

6

Work, Power and Energy

BRIEF REVIEW

Work In physics, the term work is used when a particle is displaced by the action of a force. Work is a scalar quantity. Unit is Joule (SI) and CGS unit is erg. Practical unit of work (particularly in electric consumption) is kWh. $1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$ and $1 \text{ J} = 10^7 \text{ ergs}$. Sometime eV is also used. $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. In problems of heat $1 \text{ calorie} = 4.186 \text{ J}$

$$dW = \vec{F} \cdot \vec{ds}$$

$W = \vec{F} \cdot \vec{s} = Fs \cos \theta$ if force is constant throughout.

$$W = \int F \cdot ds \text{ if force is variable}$$

Work can be positive or negative depending on the value of θ , for acute angles $\cos \theta$ is positive and hence, work is positive. For obtuse angle $\cos \theta$ may be negative making work negative. Positive work is parallel to displacement and negative work is opposite to displacement.

Work done in lifting a body up (against gravity) is positive and work done by the force of gravity (vertically downward motion) is negative.

No work will be done if the body is in static or dynamic equilibrium, i.e., $W = 0$ if $\sum F = 0$.

No work is done if displacement is zero or force is perpendicular to the displacement. Thus, work done by centripetal force and work done by moving charged particle in a magnetic field is zero i.e., $F = q(\vec{v} \times \vec{B})$ will do no work. Work done depends upon the frame of reference. With change

in frame of reference displacement may vary and hence work done could be different in different frame of references.

In a conservative field work done is path independent

$$W = \Delta PE = \int F \cdot ds$$

In a force versus displacement curve, work done is area under the graph. The algebraic sum of the area is to be found out as illustrated in fig. 6.1

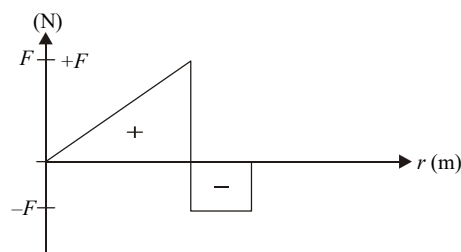


Fig. 6.1

$$W = \int_{V_1}^{V_2} P dV \text{ Area under Pressure (P) and Volume (V)}$$

curve is work done.

$W = \Delta KE$, i.e., work = change in KE . This is also called work energy theorem.

For positive work $KE_{\text{final}} > KE_{\text{initial}}$. Work energy theorem is valid for all types of forces (internal or external; conservative or non conservative).

In case of a spring $W = \frac{1}{2} kx^2$ where x is extension or compression in the spring.

$$W = \frac{1}{2} \text{ stress} \times \text{strain} \times \text{volume in elastic bodies}$$

Since work is independent of time. We define, **time rate of doing work is Power.**

$$P = \frac{dW}{dt} = \frac{d}{dt} (\vec{F} \cdot d\vec{s}) = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

Power is a scalar quantity. Its SI unit is Watt (W) or J/s. Practical unit of power is HP or bhp (british horse power)

$$1 \text{ bhp} = 746 \text{ W} = 550 \text{ ft-lb/s}$$

$$W = \int P \cdot dt \text{ or area under } P-t \text{ graph.}$$

Note *KE* can never be negative while *PE* can be both negative or positive. Potential energy is defined only for conservative forces. It does not exist for nonconservative forces.

$$\text{Elastic } PE = \frac{1}{2} kx^2 \text{ and is taken positive in all cases.}$$

$$\text{Electric } PE = \frac{q_1 q_2}{4\pi \epsilon_0 r} \text{ may be negative or positive.}$$

$$\text{Gravitational } PE = -\frac{GM_1 M_2}{r} \text{ may be negative or positive.}$$

Mechanical energy = *KE* + *PE* is conserved if internal forces are conservative and no work is done by nonconservative forces. If some of the internal forces are nonconservative mechanical energy of the system is not conserved.

$$\text{Total energy} = KE + PE + \text{internal energy.}$$

Internal energy is directly related to temperature. Larger the internal energy, higher is the temperature of the body.

Thermal energy is related to random motion of molecules while internal energy is related to motion as well as their configuration or arrangement.

$$E = mc^2 \text{ is mass energy relationship.}$$

Quantization of energy Planck has shown that the radiations emitted by a black body are quantized. Quantum nature of energy is confirmed in atomic and subatomic world. Even light energy is quantized.

SHORT CUTS AND POINTS TO NOTE

1. Work done $W = \vec{F} \cdot \vec{s} = Fs \cos \theta$ if force *F* is constant

$$W = \int F \cdot ds \quad \text{if force is variable}$$

$$W = \Delta KE \quad (\text{work energy theorem})$$

$$W = \Delta PE \quad (\text{for conservative forces})$$

$$W = \frac{1}{2} kx^2 \quad \text{in a spring}$$

$$W = \frac{1}{2} \text{ stress} \times \text{strain} \times \text{volume} \quad (\text{in elastic bodies})$$

$$W = \frac{F \cdot x}{2} \text{ where } x \text{ is extension produced in a spring or elastic bodies.}$$

$$W = \int P \, dV \text{ where } P \text{ is pressure and } V \text{ is volume.}$$

$$W = \int P \cdot dt \text{ where } P \text{ represents power}$$

$$2. \text{ Power } P = \frac{dW}{dt} = \frac{dE}{dt}$$

$$P = \vec{F} \cdot \vec{v} \quad \text{if } F \text{ is constant}$$

$$P = \int \vec{F} \cdot d\vec{v} \text{ if } F \text{ is variable or } v \text{ is variable.}$$

3. Potential energy exists only for conservative forces. Nonconservative forces do not show *PE*. If a particle moves in a circle then binding energy = *KE* + *PE* =

$$\frac{1}{2} PE = -KE. \text{ In a bound system like this } PE \text{ is negative.}$$

4. In conservative forces work done is independent of path followed. It depends only on the initial and final position. Total work done in a round trip is zero.

$$5. PE = \frac{1}{2} kx^2 \text{ (in a spring and is only positive)}$$

$$PE = \frac{-GM_1 M_2}{r^2} \quad (\text{in gravitational fields). It}$$

may be positive or negative

$$PE = mgh \quad \text{if } h \text{ is small}$$

$$PE = \frac{q_1 q_2}{4\pi \epsilon_0 r} \quad (\text{in electric fields). It may be}$$

positive or negative.

6. If a body is in static or dynamic equilibrium then *W* = 0.
7. If a force is always perpendicular to velocity then work done by this force is zero.
8. Mechanical energy = *KE* + *PE* is conserved if internal forces are conservative and do no work.
9. *KE* + *PE* is not conserved if nonconservative forces are present.

$$10. KE = \frac{p^2}{2m} \text{ where } p \text{ is momentum of the body.}$$

11. If a lighter and a heavier body have equal *KE* then heavier body has more momentum.

12. If a lighter and heavier body have equal momentum then lighter body has more KE .
13. Area under Power time graph gives work.
14. $\Delta U = \text{change in } PE = \int F \cdot dr$ for conservative forces.
15. If $\frac{dU}{dr} = 0$, body is said to be in equilibrium.
Equilibrium is stable if U is minimum; unstable if U is maximum and neutral if $U = \text{constant}$.
16. If $\frac{l}{n}$ th part of the chain hangs then the work done

to pull up the hanging chain is $\frac{mgl}{2n^2}$. [See Fig 6.2]

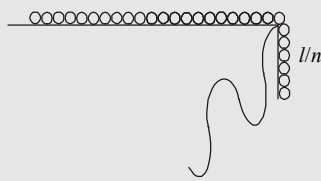


Fig. 6.2

17. If maximum displacement in a spring is to be found use $W = \frac{1}{2} kx^2$ if steady state displacement in a spring is to be found use $F = -kx$.
18. For a Rolling body $KE = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$.
19. Equation of dynamics of a body with variable mass

$$F = \frac{mdv}{dt} + \frac{dm}{dt} v$$
 if reference frame is at rest

$$F = \frac{mdv}{dt} + \frac{vdm}{dt} - \frac{udm}{dt}$$
 if reference frame is moving with a velocity u .
20. Change in total energy $\Delta E = W_{\text{ext}} + W_{\text{int}}$ (nonconservative).

CAUTION

1. Applying $W = F \cdot s$ even if force is variable.
 \Rightarrow When force is variable use $W = \int F \cdot ds$ or $W = \Delta KE$ or $W = PE$ as it suits.
2. To find work done even when the force is perpendicular to velocity.
 \Rightarrow No work will be done when force is perpendicular to velocity. For example, no work is done by centripetal force in a circular motion. No work is done by the magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$ as \vec{F} and \vec{v} are perpendicular.
3. Assuming when a body strikes another body connected to a spring, as shown in Fig. 6.3, it imparts its complete KE to the spring.

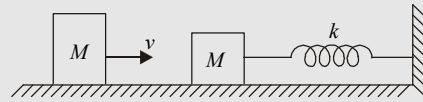


Fig. 6.3

- \Rightarrow If a body would have not been connected to the spring only then complete KE of the body would have been converted to PE stored in the spring. But if a body is connected then first conserve momentum. The KE of system after collision will be equal to PE stored in the spring.
4. To find maximum displacement in a spring applying the force equation i.e. $F = -kx$.
 $\Rightarrow F = -kx$ will provide steady state displacement. Maximum displacement is obtained when we equate energy i.e.

$$W = \frac{1}{2} kx^2.$$
5. Considering work done in a spring is always $\frac{1}{2} kx^2$.
 \Rightarrow If the force moves the block slowly and steadily then $W = \frac{1}{2} kx^2$ but if the movement of the block is very fast (or a force applied for a very short interval called sudden force/impulse) then work done by force F is $F \cdot x$.
6. Conserve energy even when nonconservative force/s are present.
 \Rightarrow If non conservative forces perform work then energy is not conserved.
7. Assuming even a rolling body has $KE = \frac{1}{2} mv^2$.
 \Rightarrow Rolling body possesses both linear KE and rotational KE . Total KE is sum of the two

$$KE_{\text{Tot}} = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2.$$
8. Assuming gravitational PE is only mgh .
 \Rightarrow The gravitational PE is mgh when distance from earth is not very large. If the distance is large employ

$$PE = -\frac{GMm}{R+h}.$$

If the attraction between two bodies is involved then

$$PE = \frac{-Gm_1m_2}{r}.$$
9. In a system of mutual forces considering only KE due to one particle is equal to ΔPE .
 \Rightarrow Consider KE due to both the particles.

10. When a vehicle is moving up an incline and the efficiency of an engine is given then applying efficiency in a wrong manner. For example, a truck of mass 20 ton is moving up an incline of 1 : 10 with velocity 10 ms^{-1} . The friction is 500 N per ton. Efficiency is 80%. Find power. Then using

$$P_{\text{net}} = 0.8 (mg \sin \theta + F_f) v.$$

$$\Rightarrow \text{Apply } P_{\text{eff}} = (mg \sin \theta + F_f) v$$

$$\text{and } P_{\text{engine}} \times (0.8) = P_{\text{eff}}$$

$$\text{or } P_{\text{engine}} = \frac{P_{\text{eff}}}{0.8} = \frac{(mg \sin \theta + F_f) v}{0.8}.$$

SOLVED PROBLEMS

1. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground and then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is

[AIEEE 2005]

- (a) 40 ms^{-1} (b) 20 ms^{-1}
(c) 10 ms^{-1} (d) $10\sqrt{30} \text{ ms}^{-1}$

Solution (a) $mgh = \frac{1}{2}mv^2$ or $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 80} = 40 \text{ ms}^{-1}$.

2. The block of mass M moving on the frictionless horizontal surface collides with a spring of spring constant K and compresses it by L . The maximum momentum of the block after collision is

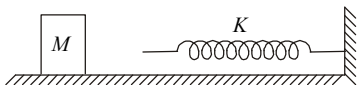


Fig 6.4

- (a) $\sqrt{MK} L$ (b) $\frac{KL^2}{2M}$
(c) Zero (d) $\frac{ML^2}{K}$

[AIEEE 2005]

Solution (a) $\frac{1}{2}KL^2 = \frac{p^2}{2M}$ or $p = \sqrt{MK} L$

3. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is

- (a) $2 S^2 Y$ (b) $\frac{S^2}{2Y}$
(c) $2Y/S^2$ (d) $S/2Y$

[AIEEE 2005]

Solution (b) $U = \frac{1}{2} \text{ stress} \times \text{strain} = \frac{1}{2} \times S \times \frac{S}{Y} = \frac{S^2}{2Y}$.

4. A body of mass m is accelerated uniformly from rest to a speed v in a time T . The instantaneous power delivered to the body as a function of time is

- (a) $\frac{mv^2}{T^2} t$ (b) $\frac{mv^2}{T^2} t^2$
(c) $\frac{mv^2}{2T^2} t$ (d) $\frac{mv^2}{2T^2} t^2$

[AIEEE 2005]

Solution (a) $P = ma$ $v = ma^2 t = m \left(\frac{v}{T}\right)^2 t$

5. A car is moving on a straight road with a speed 100 ms^{-1} . The distance at which car can be stopped is [$\mu_k = 0.5$]

- (a) 800 m (b) 1000 m
(c) 100 m (d) 400

[AIEEE 2005]

Solution (b) $D = \frac{u^2}{2g\mu_k} = \frac{100 \times 100}{2 \times 10 \times 0.5} = 1000$

6. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from them

- (a) $13.34 \times 10^{-10} \text{ J}$ (b) $3.33 \times 10^{-10} \text{ J}$
(c) $6.67 \times 10^{-11} \text{ J}$ (d) $6.67 \times 10^{-10} \text{ J}$

[AIEEE 2005]

Solution (d) $W = \Delta U = \frac{GMm}{r}$

$$= \frac{6.67 \times 10^{-11} \times 100 \times 10^{-2}}{10^{-1}} = 6.67 \times 10^{-10} \text{ J}.$$

7. A force F acting on an object varies with distance x as shown in fig 6.5. The work done by the force in moving the object from $x = 0$ to $x = 6 \text{ m}$ is

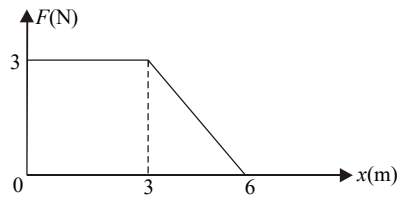


Fig 6.5

- (a) 18 J (b) 13.5 J
(c) 9 J (d) 4.5 J

[CBSE PMT 2005]

Solution (b) $W = \text{Area under } F-x \text{ graph.}$

8. A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 ms^{-1} . The KE of other mass is

- (a) 324 J (b) 486 J
(c) 256 J (d) 524 J

[CBSE PMT 2005]

Solution (b) $m_1 v_1 = m_2 v_2 \therefore v_2 = \frac{18 \times 6}{12} = 9 \text{ ms}^{-1}$

$$KE = \frac{1}{2} \times 12 (9)^2 = 486 \text{ J}$$

9. A block of mass 10 kg is moving in x -direction with a constant speed 10 ms^{-1} . It is subjected to a retarding force $F = -0.1 x \text{ J/m}$ during its travel from $x = 20 \text{ m}$ to $x = 30 \text{ m}$. Find the final KE .

- (a) 475 J (b) 450 J
(c) 275 J (d) 250 J

[AIIMS 2005]

Solution (a) $KE_f = \int_{20}^{30} F \cdot dx + KE_{\text{initial}}$

$$= \int_{20}^{30} -(0.1)x dx + \frac{1}{2} \times 10 \times 10^2$$

$$= \left[-\frac{30^2}{2} + \frac{20^2}{2} \right] (-0.1) + 500 = 475 \text{ J}$$

10. Energy required to break a bond of DNA is approximately

- (a) $\sim 1 \text{ eV}$ (b) $\sim 0.1 \text{ eV}$
(c) $\sim 0.01 \text{ eV}$ (d) $\sim 2.1 \text{ eV}$

[AIIMS 2005]

Solution (a)

11. A particle is moving with centripetal force $\frac{k}{r^2}$. Find the total energy associated.

[CBSE PMT Mains 2005]

Solution $\frac{mv^2}{r} = \frac{k}{r^2}$ or $mv^2 = \frac{k}{r}$

$$\text{Total energy} = KE + PE = -KE = -\frac{k}{2r}$$

12. A spring does not obey Hooke's law. Rather it follows, $F = kx - bx^2 + cx^3$. If the spring has natural length l and compressed length l' then find the work done.

