MATHEMATICS
JEE-MAIN (July-Attempt)
25 July (Shift-1) Paper Solution

SECTION - A

1. The total number of functions, $f : \{1, 2, 3, 4,\} \to \{1, 2, 3, 4, 5, 6\}$ such that $f(1) + f(2) = f(3)$, is equal to :

(A) 60 (B) 90 (C) 108 (D) 126

Ans. (B)

Sol. (I) $f(1)\ \epsilon\ \{1\} \to f(2)\ \epsilon\ \{1,2,3,4,5\}, f(3)$correpsponding & $f(6)$ all 6 $\Rightarrow 1 \times 5 \times 1 \times 6 = 30$

(II) $f(1)\ \epsilon\ \{2\} \to f(2)\ \epsilon\ \{1,2,3,4\}, f(3)$correpsponding & $f(6)$ all 6 $\Rightarrow 1 \times 4 \times 1 \times 6 = 24$

(III) $f(1)\ \epsilon\ \{3\} \to f(2)\ \epsilon\ \{1,2,3\}, f(3)$correpsponding & $f(6)$ all 6 $\Rightarrow 1 \times 3 \times 1 \times 6 = 18$

(IV) $f(1)\ \epsilon\ \{4\} \to f(2)\ \epsilon\ \{1\}, f(3)$correpsponding & $f(6)$ all 6 $\Rightarrow 1 \times 1 \times 1 \times 6 = 6$

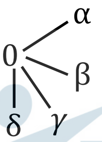
Hence Total functions $= 30 + 24 + 18 + 12 + 6 = 90$ Ans. B

2. If α, β γ, δ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to :

(A) –4 (B) –1 (C) 1 (D) 4

Ans. (B)

Sol. $x^4 + x^3 + x^2 + x + 1 = 0$



$(x - 1)(x^4 + x^3 + x^2 + x + 1) = (x - 1) \cdot 0$

$x^5 - 1 = 0 \Rightarrow \boxed{x^5 = 1} \Rightarrow \alpha^5 = \beta^5 = \gamma^5 = \delta^5 = 1$

Now $S = \alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = \Sigma\alpha^{2021}$

$S = \Sigma\alpha(\alpha^5)^{400}$

$S = \Sigma\alpha(1)$

$S = \Sigma\alpha$

$S = -1$

3. For n ∈ N, let $S_n = \left\{z \in C\ ;\ |z - 3 + 2i| = \frac{n}{4}\right\}$ and $T_n = \left\{z \in C\ ;\ |z - 2 + 3i| = \frac{1}{n}\right\}$. Then the number of elements in the set $\{n \in N\ ;\ S_n \cap T_n = \phi\}$ is :

(A) 0 (B) 2 (C) 3 (D) 4

Ans. (D)

Sol. Sn :

$|z - 3 + 2i| = \frac{n}{4}$

$S_n: (x - 3)^2 + (y + 2)^2 = \left(\frac{n}{4}\right)^2$ &

Now $S_1 \cap S_2 = \phi$

$C_1C_2 > r_1 + r_2$

Tn:

$|z - 2 + 3i| = \frac{1}{n}$

$T_n: (x - 2)^2 + (y + 3)^2 = \left(\frac{1}{n}\right)^2$

Case-II : $C_1C_2 < |r_1 - r_2|$

$\sqrt{(3 - 2)^2 + (-2 + 3)^2} < \left|\frac{n}{4} - \frac{1}{n}\right|$

$\sqrt{(3-2)^2+(-2+3)^2} > \frac{n}{4}+\frac{1}{n}$

$\left(\frac{n}{4}+\frac{1}{n}\right)^2 < 2$

$\Rightarrow N\epsilon\{1,2,3,4\}$

$\sqrt{2} < \left|\frac{n}{4} - \frac{1}{n}\right|$

⇒ n has a solution

Ans. (D) (according to NTA) But Bonus

4. The number of $\theta \in (0, 4\pi)$ for which the system of linear equations

$3(\sin 3\theta)x - y + z = 2$

$3(\cos 2\theta)x + 4y + 3z = 3$

$6x + 7y + 7z = 9$ has no solution, is :

(A) 6 (B) 7 (C) 8 (D) 9

Ans. (B)

Sol. For no solution

$$\Delta = \begin{vmatrix} 3sin3\theta & -1 & 1 \\ 3cos2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix} = 0$$

$3sin3\theta(7) + 1(21cos2\theta - 18) + 1(21cos2\theta - 24) = 0$

$21\,[sin3\theta + 2cos2\theta] = 42$

$sin3\theta + 2cos2\theta = 2$

$3sin\theta - 4sin3\theta + 2 - 4\sin^2\theta = 2$

$4\sin^3\theta + 4\sin^2\theta - 3sin\theta = 0$

$sin\theta = 0\ \&4\sin^2\theta + 4sin + 1 = 4$

$sin\theta = 0\&\ (2sin\theta + 1)^2 = 4$

$\theta\ \epsilon\ \{\pi, 2\pi, 3\pi\}$ $2sin\theta + 1 = 2$ or $2sin\theta + 1 = -2$

$sin\theta = \frac{1}{2}$ or $sin\theta = \frac{-3}{2}$

$\theta\ \epsilon\ \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}\frac{17\pi}{6}\}$ No sol.

and $\Delta_3 = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 4 & 3 \\ 9 & 7 & 7 \end{vmatrix} \Rightarrow \Delta_3 \neq 0$

