JEE–MAIN EXAMINATION – JUNE, 2022

24 June S - 01 Paper Solution

SECTION-A

1. The bulk modulus of a liquid is 3×10^{10} Nm^{-2}. The pressure required to reduce the volume of liquid by 2% is :

(A) 3×10^8 Nm^{-2} (B) 9×10^8 Nm^{-2}

(C) 6×10^8 Nm^{-2} (D) 12×10^8 Nm^{-2}

Ans. (C)

Sol. $B = 3 \times 10^{10}$

$$-\frac{\Delta V}{V} = 0.02$$

$$B = \frac{\Delta P}{-\frac{\Delta V}{v}} \Rightarrow \Delta P = -B\left(\frac{\Delta V}{V}\right)$$

$$= (3 \times 10^{10})(0.02)$$

$$= 6 \times 10^8 \, N/m^2$$

2. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In an uniform magnetic field, speed and energy remains the same for a moving charged particle.

Reason (R) : Moving charged particle experiences magnetic force perpendicular to its direction of motion.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true.

Ans. (A)

Sol. $\vec{F} = q(\vec{v} \times \vec{B})$

$\boxed{\vec{F} \perp \vec{v}}$

Work done $= \vec{F} \cdot \vec{S}$

Work done = 0

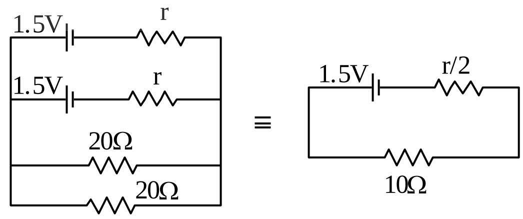
3. Two identical cells each of emf 1.5 V are connected in parallel across a parallel combination of two resistors each of resistance 20Ω. A voltmeter connected in the circuit measures 1.2 V. The internal resistance of each cell is

(A) 2.5Ω   (B) 4Ω

(C) 5Ω   (D) 10Ω

Ans. (C)

Sol.



$V = E - ir/2$

$$1.2 = 1.5 - i\left(\frac{r}{2}\right)$$

$$i\frac{r}{2} = 0.3$$

$$i = \frac{1.5}{10 + \frac{r}{2}} \Rightarrow 10i + \frac{ir}{2} = 1.5$$

$10i = 1.5 - 0.3$

$i = 0.12$ A

$$\Rightarrow \quad r = \frac{0.6}{0.12} = 5\Omega$$

4. Identify the pair of physical quantities which have different dimensions :

(A) Wave number and Rydberg's constant

(B) Stress and Coefficient of elasticity

(C) Coercivity and Magnetisation

(D) Specific heat capacity and Latent heat

Ans. (D)

Sol. $S = \frac{Q}{m\Delta T} = \frac{J}{Kg°C}$

$L = \frac{Q}{m} = \frac{J}{Kg}$

5. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be : [use g = 10 m/s²]

(A) $\frac{1}{2}\sin^{-1}\left(\frac{5t^2}{4R}\right)$ (B) $\frac{1}{2}\sin^{-1}\left(\frac{4R}{5t^2}\right)$

(C) $\tan^{-1}\left(\frac{4t^2}{5R}\right)$ (D) $\cot^{-1}\left(\frac{R}{20t^2}\right)$

Ans. (D)

Sol. $R = \frac{V^2(2\sin\theta\cos\theta)}{g}$

$t = \frac{V\sin\theta}{g} \Rightarrow V = \frac{gt}{\sin\theta}$

$\Rightarrow \quad R = \frac{g^2t^2}{\sin^2\theta}.\frac{2\sin\theta\cos\theta}{g}$

$\tan\theta = \frac{2gt^2}{R} \quad = \frac{20t^2}{R}$

$\cot\theta = \frac{R}{20t^2}$

6. A block of mass 10 kg starts sliding on a surface with an initial velocity of 9.8 ms^{-1}. The coefficient of friction between the surface and bock is 0.5. The distance covered by the block before coming to rest is : [use g = 9.8 ms^{-2}]

(A) 4.9 m (B) 9.8 m

(C) 12.5 m (D) 19.6 m

Ans. (B)