Solution $W = \int_0^{l-l'} F \cdot dx = \frac{k}{2} (l-l')^2 - \frac{b}{3} (l-l')^3 + \frac{c}{4} (l-l')^4$

13. A space shuttle of mass 86400 kg is revolving in a circular orbit of radius $6.66 \times 10^6 \text{ m}$ around the earth. It takes 90.1 minutes for the shuttle to complete one revolution. On a repair mission it moves 1 m closer to a disabled satellite every 3.0 s. Find the KE of shuttle relative to the satellite.

- (a) 4800 J (b) 480 J
(c) $2.59 \times 10^{12} \text{ J}$ (d) $2.69 \times 10^{11} \text{ J}$

Solution (a) $\frac{1}{2} m v^2 = \frac{1}{2} \times 86400 \times \left(\frac{1}{3}\right)^2 = 4800 \text{ J}$

14. A particle of mass 6 kg moves according to the law $x = 0.2 t^2 + 0.02 t^3$. Find the work done by the force in first 4 s.

- (a) 1.1231 J (b) 2.6428 J
(c) 2.1324 J (d) 1.6428 J

Solution (d) $W = \frac{1}{2} m (v_f^2 - v_i^2)$

$$v = \frac{dx}{dt} = 0.4 t + 0.06 t^2$$

$$= \frac{1}{2} \times 6 \left[(0.74)^2 - 0 \right] = 1.6428 \text{ J}$$

15. A moving electron has $KE 'K'$. When a certain amount of work is done, it moves with one quarter of its velocity in opposite direction. Find the work in terms of K .

- (a) $\frac{-15}{16} K$ (b) $\frac{-17}{16} K$
(c) $\frac{-5}{4} K$ (d) $\frac{-3}{4} K$

Solution (b) $\frac{-K}{(4)^2} = K + W$ or $W = \frac{-17}{16} K$

16. A brick of mass 1.8 kg is kept on a spring of spring constant $K = 490 \text{ N m}^{-1}$. The spring is compressed so

that after the release brick rises to 3.6 m. Find the compression in the spring.

- (a) 0.21 m (b) 0.322 m
(c) 0.414 m (d) 0.514 m

Solution (d) $\frac{1}{2} kx^2 = mgh$ or

$$x = \sqrt{\frac{2 \times 1.8 \times 10 \times 3.6}{490}} = \frac{3.6}{7} = 0.514 \text{ m}$$

17. 75 kW engine is generating full power. It is able to provide a 700 kg airplane a speed 2.5 ms^{-1} . Find the fraction of engine power used.

- (a) $\frac{1}{100}$ (b) $\frac{3}{100}$
(c) $\frac{5}{200}$ (d) $\frac{7}{300}$

Solution (d) $\frac{P_{\text{used}}}{P_{\text{supplied}}} = \frac{F \cdot V}{P_{\text{supplied}}} = \frac{700 \times 10 \times 2.5}{750 \times 10^3} = \frac{7}{300}$

18. In an ice rink a skator is moving at 3 ms^{-1} and encounters a rough patch that reduces her speed by 45% due to a friction force that is 25% of her weight. Find the length of the rough patch

- (a) 1.56 m (b) 1.46 m
(c) 1.36 m (d) 1.26 m

Solution (d) $l = \frac{v_i^2 - v_f^2}{2\mu g} = \frac{3^2(1 - (.55)^2)}{2 \times 2.5} = 1.8 [.] = 1.26 \text{ m}$

19. A pump having efficiency 75% lifts 800 kg water per minute from a 14 m deep well and throws at a speed of 18 ms^{-1} . Find the power of the pump.

- (a) 2060 W (b) 2490 W
(c) 3218 W (d) 1400 W

Solution (b) $P_{\text{eff}} = \frac{dm}{dt} gh = \frac{800}{60} \times 10 \times 14 = \frac{5600}{3} \text{ W},$

$$P_{\text{app}} = P_{\text{eff}} \times \frac{4}{3} = \frac{22400}{9} = 2488.88 \text{ W}$$

20. The heart takes and discharges 7500 l of blood in a day. Density of blood = $1.05 \times 10^3 \text{ kg m}^{-3}$. If on an average it takes to a height of 1.6 m. Find the power of the heart pump.

- (a) 1.63 W (b) 1.36 W
(c) 1.96 W (d) 2.46 W

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Solution (a) $P = \frac{dm}{dt} gh = \rho \frac{dV}{dt} gh$

$$P = \frac{1.05 \times 10^3 \times 7500 \times 10^{-3}}{24 \times 60 \times 60} \times 10 \times 1.6 = 1.63 \text{ W}$$

21. In the system shown, find the speed with which 12 kg block weight hit the ground

- (a) $2\sqrt{10} \text{ ms}^{-1}$ (b) $4\sqrt{2} \text{ ms}^{-1}$
(c) $2\sqrt{5} \text{ ms}^{-1}$ (d) 3 ms^{-1}

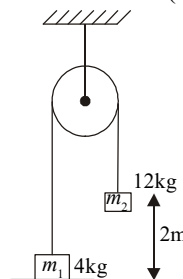


Fig 6.6

Solution (c) $m_2 gh - m_1 gh = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$$\text{or } 8 \times 10 \times 2 = \frac{1}{2} \times 16 v^2$$

$$v = 2\sqrt{5} \text{ ms}^{-1}$$

22. A skier starts at the top of a snowball with negligible speed and skis straight down the side. At what point does he lose contact with the snowball.

- (a) $\theta = \sin^{-1} \frac{2}{3}$ (b) $\theta = \cos^{-1} \frac{2}{3}$
(c) $\theta = \tan^{-1} \frac{2}{3}$ (d) none

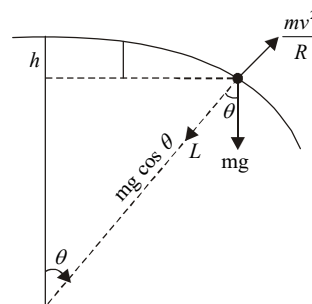


Fig 6.7

Solution (b) $\frac{mv^2}{2} = mgh = mgR(1 - \cos \theta)$

$$\text{or } \frac{mv^2}{R} = 2mg(1 - \cos \theta).$$

$$\text{Condition of losing contact } \frac{mv^2}{R} = mg \cos \theta$$

$$\text{or } 2mg(1 - \cos \theta) = mg \cos \theta \text{ or } \theta = \cos^{-1} \frac{2}{3}.$$

23. The force $F = Cy^2 \hat{j}$ with C as negative constant is ____
- (a) conservative (b) restoring
(c) nonconservative (d) none

Solution (a) $W_{\text{tot}} = W_{AB} + W_{BC} + W_{CA} = 0 + \int_0^a -Cy^2 dy +$

$$\int_a^0 -Cy^2 dy = 0$$

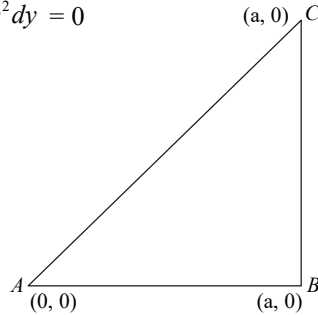


Fig 6.8

Since the work done in a round trip is zero.

24. A particle has PE vs x curve as shown in Fig. 6.9. The unstable equilibrium occurs at

- (a) A (b) B
(c) C (d) D

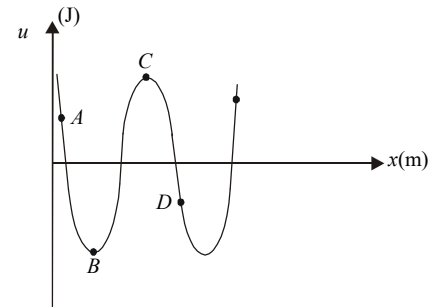


Fig 6.9

Solution (c) $\sum F = 0$ and $PE = \text{maximum}$ for unstable equilibrium.

TYPICAL PROBLEMS

25. A particle is released from the top of a quarter circle of radius 1.6 m. It stops at C, 3 m away from B. Find coefficient of friction which is present only on the horizontal surface.

- (a) 0.533 (b) 0.333
(c) 0.433 (d) none

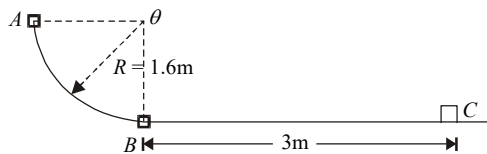


Fig 6.10

Solution (a) $\mu = \frac{2gh}{3} = \frac{1.6}{3} = 0.533$

26. A 500 g ball is released from a height of 4m. Each time it makes contact with the ground it loses 25% of its energy. Find the KE it possesses after 3rd hit.

- (a) 15 J (b) 11.25 J
(c) 8.44 J (d) none

Solution (c) $KE = mgh \left(\frac{3}{4}\right)^3 = \frac{1}{2} \times 10 \times 4 \left(\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right)$

$$= \frac{270}{32} \text{ J.}$$

27. The following data is obtained from a computer simulation for a patted baseball with mass 0.145 kg including air resistance. Find the work done by the air

on the base ball as it moved from maximum height to back to its position

t (s)	x (m)	y (m)	v_x (ms ⁻¹)	v_y (ms ⁻¹)
0	0	0	30	40
3.05	70.2	53.6	18.6	0
6.59	124.4	0	12.0	-30

- (a) 0 (b) 106.56 J
(c) 76 J (d) 213 J

Solution (b) $W = \frac{1}{2} m (v_{x1}^2 + v_{y1}^2 - v_{x2}^2 - v_{y2}^2)$

$$= \frac{1}{2} \times 1.45 (30^2 + 40^2 - 12^2 - 30^2) = 106.56 \text{ J}$$

28. A locomotive of mass m starts with a velocity $v = a\sqrt{x}$. Find the work done by all the forces acting on locomotive in first t sec.

- (a) $\frac{ma^2 t^2}{4}$ (b) $\frac{ma^4 t^2}{4}$
(c) $\frac{ma^4 t^2}{8}$ (d) $\frac{ma^4 t^2}{2}$

Solution (c) $\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{a}{2\sqrt{x}} (a\sqrt{x}) \Rightarrow h = \frac{a^2}{2}$

$$\therefore F = \frac{ma^2}{2}$$

$$S = ut + \frac{1}{2}ft^2 = 0 + \frac{a^2}{4}t^2$$

$$W = F \cdot s = \frac{ma^4t^2}{8}$$

29. The KE of a particle moving along a circle of radius R depends upon the distance covered x as $T = ax^2$. Find the force on the particle.

Solution $\frac{mv^2}{2} = ax^2$ or $v^2 = \frac{2ax^2}{m}$... (i)

differentiating (i) $2v \frac{dv}{dt} = \frac{4axv}{m}$

or acceleration $a_t = \frac{2ax}{m}$

Hence net force $= m\sqrt{a_r^2 + a_t^2}$

$$= m\sqrt{\left(\frac{2ax^2}{mR}\right)^2 + \left(\frac{2ax}{m}\right)^2}$$

$$= 2ax\sqrt{1 + \left(\frac{x}{R}\right)^2}$$

30. Two blocks of mass m_1 and m_2 are connected by a non deformed light spring resting on a horizontal table. The coefficient of friction between the blocks and table is μ . Find the minimum force applied on block 1 which will move the block 2 also. See Fig 6.11.

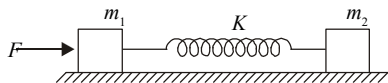


Fig 6.11

Solution If x is the compression in the spring when block m_2 is just to move then

$$Kx = \mu m_2 g \text{ ... (1) for min. force; } W = 0$$

$$F \cdot x - \frac{1}{2} Kx^2 - \mu m_1 g x = 0 \text{ or } \frac{Kx}{2} = F - \mu m_1 g \text{ ... (2)}$$

$$\text{From (1) and (2) } F = \mu g \left(m_1 + \frac{m_2}{2} \right)$$

31. A chain of mass m and length l rests on a rough table with part overhanging. The chain starts sliding down by itself if overhanging part is $\frac{l}{3}$. What will be the work performed by the friction forces acting on the chain by the moment it slides completely off the table.

(a) $\frac{Mgl}{3}$ (b) $\frac{2Mgl}{3}$

(c) $\frac{2Mgl}{9}$ (d) $\frac{Mgl}{9}$

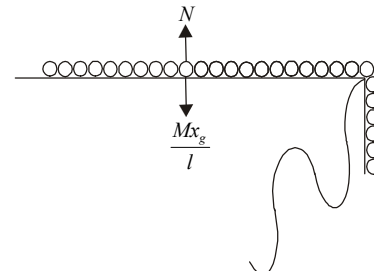


Fig 6.12

Solution (d) $\mu \frac{2}{3} mg = \frac{m}{3} g$ or $\mu = \frac{1}{2}$

Assume at any instant the length of the chain on the table is x then force of friction $= \mu N = \frac{\mu M}{l} xg$

Work done against friction $\int_0^{2/3} \mu \frac{M}{l} xg \cdot dx$

$$= \mu \frac{Mg}{l} \frac{x^2}{2} \Big|_0^{2/3} = \frac{1}{2} \frac{M}{l} \frac{g}{2} \left(\frac{4l^2}{9} \right) = \frac{Mgl}{9}$$

32. A horizontal plane supports a stationary vertical cylinder of radius R and a disc attached to the cylinder by a horizontal thread AB of length l_0 as shown in fig. 6.13. Disc is given a velocity v_0 . How long will it take to strike the cylinder. Assume no friction.

(a) $\frac{l_0}{v_0}$ (b) $\frac{l_0^2}{Rv_0}$

(c) $\frac{l_0^2}{2Rv_0}$ (d) $\frac{l_0^3}{3R^2v_0}$

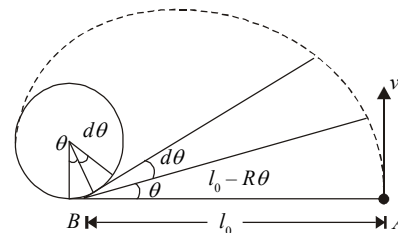


Fig 6.13

Solution (c) Let s be the total distance covered by disc.

Then $t = \frac{s}{v_0}$

At any instant $ds = (l_0 - R\theta) d\theta$

$$\therefore s = \int_0^{l_0/R} (l_0 - R\theta) d\theta = \frac{l_0^2}{R} - \frac{l_0^2}{2R} = \frac{l_0^2}{2R}$$

$$\therefore t = \frac{l_0^2}{2Rv_0}$$

33. In the system of two masses m_1 and m_2 tied through a light string passing over a smooth light pulley. Find the acceleration of COM. (Centre of mass).

(a) $\frac{(m_1 - m_2)}{m_1 + m_2} g$ (b) $\frac{(m_1 - m_2)^2}{m_1 + m_2} g$
 (c) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \frac{g}{2}$ (d) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right) \frac{g}{2}$

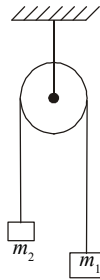


Fig 6.14

Solution (b) $a_{\text{COM}} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$ Note that $a_2 = -a_1$,

$$\therefore a_{\text{COM}} = \frac{(m_1 - m_2)a_2}{m_1 + m_2} \text{ and } a_1 = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

$$= \frac{(m_1 - m_2)^2 g}{(m_1 + m_2)^2}$$

34. Two blocks of masses m_1 and m_2 joined to a non deformed spring of length l_0 and stiffness K as shown in fig. 6.15. If a force F is applied on block of mass m_2 . Find the maximum separation between the blocks.

(a) $l_0 + \frac{m_1 F}{k(m_1 + m_2)}$ (b) $l_0 + \frac{m_2 F}{k(m_1 + m_2)}$
 (c) $l_0 + \frac{2m_2 F}{m_1 + m_2}$ (d) $l_0 + \frac{2m_1 F}{m_1 + m_2}$

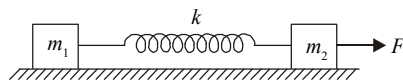


Fig 6.15

Solution (d) x_1 and x_2 be the maximum displacement in m_1 and m_2 respectively.

$$a_{\text{COM}} = \frac{F}{m_1 + m_2}$$

$$\frac{1}{2} k (x_1 + x_2)^2 = \frac{m_1 F}{m_1 + m_2} x_1 + \left(F - \frac{m_2 F}{m_1 + m_2} \right) x_2$$

$$= \frac{m_1 F}{m_1 + m_2} (x_1 + x_2)$$

or $x_1 + x_2 = \frac{2m_1 F}{k(m_1 + m_2)}$

Thus maximum separation $= l_0 + x_1 + x_2$

$$= l_0 + \frac{2m_1 F}{k(m_1 + m_2)}$$

35. A particle of mass m is moving in a circular path of constant radius r such that radial acceleration $a_r = k^2 t^2 r$. Find the power delivered to the particle by the forces acting on it.