$= 3(7) + 1(-6) + 1(-15) \neq 0$

Hence total 7 vales of θ are possible

5. If $\lim\limits_{n\to\infty}\left(\sqrt{n^2 - n - 1} + n\alpha + \beta\right) = 0$, then $8(\alpha + \beta)$ is equal to :

(A) 4 (B) – 8 (C) – 4 (D) 8

Ans. (C)

Sol. $\lim\limits_{n\to\infty}\left(\sqrt{n^2 - n - 1} + n\alpha + \beta\right) = 0$

Put $n = \frac{1}{t}$

$$\lim_{t\to 0}\left(\sqrt{\frac{1}{t^2}-\frac{1}{t}-1}+\frac{\alpha}{t}+\beta\right)=0$$

$$\lim_{t\to 0}\frac{(\sqrt{1-t-t^2}+\alpha+\beta t}{t}=0$$

$1+\alpha=0\Rightarrow\boxed{\alpha=-1}$

Diff use L-Hospital rule

$$\lim_{t\to 0}\frac{-1-2t}{2\sqrt{1-t-t^2}}+\beta=0$$

$-\frac{1}{2}+\beta=0\ \Rightarrow\boxed{\beta=\frac{1}{2}}$

Now $8(\alpha\ +\beta\ )$

$=8\left(-1+\frac{1}{2}\right)=\ \mathbf{-4}$

6. If the absolute maximum value of the function $f(x) = (x^2 - 2x + 7)\ e^{(4x^3-12x^2-180x+31)}$ in the interval [–3, 0] is $f(\alpha)$, then

(A) $\alpha = 0$ (B) $\alpha = -3$ (C) $\alpha \in (-1, 0)$ (D) $\alpha \in (-3, -1]$

Ans. (B)

Sol. $f(x)=\ (x^2-2x+7)e^{(4x^3-12x^2-180x+31)}$

$f'(x)=(2x-2)e^{(4x^3-12x^2-180x-31)}+(x^2-2x+7)e^{\left(4x^3-12x^2-180x+31\right)}(12x^2-24x-180)$

$f'(x)=e^{\left(4x^3-12x^2-180x+31\right)}\left[(2x-2)+\ (x^2-2x+7)12.(x^2-2x-15)\right]$

$f'(x)=e^{\left(4x^3-12x^2-180x+31\right)}\left[2x-2+\ 12(x^2-2x+7)(x-5)(x+3)\right]$

Now $f'(x)<\ 0\ \forall\ \epsilon\ [-3,0]$

$\Rightarrow f'(x)<0\ \forall\ x\ \epsilon\ [-3,0]$

$\Rightarrow f(x)dec.\ \ \forall\ x\ \epsilon\ [-3,0]$

f(x)max. at $\boxed{x=-3}$

7. The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point P(–2, 0) and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of y(x) is :

(A) $\frac{27}{4}$ (B) $\frac{29}{4}$ (C) $\frac{37}{4}$ (D) $\frac{9}{2}$

Ans. (A)

Sol. $f(x)=ax^3+bx^2+cx+5$

$$\left.\begin{aligned}f'(-2)=0\Rightarrow 12a-4b+c=0\\ f'(-2)=0\Rightarrow -8a+4b-2c+5=0\end{aligned}\right]\ 4a-c+5=0$$

$$\boxed{a = -\frac{1}{2}}$$

$Q : (0,5) \Rightarrow f'(0) = 3 \;\; \& \, b = -\frac{3}{4}$

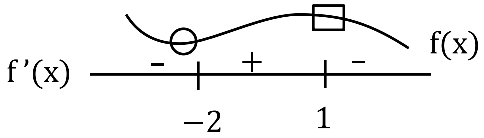
$\Rightarrow \boxed{c = 3}$

Hence $f(x) = -\frac{x^3}{2} - \frac{3x^2}{4} + 3x + 5$

$f'(x) = -\frac{3x^3}{2} - \frac{3x}{2} + 3$

$f'(x) = -\frac{3}{2}[x^2 + x - 2]$

$f'(x) = -\frac{3}{2}[x + 2][x - 1]$



$f(x)_{max} \;\; at \;\; x = 1$

$f_{max} = f(1) = -\frac{1}{2} - \frac{3}{4} + 3 + 5$

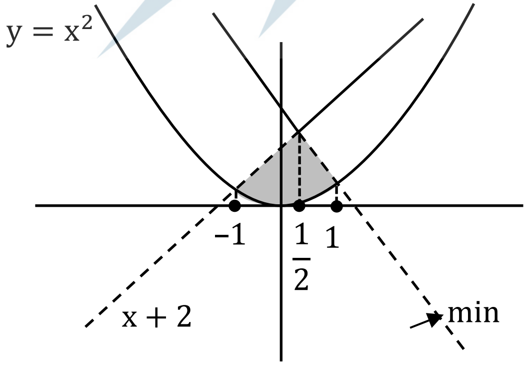
$= \frac{32-3-2}{4} = \boxed{\frac{27}{4}}$

8. The area of the region given by A = {(x, y) : $x^2 \le y \le$ min {x + 2, 4 – 3x}} is :

(A) $\frac{31}{8}$ (B) $\frac{17}{6}$ (C) $\frac{19}{6}$ (D) $\frac{27}{8}$

Ans. (B)

