JEE–MAIN EXAMINATION – JUNE, 2022

25 June S - 01 Paper Solution

SECTION-A

1. If $Z=\frac{A^2B^3}{C^4}$, then the relative error in Z will be :

(A) $\frac{\Delta A}{A}+\frac{\Delta B}{B}+\frac{\Delta C}{C}$

(B) $\frac{2\Delta A}{A}+\frac{3\Delta B}{B}-\frac{4\Delta C}{C}$

(C) $\frac{2\Delta A}{A}+\frac{3\Delta B}{B}+\frac{4\Delta C}{C}$

(D) $\frac{\Delta A}{A}+\frac{\Delta B}{B}-\frac{\Delta C}{C}$

Ans. (C)

Sol. $Z=\frac{A^2B^3}{C^4}$

In case of error

$$\frac{dZ}{Z}=\frac{2dA}{A}+\frac{3dB}{B}+\frac{4dC}{C}$$

$$\boxed{\frac{\Delta Z}{Z}=\frac{2\Delta A}{A}+\frac{3\Delta B}{B}+\frac{4\Delta C}{C}}$$

2. $\vec{A}$ is a vector quantity such that $|\vec{A}|=$ non-zero constant. Which of the following expressions is true for $\vec{A}$?

(A) $\vec{A}.\vec{A}=0$

(B) $\vec{A}\times\vec{A}<0$

(C) $\vec{A}\times\vec{A}=0$

(D) $\vec{A}\times\vec{A}>0$

Ans. (C)

Sol. $|\vec{A}|\neq 0$

$\vec{A}\times\vec{A}=|\vec{A}||\vec{A}|\sin 0°\hat{n}=0$

3. Which of the following relations is true for two unit vectors $\hat{A}$ and $\hat{B}$ making an angle θ to each other?

(A) $|\hat{A}+\hat{B}|=|\hat{A}-\hat{B}|\tan\frac{\theta}{2}$

(B) $|\hat{A}-\hat{B}|=|\hat{A}+\hat{B}|\tan\frac{\theta}{2}$

(C) $|\hat{A}+\hat{B}|=|\hat{A}-\hat{B}|\cos\frac{\theta}{2}$

(D) $|\hat{A}-\hat{B}|=|\hat{A}+\hat{B}|\cos\frac{\theta}{2}$

Ans. (B)

Sol. $|\hat{A}+\hat{B}|=\sqrt{|\hat{A}|^2+|\hat{B}|^2+2|\hat{A}||\hat{B}|\cos\theta}$

$=\sqrt{1+1+2\cos\theta}$

$=\sqrt{2(1+\cos\theta)}$

$=\sqrt{2\times 2\cos^2\frac{\theta}{2}}$

$=2\cos\frac{\theta}{2}$

$|\hat{A}-\hat{B}|=\sqrt{|\hat{A}|^2+|\hat{B}|^2-2|\hat{A}||\hat{B}|\cos\theta}$

$=\sqrt{2-2\cos\theta}$

$=2\sin\frac{\theta}{2}$

$\frac{|\hat{A}+\hat{B}|}{|\hat{A}-\hat{B}|}=\cot\frac{\theta}{2}$

4. If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle having position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be :-

(A) $3\hat{i} + 4\hat{j} - 2\hat{k}$

(B) $-10\hat{i} + 10\hat{j} + 5\hat{k}$

(C) $10\hat{i} + 5\hat{j} - 10\hat{k}$

(D) $10\hat{i} + \hat{j} - 5\hat{k}$

Ans. (B)

Sol. $\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & 4 & -2 \end{vmatrix}$$

$$= \hat{i}(-2-8) - \hat{j}(-4-6) + \hat{k}(8-3)$$

$$= -10\hat{i} + 10\hat{j} + 5\hat{k}$$

5. The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be : (Given g = acceleration due to gravity at the surface of earth)

(A) g/2

(B) g/4

(C) g/3

(D) g/9

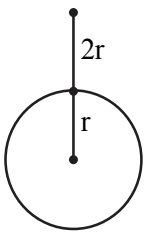
Ans. (D)

Sol. $g = \frac{Gm}{r^2}$

$g' = \frac{Gm}{(3r)^2}$

$g' = \frac{Gm}{9r^2}$

$g' = \frac{g}{9}$



6. The terminal velocity (v_t) of the spherical rain drop depends on the radius (r) of the spherical rain drop as:-

(A) $r^{1/2}$ (B) r

(C) r^2 (D) r^3

Ans. (C)