Sol. $a = -\mu g = -0.5 \times 9.8 = -4.9 m/s^2$

$d = \frac{v^2}{2a} = \frac{9.8 \times 9.8}{2(4.9)}$

= 9.8 m

7. A boy ties a stone of mass 100 g to the end of a 2 m long string and whirls it around in a horizontal plane. The string can withstand the maximum tension of 80 N. If the maximum speed with which the stone can revolve is $\frac{K}{\pi}$ rev. / min . The value of K is : (Assume the string is massless and unstretchable)

(A) 400 (B) 300

(C) 600 (D) 800

Ans. (C)

Sol. $T = M\omega^2R$

$T = 80\text{ N} \quad M = 0.1 \quad \omega = ? \quad R = 2m$

$80 = 0.1\ \omega^2(2)$

$\omega^2 = 400$

$\omega = 20$

$2\pi f = 20$

$f = \frac{10}{\pi}\frac{rev}{s}$

$= \frac{600}{\pi}\frac{rev}{min}$

8. A vertical electric field of magnitude 4.9×10^5 N/C just prevents a water droplet of a mass 0.1 g from falling. The value of charge on the droplet will be : (Given g = 9.8 m/s^2)

(A) 1.6×10^{-9} C (B) 2.0×10^{-9} C

(C) 3.2×10^{-9} C (D) 0.5×10^{-9} C

Ans. (B)

Sol. Mg = qE

$$(0.1\times10^{-3})(9.8) = 4.9\times10^5 q$$

$$\frac{2\times10^{-4}}{10^5} = q$$

$$\boxed{q = 2\times10^{-9}C}$$

9. A particle experiences a variable force $\vec{F} = (4x\hat{i} + 3y^2\hat{j})$ in a horizontal x-y plane. Assume distance in meters and force is newton. If the particle moves from point (1, 2) to point (2, 3) in the x-y plane, the Kinetic Energy changes by

(A) 50.0 J (B) 12.5 J

(C) 25.0 J (D) 0 J

Ans. (C)

Sol. $F = 4x\hat{i} + 3y^2\hat{j}$

WD = ΔKE

$$W = \int \vec{F}.(dx\hat{i} + dy\hat{j})$$

$$= \int_1^2 4xdx + \int_2^3 3y^2dx$$

$$= (2x^2)_1^2 + (y^3)_2^3$$

$$= (8-2) + (27-8)$$

$$= 6 + 19 = 25J$$

10. The approximate height from the surface of earth at which the weight of the body becomes $\frac{1}{3}$ of its weight on the surface of earth is : [Radius of earth R = 6400 km and $\sqrt{3} = 1.732$]

(A) 3840 km (B) 4685 km

(C) 2133 km (D) 4267 km

Ans Ans. (B)

Sol. $Mg' = \frac{M}{3}g$

$$g' = \frac{g}{3}$$

$$g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{3}$$

$$\frac{R}{R+h} = \frac{1}{\sqrt{3}}.$$

$$h = (\sqrt{3}-1)R$$

$$= (1.732-1)6400$$

$$\boxed{h = 4685km}$$

11. A resistance of 40 Ω is connected to a source of alternating current rated 220 V, 50 Hz. Find the time taken by the current to change from its maximum value to rms value :

(A) 2.5 ms (B) 1.25 ms

(C) 2.5 s (D) 0.25 s

Ans. (A)

Sol. Considering sinusoidal AC.

Phase at maximum value $= \frac{\pi}{2}$

Phase at rms value $= \frac{3\pi}{4}$

Thus phase change $= \frac{3\pi}{4} - \frac{\pi}{2} = \frac{\pi}{4}$

Now $\omega = 2\pi f$

$= 2\pi \times 50$

$= 100\pi$

time taken $t = \frac{\theta}{\omega} = \frac{\pi/4}{100\pi} = \frac{1}{400}s$

$t = 2.5 \times 10^{-3} = 2.5$ ms

12. The equations of two waves are given by :

$$y_1 = 5\sin 2\pi(x - vt)\text{cm}$$

$$y_2 = 3\sin 2\pi(x - vt + 1.5)\text{cm}$$

These waves are simultaneously passing through a string. The amplitude of the resulting wave is