Sol. $x^2 \le y \le \min\{x + 2, 4 - 3x\}$



Area $= \int_{-1}^{1/2}(x + 2 - x^2)dx + \int_{1/2}^{1}(4 - 3x - x^2)dx$

Area $= \frac{x^2}{2} + 2x\Big|_{-1}^{\frac{1}{2}} + 4x\frac{-3x^2}{2}\Big|_{\frac{1}{2}}^{1} - \frac{x^3}{3}\Big|_{-1}^{1}$

$= \left(\frac{1}{8} + 1\right) - \left(\frac{1}{2} - 2\right) + \left(4 - \frac{3}{2}\right) - \left(2 - \frac{3}{8}\right) - \left(\frac{1}{3} - \left(-\frac{1}{3}\right)\right)$

$\frac{1}{8} + 1 + \frac{3}{2} + 2 - \frac{3}{2} + \frac{3}{8} - \frac{2}{3}$

$= \frac{1}{8} + 3 + \frac{3}{8} - \frac{2}{3}$

$= \frac{1}{2} + 3 - \frac{2}{3}$

$= \frac{7}{2} - \frac{2}{3}$

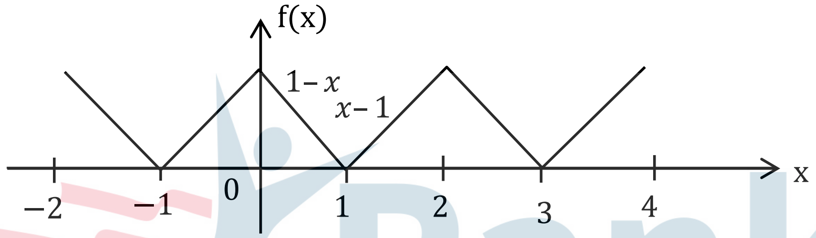
$\frac{(21-4)}{6} = \frac{17}{6}$

9. For any real number x, let [x] denote the Largest integer less than equal to x. Let f be a real valued function defined on the interval [– 10, 10] by f(x) = $\begin{cases} x - [x], if\ [x]\ is\ odd \\ 1 + [x] - x,\ if\ [x]\ is\ even. \end{cases}$ Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x) cos\,\pi x\, dx\, is$:

(A) 4 (B) 2 (C) 1 (D) 0

Ans. (A)

Sol. $f(x) = \left[\begin{array}{lll} \{x\} & [x] & \text{odd} \\ 1 - \{x\} & [x] & \text{even} \end{array}\right.$



$f(x)$ is periodic with perod 2 and even ins

Hence $I = \frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x$

↓ Jack property (P–7) & (P–4)

$I = \frac{\pi^2}{20} 2.5 \int_0^2 f(x)\cos\pi x$

$I = \pi\left[\int_0^1 (1-x)cos\pi x dx + \int_1^2 (x-1)cos\pi x dx\right]$

Using by prop.

$I = \pi^2 \cdot \frac{4}{\pi^2} \quad \Rightarrow \boxed{I = 4}$

10. The slope of the tangent to a curve C : y = y(x) at any point (x, y) on it is $\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$, then e^{α} is equal to :

(A) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$ (B) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$ (C) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ (D) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

Ans. (B)

Sol. $\frac{dy}{dx} = \frac{2e^{2x}-6x^{-x}+9}{2+9e^{-2x}}$

$$\int_{\frac{1}{2}+\frac{\pi}{2\sqrt{2}}}^{\frac{1}{2}e^{2\alpha}} dy = \int_0^{\alpha} \frac{2e^{2x} - 6e^{-x}+9}{2+9e^{-2x}} dx$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \int_0^{\alpha} \frac{(2e^{2x}-6e^{-x}+9)}{(2+9e^{-2x})} \frac{e^{2x}}{e^{2x}} dx$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \int_0^{\alpha} \frac{2e^{4x}+9e^{2\pi}-6e^{x}}{2e^{2x}+9} dx$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \int_1^{e^{\alpha}} \frac{2t^3+9t-6}{2t^2+9} dt$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \int_1^{e^{\alpha}} \left(t - \frac{6}{2t^2+9}\right) dt$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \frac{t^2}{2} \int_1^{e^{\alpha}} -3 \int_1^{e^{\alpha}} \frac{dt}{t^2+\frac{9}{2}}$$

$$\frac{e^{2\alpha}-1}{2} - \frac{\pi}{2\sqrt{2}} = \frac{e^{2\alpha}-1}{2} - 3(\frac{\sqrt{2}}{3}\tan^{-1}\left(\frac{\sqrt{2}}{3}\right))$$

$$\frac{\pi}{2\sqrt{2}} = \sqrt{2}\left[\tan^{-1}(\frac{\sqrt{2}e^{\alpha}}{3}) - \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)\right]$$

$$\frac{\pi}{4} = \tan^{-1}\left(\frac{(\sqrt{2}e^{\alpha}-\sqrt{2})3}{9+2e^{\alpha}}\right)$$

$$\frac{3\sqrt{2}(e^{\alpha}-1)}{2e^{\alpha}+9} = 1 \Rightarrow (e^{\alpha}-1)3\sqrt{2} = 2e^{\alpha}+9$$

$$e^{\alpha}\left(3\sqrt{2}-2\right) = 9+3\sqrt{2}$$

$$e^{\alpha} = \frac{3\left(3+\sqrt{2}\right)}{\sqrt{2}(3-\sqrt{2})}$$

11. The general solution of the differential equation $(x - y^2)dx + y(5x + y^2)dy = 0$ is :

(A) $(y^2 + x)^4 = C|(y^2 + 2x)^3|$

(B) $(y^2 + 2x)^4 = C|(y^2 + x)^3|$

(C) $|(y^2 + x)^3| = C\,(2y^2 + x)^4$

(D) $|(y^2 + 2x)^3| = C(2y^2 + x)^4$

Ans. (A)