Sol. $v_t = \frac{2}{9}\frac{gr^2(\rho_p - \rho_l)}{\eta}$; $v_t \propto r^2$

7. The relation between root mean square speed (v_{rms}) and most probable speed (v_p) for the molar mass M of oxygen gas molecule at the temperature of 300 K will be :-

(A) $v_{rms} = \sqrt{\frac{2}{3}}v_p$ (B) $v_{rms} = \sqrt{\frac{3}{2}}v_p$

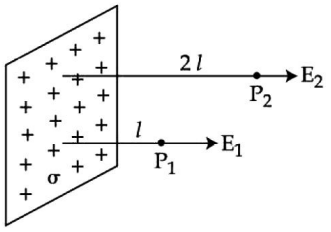
(C) $v_{rms} = v_p$ (D) $v_{rms} = \sqrt{\frac{1}{3}}v_p$

Ans. (B)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M}}$ and $v_{mp} = \sqrt{\frac{2RT}{M}}$

Thus $v_{rms} = \sqrt{\frac{3}{2}}v_{mp}$

8. In the figure, a very large plane sheet of positive charge is shown. P_1 and P_2 are two points at distance l and $2l$ from the charge distribution. If σ is the surface charge density, then the magnitude of electric fields E_1 and E_2 at P_1 and P_2 respectively are :



(A) $E_1 = \sigma/\varepsilon_0, E_2 = \sigma/2\varepsilon_0$

(B) $E_1 = 2\sigma/\varepsilon_0, E_2 = \sigma/\varepsilon_0$

(C) $E_1 = E_2 = \sigma/2\varepsilon_0$

(D) $E_1 = E_2 = \sigma/\varepsilon_0$

Ans. (C)

Sol. As the sheet is very large $\vec{E}$ is independent of distance from it.

Thus $E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$

9. Match List-I with List-II

List-I	List-II
(A) AC generator	(I) Detects the presence of current in the circuit
(B) Galvanometer	(II) Converts mechanical energy into electrical energy
(C) Transformer	(III) Works on the principle of resonance in AC circuit
(D) Metal detector	(IV) Changes an alternating voltage for smaller or greater value

Choose the **correct answer** from the options given below :-

(A) (A)–(II), B–(I), (C)–(IV), (D)–(III)

(B) (A)–(II), B–(I), (C)–(III), (D)–(IV)

(C) (A)–(III), B–(IV), (C)–(II), (D)–(I)

(D) (A)–(III), B–(I), (C)–(II), (D)–(IV)

Ans. (A)

Sol. AC generator converts mechanical energy into electrical energy. Galvanometer shows deflection when current passes through it so it is used to show presence of current in any wire.

Transformer is used to step up or step down the voltage. Metals detectors contain inductor coils and use principle of induction and resonance in AC circuit.

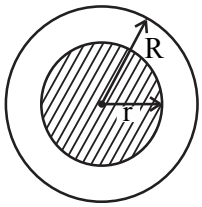
10. A long straight wire with a circular cross-section having radius R, is carrying a steady current I. The current I is uniformly distributed across this cross-section. Then the variation of magnetic field due to current I with distance r (r < R) from its centre will be :-

(A) $B \propto r^2$ (B) $B \propto r$

(C) $B \propto \frac{1}{r^2}$ (D) $B \propto \frac{1}{r}$

Ans. (B)

Sol. Use Ampere's law



$B.2\pi r = \mu_0 . \frac{I}{\pi R^2} . \pi r^2$

Thus $B \propto r$

11. If wattless current flows in the AC circuit, then the circuit is

(A) Purely Resistive circuit

(B) Purely Inductive circuit

(C) LCR series circuit

(D) RC series circuit only

Ans. (B)

Sol. Purely Inductive circuit

$\theta = \frac{\pi}{2}$

$\cos\frac{\pi}{2} = 0$

Average power = 0

12. The electric field in an electromagnetic wave is given by $E = 56.5 \sin\omega(t - x/c)\, NC^{-1}$. Find the intensity of the wave if it is propagating along x-axis in the free space. (Given $\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$)

(A) 5.65 Wm^{-2} (B) 4.24 Wm^{-2}

(C) 1.9×10^{-7} Wm^{-2} (D) 56.5 Wm^{-2}

Ans. (B)

Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

$I = \frac{1}{2} \times (8.85 \times 10^{-12})(56.5)^2 \times (3 \times 10^8)$

= 4.24 Wm^{-2}.