(A) 2 cm (B) 4 cm

(C) 5.8 cm (D) 8 cm

Ans. (A)

Sol. $A_1 = 5 \quad A_2 = 3$

$\Delta\theta = 2\pi(1.5) = 3\pi$

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(3\pi)}$

$= |A_1 - A_2|$

= 2cm

13. A plane electromagnetic wave travels in a medium of relative permeability 1.61 and relative permittivity 6.44. If magnitude of magnetic intensity is 4.5×10^{-2} Am^{-1} at a point, what will be the approximate magnitude of electric field intensity at that point ?

(Given : permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}, speed of light in vacuum $c = 3 \times 10^8$ ms^{-1})

(A) 16.96 Vm^{-1} (B) 2.25×10^{-2} Vm^{-1}

(C) 8.48 Vm^{-1} (D) 6.75×10^6 Vm^{-1}

Ans. (BONUS)

Sol. $\mu_r = 1.61 \qquad \in_r = 6.44$

$B = 4.5 \times 10^{-2}$

E = ?

$C = \frac{1}{\sqrt{\mu_0 \in_0}} \quad V = \frac{1}{\sqrt{\mu \in}}$

$\frac{C}{V} = \sqrt{\mu_r \in_r} = \sqrt{1.61 \times 6.44}$

$\frac{E}{B} = V = \frac{3 \times 10^8}{\sqrt{1.61 \times 6.44}} = 9.32 \times 10^7 \text{ m/s}$

$E = 4.5 \times 10^{-2} \times 9.32 \times 10^7$

$= 4.2 \times 10^6$

14. Choose the correct option from the following options given below :

(A) In the ground state of Rutherford's model electrons are in stable equilibrium. While in Thomson's model electrons always experience a net-force.

(B) An atom has a nearly continuous mass distribution in a Rutherford's model but has a highly non-uniform mass distribution in Thomson's model

(C) A classical atom based on Rutherford's model is doomed to collapse.

(D) The positively charged part of the atom possesses most of the mass in Rutherford's model but not in Thomson's model.

Ans. (C)

Sol. According to Rutherford, e^- revolves around nucleus in circular orbit. Thus e^- is always accelerating (centripetal acceleration). An accelerating change emits EM radiation and thus e^- should loose energy and finally should collapse in the nucleus.

15. Nucleus A is having mass number 220 and its binding energy per nucleon is 5.6 MeV. It splits in two fragments 'B' and 'C' of mass numbers 105 and 115. The binding energy of nucleons in 'B' and 'C' is 6.4 MeV per nucleon. The energy Q released per fission will be :

(A) 0.8 MeV (B) 275 MeV

(C) 220 MeV (D) 176 MeV

Ans. (D)

Sol. $Q = (B.E)_P - (B.E)_R$

$= (105 + 115)(6.4) - (220)(5.6)$

= 176 MeV

16. A baseband signal of 3.5 MHz frequency is modulated with a carrier signal of 3.5 GHz frequency using amplitude modulation method. What should be the minimum size of antenna required to transmit the modulated signal ?

(A) 42.8 m (B) 42.8 mm

(C) 21.4 mm (D) 21.4 m

Ans. (C)

Sol. $f_c = 3.5\text{GHz} \quad f_m = 3.5\text{MHz}$

Side band frequencies are $f_c - f_m$ & $f_c + f_m$. which are almost f_c

$$\lambda = \frac{c}{f_c}$$

Minimum length of antenna =

$$\frac{c}{f_c 4} = \frac{\lambda}{4} = \frac{3\times10^8}{3.5\times10^9\times4}$$

= 21.4 mm

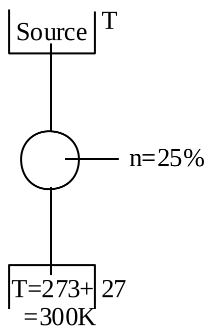
17. A Carnot engine whose heat sinks at 27°C, has an efficiency of 25%. By how many degrees should the temperature of the source be changed to increase the efficiency by 100% of the original efficiency ?