(a) $2\pi m k^2 r^2 t$ (b) $m k^2 r^2 t$
 (c) $\frac{1}{3} m k^4 r^2 t^3$ (d) 0

Solution (b) $\frac{v^2}{r} = k^2 t^2 r$

or

$$v = k t r \quad F = \frac{mdv}{dt} = m k r \quad \text{and}$$

Power $P = \vec{F} \cdot \vec{v}$

$$P = m k r (t k r) = m k^2 r^2 t$$

36. You lift a suitcase from the floor and keep it on a table. The work done by you on the suitcase does not depend on.

(a) the path taken by the suitcase.
 (b) the time taken by you in doing work.
 (c) weight of the suitcase.
 (d) initial and final position.

Solution (a) and (b) gravitational force is conservative; work done is independent of path and time.

37. A particle of mass m is attached to a light string of length l . The other end of which is fixed. Initially the string is kept horizontal and then particle is given an upward velocity v so that it is just able to complete the circle. Then

(a) the string becomes slack when the particle reaches the highest point.
 (b) the velocity of the particle becomes zero at the highest point.

- (c) the KE of the ball at highest position was $\frac{1}{2}mv^2 = mgl$.
- (d) the particle again passes through initial position

Solution (a), (d)

38. In the Fig. 6.16 pulley is light and smooth. Thread is massless. On applying force F , KE increases by 20 J in 1 s.

- (a) tension in the string is Mg .
- (b) the tension in the string is F .
- (c) work done by the tension in 1 s is 20 J.
- (d) the work done by the force of gravity is 20 J in 1 s.

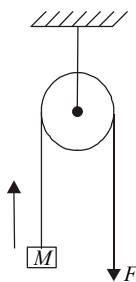


Fig 6.16

Solution (b)

39. In a factory, 2000 kg metal is lifted by 12 m in 1 minute by a crane. The minimum horse power of the engine is

- (a) 320 bhp (b) 32 bhp
- (c) 5.3 bhp (d) 6.4 bhp

Solution (c) $P = \frac{dmgh}{dt} = \frac{2000}{60} \times 9.8 \times 12$

$$P \text{ (bhp)} = \frac{400 \times 9.8}{746} = 5.3 \text{ bhp}$$

40. One end of a light spring of stiffness k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. The work done by the spring on displacement x is $\frac{1}{2}kx^2$. The possible cases may be

- (a) the spring was initially compressed by x and finally it has acquired natural state.
- (b) the spring was initially stretched by x and finally in natural state.
- (c) the spring was initially in natural state and finally compressed by x .
- (d) the spring was initially in natural state and finally stretched by x .

Solution (a), (b). In (c) and (d) we do the work on spring.

PASSAGE 1

Read the following passage and answer the questions given at the end.

Consider a man standing on frictionless roller skates on a level surface facing a rigid wall. He pushes against the wall, setting himself in motion to the right. The forces acting on him are his weight W , upward normal forces n_1 and n_2 exerted by the ground on his skates, and the horizontal force \vec{F} exerted by him on the wall. There is no vertical displacement. Hence W , n_1 and n_2 do no work. F is horizontal force that accelerates him right but parts of the body where that force is applied (the hands of the man) do not move. Thus, force F does no work.

1. Where does the man's kinetic energy come from?

- (a) from his muscle power
- (b) from the force he applied on the wall
- (c) whatever he ate.
- (d) from muscular motion

Solution (d) We can not represent man as a point mass. Different parts of the body must have different motions. Though his hands are stationary but the forces may appear at other body parts and do work on another part. This happens in this case.

2. Can we consider work energy theorem as a generalization in Physics for any system?

- (a) Yes
- (b) No, in some cases it fails particularly in thermodynamics
- (c) can not say.

Solution (b) In many cases we can apply. But in thermodynamics

$$dQ = dU + dW$$

PASSAGE 2

Read the following passage and answer the questions given at the end.

At a point where $u(x)$ has a minimum value such as $x = x_0$, the

slope of the curve is zero, i.e., $F(x_0) = \left. \frac{-du}{dx} \right|_{x=x_0} = 0$. A particle

at rest at this point will remain at rest. If the particle is slightly

displaced from x_0 then $F(x) = \frac{-du}{dx}$ tends to bring it back and particle will oscillate about the equilibrium point. Such an equilibrium is called stable equilibrium.

At a point where $u(x)$ has a maximum value say at $x = x_1$, the slope of the curve is zero, i.e., $F(x_1) = \left. \frac{-du}{dx} \right|_{x=x_1} = 0$.

A particle at rest at this point will remain at rest. However, if the particle is displaced even slightly from this point, the force $\left. \frac{-du}{dx} \right|_{x=x_1} = F(x_1)$ will push it further apart from equilibrium point. Such an equilibrium is termed as unstable equilibrium.

- Assume at $x = x_2$, $u(x)$ is constant. Slope $\frac{-du}{dx} = 0$. The particle is displaced slightly from $x = x_2$. Then
 - particle will return back to x_2 after oscillating.
 - particle will move further away from x_2 .
 - particle will stay at x_2 .
 - particle will come to a point where $\frac{-du}{dx}$ is maximum or minimum.

Solution (b) Since there is no repelling or restoring force at this point, particle stays there at the displaced position.

- Assume at any point $u(x) = 0$. Which kind of equilibrium exists at this point?
 - stable
 - unstable
 - dynamic
 - static
 - none

Solution (e) Since the particle does not possess any energy initially. It is very difficult to displace it.

- A particle is at D and is slightly displaced
 - it will execute SHM and return to D finally.
 - it will oscillate (but not SHM) and return to D finally.
 - it will crossover E and stay at F .
 - it will stay at E .

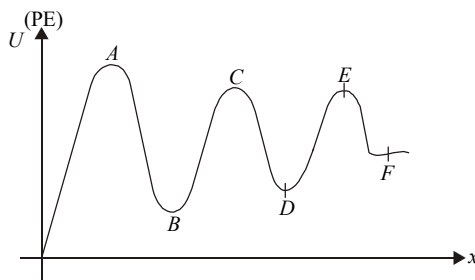


Fig 6.17

Solution (a)

PASSAGE 3

Read the following passage and answer the questions given at the end.

On a winter day in Srinagar (Jammu & Kashmir), a warehouse worker is shoving boxes up a rough plank inclined at an

angle α above the horizontal. The plank is partially covered with ice, with more ice near the bottom of the plank than near the top, so that the coefficient of friction increases with the distance x along the plank $\mu = Ax$, where A is positive constant. Bottom of the plank is considered at $x = 0$. Coefficient of kinetic and static friction for this plank are equal and equal to μ . The worker shoves the box up the plank so that it leaves the bottom of the plank with velocity V_0 .

- Find the velocity V_0 that the box reaches the height h .

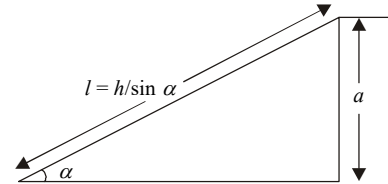


Fig 6.18

- $\sqrt{2gh}$
- $\sqrt{\frac{gA \cos \alpha h^2}{\sin^2 \alpha}}$
- $\sqrt{2gh + \frac{gA \cos \alpha h^2}{\sin^2 \alpha}}$
- none

- Find the condition that if box once comes to rest, it remains at rest (somewhere in between the top & bottom).

- $V_0^2 = \frac{2g \sin^2 \alpha}{A \cos \alpha}$
- $\frac{3g \sin^2 \alpha}{A \cos \alpha}$
- $\frac{2g \cos^2 \alpha}{A \sin \alpha}$
- $\frac{3g \cos^2 \alpha}{A \sin \alpha}$

Solution

$$1(c) \quad \frac{1}{2} m v_0^2 = mgh + \int_0^{h/\sin \alpha} \mu mg \cos \alpha dx, \quad l = \frac{h}{\sin \alpha}$$

$$\frac{v_0^2}{2} = gh + gA \cos \alpha \int_0^{h/\sin \alpha} x dx$$

$$v_0 = \sqrt{2gh + gA \cos \alpha \frac{h^2}{\sin^2 \alpha}}$$

- The box will come to rest if final velocity becomes zero before reaching highest point.

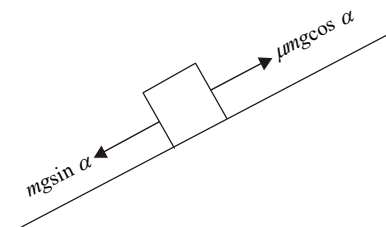


Fig 6.19

$$\frac{mv_0^2}{2} = mg \sin \alpha x + \int_0^x \mu mg \cos \alpha dx$$

$$\text{or } v_0^2 = 2g \sin \alpha x + \mu g \cos \alpha x^2$$

it will remain at rest if the net force at that place is zero
i.e.,

$$\mu mg \cos \alpha = mg \sin \alpha$$

$$\text{or } \mu x \cos \alpha = \sin \alpha \text{ or } x = \frac{\sin \alpha}{\mu \cos \alpha}$$

$$\begin{aligned} \therefore v_0^2 &= 2g \sin \alpha \left(\frac{\sin \alpha}{\mu \cos \alpha} \right) + \mu g \cos \alpha \left(\frac{\sin \alpha}{\mu \cos \alpha} \right)^2 \\ &= \frac{3g \sin^2 \alpha}{\mu \cos \alpha} \end{aligned}$$

PASSAGE 4

Read the following passage and answer the questions given at the end.

A diver jumps off a high board into a swimming pool. Before he jumps the diver bounces on end of the board he hits the water moving pretty fast. As the diver plummets towards the water, he loses speed. The gravitational potential energy and elastic potential energy is converted to his kinetic energy. This energy is lost in doing work against the water resistance. Energy transformation occurs at every step but energy is not lost. We say energy is conserved from one form to the other.

- As the diver enters the water the work done by water on the diver is ----- and work done by gravity on him is -----
 (a) negative, negative (b) negative, positive
 (c) positive, positive (d) positive, negative
- In which form the diver's KE is transformed in water.
 (a) electrical (b) heat
 (c) viscous (d) surface tension

Solution

- (b) 2. (b)

QUESTIONS FOR PRACTICE

- A motorcycle of mass m resting on a frictionless road moves under the influence of a constant force F . The work done by this force in moving the motorcycle is given by $F^2 t^2 / 2m$, where t is the time in seconds.

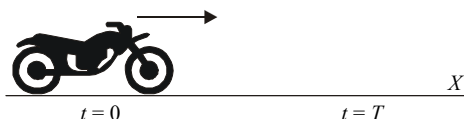


Fig 6.20

Ratio of instantaneous power to average power of the motorcycle in $t = T$ second is

- 1 : 1
 - 2 : 1
 - 3 : 2
 - 1 : 2
- Two ends of a uniform garland of mass m and length L are hanging vertically as shown. At instant t the end Z is released. If y is the distance moved by this end in time dt , change in momentum of an element of length dy is given as

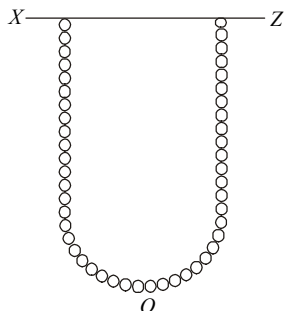


Fig 6.21

$$(a) \frac{m}{L} dy (2gy)^{\frac{1}{2}} \text{ (down ward)}$$

$$(b) \frac{m}{L} dy (2gy)^{\frac{1}{2}} \text{ (upward)}$$

$$(c) \frac{-m}{L} dy dy \sqrt{gy} \text{ (upward)}$$

$$(d) \frac{m}{L} dy (gy)^{\frac{1}{2}} \text{ (upward)}$$

- A compressed spring of spring constant k releases a ball of mass m . If the height of spring is h and the spring is compressed through a distance x , the horizontal distance covered by ball to reach ground is

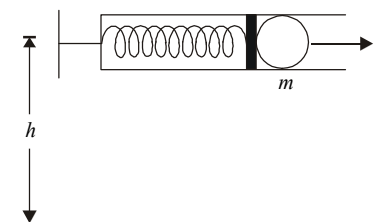


Fig 6.22

$$(a) x \sqrt{\frac{kh}{mg}}$$

$$(b) \frac{xkh}{mg}$$

$$(c) x \sqrt{\frac{2kh}{mg}}$$

$$(d) \frac{mg}{x\sqrt{kh}}$$

4. A mass m starting from A reaches B of a frictionless track. On reaching A it pushes the track with a force equal to x times its weight, then applicable relation is

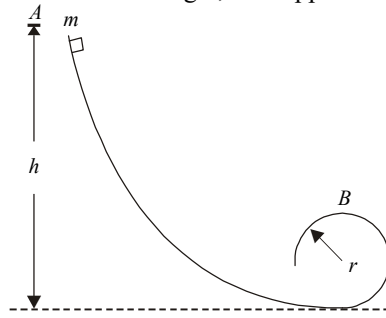


Fig 6.23

- (a) $h = \frac{(x+5)}{2} r$ (b) $h = \frac{x}{2} r$
 (c) $h = r$ (d) $h = \left(\frac{x+1}{2}\right) r$

5. Coefficient of friction between a tool and grinding wheel is μ . Power developed in watt by the wheel of radius r running at n revolutions per second when tool is pressed to the wheel with F' kgf is

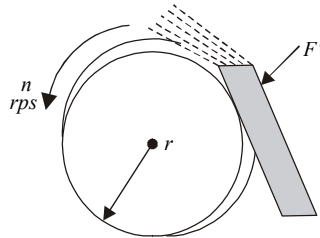


Fig 6.24

- (a) $\mu F' r (2\pi n)$ (b) $\mu F' g r (2\pi n)$
 (c) $\mu F' r$ (d) $\mu F' g$
6. A minute particle resting at a frictionless surface is acted upon by a constant horizontal force. Neglecting frictional force the graph between work done on the particle w and speed of particle u is represented by

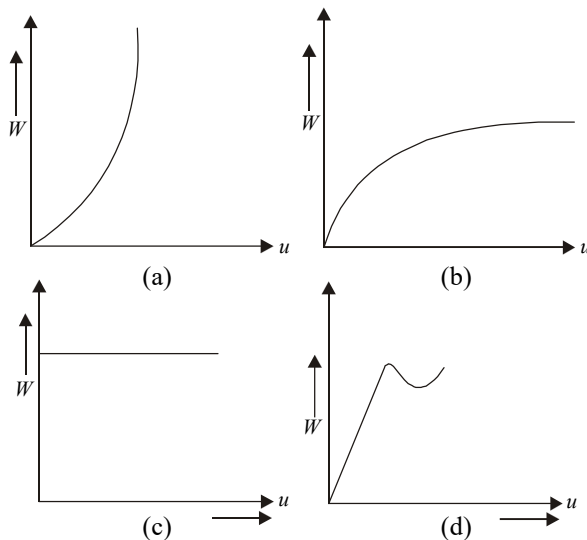


Fig 6.25

7. A particle of mass m slides along curved – flat – curved track. The curved portions of the track are smooth. If the particle is released at the top of one of the curved portions the particle comes to rest at flat portion of length l and of coefficient of kinetic friction $= \mu_{\text{kinetic}}$ after covering length of

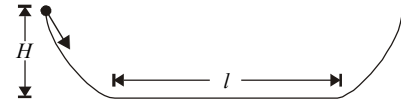


Fig 6.26

- (a) $l/3\mu_{\text{kinetic}}$ (b) $H/2\mu_{\text{kinetic}}$
 (c) $l/6$ (d) $\frac{H}{\mu_{\text{kinetic}}}$
8. A light rigid rod of length L has a bob of mass M attached to one of its end just like a simple pendulum. Speed at the lowest point when it is inverted and released is

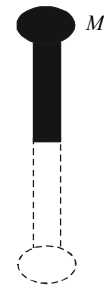


Fig 6.27

- (a) \sqrt{gL} (b) $\sqrt{2gL}$
 (c) $2\sqrt{gL}$ (d) $\sqrt{5gL}$
9. Astronauts Mr. X and Mr. Y float in gravity zero space with no relative velocity to one another. Mr. Y throws a mass of 5 kg towards X with speed 2 ms^{-1} . If Mr. X catches it the changes in velocity of X and Y are