13. The two light beams having intensities I and 9I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point P and π at point Q. Then the difference between the resultant intensities at P and Q will be :

(A) 2 I (B) 6 I

(C) 5 I (D) 7 I

Ans. (B)

Sol. $I_P = I + 9I + 2\sqrt{I \times 9I}\cos\frac{\pi}{2}$

$I_P = 10\ I$

$I_Q = I + 9I + 2\sqrt{I \times 9I}\cos\pi$

$= 10\ I - 6I = 4I$

$\therefore I_P - I_Q = 10I - 4I = 6I$

14. A light wave travelling linearly in a medium of dielectric constant 4, incident on the horizontal interface separating medium with air. The angle of incidence for which the total intensity of incident wave will be reflected back into the same medium will be (Given : relative permeability of medium $\mu_r = 1$)

(A) 10° (B) 20°

(C) 30° (D) 60°

Ans. (D)

Sol. For total internal reflection, $i > \theta_C$

$\Rightarrow \sin i > \sin\theta_C$

$\Rightarrow \sin i > \frac{\mu_R}{\mu_D}$ .....(1)

Also $\mu = \sqrt{\mu_r \in_r}$

$\frac{\mu_R}{\mu_D} = \frac{\sqrt{1\times1}}{\sqrt{4\times1}} = \frac{1}{2}$

From (1), $\sin i > \frac{1}{2} \Rightarrow i > 30°$, i = 60°

15. Given below are two statements :-

Statement I : Davisson-Germer experiment establishes the wave nature of electrons.

Statement II : If electrons have wave nature, they can interfere and show diffraction.

In the light of the above statements choose the **correct answer** from the options given below:-

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Ans. (A)

Sol. In Davisson-Germer experiment the electrons exhibit diffraction there by proving that electrons have wave nature. Hence both statement are correct.

Sol. Both the options are correct by concept.

16. The ratio for the speed of the electron in the 3rd orbit of He^+ to the speed of the electron in the 3rd orbit of hydrogen atom will be :-

(A) 1 : 1 (B) 1 : 2

(C) 4 : 1 (D) 2 : 1

Ans. (D)

Sol. $v \propto \frac{Z}{n} \propto Z$ (n = constant)

$\Rightarrow \frac{v_{He^+}}{v_H} = \frac{Z_{He^+}}{Z_H} = \frac{2}{1}$

17. The photodiode is used to detect the optiocal signals. These diodes are preferably operated in reverse biased mode because.

(A) fractional change in majority carriers produce higher forward bias current

(B) fractional change in majority carriers produce higher reverse bias current

(C) fractional change in minority carriers produce higher forward bias current

(D) fractional change in minority carriers produce higher reverse bias current

Ans. (D)

Sol. Very small change in minority charge carriers produces high value of reverse bias current.

18. A signal of 100 THz frequency can be transmitted with maximum efficiency by :

(A) Coaxial cable

(B) Optical fibre

(C) Twisted pair of copper wires

(D) Water

Ans. (B)

Sol. Optical fibre frequency range is 1 THz to 1000 THz.

19. The difference of speed of light in the two media A and B ($v_A - v_B$) is 2.6×10^7 m/s. If the refractive index of medium B is 1.47, then the ratio of refractive index of medium B to medium A is : (Given : speed of light in vacuum $c = 3 \times 10^8$ ms^{-1})

(A) 1.303

(B) 1.318

(C) 1.13

(D) 0.12

Ans. (C)

Sol. $v = \frac{c}{\mu}$

$$\Rightarrow v_B = \frac{3\times10^8}{1.47} = 2.04\times10^8 = 20.4\times10^7 \text{ m/s}$$

$$\because v_A - v_B = 2.6\times10^7 \text{ m/s}$$

$$\therefore v_A = (20.4+2.6)\times10^7 = 23\times10^7 \text{ m/s}$$

$$\therefore \frac{\mu_B}{\mu_A} = \frac{v_A}{v_B} = \frac{23\times10^7}{20.4\times10^7} = 1.13$$

20. A teacher in his physics laboratory allotted an experiment to determine the resistance (G) of a galvanometer. Students took the observations for $\frac{1}{3}$ deflection in the galvanometer. Which of the below is **true** for measuring value of G?

(A) $\frac{1}{3}$ deflection method cannot be used for determining the resistance of the galvanometer.