Sol. $(x - y^2)\,dx + y(5x+y^2)dy = 0$

$y\frac{dy}{dx} = \frac{y^2 - x}{y^2+5x}$ Let $y^2 = t$

$\frac{1}{2}\frac{dt}{dx} = \frac{t - x}{t + 5x}$ | HDE

$t = vx$

$v + x\frac{dv}{dx} = 2\frac{(v-1)}{(v+5)}$

$x\frac{dv}{dx} = \frac{2v - 2 - v^2 - 5v}{v+5}$

$\int \frac{(v+5)dv}{v^2+3v+2} = -\int \frac{1}{x}dx$

$\int \left(\frac{4}{v+1} - \frac{3}{v+2}\right)dv = -\int \frac{dx}{x}$

$4\,\ell_n(v+1) - 3\ell_n\,(v+2) = -\,l_n x + C$

$\ell n\left(\frac{(\nu+1)^4}{(\nu+2)^3}\cdot x\right) = C$

$(y^2 + x)^4 = C(y^2 + 2x)^3$ **Ans. A**

12. A line, with the slope greater than one, passes through the point A(4, 3) and intersects the line x – y – 2 = 0 at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line :

(A) 2x + y = 9 (B) 3x – 2y = 7 (C) x + 2y = 6 (D) 2x – 3y = 3

Ans. (C)

Sol. m > 1 & A : (4, 3)

L : y – 3 = m(x – 4) & L_1 : x – y = 2

Let - m_L = m = tan θ & B on L_1

$\Rightarrow$ B : (λ , λ – 2)

Given AB = $\frac{\sqrt{29}}{3}$ $\Rightarrow$ $\sqrt{(\lambda-4)^2+(\lambda-2-3)^2} = \frac{\sqrt{29}}{3}$

$\Rightarrow$ $(\lambda - 4)^2 + (\lambda - 5)^2 = \frac{29}{3}$

$\Rightarrow$ $\lambda = \frac{51}{9}$ $\lambda = \frac{10}{3}$

$\Rightarrow$ B : $\left(\frac{51}{9}, \frac{33}{9}\right)$ or $\left(\frac{10}{3}, \frac{4}{3}\right)$

Now check options **Ans. C**

13. Let the locus of the centre (α, β), β > 0, of the circle which touches the circle $x^2 + (y - 1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line y = 4 is :

(A) $\frac{32\sqrt{2}}{3}$ (B) $\frac{40\sqrt{2}}{3}$ (C) $\frac{64}{3}$ (D) $\frac{32}{3}$

Ans. (C)

Sol. C : (α , β) & radius = r

S : $(x - \alpha)^2 + (y - \beta)^2 = r^2$

S touches externally S_1

$\Rightarrow$ $CC_1 = r + r_1$

$\alpha^2 + (\beta - 1)^2 = (1 + r)^2$ ... (1)

S touches x-axis

$\Rightarrow$ y coordinates of center = radius of circle

$\Rightarrow \beta = r$

Put in (1)

$\alpha^2 + \beta^2 - 2\beta + x = \alpha + \beta^2 + 2\beta$

$\alpha^2 = 4\beta$

$\Rightarrow$ Locus in $\boxed{x^2 = 4y}$

Area $= \frac{2}{3}$ [PQRS] $= \frac{2}{3}$ [8 × 4] $= \mathbf{\frac{64}{3}}$ **Ans. C**

14. Let P be the plane containing the straight line $\frac{x-3}{9}=\frac{y+4}{-1}=\frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and $\frac{x}{3}=\frac{y}{7}=\frac{z}{8}$. If d is the distance of P from the point (2, – 5, 11), then d^2 is equal to :

(A) $\frac{147}{2}$ (B) 96 (C) $\frac{32}{3}$ (D) 54

Ans. (C)

Sol. Point on plane is (3, –4, 7)

$n_p = \vec{V}_{L1} \times (\vec{V}_{L2} \times \vec{V}_{L3})$

$= (9\hat{\imath} - \hat{\jmath} - 5\hat{k}) \times ((2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) \times (3\hat{\imath} + 7\hat{\jmath} + 8\hat{k}))$

$= \langle 9, -1, -5\rangle \times \langle -11, -1, 5\rangle$

$$\vec{n}_p = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 9 & -1 & -5 \\ -11 & -1 & 5 \end{vmatrix} = \langle -10, 10, -20\rangle$$

$\vec{n}_p = \langle 1, -1, 2\rangle$

Eq. of Plane : $1(x - 3) - 1(y + 4) + 2(z - 7) = 0$

$P : x - y + 2z = 21$

Now d of (2, –5, 11) from plane

$$d = \left(\frac{2+5+22-21}{\sqrt{1+1+4}}\right)$$

$$d = \left(\frac{8}{\sqrt{6}}\right)$$

$d = \frac{64}{6} = \frac{32}{3}$ = **Ans.C (NTA Ans. D)**

15. Let ABC be a triangle such that $\overrightarrow{BC}=\vec{a}, \overrightarrow{CA}=\vec{b}, \overrightarrow{AB}=\vec{c}, |\vec{a}| = 6\sqrt{2}, |\vec{b}| = 2\sqrt{3}$ and $\vec{b}\cdot\vec{c} = 12$. Consider the statements :