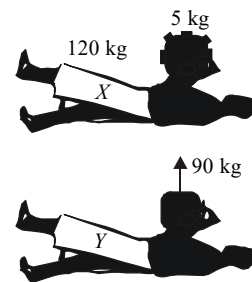


Fig 6.28

- (a) $0.21 \text{ ms}^{-1}, 0.80 \text{ ms}^{-1}$
 (b) $0.80 \text{ ms}^{-1}, 0.21 \text{ ms}^{-1}$
 (c) $0.12 \text{ ms}^{-1}, 0.08 \text{ ms}^{-1}$
 (d) $0.08 \text{ ms}^{-1}, 0.12 \text{ ms}^{-1}$
10. A particle is dropped from a height h . A constant horizontal velocity is given to the particle. Taking g

to be constant everywhere, kinetic energy E of the particle with reference to time t is correctly shown in

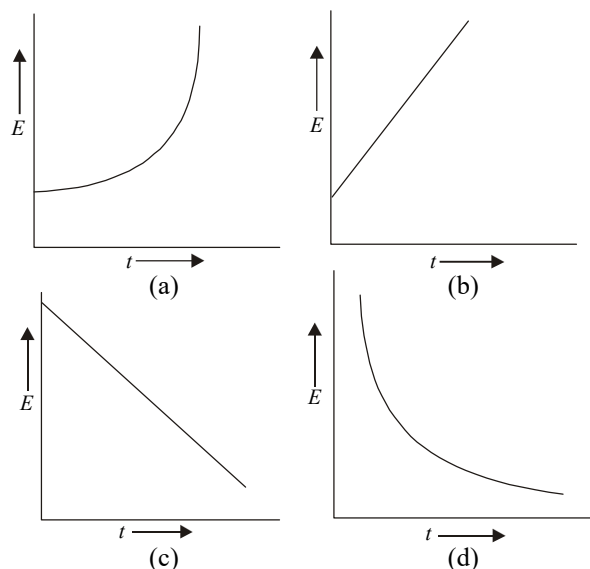


Fig 6.29

11. An aeroplane is flying with a velocity U_p . The plane takes in volume V of air of mass μ per second to burn mass m of fuel every second. The exhaust relative to plane has a speed of U_E . b.h.p. of engine of the aeroplane is

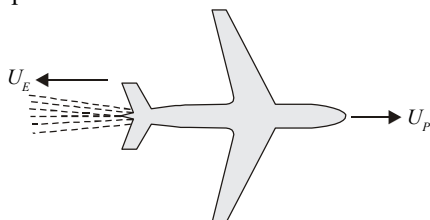


Fig 6.30

- (a) $\frac{U_E^2(\mu + m) - U_p^2(\mu - m)}{776}$
 (b) $\frac{U_E(\mu + m) - U_p(\mu - m)}{746}$
 (c) $\frac{U_E U_p(\mu + m) - U_p^2 \mu}{746}$
 (d) $\frac{U_p^2(\mu + m) - U_p U_E(\mu - m)}{746}$
12. A vehicle of mass M is accelerated on a horizontal frictionless road under a force changing its velocity from u to v in distance S . A constant power P is given by the engine of the vehicle, then $v =$

- (a) $\left(u^3 + \frac{2PS}{M}\right)^{1/3}$ (b) $\left(\frac{PS}{M} + u^3\right)^{1/2}$
 (c) $\left(\frac{PS}{M} - u^2\right)^{1/3}$ (d) $\left(\frac{3PS}{M} + u^3\right)^{1/3}$

13. Two frames, one stationary and the other moving, are initially coincident. Two observers in the two frames observe a body initially at rest in the coincident frame. A constant force F starts acting on the body along horizontal axis when moving frame starts separation from fixed frame. Work done ' W ' as observed by stationary frame and W' as observed from moving frame are compared to each other as

- (a) $W = W'$ (b) $W \neq W'$
 (c) $W = \frac{1}{2} W'$ (d) $W = 2W'$

14. A proton (mass 1.67×10^{-27} kg) is accelerated along a straight line by $3.6 \times 10^{15} \text{ ms}^{-2}$. If the length covered is 3.5 cm and initial speed of proton was $2.4 \times 10^7 \text{ ms}^{-1}$, the gain in kinetic energy is

- (a) 1.32 MeV (b) 1.23 MeV
 (c) 2.13 MeV (d) 3.12 MeV

15. A mass m is thrown vertically upward into air with initial speed u . A constant force F due to air resistance acts on the mass during its travel. Taking into account the work done against air drag the maximum distance covered by the mass to reach the top is

- (a) $\frac{u^2}{2g}$ (b) $\frac{u^2}{2g + (2F/m)}$
 (c) $\frac{u^2}{2g + F/m}$ (d) $\frac{u^2}{g + F/m}$

16. A 20 g bullet passes through a plate of mass 1 kg and finally comes to rest inside another plate of mass 2980 g. It makes the plates move from rest to same velocity. The percentage loss in velocity of bullet between the plate is

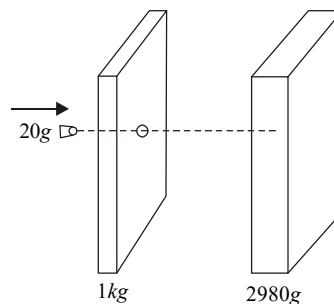


Fig 6.31

- (a) 0 (b) 50 %
 (c) 75 % (d) 25 %

17. A particle of mass 1 kg has potential energy, P.E. = $3x + 4y$. At $t = 0$, the particle is at rest at (6, 4). Work done by external force to displace the particle from rest to the point of crossing the x axis is

- (a) 25 J (b) 20 J
(c) 15 J (d) 52 J

18. A body is lifted over route I and then route II such that force is always tangent to the path. Coefficient of friction is same for both the paths. Work done

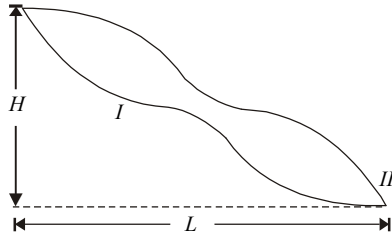


Fig 6.32

- (a) on both routes is same.
(b) on route I is more.
(c) on route II is more.
(d) on both routes is zero.
19. Interaction between atoms of a diatomic molecule is described by the relation P.E. (x) = $\frac{\alpha}{x^{12}} - \frac{\beta}{x^6}$, where P.E. is potential energy. If the two atoms enjoy stable equilibrium, the distance between them would be
- (a) $\left(\frac{\alpha}{\beta}\right)^{1/6}$ (b) $\left(\frac{2\alpha}{\beta}\right)^{1/6}$
(c) $\left(\frac{2\beta}{\alpha}\right)^{1/6}$ (d) $\left(\frac{\beta}{\alpha}\right)^{1/6}$
20. A hammer of mass M falls from a height h repeatedly to drive a pile of mass m into the ground. The hammer makes the pile penetrate in the ground to a distance d in single blow. Opposition to penetration is given by

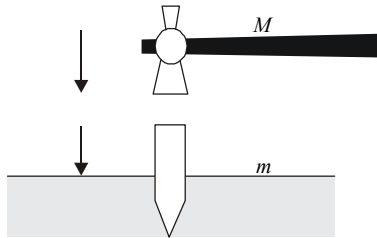


Fig 6.33

- (a) $\frac{m^2 gh}{M + md}$ (b) $\frac{m^2 gh}{(M + m)d} + (M + m)g$
(c) $\frac{M^2 gh}{M + md}$ (d) $\frac{m^2 gh}{(m + M)d} - (M + m)g$
21. The height h from which a car of mass m has to fall to gain the kinetic energy equivalent to what it would have gained when moving with a horizontal velocity of $(u + v)$ is given by

- (a) $\frac{v}{2g}$ (b) $\frac{v^2}{2g}$
(c) $\frac{(u + v)^2}{2g}$ (d) $\frac{(u + v)^2}{g}$

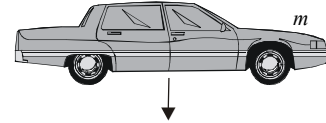


Fig 6.34

22. A gun requiring the support of shoulder shows intensive recoil when fired with



Fig 6.35

- (a) the butt held tightly with the shoulder.
(b) the butt held loosely against the shoulder.
(c) butt held loosely or tightly against the shoulder.
(d) a bullet of greater mass.
23. A mass m is allowed to fall on a pedestal fixed on the top of a vertical spring. If the height of the mass was H from the pedestal and the compression of the spring is d then the spring's force factor is given by

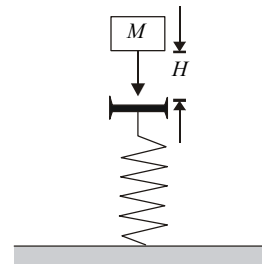


Fig 6.36

- (a) $Mg \frac{(H + d)}{d^2}$ (b) $2 Mg \frac{(H - d)}{d^2}$
(c) $\frac{Mg}{2} \frac{H}{d^2}$ (d) $2 Mg \frac{(H + d)}{d^2}$
24. How high can a man weighing m kg climb using the energy from a chocolate producing 100 calories in him? Let his efficiency be η

- (a) $\frac{420\eta}{mg}$ metre (b) $\frac{4.20\eta}{mg}$ metre
 (c) $\frac{\eta}{mg}$ metre (d) $42 \eta mg$ metre

25. A toy car moves up a ramp under the influence of force F plotted against displacement. The maximum height attained is given by

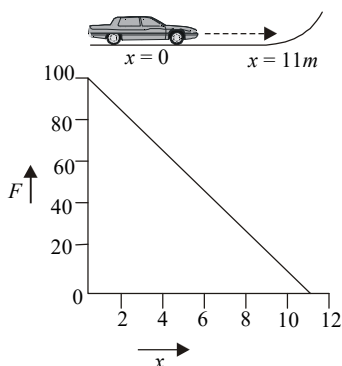


Fig 6.37

- (a) $y_{\max} = 20$ m (b) $y_{\max} = 15$ m
 (c) $y_{\max} = 10$ m (d) $y_{\max} = 5$ m.

26. A block is placed on a plank which is placed on a horizontal plane. A massless but elastic string deviates by an angle θ with vertical when a force F is applied to the plank to shift it to the right making the block slide over it. If F_p and F_b are frictional forces between plank and plane and between block and plane respectively then work done by applied force is given by

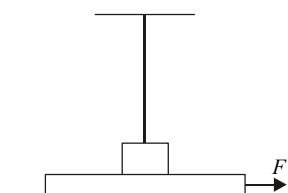


Fig 6.38

- (a) work done against friction acting on plank + energy in the elastic string - work done by friction acting on block.
 (b) work done against frictions acting on plank and block + elastic energy.
 (c) elastic energy of string.
 (d) difference of work done against friction acting on plank and block.

27. A truck tows a jeep of mass 1200 kg at a constant speed of 10 ms^{-1} on a level road. The tension in the coupling is 1000 N. If they ascend an inclined plane of 1 in 6 with same velocity, the tension in coupling is

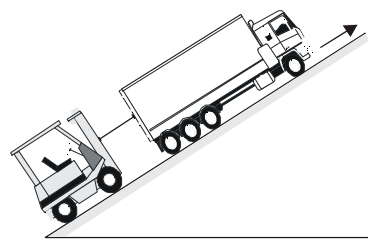


Fig 6.39

- (a) 2000 N (b) 2960 N
 (c) 1680 N (d) 1000 N

28. A screw jack of pitch 5 mm is used to lift the tyre of a vehicle of load 200 kg with the help of a handle of length 0.5 m. Neglecting the friction force between screw and nut of the jack, the least force required to raise the load is

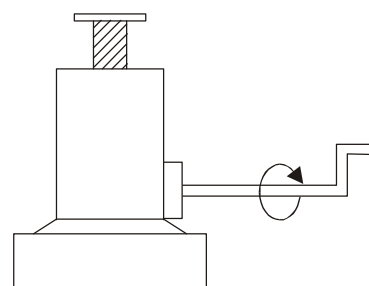


Fig 6.40

- (a) 1.2 N (b) 2.2 N
 (c) 3.2 N (d) 4.2 N

29. A light rod of length L can revolve in a vertical circle around point O . The rod carries two equal masses of mass m each such that one mass is connected at the end of the rod and the second mass is fixed at the middle of the rod. u is the velocity imparted to the end P to deflect the rod to the horizontal position. Again mass m in the middle of the rod is removed and mass at end P is doubled. Now v is the velocity imparted to end P to deflect it to the horizontal position. Then v/u is

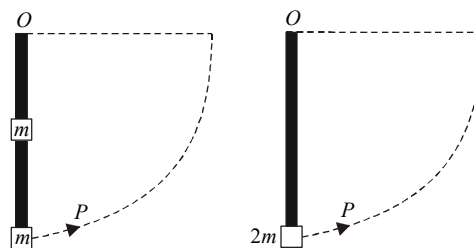


Fig 6.41

- (a) $(6/5)^{\frac{1}{2}}$ (b) 1
 (c) 2.5 (d) $(5/6)^{\frac{1}{2}}$

30. A coconut of mass m falls from the tree through a vertical distance of s and could reach ground with a

velocity of $v \text{ ms}^{-1}$ due to air resistance. Work done by air resistance is

- (a) $-\frac{m}{2}(2gs - v^2)$ (b) $-\frac{1}{2}mv^2$
 (c) mgs (d) $mv^2 + 2mgs$

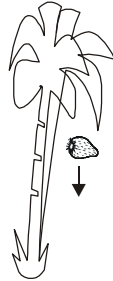


Fig 6.42

31. A body of mass m is in vertical motion under the influence of gravity then

- (a) work will be negative and kinetic energy increases during fall.
 (b) kinetic energy decreases when body is projected up and work will be negative.
 (c) kinetic energy increases when body is projected up and work will be positive.
 (d) work will be positive and kinetic energy decreases during fall.

32. A smooth chain PQ of mass M rests against a $\frac{1}{4}$ th circular and smooth surface of radius r . If released, its velocity to come over the horizontal part of the surface is

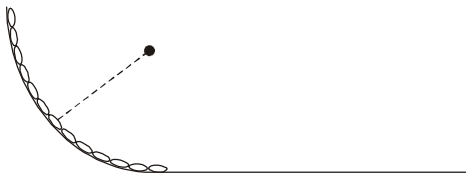


Fig 6.43

- (a) $\sqrt{2gr} \times \frac{1}{4}$ (b) $\sqrt{2gr\left(1 - \frac{1}{\pi}\right)}$
 (c) $\sqrt{2gr\left(1 - \frac{2}{\pi}\right)}$ (d) $\sqrt{gr\left(1 - \frac{2}{\pi}\right)}$

33. A block of mass m has initial velocity u having direction $+x$ axis. The block stops after covering distance S causing similar extension in the spring of spring constant K holding it. If μ is the kinetic friction between the block and the surface on which it was moving, the distances S is given by

- (a) $\frac{1}{K} \mu^2 m^2 g^2$
 (b) $\frac{1}{K} (mKu^2 - \mu^2 m^2 g^2)^{\frac{1}{2}}$
 (c) $\frac{1}{K} (\mu^2 m^2 g^2 + mKu^2 + \mu mg)^{\frac{1}{2}}$
 (d) $\frac{1}{K} (\mu^2 m^2 g^2 - mKu^2 + \mu mg)^{\frac{1}{2}}$

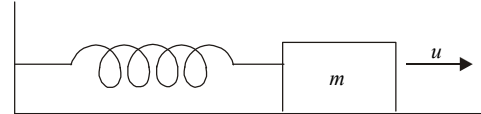


Fig 6.44

34. A rigid cord and an elastic cord support two small spheres of same mass M . The are deflected from the mean position through an angle of 90° . When the spheres pass through the mean position the lengths of the two cords become same. If v_1 is the velocity of the sphere attached to rigid cord and v_2 is the velocity of the other sphere, then

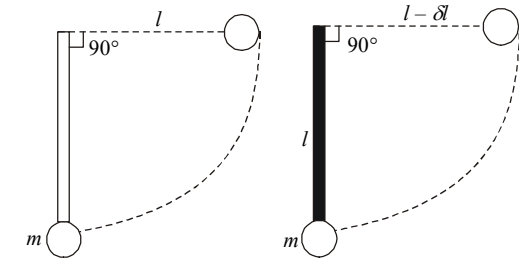


Fig 6.45

- (a) $v_1 = v_2$ (b) v_1 is more than v_2
 (c) $v_1 < v_2$ (d) $v_1 = v_2 = 0$
35. A streamlined and almost symmetrical car of mass M has centre of gravity at a distance p from the rear wheel, q from the front wheel and h from the road. If the car has required power and friction, the maximum acceleration developed without tipping over towards back is

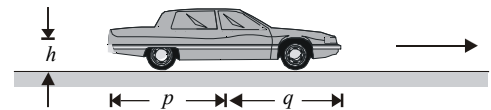


Fig 6.46

- (a) $\frac{qg}{h}$ (b) hgq
 (c) $\frac{hMg}{q}$ (d) $\frac{pq}{hg}$

36. A pipe of uniform crosssectional of area a has a right angled bend. Impure water of density δ flows through the horizontal portion of the pipe. The distance covered by water per second at the bend to keep the pipe in equilibrium against an applied force F at the bend is