(B) $\frac{1}{3}$ deflection method can be used and in this case the G equals to twice the value of shunt resistance(s).

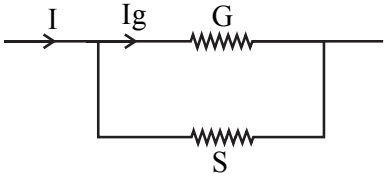
(C) $\frac{1}{3}$ deflection method can be used and in this case, the G equals to three times the value of shunt resistance(s)

(D) $\frac{1}{3}$ deflection method can be used and in this case the G value equals to the shunt resistance(s).

Ans. (B)

Sol. In galvanometer

$$\Rightarrow (I - I_g)S = I_g G$$



$$\frac{I_g}{I} = \frac{S}{S+G}$$

$$\Rightarrow \frac{1}{3} = \frac{S}{S+G} \Rightarrow S+G = 3S \Rightarrow G = 2S$$

SECTION-B

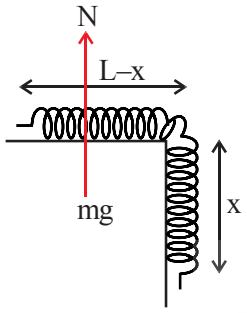
1. A uniform chain of 6 m length is placed on a table such that a part of its length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is _____m.

Ans. 2

Sol. Mass per unit length = λ

$$N = mg = \lambda(L-x)g$$

$$fs_{max} = \mu_s N$$



$$fs_{max} = (0.5)(\lambda)(L-x)g$$

And also $fs_{max} = m_x g$

$$0.5\lambda(L-x)g = \lambda x g$$

$$\frac{L-x}{2} = x$$

$$\frac{L}{2} = \frac{3x}{2} \Rightarrow x = \frac{L}{3} = \frac{6}{3} = 2m$$

2. A 0.5 kg block moving at a speed of 12 ms^{-1} compresses a spring through a distance 30 cm when its speed is halved. The spring constant of the spring will be _____ Nm^{-1}.

Ans. 600

Sol. $U_i + K_i = U_f + K_f$

$$\Rightarrow 0 + \frac{1}{2}m(12)^2 = \frac{1}{2}K(0.3)^2 + \frac{1}{2}m(6)^2$$

$$\Rightarrow 0.5(12^2 - 6^2) = K(0.3)^2$$

K = 600 N/m

3. The velocity of upper layer of water in a river is 36 kmh^{-1}. Shearing stress between horizontal layers of water is 10^{-3} Nm^{-2}. Depth of the river is _____m. (Co-efficiency of viscosity of water is 10^{-2} Pa.s)

Official Ans. by NTA 100

Allen Ans. 100

Sol. $$F = \eta A \frac{\Delta v_x}{\Delta y}$$

$$\frac{F}{A} = \eta \frac{\Delta v_x}{\Delta y}$$

$$\Rightarrow 10^{-3} = 10^{-2} \times \frac{36 \times 1000}{h \times 3600}$$

$$\Rightarrow h = 10^{-2} \times \frac{36 \times 1000}{10^{-3} \times 3600} = 100 \text{ m}$$

4. A steam engine intakes 50g of steam at 100°C per minute and cools it down to 20°C. If latent heat of vaporization of steam is 540 cal g^{-1}, then the heat rejected by the steam engine per minute is _____ $\times 10^3$ cal.

Ans. 31

Sol. Heat rejected = $mL_f + mS\Delta T$

= (50 × 540) + 50 (1) (100 – 20)

= 31000 Cal

= 31 × 10^3 Cal

5. The first overtone frequency of an open organ pipe is equal to the fundamental frequency of a closed organ pipe. If the length of the closed organ pipe is 20 cm. The length of the open organ pipe is _____ cm.