(A) Increases by 18°C (B) Increase by 200°C

(C) Increase by 120°C (D) Increase by 73°

Ans. (B)

Sol.



$$1 - \frac{300}{T} = 0.25$$

$$\frac{300}{T} = 0.75$$

T = 400K

If efficiency increased by 100% then new efficiency ⇒ n' = 50%

$$1 - \frac{300}{T'} = 0.5$$

$T' = 600K$

Increase in temp = 600 – 400

= 200 K or 200°C

18. A parallel plate capacitor is formed by two plates each of area $30\pi\ \text{cm}^2$ separated by 1 mm. A material of dielectric strength $3.6 \times 10^7\ \text{Vm}^{-1}$ is filled between the plates. If the maximum charge that can be stored on the capacitor without causing any dielectric breakdown is 7×10^{-6} C, the value of dielectric constant of the material is :

$$\left\{\text{Use} : \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{Nm}^2\text{C}^{-2}\right\}$$

(A) 1.66 (B) 1.75

(C) 2.25 (D) 2.33

Ans. (D)

Sol. $$K = \frac{q}{A \in_0 E} = \frac{7\times10^{-6}}{30\pi\times10^{-4}\times\frac{1}{4\pi\times9\times10^9}\times3.6\times10^7}$$

$$K = \frac{36\times7}{30\times3.6} = 2.33$$

19. The magnetic field at the centre of a circular coil of radius r, due to current I flowing through it, is B. The magnetic field at a point along the axis at a distance $\frac{r}{2}$ from the centre is :

(A) B/2 (B) 2B

(C) $\left(\frac{2}{\sqrt{5}}\right)^3 B$ (D) $\left(\frac{2}{\sqrt{3}}\right)^3 B$

Ans. (C)

Sol. $$B_C = \frac{\mu_0 I}{2r}, B_a = \frac{\mu_0 I r^2}{2(x^2+r^2)^{3/2}}$$

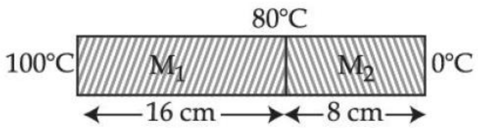
At $x = \frac{r}{2}$

$$B_a = \frac{\mu_0 I r^2}{2\left(\frac{r^2}{4}+r^2\right)^{3/2}}$$

$$= \frac{\mu_0 I r^2}{2\left(\frac{5}{4}r^2\right)^{3/2}} = \frac{\mu_0 I}{2r}\left(\frac{4}{5}\right)^{3/2}$$

$$= \frac{\mu_0 I}{2r}\left(\frac{2}{\sqrt{5}}\right)^3$$

20. Two metallic blocks M_1 and M_2 of same area of cross-section are connected to each other (as shown in figure). If the thermal conductivity of M_2 is K then the thermal conductivity of M_1 will be : [Assume steady state heat conduction]



(A) 10 K (B) 8 K

(C) 12.5 K (D) 2 K

Ans. (B)

Sol. $\Delta T \ \alpha \ R \ \alpha \ \frac{\ell}{k}$,

$$\frac{\Delta T_1}{\Delta T_2} = \frac{\ell_1}{k_1} \times \frac{k_2}{\ell_2} = \frac{16}{k_1} \times \frac{k}{8}$$

$$\frac{20}{80} = \frac{16}{k_1} \times \frac{k}{8} \rightarrow k_1 = 8k$$

SECTION-B

1. 0.056 kg of Nitrogen is enclosed in a vessel at a temperature of 127°C. The amount of heat required to double the speed of its molecules is _____ k cal. (Take R = 2 cal $mole^{-1}K^{-1}$)

Ans. (12)

Sol. 0.056 kg N_2 = 56 gm of N_2 = 2 mole of N_2

$T_1 = 400$ K, $v \alpha \sqrt{T}$ so $T_2 = 4T_1 = 1600K$

$$Q = \frac{f}{2} nR\Delta T$$

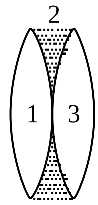
f = 5

Q = 12 k cal

2. Two identical thin biconvex lenses of focal length 15 cm and refractive index 1.5 are in contact with each other. The space between the lenses is filled with a liquid of refractive index 1.25. The focal length of the combination is ______ cm.