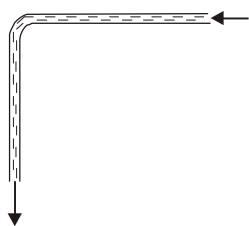


Fig 6.47

- (a) $\frac{\delta a F}{\sqrt{2}}$ (b) $\sqrt{2} \frac{\delta}{F}$
 (c) $\left(\frac{\sqrt{2} F}{\delta a}\right)^{\frac{1}{2}}$ (d) $\left(\frac{F}{\sqrt{2} \delta a}\right)^{\frac{1}{2}}$

37. Spring of constant k fixed to its front wall. A boy stretches this spring by distance x and in the mean time the compartment moves by a distance s . The work done by boy with reference to earth is

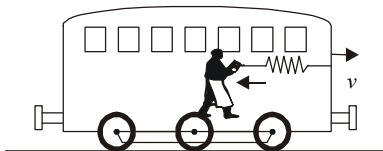


Fig 6.48

- (a) $\frac{1}{2} kx^2$ (b) $\frac{1}{2} (kx) (s+x)$
 (c) $\frac{1}{2} kxs$ (d) $\frac{1}{2} kx (s+x+s)$

38. Six identical uniform rods PQ, QR, RS, ST, TU and UP each weighing W are freely joined at their ends to form a hexagon. The rod PQ is fixed in a horizontal position and middle points of PQ and ST are connected by a vertical string. The tension in string is

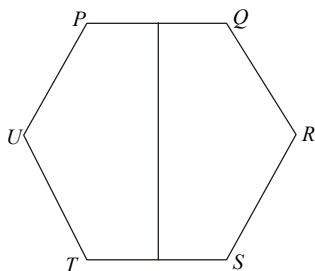


Fig 6.49

- (a) W (b) $3W$
 (c) $2W$ (d) $4W$

39. N similar slabs of cubical shape of edge b are lying on ground. Density of material of slab is δ . Work done to arrange them one over the other is

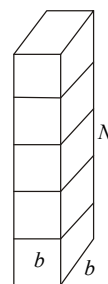


Fig 6.50

- (a) $(N^2 - 1) b^3 \rho g$ (b) $(N - 1) b^4 \rho g$
 (c) $\frac{1}{2} (N^2 - N) b^4 \rho g$ (d) $(N^2 - N) b^4 \rho g$

40. A floor wiper makes an angle $(90 - \theta)$ with horizontal floor. μ_s and μ_k are coefficients of static and kinetic frictions between the wiper and the floor. The wiper can not be used to wipe the floor for $\theta < \theta_0$ even if a large force directed along the handle aiming towards the centre is applied. Angle θ_0 is given by

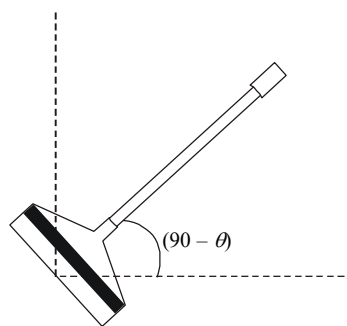


Fig 6.51

- (a) $\cos^{-1} \mu_k$ (b) $\tan^{-1} \mu_s$
 (c) $\tan^{-1} \mu_k$ (d) $\cot^{-1} \mu_s$

41. For the equilibrium of the system of a toy shown the relation between force F_1 and F_2 when the members are arranged to form three identical rhombuses is

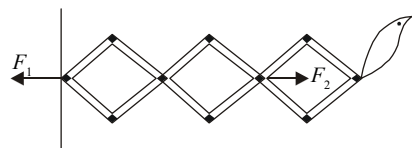


Fig 6.52

- (a) $F_1 = F_2$ (b) $F_2 = \frac{F_1}{3}$
 (c) $F_2 = \frac{F_1}{2}$ (d) $F_1 = \frac{F_2}{3}$

42. An elastic cord of constant K and length L is hung from point A having a massless lock at the other end. A smooth ring of mass M falls from point A , the maximum elongation of cord is

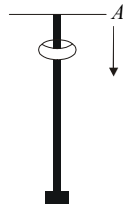


Fig 6.53

- (a) $\frac{Mg}{K} \left(1 + \frac{1+2KL}{Mg} \right)^{1/2}$
- (b) $\frac{Mg}{K} \left(1 - \left(1 - \frac{2KL}{Mg} \right)^{1/2} \right)$ (c) $\frac{MgL}{K}$
- (d) $\frac{Mg}{K} \left(1 + \left(1 + \frac{2KL}{Mg} \right)^{1/2} \right)$

43. A particle of mass M rests on a straight groove along which it is constrained to move. A perfectly elastic rubber band of natural length l and uniform area of cross-section is attached with the particle. The other end of the band is suspended from a rigid support. A force $K(l'^2 - l^2)^{1/2}$ is required to stretch the band to a length l' . The particle is moved to a distance S (where $S \ll l$) and then released. Taking $K = \frac{Mg}{S}$ and μ as the coefficient of friction between the particle and the groove, the velocity of particle when passing through the initial position is

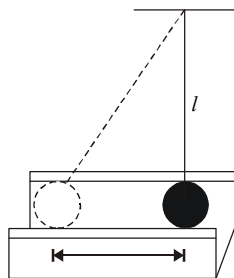


Fig 6.54

- (a) $\frac{gS}{3l} (2S - 3\mu l)^{1/2}$ (b) $\left[\frac{gS}{3l} (2S - 3\mu l) \right]^{1/2}$
- (c) $\frac{gS}{l} (3S - 2\mu l)^{1/2}$ (d) $\left[\frac{gS}{2l} (3S - 2\mu l) \right]^{1/2}$
44. In a two step pulley arrangement meant for a load W , the ratio between force P and W in equilibrium position when radii of pulleys are r_1 and r_2
- (a) $\frac{r_2}{r_1 - r_2}$ (b) $\frac{r_1 - r_2}{r_2}$
- (c) $\frac{r_1}{r_2 - r_1}$ (d) $\frac{r_2 - r_1}{r_1}$

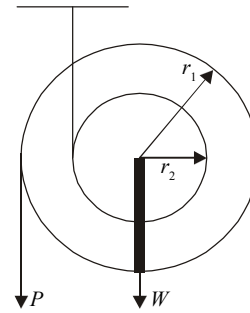


Fig 6.55

45. A rope brake is fitted to a flywheel of diameter = 1m. The flywheel runs at 220 r.p.m. It is required to absorb 5.25 kW of brake power. Difference in the two pulls ($T - S$) is

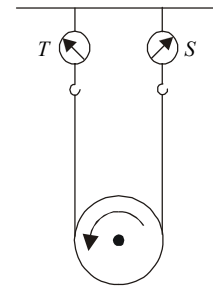


Fig 6.56

- (a) 456 N (b) 654 N
- (c) 564 N (d) 465 N
46. A body of mass 2 kg is being dragged with a uniform velocity of 2 ms^{-1} on a horizontal plane. The coefficient of friction between the body and the surface is 0.2. Work done in 5s is
- (a) 39.2 J (b) 9.32 J
- (c) 23.9 J (d) 93.2 J

(Based on I.I.T. 1980)

47. A small block of mass m is released from rest from point D and slides down DGF and reaches the point F with speed v_F . The coefficient of kinetic friction between block and both the surfaces DG and GF is μ , the velocity v_F is

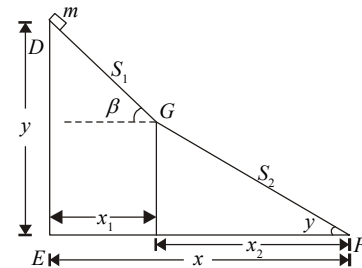


Fig 6.57

- (a) $\sqrt{2g(y-x)}$ (b) $\sqrt{2g(y-\mu x)}$
- (c) $\sqrt{2gy}$ (d) $\sqrt{2g(y^2 + x^2)}$

(Based on I.I.T. 1980)

48. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ (b) $t^{3/4}$
(c) $t^{3/2}$ (d) t^3

(I.I.T. 1984)

49. A particle of mass $4m$ which is at rest explodes into three fragments. Two of the fragments each of mass m are found to move with a speed v each in mutually perpendicular directions. The energy released in the process of explosion is

(a) $\frac{3}{2}mv^2$ (b) $3mv^2$
(c) $2mv^2$ (d) $\frac{1}{2}mv^2$

(Based on I.I.T. 1987)

50. An engine of mass m is moving up a slope of inclination θ at a speed v . The coefficient of friction between engine and the rail is μ . If the engine has an efficiency η then the energy spent by engine in time t is

(a) $\eta mg (\sin \theta + \mu \cos \theta) vt$
(b) $\frac{mg(\mu \cos \theta)vt}{\eta}$
(c) $\frac{mg(\sin \theta + \mu \cos \theta)vt}{\eta}$
(d) $\frac{mg}{2} \left(\frac{\sin \theta}{\eta} \right) vt$

(Based on Roorkee 1987)

51. A person decides to use his bath tub water to generate electric power to run a 40 W bulb. The bath tub is located at a height of h m from ground and it holds V litres of water. He installs a water driven wheel generator on ground. The rate at which water should drain from bath tub to light the bulb if efficiency of machine be 90% is

(a) $\frac{11.11}{\rho gh}$ (b) $44.44 \rho gh$
(c) $\frac{44.44}{\rho gh}$ (d) $\frac{22.22}{\rho gh}$

(Based on Roorkee 1990)

52. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic

energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. The recoil speed of emitting atom assuming it to be at rest before ionisation is (ionisation potential of hydrogen is 13.6 eV).

(a) 0.18 ms^{-1} (b) 1.80 ms^{-1}
(c) 8.10 ms^{-1} (d) 0.81 ms^{-1}

53. Two identical balls A and B of mass m kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid pipe bent in the form of a circle as shown in figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius r metre. Each spring has a natural length of $r\pi$ metre and spring constant K . Initially, both the balls are displaced by an angle θ radian with respect to diameter PQ of the circles and released from rest. The speed of ball A when A and B are at the two ends of dia PQ is

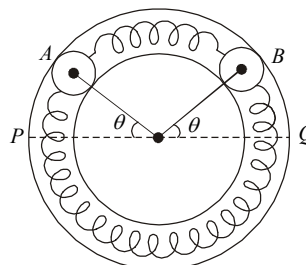


Fig 6.58

(a) $R\theta \sqrt{\frac{m}{K}}$ (b) $2R\theta \sqrt{\frac{K}{m}}$
(c) $2R\theta \sqrt{\frac{m}{K}}$ (d) $2R\theta \sqrt{\frac{K}{m}}$

(Based on I.I.T. 1993)

54. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt$, where k is a constant. The power delivered to the particle by force acting on it is

(a) $2\pi mk^2r^2$ (b) mk^2r^2t
(c) $\frac{mk^4r^2t^5}{3}$ (d) zero

(I.I.T. 1994)

55. A cart is moving along x direction with a velocity of 4 ms^{-1} . A person on the cart throws a stone with a velocity of 6 ms^{-1} relative to himself. In the frame of reference of the cart the stone is thrown in $y-z$ plane making an angle of 30° with vertical z axis. At the highest point of its trajectory the stone hits an object of equal mass hung vertically from branch of a tree by means of a string of length L . The stone gets embedded in the object. The speed of combined mass immediately after the embedding with reference to an observer on the ground is

- (a) 2.5 ms^{-1} (b) 1.5 ms^{-1}
 (c) 5.2 ms^{-1} (d) 3.5 ms^{-1} .

(I.I.T. 1997)

56. In Q. 55 the length l of string such that tension in string becomes zero when string becomes horizontal during subsequent motion of combined mass is

- (a) 0.23 m (b) 0.32 m
 (c) 0.13 m (d) 0.27 m

(I.I.T. 1997)

57. A stone tied to a string of length L is whirled in a vertical circle with the other end of string at the centre. At a certain instant of time the stone is at its lowest position and has a speed u . The magnitude of change in velocity as it reaches a position where string is horizontal is

- (a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$
 (c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

(I.I.T. 1998)

58. A force $F = -K(y \hat{i} + x \hat{j})$, (where K is a +ve constant) acts on a particle moving in xy plane starting from origin, the particle is taken along the positive x -axis to the point $(a, 0)$ and then parallel to y axis to the point $(a, 0)$. The total work done by force F on the particle is

- (a) $-2Ka^2$ (b) $2Ka^2$
 (c) $-Ka^2$ (d) Ka^2

(I.I.T. 1998)

59. A particle free to move along x axis has potential energy given as $U_{(x)} = k(1 - \exp(-x^2))$ for $-a \leq x \leq +\infty$ where k is a positive constant of appropriate dimensions. Then

- (a) at points away from the origin, the particle is in unstable equilibrium.
 (b) for any finite non-zero value of x , there is a force directed away from the origin.
 (c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin.
 (d) if its total mechanical energy is $k/2$, it has its maximum value at origin.

(I.I.T. 1999)

60. A particle which is constrained to move along the x -axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is

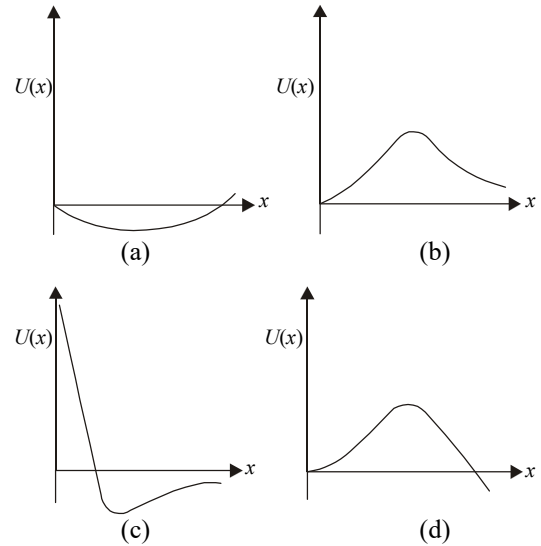


Fig 6.59

(I.I.T. Screening 2002)

61.

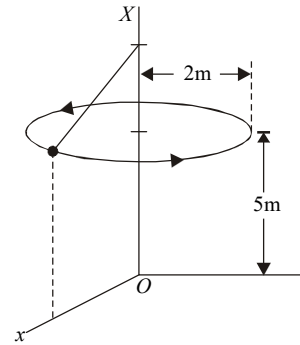


Fig 6.60

The upper end of the string of a simple pendulum is fixed to a vertical z -axis, and set in motion such that the bob moves along a horizontal circular path of radius 2 m, parallel to the xy plane, 5 m above the origin. The bob has a speed of 3 m/s. The string breaks when the bob is vertically above the x -axis, and it lands on the xy plane at a point (x, y) .

- (a) $x = 2 \text{ m}$ (b) $x > 2 \text{ m}$
 (c) $y = 3 \text{ m}$ (d) $y = 5 \text{ m}$

62.

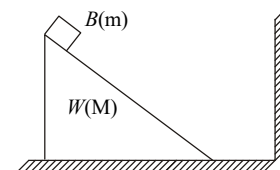


Fig 6.61

In the figure 6.61, the block B of mass m starts from rest at the top of a wedge W of mass M . All surfaces are smooth. W can slide on the ground. B slides down onto the ground, moves along it with a speed v , has an elastic collision with the wall, and climbs back onto W .

- (a) B will reach the top of W again.
- (b) From the beginning, till the collision with the wall, the centre of mass of ' B plus W ' does not move horizontally.
- (c) After the collision, the centre of mass of ' B plus W ' moves with the velocity $\frac{2mv}{m+M}$.
- (d) When B reaches its highest position on W , the speed of W is $\frac{2mv}{m+M}$.

63.

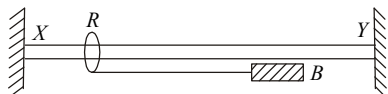


Fig 6.62

The ring R in the arrangement shown can slide along a smooth, fixed, horizontal rod XY . It is attached to the block B by a light string. The block is released from rest, with the string horizontal.