(S1) : $|(\vec{a}\times\vec{b}) + (\vec{c}\times\vec{b})| - |\vec{c}| = 6(2\sqrt{2}-1)$

(S2) : $\angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Then

(A) both (S1) and (S2) are true (B) only (S1) is true

(C) only (S2) is true (D) both (S1) and (S2) are false

Ans. (C)

Sol. $\vec{a} + \vec{b} + \vec{c} = 0$

$-\vec{a} = \vec{b} + \vec{c}$

$|\vec{a}|^2 = |\vec{b} + \vec{c}|^2$

$a^2 = b^2 + c^2 + 2\,\vec{b}\cdot\vec{c}$

$c^2 = 72 - 12 - 24 = 36$

$|\vec{c}| = 6$

S-I : $|(\vec{a}\times\vec{b}) + (\bar{c}\times\vec{b})| - |\vec{c}|$

$= |(\vec{a} + \vec{c}) \times \vec{b}| - |\vec{c}|$

$= |- \vec{b} \times \vec{b}| - |\vec{c}|$

$= - |\vec{c}|$

$= - 6 \quad \neq \quad 6(2\sqrt{2} - 1)$ **S-I False**

S-II : $\bar{a} + \bar{b} + \bar{c} = 0$

$\bar{a} + \bar{b} = - \bar{c}$

$= |\bar{a} + \bar{b}|^2 - |\bar{c}|^2$

$= |72 + 12 + 24\sqrt{6}\cos(\pi - \theta)|$

$= - \cos\theta = \frac{-2}{\sqrt{6}} \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$ **S-II True**

Ans. C **(NTA Ans. D)**

16. If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is :

(A) $\frac{33}{2^{32}}$ (B) $\frac{33}{2^{29}}$ (C) $\frac{33}{2^{28}}$ (D) $\frac{33}{2^{27}}$

Ans. (C)

Sol. Mean = np variance = npq

$np + npq = 24$ & $np \cdot npq = 128$

$$\left.\begin{array}{r} np(1+q) = 24 \\ (np)^2 q = 128 \end{array}\right] \Rightarrow \frac{(1+q)^2}{q} = \frac{(24)^2}{128}$$

$1 + q^2 + 2q = q\left[\frac{24 \times 24}{128}\right]$

$1 + q^2 + 2q = \frac{9q}{2}$

$2q^2 - 5q + 2 = 0$

$$(2q-1)(q-2) = \left[\begin{array}{l} q = \frac{1}{2} \\ q = 2 \end{array}\right.$$

Now $np = \frac{24}{1 + \frac{1}{2}} = \boxed{np = 16}$ & $p + q = 1$

$\boxed{n = 32}$ $\boxed{p = \frac{1}{2}}$

Prob. (1 (suvn)) + 2(suvn)

$= {}^nC_1 \cdot (p)^1 (q)^{n-1} + {}^nC_2 \cdot (p)^2 (q)^{n-2}$

$= npq^{n-1} + \frac{n(n-1)P^2}{2} q^{n-2}$

$= 32 \cdot \frac{1}{2}\left(\frac{1}{2}\right)^{31} + \frac{32 \cdot 31}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{30}$

$= \frac{33}{2^{28}}$ Ans. C

17. If the numbers appeared on the two throws of a fair six faced die are α and, β, then the probability that $x^2 + \alpha x + \beta > 0$, for all $x \in R$, is :

(A) $\frac{17}{36}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{19}{36}$

Ans. (A)

Sol. $x^2 + \alpha x + \beta > 0 \quad \forall \quad x \in R$

$\Rightarrow \quad D < 0$

$\Rightarrow \quad \alpha^2 - 4\beta < 0$

$\Rightarrow \quad \alpha^2 < 4\beta$

n(S) : 6 × 6

n(A) :

α	β
1	1, 2, 3, 4, 5, 6
2	2, 3, 4, 5, 6
3	3, 4, 5, 6
4	5, 6

n(A) = 17

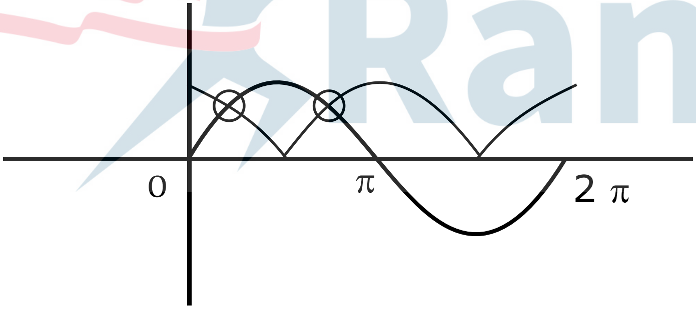
$p(A) = \frac{n(A)}{n(S)} = \frac{17}{36}$ **Ans. A**

18. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is :

(A) 4 (B) 6 (C) 8 (D) 12

Ans. (C)

Sol. $|\cos x| = \sin x$



Total solu. : $\boxed{8}$ **Ans. C**

19. A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that QR = 15 m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is :