Ans. (10)

Sol.



$$\frac{1}{f_1} = \frac{1}{15} = \left(\frac{3}{2} - 1\right)\left[\frac{2}{R}\right]$$

$$\frac{1}{R} = \frac{1}{15}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= \frac{1}{15} + \left(\frac{5}{4} - 1\right)\left[\frac{-2}{R}\right] + \frac{1}{15}$$

$$= \frac{1}{15} - \frac{1}{30} + \frac{1}{15}$$

$$= \frac{2 - 1 + 2}{30}$$

$$= \frac{3}{30} = \frac{1}{10}$$

= 10

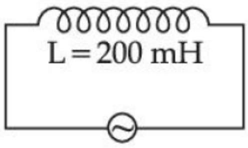
3. A transistor is used in common-emitter mode in an amplifier circuit. When a signal of 10 mV is added to the base-emitter voltage, the base current changes by 10 μA and the collector current changes by 1.5 mA. The load resistance is 5 kΩ. The voltage gain of the transistor will be _____ . **Ans. (750)**

Sol. $r_i = \frac{10mV}{10\mu A} = 10^3 \Omega$

$$\beta = \frac{1.5mA}{10\mu A} = 150$$

$$A_V = \left(\frac{R_0}{r_i}\right)\beta = \left(\frac{5000}{1000}\right) \times 150 = 750$$

4. As shown in the figure an inductor of inductance 200 mH is connected to an AC source of emf 220 V and frequency 50 Hz. The instantaneous voltage of the source is 0 V when the peak value of current is $\frac{\sqrt{a}}{\pi}$A. The value of a is _______.



Ans Ans. (242)

Sol. $f = 50Hz$

$X_L = 2\pi fL$

$= 2\pi(50)(200\times10^{-3})$

$= 20\pi\Omega$

$i_0 = \frac{V_0}{X_L} \Rightarrow \frac{V_{rms}\sqrt{2}}{X_L}$

$= \frac{(220)\sqrt{2}}{20\pi} = \frac{11\sqrt{2}}{\pi}$

$\boxed{i_o = \frac{\sqrt{242}}{\pi}}$

5. Sodium light of wavelengths 650 nm and 655 nm is used to study diffraction at a single slit of aperture 0.5 mm. The distance between the slit and the screen is 2.0 m. The separation between the positions of the first maxima of diffraction pattern obtained in the two cases is _______ × 10^{-5} m.

Ans. (3)

Sol. $a\sin\theta = \frac{\quad}{2}$

$\frac{y}{L} = \theta = \frac{3\lambda}{2a}$ $\qquad$ L = 2m

$y_1 = \frac{3\lambda_1 L}{2a}$ $\qquad$ λ_2 = 655 nm

$y_2 = \frac{3\lambda_2 L}{2a}$ $\qquad$ λ_1 = 650 nm

a = 0.5 nm

$\Delta y = y_2 - y_1 = \frac{3(\lambda_2-\lambda_1)}{2a}L$

$= \frac{3(655-650)}{2\times0.5\times10^{-3}}\times2\times10^{-9}$

$= \frac{3\times5\times2}{1\times10^{-3}}\times10^{-9}$

$= 3\times10^{-5}$

6. When light of frequency twice the threshold frequency is incident on the metal plate, the maximum velocity of emitted election is v_1. When the frequency of incident radiation is increased to five times the threshold value, the maximum velocity of emitted electron becomes v_2. If $v_2 = x\ v_1$, the value of x will be ______.