- (a) One point in the string will have only vertical motion.
- (b) R and B will always have momenta of the same magnitude.
- (c) When the string becomes vertical, the speeds of R and B will be inversely proportional to their masses.
- (d) R will lose contact with the rod at some point.
64. A strip of wood of mass M and length l is placed on a smooth horizontal surface. An insect of mass m starts at one end of the strip and walks to the other end in time t , moving with a constant speed.
- (a) The speed of the insect as seen from the ground is $< \frac{l}{t}$.
- (b) The speed of the strip as seen from the ground is $\frac{l}{t} \left(\frac{M}{M+m} \right)$.
- (c) The speed of the strip as seen from the ground is $\frac{l}{t} \left(\frac{m}{M+m} \right)$.
- (d) The total kinetic energy of the system is $\frac{1}{2} (m + M) \left(\frac{l}{t} \right)^2$.
65. A charged particle X moves directly towards another charged particle Y . For the ' X plus Y ' system, the total momentum is p and the total energy is E .

- (a) p and E are conserved if both X and Y are free to move.
- (b) (A) is true only if X and Y have similar charges.
- (c) If Y is fixed, E is conserved but not p .
- (d) If Y is fixed, neither E nor p is conserved.

66. In a one-dimensional collision between two particles, their relative velocity is \vec{v}_1 before the collision and \vec{v}_2 and the collision.

- (a) $\vec{v}_1 = \vec{v}_2$ if the collision is elastic.
- (b) $\vec{v}_1 = -\vec{v}_2$ if the collision is elastic.
- (c) $|\vec{v}_2| = |\vec{v}_1|$ in all cases.
- (d) $\vec{v}_1 = -k \vec{v}_2$ in all cases, where $k \geq 1$.

67. A sphere A moving with a speed u and rotating with an angular velocity ω , makes a head-on elastic collision with an identical stationary sphere B . There is no friction between the surfaces of A and B . Disregard gravity.

- (a) A will stop moving but continue to rotate with an angular velocity ω .
- (b) A will come to rest and stop rotating.
- (c) B will move with a speed u without rotating.
- (d) B will move with a speed u and rotate with an angular velocity ω .

68. In an elastic collision between spheres A and B of equal mass but unequal radii, A moves along the x -axis and B is stationary before impact. Which of the following is possible after impact?

- (a) A comes to rest.
- (b) The velocity of B relative to A remains the same in magnitude but reverses in direction.
- (c) A and B move with equal speeds, making an angle of 45° each with the x -axis.
- (d) A and B move with unequal speeds, making angles of 30° and 60° with the x -axis respectively.

69. In a one-dimensional collision between two identical particles A and B , B is stationary and A has momentum p before impact. During impact, B gives impulse J to A .

- (a) The total momentum of the ' A plus B ' system is p before and after the impact, and $(p - J)$ during the impact.
- (b) During the impact, A gives impulse J to B .
- (c) The coefficient of restitution is $\frac{2J}{p} - 1$.
- (d) The coefficient of restitution is $\frac{J}{p} + 1$.

70. When a cannon shell explodes in mid-air,
- the momentum of the system is conserved in all cases
 - the momentum of the system is conserved only if the shell was moving horizontally
 - the kinetic energy of the system either remains constant or decreases
 - the kinetic energy of the system always increases
71. A cannon, shell is fired to hit a target at a horizontal distance R . However, it breaks into two equal parts at its highest point. One part (A) returns to the cannon. The other part
- will fall at a distance of R beyond the target.
 - will fall at a distance of $3R$ beyond the target.
 - will hit the target.
 - have nine times the kinetic energy of A .
72. A particle moving with a speed v changes direction by an angle θ , without change in speed.
- The change in the magnitude of its velocity is zero.
 - The change in the magnitude of its velocity is $2v \sin(\theta/2)$.
 - The magnitude of the change in its velocity is $2v \sin(\theta/2)$.
 - The magnitude of the change in its velocity is $v(1 - \cos \theta)$.
73. A block of weight 9.8 N is placed on a table. The table surface exerts an upward force of 10 N on the block. Assume $g = 9.8 \text{ m/s}^2$.
- The block exerts a force of 10 N on the table.
 - The block exerts a force of 19.8 N on the table.
 - The block exerts a force of 9.8 N on the table.
 - The block has an upward acceleration.
74. The blocks B and C in the figure have mass m each. The strings AB and BC are light, having tensions T_1 and T_2 respectively. The system is in equilibrium with a constant horizontal force mg acting on C .

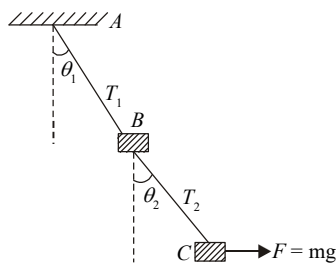


Fig 6.63

- $\tan \theta_1 = 1/2$
- $\tan \theta_2 = 1$
- $T_1 = \sqrt{5} mg$
- $T_2 = \sqrt{2} mg$

75. A particle of mass 70 g , moving at 50 cm/s , is acted upon by a variable force opposite to its direction of motion. The force F is shown as a function of time t .

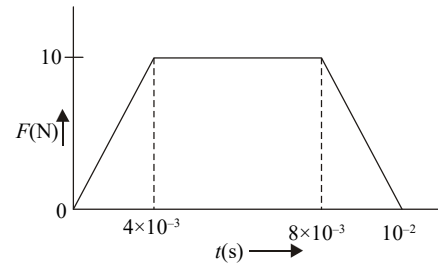


Fig 6.64

- Its speed will be 50 cm/s after the force stops acting.
 - Its direction of motion will reverse.
 - Its average acceleration will be 1 m/s^2 during the interval in which the force acts.
 - Its average acceleration will be 10 m/s^2 during the interval in which the force acts.
76. A monkey of mass $m \text{ kg}$ slides down a light rope attached to a fixed spring balance, with an acceleration a . The reading of the spring balance is $W \text{ kg}$. [g = acceleration due to gravity].
- The force of friction exerted by the rope on the monkey is $m(g - a) \text{ N}$.
 - $m = \frac{Wg}{g - a}$
 - $m = W \left(1 + \frac{a}{g} \right)$
 - The tension in the rope is $Wg \text{ N}$.
77. A block of weight W is suspended from a spring balance. The lower surface of the block rests on a weighing machine. The spring balance reads W_1 and the weighing machine reads W_2 . (W, W_1, W_2 are in the same unit).
- $W = W_1 + W_2$ if the system is at rest.
 - $W > W_1 + W_2$ if the system moves down with some acceleration.
 - $W_1 > W_2$ if the system moves up with some acceleration.
 - No relation between W_1 and W_2 can be obtained with the given description of the system.
78. A simple pendulum with a bob of mass m is suspended from the roof of a car moving with a horizontal acceleration a .
- The string makes an angle of $\tan^{-1}(a/g)$ with the vertical.
 - The string makes an angle of $\tan^{-1} \left(1 - \frac{a}{g} \right)$ with the vertical.

(c) The tension in the string is $m\sqrt{a^2 + g^2}$.

(d) The tension in the string is $m\sqrt{g^2 - a^2}$.

79. Two masses M and m ($M > m$) are joined by a light string passing over a smooth light pulley.

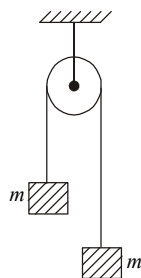


Fig 6.65

- (a) The acceleration of each block is $\left(\frac{M-m}{M+m}\right)g$.
- (b) The tension in the string is $\frac{2Mmg}{M+m}$.
- (c) The centre of mass of the ' M plus m ' system moves down with an acceleration of $g\left(\frac{M-m}{M+m}\right)^2$.
- (d) The tension in the string by which the pulley is attached to the roof is $(M+m)g$.

80. In the previous question, the blocks are allowed to move for some time, after which M is stopped momentarily (brought to rest and released at once). After this,

- (a) both blocks will move with the same acceleration.
- (b) the strings will become taut (under tension) again when the blocks acquire the same speed.
- (c) the string will become taut again when the blocks cover equal distances.
- (d) at the instant when the string becomes taut again, there may be some exchange of impulse between the string and blocks.

81. A block of mass 1 kg moves under the influence of external forces on a rough horizontal surface. At some instant, it has a speed of 1 m/s due east and an acceleration of 1 m/s² due north. The force of friction acting on it is F .

- (a) F acts due west.
- (b) F acts due south.
- (c) F acts in the south-west direction.
- (d) The magnitude of F cannot be found from the given data.

82. A long block A is at rest on a smooth horizontal surface. A small block B , whose mass is half of A , is placed on A at one end and projected along A with some velocity u . The coefficient of friction between the blocks is μ .

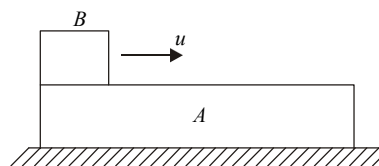


Fig 6.66

- (a) The blocks will reach a final common velocity $\frac{u}{3}$.
- (b) The work done against friction is two-thirds of the initial kinetic energy of B .
- (c) Before the blocks reach a common velocity, the acceleration of A relative to B is $\frac{2}{3}\mu g$.
- (d) Before the blocks reach a common velocity the acceleration of A relative to B is $\frac{3}{2}\mu g$.

83. A 10-kg block is placed on a horizontal surface. The coefficient of friction between them is 0.2. A horizontal force $P = 15$ N first acts on it in the eastward direction. Later, in addition to P a second horizontal force $Q = 20$ N acts on it in the northward direction.

- (a) The block will not move when only P acts, but will move when both P and Q act.
- (b) If the block moves, its acceleration will be 0.5 m/s².
- (c) When the block moves, its direction of motion will be $\tan^{-1}\left(\frac{4}{3}\right)$ east of north.
- (d) When both P and Q act, the direction of the force of friction acting on the block will be $\tan^{-1}\left(\frac{3}{4}\right)$ west of south.

84. A block of mass m is placed on a rough horizontal surface. The coefficient of friction between them is μ . An external horizontal force is applied to the block and its magnitude is gradually increased. The force exerted by the block on the surface is R .

- (a) The magnitude of R will gradually increase.
- (b) $R \leq mg\sqrt{\mu^2 + 1}$.
- (c) The angle made by R with the vertical will gradually increase.
- (d) The angle made by R with the vertical $\leq \tan^{-1} \mu$.

85. A man pulls a block heavier than himself with a light rope. The coefficient of friction is the same between the man and the ground, and between the block and the ground.

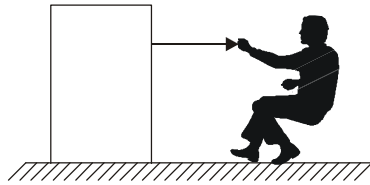


Fig 6.67

- (a) The block will not move unless the man also moves.
- (b) The man can move even when the block is stationary.
- (c) If both move, the acceleration of the man is greater than the acceleration of the block.
- (d) None of the above assertions is correct.
86. A car C of mass m_1 rests on a plank P of mass m_2 . The plank rests on a smooth floor. The string and pulley are ideal. The car starts and moves towards the pulley with acceleration.

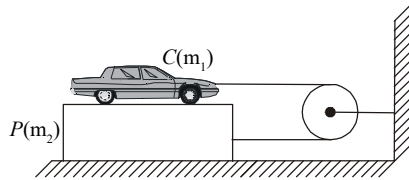


Fig 6.68

- (a) If $m_1 > m_2$, the string will remain under tension.
- (b) If $m_1 < m_2$, the string will become slack.
- (c) If $m_1 = m_2$, the string will have no tension, and C and P will have accelerations of equal magnitude.
- (d) C and P will have accelerations of equal magnitude if $m_1 \geq m_2$.
87. A man tries to remain in equilibrium,

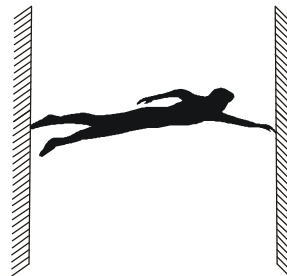


Fig 6.69

- (a) he must exert equal forces on the two walls.
- (b) the forces of friction at the two walls must be equal.
- (c) friction must be present on both walls.
- (d) the coefficients of friction must be the same between both walls and the man.
88. Two men of unequal masses hold on to the two sections of a light rope passing over a smooth light pulley. Which of the following are possible?

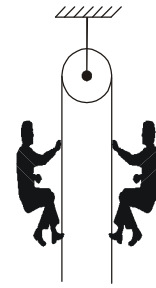


Fig 6.70

- (a) The lighter man is stationary while the heavier man slides with some acceleration.
- (b) The heavier man is stationary while the lighter man climbs with some acceleration.
- (c) The two men slide with the same acceleration in the same direction.
- (d) The two men slide with accelerations of the same magnitude in opposite directions.
89. Two blocks A and B of the same mass are joined by a light string and placed on a horizontal surface. An external horizontal force P acts on A . The tension in the string is T . The forces of friction acting on A and B are F_1 and F_2 respectively. The limiting value of F_1 and F_2 is F_0 . As P is gradually increased,

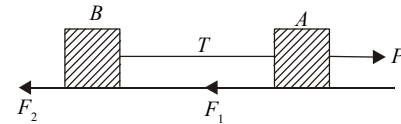


Fig 6.71

- (a) for $P < F_0$, $T = 0$
- (b) for $F_0 < P < 2F_0$, $T = P - F_0$
- (c) for $P > 2F_0$, $T = \frac{P}{2}$
- (d) none of the above
90. A block is placed at the bottom of an inclined plane and projected upwards with some initial speed. It slides up the incline, stops after time t_1 , and slides back in a further time t_2 . The angle of inclination of the plane with the horizontal is θ and the coefficient of friction is μ .

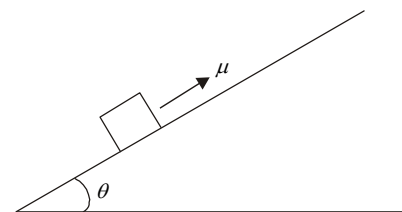


Fig 6.72

- (a) $t_1 > t_2$
- (b) $t_1 < t_2$

- (c) The retardation of the block while moving up is $g(\sin \theta + \mu \cos \theta)$.
- (d) The acceleration of the block while moving down is $g(\sin \theta - \mu \cos \theta)$.
91. The two blocks A and B of equal mass are initially in contact when released from rest on the inclined plane. The coefficients of friction between the inclined plane and A and B are μ_1 and μ_2 respectively.

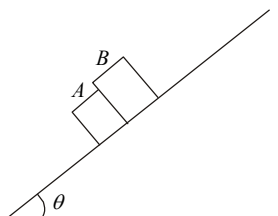


Fig 6.73

- (a) If $\mu_1 > \mu_2$, the blocks will always remain in contact.
- (b) If $\mu_1 < \mu_2$, the blocks will slide down with different accelerations.
- (c) If $\mu_1 > \mu_2$, the blocks will have a common acceleration $\frac{1}{2}(\mu_1 + \mu_2)g \sin \theta$.
- (d) If $\mu_1 < \mu_2$, the blocks will have a common acceleration $= \frac{\mu_1 \mu_2 g}{\mu_1 + \mu_2} \sin \theta$.

92. A ball of mass m is attached to the lower end of a light vertical spring of force constant k . The upper end of the spring is fixed. The ball is released from rest with the spring at its normal (unstretched) length, and comes to rest again after descending through a distance x .
- (a) $x = mg/k$
- (b) $x = 2 mg/k$
- (c) The ball will have no acceleration at the position where it has descended through $x/2$.
- (d) The ball will have an upward acceleration equal to g at its lowermost position.
93. The total work done on a particle is equal to the change in its kinetic energy.
- (a) always.
- (b) only if the forces acting on it are conservative.
- (c) only if gravitational force alone acts on it.
- (d) only if elastic force alone acts on it.
94. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
- (a) its velocity is constant.
- (b) its acceleration is constant.

- (c) its kinetic energy is constant.
- (d) it moves in a circular path.

95. Consider two observers moving with respect to each other at a speed v along a straight line. They observe a block of mass m moving a distance l on a rough surface. The following quantities will be same as observed by the two observers.
- (a) kinetic energy of the block at time t .
- (b) work done by friction.
- (c) total work done on the block.
- (d) acceleration of the block.
96. You lift a suitcase from the floor and keep it on a table. The work done by you on the suitcase does not depend on
- (a) the path taken by the suitcase.
- (b) the time taken by you in doing so.
- (c) the weight of the suitcase.
- (d) your weight.
97. No work is done by a force on an object if
- (a) the force is always perpendicular to its velocity.
- (b) the force is always perpendicular to its acceleration.
- (c) the object is stationary but the point of application of the force moves on the object.
- (d) the object moves in such a way that the point of application of the force remains fixed.
98. A particle of mass m is attached to a light string of length l , the other end of which is fixed. Initially the string is kept horizontal and the particle is given an upward velocity v . The particle is just able to complete a circle.
- (a) The string becomes slack when the particle reaches its highest point.
- (b) The velocity of the particle becomes zero at the highest point.
- (c) The kinetic energy of the ball in initial position was $\frac{1}{2}mv^2 = mgl$.
- (d) The particle again passes through the initial position.
99. The kinetic energy of a particle continuously increases with time.
- (a) The resultant force on the particle must be parallel to the velocity at all instants.
- (b) The resultant force on the particle must be at an angle less than 90° all the time.
- (c) Its height above the ground level must continuously decrease.