(A) $5(2\sqrt{3}+3)m$ (B) $5(\sqrt{3}+3)m$ (C) $10(\sqrt{3}+1)m$ (D) $10(2\sqrt{3}+1)m$

Ans. (A)

Sol. In Δ PQA

$\text{Tan } 45° = \frac{PQ}{QA}$

QA = (h + 15) cot 75° ... (1)

Now   In Δ ARQ $\tan 60° = \frac{RQ}{QA}$

$QA = 15 \cdot \cot 60°$ ... (2)

From Eq. (1) & (2)

$(h + 15) \cot 75° = 15 \cot 60°$

$h = \frac{15(\cot 60° - \cot 75°)}{\cot 75°}$

$h = 15\, \frac{(\frac{1}{\sqrt{3}} - (2-\sqrt{3}))}{(2-\sqrt{3})}$

$h = \frac{15(1 - 2\sqrt{3} + 3)}{(2-\sqrt{3})\sqrt{3}}$

$h = \frac{15}{\sqrt{3}} \times 2$ $PQ = h + 15$

$h = 10\sqrt{3}$ $PQ = 5\,(2\sqrt{3} + 3)$ **Ans.A**

20. Which of the following statements is a tautology ?

(A) $((\sim p) \vee q) \Rightarrow p$ (B) $p \Rightarrow ((\sim p) \vee q)$ (C) $((\sim p) \vee q) \Rightarrow q$ (D) $q \Rightarrow ((\sim p) \vee q)$

Ans. (D)

Sol.

p	q	~p	~q	~p v q
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Check option   **Ans. D**

SECTION - B

21. Let $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ and B = A – I. If $\omega = \frac{\sqrt{3}i-1}{2}$, then the number of elements in the set $\{n \in \{1, 2, \ldots, 100\}: A^n + (\omega B)^n = A + B\}$ is equal to ______.

Ans. (17)

Sol. $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} = B + I = \underbrace{\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}}_{B} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I}$

$A^2 = (B+I)^2 = B^2 + 2B + I$ $B^2 = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$

$A^2 = B + I = A$ $B^2 = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} = -B$

$A^3 = B + I = A$ $B^3 = -B^2 = B$

| $B^4 = -B$

$A^n = A$ $B^5 = B$

$\Rightarrow A^n + (WB)^n = A + W^nB^n = A + B$

$\Rightarrow$ n must be odd & mult. of 3

$\Rightarrow n \in \{3, 9, 15, 21, \ldots\ldots, 99\}$

Total value of n = $\boxed{17}$ Ans.

22. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is ______.

Ans. **1492**

Sol. MANKIND

ADIKMNN

A _______________ $= \frac{6!}{2!} = 360$

D _______________ $= \frac{6!}{2!} = 360$

I _______________ $= \frac{6!}{2!} = 360$

K _______________ $= \frac{6!}{2!} = 360$

MAD ____________ $= \frac{4!}{2!} = 12$

MAI ____________ $= \frac{4!}{2!} = 12$

MAK ____________ $= \frac{4!}{2!} = 12$

MAND __________ $= 3! = 6$

MANI ______ ___ $= 3! = 6$

MANKD ___ ___ $= 2! = 2$

MANKID ___ ___ $= 1! = 1$

MANKIND $= 1! = 1$

1492 Ans.

23. If the maximum value of the term independent of t in the expansion of $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$, is K, then 8 K is equal to ______.

Ans. 6006

Sol. $y = \left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$

$T_{r+1} = {}^{15}C_r \, (t^2x^{1/5})^{15-r} \cdot \left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$

$= {}^{15}C_r \, t^{30-3r} \cdot x^{\frac{15-r}{5}} \cdot (1-x)^{r/10}$

For term ind. Of $t \Rightarrow 30 - 3r = 0 \Rightarrow r = 10$

$T_{11} = {}^{15}C_{10}\ \ x^1(1-x)^1 = {}^{15}C_{10}\,(x - x^2)$

$T_{11} = {}^{15}C_{10}\ \ \left[\frac{1}{4} - \left(x - \frac{1}{2}\right)^2\right]$

$(T_{11})_{\max} = \ {}^{15}C_{10}\,\frac{1}{4}$ at $x = \frac{1}{2}$

$$K = \frac{15\cdot 14\cdot 13\cdot 12\cdot 11}{4\times 5!}$$

$\Rightarrow \qquad \boxed{\mathbf{8K = 6006}}$ Ans.

24. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p},\frac{1}{q},\frac{1}{r},\frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ equal to ______.

Ans. 38

Sol. $x^2 - 8ax + 2a = 0 \left\langle {p \atop r} \right.$ & $\qquad x^2 + 12bx + 6b = 0 \left\langle {q \atop s} \right.$

$p + r = 8a \qquad\qquad q + p = -12b$

$pr = 2a \qquad\qquad qp = 6b$

$\frac{1}{p} + \frac{1}{r} = 4 \qquad\qquad \frac{1}{q} + \frac{1}{p} = -2$

Now $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$, ax is AP with common diff. = d & first term = α

$\frac{1}{p} + \frac{1}{r} = 4 \quad \Rightarrow \quad \alpha + \alpha + 2d = 4 \quad \Rightarrow \quad \alpha + d = 2$

s"y $\quad \frac{1}{q} + \frac{1}{s} = -2 \quad \Rightarrow \quad \alpha + d + \alpha + 3d = -2 \quad \Rightarrow$

$$\begin{array}{l} \alpha + 2d = -1 \\ - \quad - \qquad - \\ \hline \quad -\ d \ = \ 3 \\ \boxed{d = -3} \end{array}$$