Ans. 80

Sol. $$f_1 = \frac{2v}{2l_1}$$

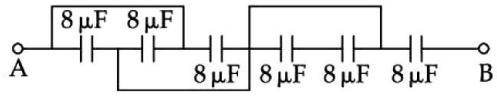
$$f_2 = \frac{v}{4l_2}$$

$f_1 = f_2$

$$= \frac{2v}{2l_1} = \frac{v}{4l_2}$$

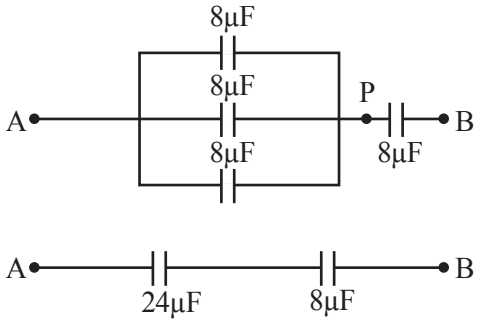
$l_1 = 4l_2 = 80$ cm

6. The equivalent capacitance between points A and B in below shown figure will be _____μF.



Ans. 6

Sol. Two capacitors are short circuited



Finally equivalent capacitance

$$=\frac{24\times8}{24+8}=\frac{24\times8}{32}=6\mu F$$

7. A resistor develops 300 J of thermal energy in 15s, when a current of 2A is passed through it. If the current increases to 3A, the energy developed in 10s is _____ J.

Ans. 450

Sol. $H = i^2Rt$

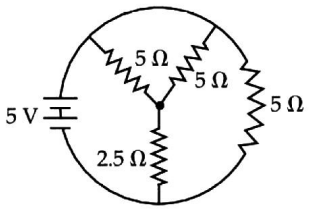
$300 = 2^2 \times R \times 15$

$$\Rightarrow R=\frac{300}{60}=5\Omega$$

Now, for i = 3A, t = 10s, R = 5Ω

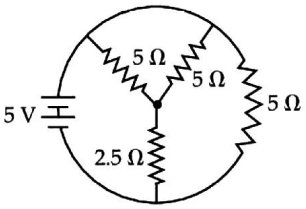
$H = 3^2 \times 5 \times 10 = 450$ J

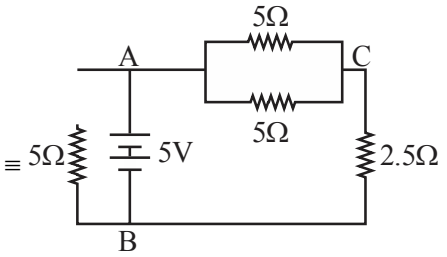
8. The total current supplied to the circuit as shown in figure by the 5V battery is _____A

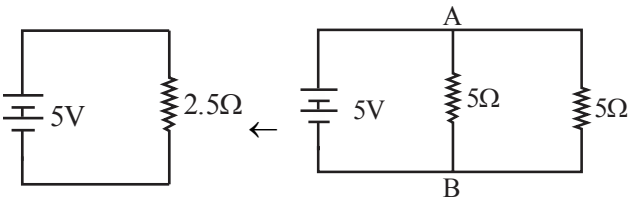


Ans. 2

Sol.







Current supplied by 5V battery

$$=\frac{5V}{2.5\Omega}=2A$$

9. The current in a coil of self inductance 2.0 H is increasing according to $I = 2\sin(t^2)$A. The amount of energy spent during the period when current changes from 0 to 2A is _____ J.

Ans. 4

Sol. $I=2\sin(t^2)\Rightarrow dI=4t\sin(t^2)\,dt$

If $I=0\Rightarrow t=0$

and $I=2\Rightarrow 2=2\sin t^2$

$$\Rightarrow t=\sqrt{\frac{\pi}{2}}$$

$$E=\int LI\,dI$$

$$=\int 2\times2\sin(t^2)\times4t\cos(t^2)\,dt$$

$$=8\int_0^{\sqrt{\pi/2}} t\sin(2t^2)\,dt$$

$$=2\left[-\cos(2t^2)\right]_0^{\sqrt{\pi/2}}$$

$$=2\left[-\cos\pi+\cos0\right]=4$$

10. A force on an object of mass 100g is $(10\hat{i}+5\hat{j})$ N. The position of that object at t = 2s is $(a\hat{i}+b\hat{j})$ m after starting from rest. The value of $\frac{a}{b}$ will be _____

Ans. 2

Sol. $\vec{F}=10\hat{i}+5\hat{j}$

m = 100 g = 0.1 kg

$\vec{a}=\frac{\vec{F}}{m}=100\hat{i}+50\hat{j}$

$\vec{S}=\vec{u}t+\frac{1}{2}\vec{a}t^2=\frac{1}{2}\vec{a}t^2(\text{as } \vec{u}=0)$

$=\frac{1}{2}(100\hat{i}+50\hat{j})2^2$

$=200\hat{i}+100\hat{j}$

$=a\hat{i}+b\hat{j}$

$a=200,\ b=100$

$\therefore \frac{a}{b}=2$