- (d) The magnitude of its linear momentum is increasing continuously.
- 100.** One end of a light spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement, the work done by the spring is $\frac{1}{2} kx^2$. The possible cases are
- the spring was initially compressed by a distance x and was finally in its natural length.
 - it was initially stretched by a distance x and finally was in its natural length.
 - it was initially in its natural length and finally in a compressed position.
 - it was initially in its natural length and finally in a stretched position.
- 101.** A block of mass M is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F . The kinetic energy of the block increases by 20 J in 1 s.
- The tension in the string is Mg .
 - The tension in the string is F .
 - The work done by the tension on the block is 20 J in the said 1 s.
 - The work done by the force of gravity is 20 J in the said 1 s.
- 102.** A man and a child are holding a uniform rod of length L in the horizontal direction in such a way that one fourth weight is supported by the child. If the child is at one end of the rod then the distance of man from another end will be—
- $\frac{3L}{4}$
 - $\frac{L}{4}$
 - $\frac{L}{3}$
 - $\frac{2L}{3}$
- 103.** A perfectly hard billiard ball of kinetic energy E_k collides with another similar ball at rest. After the collision the kinetic energy of the ball becomes $E'k$. Then—
- $E'k = Ek$
 - $E'k > Ek$
 - $E'k < Ek$
 - $E'k = E_k^2$
- 104.** The law of conservation of momentum can be perfectly applied to the collision between two particles if the time of collision is—
- very large.
 - very short.
 - less.
 - will depend upon the special circumstances.
- 105.** A body covers a distance of 10 meter under the influence of 5 N force. If the work done is 25 Joule then the angle between the force and the displacement will be—
- 0°
 - 30°
 - 60°
 - 90°
- 106.** A bullet of mass P is fired with velocity Q in a large body of mass R . The final velocity of the system will be—
- $\frac{R}{P+R}$
 - $\frac{PQ}{P+R}$
 - $\frac{(P+Q)}{R}$
 - $\frac{P+R}{P}Q$
- 107.** A sphere of mass m moving with a constant velocity collides with another stationary sphere of same mass. The ratio of velocities of two spheres after collision will be, if the co-efficient of restitution is e —
- $\frac{1-e}{1+e}$
 - $\frac{e-1}{e+1}$
 - $\frac{1+e}{1-e}$
 - $\frac{e+1}{e-1}$
- 108.** An electric motor produces a tension of 4500 N in a load lifting cable and rolls it at the rate of 2m/s. The power of the motor is—
- 9 KW
 - 15 KW
 - 225 KW
 - 9×10^3 HP
- 109.** A body of mass m is accelerated to velocity v in time t' . The work done by the force as a function of time t will be—
- $\frac{m}{2} \frac{v^2 t^2}{t'^2}$
 - $\frac{1}{2} \left(\frac{mv}{t'} \right)^2 t^2$
 - $\frac{m}{2} \frac{v}{t'} t^2$
 - $\frac{mvt^2}{2t'}$
- 110.** A ball falls from a height of 5m and strikes the roof of a lift. If at the time of collision, lift is moving in the upward direction with a velocity of 1m/s, then the velocity with which the ball rebounds after collision will be—
- 11 m/s downwards
 - 12 m/s upwards
 - 13 m/s upwards
 - 12 m/s downwards
- 111.** If the force acting on a particle is zero then the quantities which are conserved are—
- momentum and angular momentum.
 - momentum and mechanical energy.
 - momentum and charge.
 - angular momentum and mechanical energy.
- 112.** The co-efficient of restitution depends upon—

- (a) the masses of the colliding bodies.
 (b) the direction of motion of the colliding bodies.
 (c) the inclination between the colliding bodies
 (d) the materials of the colliding bodies.
113. The value of e for plastic bodies is—
 (a) 1 (b) zero
 (c) 8 (d) arbitrary
114. A motor of 100 H.P. is moving with a constant velocity of 72 km/hour. The forward force exerted by the engine on the car is—
 (a) $3.73 \times 10^3 \text{ N}$ (b) $3.73 \times 10^2 \text{ N}$
 (c) $3.73 \times 10^1 \text{ N}$ (d) None of the above
115. Uniform constant retarding force is applied in order to stop a truck. If its speed is doubled then the distance travelled by it will be—
 (a) four times (b) double
 (c) half (d) same
116. The kinetic energy of a man is half the kinetic energy of a boy of half of his mass. If the man increases his speed by 1 m/s then his kinetic energy becomes equal to that of the boy. The ratio of the velocity of the boy and that of the man is—
 (a) $\frac{2}{1}$ (b) $\frac{1}{2}$
 (b) $\frac{3}{4}$ (d) $\frac{4}{3}$
117. In the above question the initial velocity of the man will be—
 (a) 3.571 m/s (b) 2.415 m/s
 (c) 5.718 m/s (d) 4.127 m/s
118. Two elastic bodies P and Q having equal masses are moving along the same line with velocities of 16 m/s and 10 m/s respectively. Their velocities after the elastic collision will be in m/s—
 (a) 0 and 25 (b) 5 and 20
 (c) 10 and 16 (d) 20 and 5
119. A squirrel of mass m is moving on a disc of mass M and radius R , in a circle of radius $R/2$ with angular velocity ω (Fig. 4.39). The angular frequency with which the disc will rotate in the opposite direction will be—

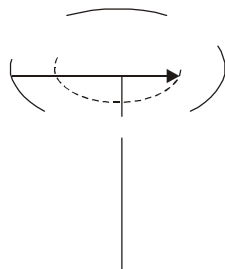


Fig 6.74

- (a) $\frac{m\omega}{2M}$ (b) $\frac{m\omega}{M}$
 (c) $\frac{2m\omega}{M}$ (d) $\frac{2M\omega}{m}$

120. A ship of mass $3 \times 10^7 \text{ Kg}$ is initially at rest. It is being pulled by a force of $5 \times 10^4 \text{ N}$ through a distance 3 m . If the air resistance is negligible, then the speed of the ship will be—
 (a) 5 m/s (2) 1.5 m/s
 (c) 60 m/s (d) 0.1 m/s
121. A metal ball does not rebound when struck on a wall, whereas a rubber ball of same mass when thrown with the same velocity on the wall rebounds. From this it is inferred that—
 (a) change in momentum is same in both.
 (b) change in momentum in rubber ball is more.
 (c) change in momentum in metal ball is more.
 (d) initial momentum of metal ball is more than that of rubber ball.
122. The unit of the co-efficient of restitution is—
 (a) m/s (b) s/m
 (c) $m \times s$ (d) none of the above.
123. A particle moves in a potential region given by $U = 8x^2 - 4x + 400 \text{ Joule}$. Its state of equilibrium will be—
 (a) $x = 25 \text{ m}$ (b) $x = 0.25 \text{ m}$
 (c) $x = 0.025 \text{ m}$ (d) $x = 2.5 \text{ m}$
124. Keeping in view the law of conservation of momentum, which of the following figures is incorrect?

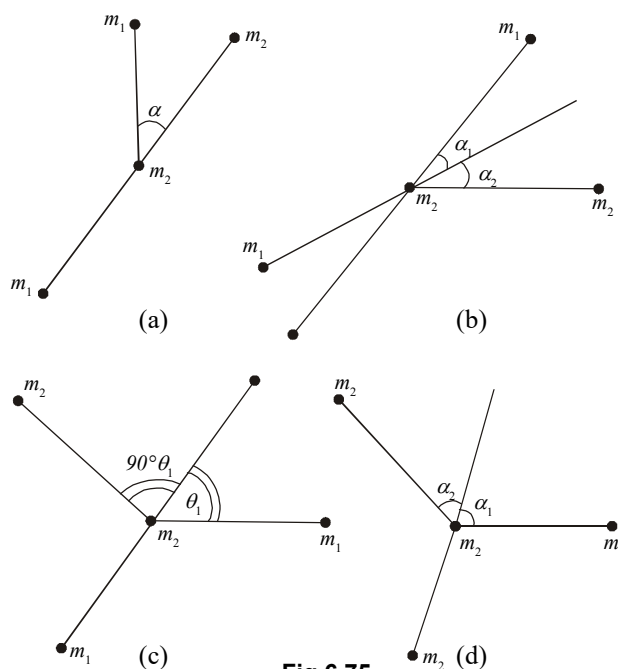


Fig 6.75

125. The graph between $\sqrt{E_k}$ and $1/p$ is (E_k = kinetic energy and p = momentum)

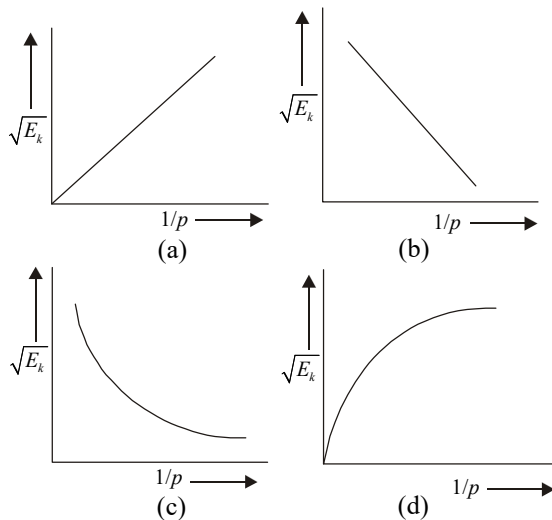


Fig 6.76

126. The graph between U and X in the state of stable equilibrium will be—

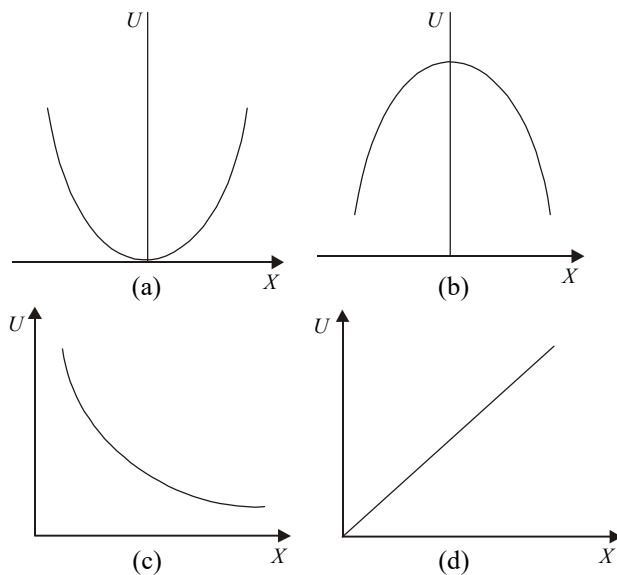


Fig 6.77

127. Two masses $m_1 = 2\text{ kg}$ and $m_2 = 5\text{ kg}$ are moving on a directionless surface with velocities 10 m/s and 3 m/s respectively m_2 is ahead of m_1 . An ideal spring of spring constant $k = 1120\text{ N/m}$ is attached on the back side of m_2 . The maximum compression of the spring will be, if on collision the two bodies stick together—

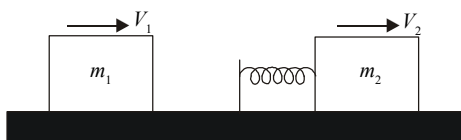


Fig 6.78

- (a) 0.51 m (b) 0.062 m
(c) 0.25 m (d) 0.72 m

128. A ball with velocity 9 m/s collides with another similar stationary ball. After the collision the two balls move in directions making an angle of 30° with the initial direction. The ratio of the speeds of balls after the collision will be—

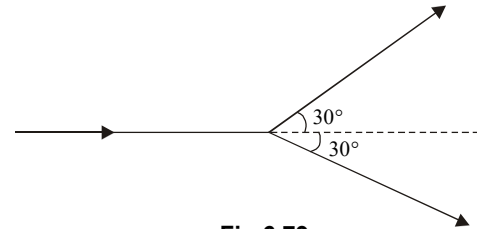


Fig 6.79

- (a) $\frac{v_1}{v_2} = 1$ (b) $\frac{v_1}{v_2} > 1$
(c) $\frac{v_1}{v_2} < 1$ (d) $\frac{v_1}{v_2} = 0$

PASSAGE 1

Read the following passage and answer the questions given at the end.

Your physics teacher who gives you tuitions asks you to do experiments on the springs which do not follow Hooke's law faithfully. He gives you a spring and asks to fix one end. Connect a mass M on the other end (The system is on a smooth table). The spring is stretched by l and released. Then he asks you to bring another identical block and provide velocity v so that it pushes the spring by x . The spring follows the law $F = kx - rx^2$. The teacher asks you to find

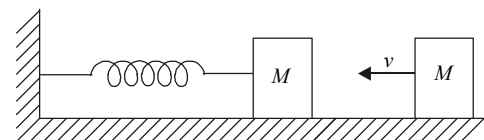


Fig 6.80

1. The velocity of the block [in case (i)] when the block has reached mid point after the release.

(a) $\sqrt{\frac{l^2}{4m} \left(3k - \frac{7rl}{3} \right)}$ (b) $\sqrt{\frac{l^2}{2m} \left(3k - \frac{7rl}{3} \right)}$

(c) $\sqrt{\left(\frac{3kl^2}{8} + \frac{7rl^3}{24} \right) \frac{1}{M}}$ (d) none

2. The maximum compression in the spring in case (ii) x is the compression so that it satisfies

(a) $\frac{kx^2}{2} - \frac{rx^3}{2} = \frac{Mv^2}{2}$

(b) satisfies $\frac{kx^2}{2} - \frac{rx^3}{3} = Mx^2$

(c) $\frac{kx^2}{2} - \frac{rx^3}{3} = \frac{Mv^2}{4}$ (d) none

3. The work done by the spring in case (i)

(a) $\frac{kl^2}{2} - \frac{rl^3}{24}$ (b) $\frac{kl^2}{2} + \frac{rl^2}{24}$

(c) $\frac{kl^3}{6} - \frac{rl^2}{8}$ (d) $\frac{kl^3}{8} - \frac{rl^2}{6}$

Solution 1. (a) $\int_0^l F \cdot dx = \int_0^{l/2} F \cdot dx + \frac{1}{2} Mv^2$ (conserve energy)

or $\frac{1}{2} Mv^2 = \left[\frac{kx^2}{2} - \frac{rx^3}{3} \right]_0^l - \left[\frac{kx^2}{2} - \frac{rx^3}{3} \right]_0^{l/2}$

or $v = \sqrt{\left(\frac{3kl^2}{8} - \frac{7rl^3}{3 \times 8} \right) \times \frac{2}{M}}$

Solution 2(c) $2 Mv^1 = Mv$ or $v' = \frac{v}{2}$

$$\frac{1}{2} kx^2 - \frac{rx^3}{3} = \frac{1}{2} (2M) \left(\frac{v}{2} \right)^2$$

The compression produced is x such that

$$\frac{rx^3}{3} - \frac{kx^2}{2} + \frac{Mv^2}{4} = 0$$

Solution 3(a) $W = \frac{kl^2}{2} - \frac{rl^3}{24}$

PASSAGE 2

Read the following passage and answer the questions given at the end.

The brothers of *Iota, Eta, Pi* fraternity build a platform, supported at all four corners by vertical springs in the basement of their frat house. A brave fraternity brother wearing a football helmet stands in the middle of the platform, his weight compresses the springs by 0.18m. Then four of his fraternity brothers pushing down at the corners of the platform, compress the springs by another 0.53 m until the top of the brave brother helmet is 0.9 m below the basement ceiling. They then simultaneously release the platform. Ignore the masses of spring and platform. The dean of students suggests them to perform the stunt at other planet.

1. Find the velocity with which fraternity brother's helmet hits the ceiling.

- (a) 3.14 ms^{-1} (b) 2.89 ms^{-1}
(c) 2.93 ms^{-1} (d) 4.13 ms^{-1}

2. If the ceiling would not have been there, how high he would have gone?

- (a) 2.4 m (b) 2.1 m
(c) 1.7 m (d) 1.4 m

Physics by Saurabh Maurya (IIT-BHU)

3. How high will he reach if he performs the stunt at a planet which has g equal to half the acceleration due to gravity on the earth?