& $\quad \boxed{\alpha = 5}$

Now $\quad \frac{1}{p} = 5, \frac{1}{q} = 2, \frac{1}{r} = -1, \frac{1}{s} = -4$

So $\quad 2a = pr \quad \Rightarrow \quad 2a = \frac{1}{5}\cdot\frac{1}{-1} \quad \Rightarrow \quad a = \frac{1}{-10}$

$6b = qs \quad \Rightarrow \quad 6b = \frac{1}{2}\cdot\left(\frac{1}{-4}\right) \quad \Rightarrow \quad b = \frac{-1}{48}$

Hence $a^{-1} - b^{-1} = -10 + 48 = \boxed{\mathbf{38}}$ Ans.

25. Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \,.\, b_n$ is equal to ______.

Ans. 27560

Sol. $a_1 = b_1 = 1, \qquad \underbrace{a_n = a_{n-1} + 2}_{AP} \qquad \& \qquad b_n = a_n + b_{n-1}\ \forall \qquad n \geq 2$

$a_1 = 1, a_2 = 3, a_3 = 5, \ldots\ldots, a_n = (2n - 1)$

Now $b_2 = a_2 + b_1 = 3 + 1 = 4$

$b_3 = a_3 + b_2 = 5 + 4 = 9$

$b_4 = a_4 + b_3 = 7 + 9 = 16$

$b_5 = a_5 + b_4 = 9 + 16 = 25$

$\Rightarrow \sum_{n=1}^{15} a_n \cdot b_n = 1 \cdot 1^2 + 3.2^2 + 5.3^2 + 7.4^2 + \ldots. + 29.15^2$

$$\Rightarrow S_{15} = \sum_{n=1}^{15}[(2n-1)\cdot n^2]$$

$$= 2\sum_{n=1}^{15} n^3 - \sum_{n=1}^{15} n^2$$

$$= 2 \cdot \left(\frac{15\cdot 16}{2}\right)^2 - \frac{15 \cdot 16 \cdot 31}{6}$$

$$= 28800 - 1240$$

$$= \boxed{27560}\ \textbf{Ans.}$$

26. Let f(x) = $\begin{cases} |4x^2 - 8x + 5|, \text{ } if 8x^2 - 6x + 1 \geq 0 \\ [4x^2 - 8x + 5], \text{ } if\ 8x^2 - 6x + 1 < 0, \end{cases}$ where [α] denotes the greatest integer less than or equal to α. Then the number of points in R where f is not differentiable is _______.

Ans. 3

Sol.

$$f(x) = \begin{bmatrix} |4x^2 - 8x + 5| & 8x^2 - 4x - 2x + 1 \geq 0 \\ [4x^2 - 8x + 5] & 8x^2 - 4x - 2x + 1 < 0 \end{bmatrix}$$

$$f(x) = \begin{bmatrix} 4(x-1)^2 + 1 & x \leq \frac{1}{4} \cup x < \frac{1}{2} \\ [4(x-1)^2 + 1] & \frac{1}{4} < x < \frac{1}{2} \end{bmatrix}$$

$$f(x) = \begin{vmatrix} 4(x-1)^2 + 1 & x \leq \frac{1}{4} \cup x < \frac{1}{2} \\ 2 & 1 - \frac{1}{\sqrt{2}} \leq x < \frac{1}{2} \\ 3 & \frac{1}{4} < x < 1 - \frac{1}{\sqrt{2}} \end{vmatrix}$$

$\Rightarrow$ f(x) is not diff. at x $\in \{\frac{1}{4}, 1 - \frac{1}{\sqrt{2}}, \frac{1}{2}\}$ $\boxed{3}$ Ans.

27. If $\lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}}$ [(nk + 1) + (nk + 2)] + ... + (nk + n)] = 33. $\lim_{n\to\infty} \frac{1}{n^{k+1}}$. [$1^k + 2^k + 3^k + \ldots. + n^k$], then the integral value of k is equal to ______.

Ans. **5**

Sol.

$$\lim_{n\to\infty} \frac{1}{n^2}\left(\frac{n+1}{n}\right)^{K-1} \cdot [(nK+1) + (nK+2) + \ldots + (nK+n)]$$

$$= \lim_{n\to\infty} \frac{1}{n} \cdot \left[1 + \frac{1}{n}\right]^{k-1} \left[\Sigma\left(K + \frac{r}{n}\right)\right]$$

$= 33 \lim_{n\to\infty} \frac{1}{n}\left[\sum\left(\frac{r}{n}\right)^K\right]$

$\int_0^1 (K+x)dx = 33\int_0^1 x^K dx$

$K + \frac{1}{2} = 33\left(\frac{1}{K+1}\right)$

$(2k+1)(K+1) = 66$

$2K^2 + 3K - 65 = 0$

$2k^2 + 13K - 10K - 65 = 0$

$(K-5)(2K+13) = 0 \Rightarrow \boxed{K = 5}$ **Ans.**

28. Let the equation of two diameters of a circle $x^2 + y^2 - 2x + 2fy + 1 = 0$ be $2px - y = 1$ and $2x + py = 4p$. Then the slope $m \in (0, \infty)$ of the tangent to the hyperbola $3x^2 - y^2 = 3$ passing through the center of the circle is equal to ______.

