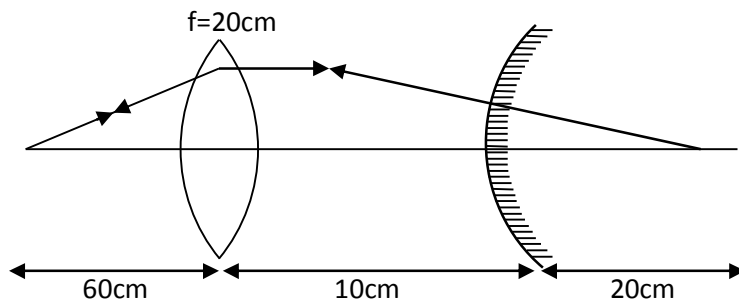
$$4Cv = v'(12C)$$

$$v' = \frac{v}{3}$$

$$v' = \frac{18}{3} = 6$$

29. A convex lens of focal length 20 cm is placed in front of a convex mirror with principle axis coinciding each other. The distance between the lens and mirror is 10 cm . A point object is placed on principal axis at a distance of 60 cm from the convex lens. The image formed by combination coincides the object itself. The focal length of the convex mirror is _____ cm .

Sol. 10



lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, $U = -60$, $f = 20$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{U} \Rightarrow v = \frac{fU}{U+f} = \frac{(20)(-60)}{-60+20} = \frac{(20)(-60)}{-40} = 30 \text{ cm}$$

$$v = 30 \text{ cm}$$

According to question final image formed at combination coincides the object itself, so this is happened when ray incident on the centre of curvatuer of convex mirror.

So, $U = 20 \text{ cm}$ and V also be 20 cm , radius of curvature of mirror is

$$R = 20 \text{ cm}$$

$$f = R/2 = 10 \text{ cm}$$

30. Magnetic flux (in weber) in a closed circuit of resistance 20Ω varies with time $t(\text{s})$ as $\phi = 8t^2 - 9t + 5$. The magnitude of the induced current at $t = 0.25 \text{ s}$ will be _____ mA.

Sol. 250

$$\phi(t) = 8t^2 - 9t + 5$$

$$\frac{d\phi(t)}{dt} = 16t - 9$$

$$e = \left| -\frac{d\phi(t)}{dt} \right| = | -16(0.25) + 9 | \quad t = 0.25$$

$$e = 5 \text{ v}$$

$$e = IR$$

$$5\text{v} = I(20\Omega) \Rightarrow I = \frac{5}{20} = \frac{1}{4} = 0.25 \text{ Amp.}$$

$$I = 250 \text{ mA}$$