Ans. (2)

Sol. $hv = hv_{th} + \frac{1}{2}mv^2$

$v = 2v_{th}$

$2hv_{th} = hv_{th} + \frac{1}{2}mv_1^2$ ….. (1)

$v = 5v_{th}$

$5hv_{th} = hv_{th} + \frac{1}{2}mv_2^2$ ….(2)

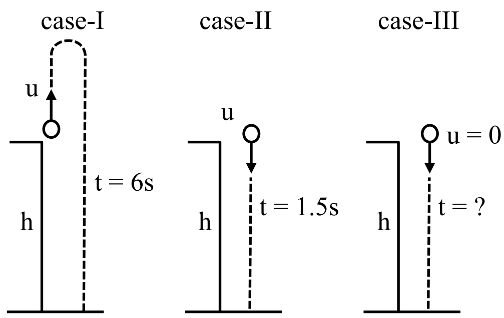
$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{hv_{th}}{4hv_{th}}$

$\left(\frac{v_1}{v_2}\right)^2 = \frac{1}{4} \Rightarrow \boxed{v_2 = 2v_1}$

7. From the top of a tower, a ball is thrown vertically upward which reaches the ground in 6 s. A second ball thrown vertically downward from the same position with the same speed reaches the ground in 1.5 s. A third ball released, from the rest from the same location, will reach the ground in ________ s.

Ans. (3)

Sol. Let height of tower be h and speed of projection in first two cases be u.



For case-I : 2[nd] equation $s = ut + \frac{1}{2}at^2$

$$h = -u(6) + \frac{1}{2}g(6)^2$$

H = – 6u + 18 g …. (i)

For case-II : $h = u(1.5) + \frac{1}{2}g(1.5)^2$

$$h = 1.5u + \frac{2.25g}{2} \quad \ldots \text{(ii)}$$

Multiplying equation (ii) by 4 we get

4h = 6u + 4.5 g …. (iii)

equation (i) + equation (iii) we get 5h = 22.5g

h = 4.5g … (iv)

For case-III :

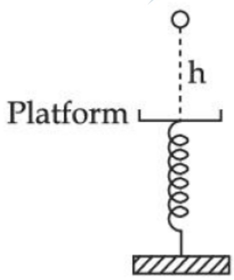
$$h = 0 + \frac{1}{2}gt^2 \quad \ldots \text{(v)}$$

Using equation (4) & equation (5)

$$4.5g = \frac{1}{2}gt^2$$

$t^2 = 9 \Rightarrow t = 3s$

8. A ball of mass 100 g is dropped from a height h = 10 cm on a platform fixed at the top of vertical spring (as shown in figure). The ball stays on the platform and the platform is depressed by a distance $\frac{h}{2}$. The spring constant is ________ Nm^{-1}. (Use $g = 10\ ms^{-2}$)



Ans. (120)

Sol. By energy conservation

PE = KE

$$mg\left(H + \frac{H}{2}\right) = \frac{1}{2}kx^2 \quad \left(x = \frac{H}{2}\right)$$

$$0.100 \times 10 \times \frac{3}{2}(0.10) = \frac{1}{2}k(0.05 \times 0.05)$$

$$k = \frac{3 \times 0.10}{0.05 \times 0.05}$$

$$= \frac{3 \times 1000}{25} = 120\ N/m$$

9. In a potentiometer arrangement, a cell gives a balancing point at 75 cm length of wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emf's of two cells respectively is 3 : 2, the difference in the balancing length of the potentiometer wire in above two cases will be ______ cm.

Ans. (25)

Sol. $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$

$$\frac{3}{2} = \frac{75cm}{\ell_2}$$

$$\ell_2 = 50cm$$

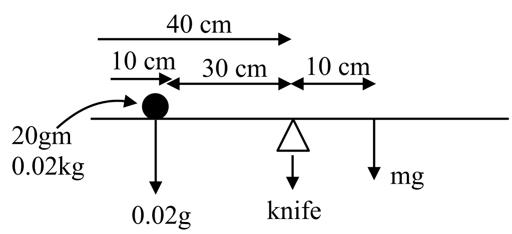
$$\ell_1 - \ell_2 = 75 - 50$$

= 25 cm

10. A metre scale is balanced on a knife edge at its centre. When two coins, each of mass 10 g are put one on the top of the other at the 10.0 cm mark the scale is found to be balanced at 40.0 cm mark. The mass of the metre scale is found to be x × 10^{-2} kg. The value of x is

Ans. (6)

Sol. Let mass of meter scale be m.



Balancing torque about knife edge

$(0.02g) \times (30 \times 10^{-2}) = mg \times (10 \times 10^{-2})$

$m = 0.06\ kg = 6 \times 10^{-2}\ kg$