Ans. 2

Sol. $S : x^2 + y^2 - 2x + 2fy + 1 = 0$

$d_1 : 2px - y = 1$

$d_2 : 2x + py = 4p$

Center : $(1, -f)$ lies on

$d_1 \Rightarrow 2p + f = 1 \quad \Rightarrow \quad 2p^2 + pf = p$

$d_2 \Rightarrow 2 - pf = 4p \quad\quad \underline{2 \quad - pf = 4p}$

$2p^2 + 2 = 5p$

$2p^2 - 5p + 2 = 0$

$2p^2 - 4p - p + 2 = 0$

$(2p-1)(p-2) = 0$

$P = \frac{1}{2}$ & $p = 2$

$\Downarrow \quad\quad \Downarrow$

$f = 0 \quad\quad f = -3$

$H : \frac{x^2}{1} - \frac{y^2}{3} = 1$ & Center are $\begin{cases} C_1 : (1, 0) \\ C_2 : (1, 3) \end{cases}$

Now tangent of slope m & passes center

$T : y = mx \pm \sqrt{m^2 - 3}$

Pass (1, 0) & Pass (1, 3)

$\Rightarrow \quad m \pm \sqrt{m^2-3} = 0 \quad\quad 3 - m = \pm\sqrt{m^2-3}$

$m^2 - 3 = m^2 \quad\quad (m-3)^2 = (m^2 - 3)$

Not possible $\quad\quad m^2 + 9 - 6m = m^2 - 3$

$6m = 12$

$\boxed{m = 2}$ Ans.

29. The sum of diameters of the circles that touch (i) the parabola $75x^2 = 64(5y - 3)$ at the point $\left(\frac{8}{5},\frac{6}{5}\right)$ and (ii) the y-axis, is equal to _______.

Ans. 10

Sol. Circle touches (ii) y-axis

$\Rightarrow$ $S : (x - r)^2 + (y - B)^2 = r^2$

Circle also touches Parabola

$\Rightarrow$ $P : x^2 = \frac{64}{75}(5y - 3)$ at $A\left(\frac{8}{5}, \frac{6}{5}\right)$

Now a Lies on s = 0

$$\left(\frac{8}{5}-r\right)^2+\left(\frac{8}{5}-\beta\right)^2=r^2 \quad \text{... (1)}$$

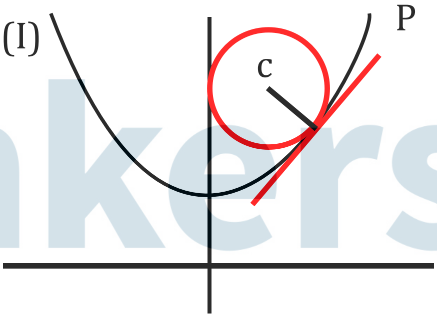
acc. to figure

$m_T|_A^P \cdot m_{AC} = -1$

$$\left(2\cdot\frac{8}{5}\cdot\frac{75}{64}\cdot\frac{1}{5}\right)=\left(\frac{\frac{6}{5}-\beta}{\frac{8}{5}-r}\right)=-1$$

$$\left(\frac{6}{5}-\beta\right)=\left(\frac{8}{5}-r\right)\left(\frac{-4}{3}\right)$$

Put in (1)



$$\left(\frac{8}{5}-r\right)^2+\left(-\frac{4}{3}\left(\frac{8}{5}-r\right)\right)^2=r^2$$

$$\left(\frac{8}{5}-r\right)^2+\left[1+\frac{16}{9}\right]=r^2$$

$$\left(\frac{\frac{8}{5}-r}{r}\right)^2\left(\frac{25}{9}\right)=1$$

$\frac{8-5r}{5r}=\frac{3}{5}$ & $\frac{8-5r}{5r}=\frac{-3}{5}$

40 – 25r = 15r & 40 – 25r = –15r

$r_1 = 1$ 40 = 10r $\Rightarrow$ $\boxed{r_2 = 4}$

sum of diameter = $2r_1 + 2r_2$ = $\boxed{10}$ **Ans.**

30. The line of shortest distance between the lines $\frac{x-2}{0}=\frac{y-1}{1}=\frac{z}{1}$ and $\frac{x-3}{2}=\frac{y-5}{2}=\frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane P : ax – y – z = 0, (a > 0). If the image of the point (1, 1, – 5) in the plane P is (α, β, γ), then α + β – γ is equal to ______.

Ans. BONUS

Sol. DR's of line of shortest distance

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 1\\ 2 & 2 & 1\end{vmatrix} = -\hat{i} + 2\hat{j} - 2\hat{k}$$

Angle between line and plane is $\cos^{-1}\sqrt{\frac{2}{27}} = \alpha$

$\text{Cos } \alpha = \sqrt{\frac{2}{27}}, \ \sin \alpha = \frac{5}{3\sqrt{3}}$

DR's normal to plane (1, –1, –1)

$\text{Sin } \alpha = \left| \frac{-a-2+2}{\sqrt{4+4+1}\sqrt{a^2+1+1}} \right| = \frac{5}{3\sqrt{3}}$

$\sqrt{3}\ |a| = 5\sqrt{a^2+2}$

$3a^2 = 25a^2 + 50$

Ans. No value of (a) [Bonus]