- (a) 1.4 m (b) 2 m
(c) $\sqrt{3}$ m (d) none of these

Solution 1(a) $v^2 = u^2 - 2gh$

or $v = \sqrt{2g(1.4 - .9)} = 3.14 \text{ ms}^{-1}$

Solution 2 (d) $k(.18) = Mg$

or $k = \frac{Mg}{.18}$

$$\left(\frac{kx^2}{2} \right) = Mgh, \frac{mg(.11)^2}{2 \times .18} = mgh \text{ or } h = 1.4 \text{ m}$$

Solution 3 (a) The height reached will not be affected by g if compression of the string remains same.

PASSAGE 3

Read the following passage and answer the questions given at the end.

In an experiment a proton of mass $1.67 \times 10^{-27} \text{ kg}$ is propelled at an initial velocity $3 \times 10^5 \text{ ms}^{-1}$ directly towards a uranium nucleus 5 m away. The proton is repelled by a force

$$F = \frac{2.12 \times 10^{-26}}{x^2} \text{ Nm}^{-2} \text{ where } x \text{ is separation between two}$$

objects. Assume that uranium is at rest. As the proton approaches the uranium atom, repulsive force slows down the proton until it comes momentarily to rest, after which proton moves away from the nucleus.

1. What is the speed of the proton when it is 8 A^0 away from the nucleus?

- (a) $1.85 \times 10^5 \text{ ms}^{-1}$ (b) $1.85 \times 10^4 \text{ ms}^{-1}$
(c) $1.85 \times 10^3 \text{ ms}^{-1}$ (d) $1.85 \times 10^2 \text{ ms}^{-1}$

2. Find the closest distance of approach.

- (a) 1.4 A^0 (b) 2.8 A^0
(c) 2.8 pm (d) 2.8 fm

3. What is the velocity of proton when it reaches back 5 m away?

- (a) $1.8 \times 10^5 \text{ ms}^{-1}$ (b) $2.3 \times 10^5 \text{ ms}^{-1}$
(c) $3 \times 10^5 \text{ ms}^{-1}$ (d) $2.7 \times 10^5 \text{ ms}^{-1}$

Solution 1. (a) $\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 - \int_5^{8 \times 10^{-10}} \frac{F \cdot dx}{m}$

or $v_f^2 = v_i^2 - \frac{2.12 \times 10^{-26}}{mx} \Big|_5^{8 \times 10^{-10}}$

or $v_f^2 = (3 \times 10^5)^2 - \frac{2.12 \times 10^{-26}}{1.67 \times 10^{-27} \times 9 \times 10^{10}} = 9 \times 10^{10}$

$$-5.58 \times 10^{10} = 3.42 \times 10^{10}$$

or $v = 1.85 \times 10^5 \text{ ms}^{-1}$

Solution 2(b) $\frac{1}{2} m v_f^2 = \frac{2.12 \times 10^{-26}}{x}$

or $x = \frac{2 \times 2.12 \times 10^{-26}}{1.67 \times 10^{-27} \times 9 \times 10^{10}} = 2.8 \times 10^{-10} \text{ m}$

Solution 3(c)

PASSAGE 4

Read the following passage and answer the questions given at the end.

An air plane in flight is subject to an air resistance force proportional to square of its speed v . But there is an additional resistive force because the air plane has wings. Air flowing over the wings is pushed down and slightly forward. From Newton's third law the air exerts a force on the wings and the airplane in upward and in backward direction. The upward force is the lift force that keeps the airplane aloft and backward force is called induced drag. At flying speeds induced drag is inversely proportional to v^2 so that total air resistance can be expressed as $F = \alpha v^2 + \beta/v^2$ where α and β are positive constants. Their values depend upon shape and size of the airplane and density of air. For a cessna 150, a small single engine airplane $\alpha = 0.3 \text{ N s}^2 \text{ m}^{-2}$ and $\beta = 3.5 \times 10^5 \text{ N m}^2 \text{ s}^{-2}$. In steady flight engine must provide a forward force that exactly balances the air resistance.

1. Find the speed at which the airplane will have maximum range.

(a) $\leq 42.6 \text{ ms}^{-1}$

(b) $\leq 33 \text{ ms}^{-1}$

(c) $\geq 42.6 \text{ ms}^{-1}$

(d) none

2. Find the speed at which the airplane will have the maximum endurance (remain in air for the longest period).

(a) $v > 42.6 \text{ ms}^{-1}$

(b) $v = 33 \text{ ms}^{-1}$

(c) $33 \text{ ms}^{-1} < v < 42.6 \text{ ms}^{-1}$ (d) $< 33.0 \text{ ms}^{-1}$

Solution $F_{\text{air}} = .3v^2 + \frac{3.5 \times 10^5}{v^2}$

For v , to be maximum (Range maximum) air drag be

minimum and $\frac{d^2 F}{dv^2}$ should be negative

$$\therefore \frac{dF}{dv} = 0$$

$$.6v + (-2)(3.5 \times 10^5) v^{-3} = 0$$

or $v^4 = \frac{2 \times 3.5 \times 10^5}{.6}$

or $v = 33 \text{ ms}^{-1}$

$$\frac{d^2 F}{dv^2} = .6 + 6(3.5 \times 10^5) v^{-4}$$

or $v > \left(\frac{6 \times 3.5 \times 10^5}{.6} \right)^{1/4}$

$$v_{\text{max range}} = 42.6 \text{ ms}^{-1} \quad v > (350 \times 10^4)^{1/4}$$

$$v_{\text{endurance}} = 33 \text{ ms}^{-1} \quad v > 42.6 \text{ ms}^{-1}$$

Answers to Questions for Practice

1. (b)	2. (d)	3. (c)	4. (a)	5. (b)	6. (a)	7. (d)
8. (c)	9. (d)	10. (a)	11. (c)	12. (d)	13. (b)	14. (a)
15. (b)	16. (d)	17. (a)	18. (a)	19. (b)	20. (b)	21. (c)
22. (b)	23. (d)	24. (a)	25. (c)	26. (a)	27. (b)	28. (c)
29. (d)	30. (a)	31. (a)	32. (c)	33. (d)	34. (b)	35. (a)
36. (d)	37. (a)	38. (b)	39. (c)	40. (b)	41. (d)	42. (d)
43. (a)	44. (a)	45. (a)	46. (a)	47. (b)	48. (a)	49. (a)
50. (c)	51. (c)	52. (d)	53. (b)	54. (b)	55. (a)	56. (b)
57. (a)	58. (c)	59. (d)	60. (d)	61. (a,c)	62. (b,c,d)	63. (a,c)
64. (a,c)	65. (a,c)	66. (b,c,d)	67. (a,c)	68. (a,b,c,d)	69. (b,c)	70. (a,d)
71. (a,d)	72. (a,c)	73. (a,d)	74. (a,b,c,d)	75. (a,b)	76. (a,b,d)	77. (a,b,d)
78. (a,d)	79. (a,c)	80. (a,b,c)	81. (a,c,d)	82. (a,d)	83. (a,b,d)	84. (a,b,d)
85. (a,b,c,d)	86. (a,b,c)	87. (a,b,c,d)	88. (a,b,d)	89. (a,b,c)	90. (b,c,d)	91. (a,b)
92. (b,c,d)	93. (a)	94. (c,d)	95. (d)	96. (a,b,d)	97. (a,c,d)	98. (a,d)
99. (b,d)	100. (a,b)	101. (b)	102. (c)	103. (c)	104. (b)	105. (c)
106. (b)	107. (a)	108. (a)	109. (a)	110. (b)	111. (a)	112. (d)
113. (b)	114. (a)	115. (a)	116. (a)	117. (b)	118. (c)	119. (a)
120. (d)	121. (b)	122. (d)	123. (b)	124. (a)	125. (c)	126. (a)
127. (c)	128. (a)					

EXPLANATION

1(b) For instantaneous power

$$dP = \frac{dW}{dt} = \frac{dF^2 t^2}{2m} = \frac{F^2}{2m} \cdot 2t$$

or
$$dP = \frac{2F^2 T}{2m}$$

But average power = $\frac{F^2(T^2 - 0)}{2mT} = \frac{F^2 T}{2m}$

\therefore Instantaneous power : average power = 2 : 1.

2(d) Let $XO = OZ = \frac{L}{2}$. When end Z goes down by y then velocity of bend

$$v = \sqrt{2g \frac{y}{2}}$$

because bend falls through $y/2$.

$$= - \left(\frac{m}{L} dy \right) v$$

$$= \frac{-m}{L} dy (gy)^{1/2} \quad (\text{downward})$$

$$= \frac{m}{L} (gy)^{\frac{1}{2}} dy \quad (\text{upward})$$

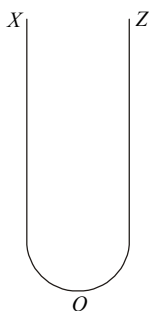


Fig 6.81

3(c) Spring energy = kinetic energy

$$i.e. \frac{1}{2} kx^2 = \frac{1}{2} mv^2 \quad \text{or } v = x \sqrt{\frac{k}{m}}$$

Time taken by ball to reach ground, t

$$= \sqrt{\frac{2h}{g}} \quad (\text{from } S = ut + \frac{1}{2} at^2)$$

\therefore Horizontal distance covered = vt

$$= x \sqrt{\frac{k}{m}} \sqrt{\frac{2h}{g}} = x \sqrt{\frac{2kh}{mg}}$$

4(a) K.E. of block at B = P.E. at A – P.E. at B

$$i.e. \frac{1}{2} mv^2 = mgh - mg^2 r = mg(h - 2r)$$

$$i.e. v^2 = 2g(h - 2r) \quad \dots\dots(i)$$

$$\text{Also } \frac{mv^2}{r} - mg = x mg$$

$$\text{or } v^2 = (x + 1) rg \quad \dots\dots(ii)$$

Equating (i) and (ii)

$$2g(h - 2r) = (x + 1) rg$$

$$\text{or } 2gh = (x + 1) gr + 4gr \\ = (x + 5) gr$$

$$\text{or } h = \left(\frac{x + 5}{2} \right) r$$

5(b) Using $P = Fv$, we get

$$P = Fr\omega \quad (\because v = r\omega)$$

$$\text{Here } F = \mu N$$

(where N is normal reaction)

$$= \mu F' g \quad (\because 1 \text{ kgf} = g \text{ newton})$$

$$\therefore P = \mu F' gr\omega = 2\pi \mu n F' gr \quad (\because \omega = 2\pi)$$

6. (a) Work energy relationship $i.e. W = \frac{1}{2} mu^2$

indicates that the graph between w and u should be a *parabola* towards w axis.

7(d) use P.E. = Work done

$$mgH = \mu_{\text{kinetic}} mgS$$

$$\text{or } S = \frac{H}{\mu_{\text{kinetic}}}$$

The particle covers the length l or not, or covers it repeatedly is determined by the above relation.

8(c) P.E. with reference to lowest point = $Mg(2L)$

From conservation of energy,

$$\frac{1}{2} Mv^2 = 2MgL \quad i.e. v = 2\sqrt{gL}$$

9(d) Using conservation of momentum,

$$(120 + 5) v_x = 5 \times 2$$

$$\text{or } v_x = \frac{10}{125} = 0.08 \text{ ms}^{-1}$$

$$\text{Also } (90 - 5) v_y = 5 \times 2$$

$$\text{or } v_y = \frac{10}{85} = 0.12 \text{ ms}^{-1}$$

$$10. \text{ (a) Kinetic energy, } E = \frac{1}{2} m v^2$$

$$\text{here } v^2 = u^2 + (gt)^2$$

(\because vertical component of velocity is gt)

$$E = \frac{1}{2} m (u^2 + gt^2)$$

Thus, curve should be a parabola of the type shown by (A).

11(c) The resultant force = forward thrust – backward thrust
and forward thrust = $U_E \times (\mu + m)$

$$\text{Backward thrust} = U_p \times \mu$$

\therefore Resultant thrust,

$$F = U_E (\mu + m) - U_p \mu$$

$$\text{Power} = F \times v$$

$$= \left[U_E \frac{(\mu + m) - U_p \mu}{746} \right] U_p$$

$$= \frac{U_E U_p (\mu + m) - U_p^2 \mu}{746}$$

$$12(d) \text{ Using } P = Fv = M \left(\frac{dv}{dt} \right) v$$

$$\text{i.e. } v^2 dv = \frac{P}{M} v dt = \frac{P}{M} dS$$

$$\text{Integrating } \int_u^v v^2 dv = \frac{P}{M} \int_0^S dS$$

$$v^3 - u^3 = \frac{3PS}{M}$$

$$\text{or } v = \left[\frac{3PS}{M} + u^3 \right]^{1/3}$$

13(b) Let u be the speed of moving frame and horizontal distance covered at any instant from moving frame is x then distance from stationary frame is $(x + ut)$

Work done w.r.t. stationary frame W

$$= F(x + ut) = Fx + Fut$$

But Fx is the work done w.r.t. moving frame

$$\therefore W = W' + Fut$$

$$\text{i.e. } W \neq W'$$

14(a) Using $v^2 - u^2 = 2aS$, we get

$$v = (u^2 + 2aS)^{1/2}$$

$$= (2.4 \times 10^7) + 2 \times 3.6 \times 10^{15} \times 0.035$$

$$= 2.88 \times 10^7 \text{ ms}^{-1}$$

$$\text{Change in K.E.} = \frac{1}{2} m (v^2 - u^2)$$

$$= \frac{1}{2} (1.67 \times 10^{-27}) (2.88 \times 10^7 - 2.4 \times 10^7)$$

$$= 2.11 \times 10^{-13} \text{ J}$$

$$= 1.32 \times 10^6 \text{ eV} = 1.32 \text{ MeV}$$

15(b) Loss in K.E. = Gain in P.E. + Work done against air drag

$$\text{i.e. } \frac{1}{2} mu^2 = mgh + Fh = h(mg + F)$$

$$\text{or } h = \frac{u^2}{2(g + F/m)}$$

$$= \frac{u^2}{2g + 2F/m}$$

16(d) Using conservation of momentum

$$0.02 u = 0.02 v + 1 V_1 \quad \dots\dots(i)$$

(where V_1 be the velocity of plate of 1 kg)

$$\text{and } 0.02 v = (2.98 + 0.02) V_1 \quad \dots\dots(ii)$$

(\because plate of 3 kg has also same velocity i.e. V_1)

$$\text{or } V_1 = 0.02 \frac{v}{3}$$

Substituting

$$\therefore 0.02 v = 0.02 v + 0.02 \frac{v}{3}$$

$$\text{or } v = \frac{4v}{3} \text{ or } v = \frac{3u}{4}$$

$$\% \text{age loss in velocity} = \frac{v - u}{u} \times 100$$

$$= \frac{v - \frac{3}{4}v}{v} \times 100 = 25\%$$

17(a) The particle was initially at rest at (6, 4)

$$\therefore x = 6 + \frac{1}{2} a_x t^2$$

$$\text{and } y = 4 + \frac{1}{2} a_y t^2 \quad \dots\dots(i)$$

Acceleration

$$\vec{a} = \frac{\vec{F}}{m} = -(3\hat{i} + 4\hat{j})$$

$$\left(Q F_x = \frac{-dP.E.}{dx} \text{ and } F_y = \frac{-dP.E.}{dx} \right)$$

$$i.e. a = (3^2 + 4^2)^{1/2} = 5 \text{ ms}^{-2} \quad \dots\dots(ii)$$

Substituting the value of a from (ii) in (i)

$$x = \left(6 - \frac{3}{2}t^2 \right)$$

$$\text{and } y = (4 - 2t^2)$$

Now $y = 0$ when particle crosses x - axis

$$\therefore t = \sqrt{2} \text{ s}$$

$$\text{Displacement } S = \frac{1}{2} at^2$$

$$= \frac{1}{2} \times 5 \times 2 = 5 \text{ m}$$

$$\text{Then work done} = F.S. = 5 \times 5 = 25 \text{ J}$$

$$18(a) \text{ Work done, } W = mgh + \mu mgl$$

which is a constant.

$$19(b) \text{ Potential energy is minimum if equilibrium is stable.}$$

$$\frac{d}{dx} [\text{P.E.}(x)] = 0$$

$$i.e. \frac{d}{dx} \left[\frac{\alpha}{x^{12}} - \frac{\beta}{x^6} \right] = 0$$

$$i.e. \frac{-12\alpha}{x^{13}} + \frac{6\beta}{x^7} = 0$$

$$i.e. [-2\alpha + \beta x^6] \frac{6}{x^{13}} = 0$$

$$i.e. (-2\alpha + \beta x^6) = 0$$

$$i.e. x = \left(\frac{2\alpha}{\beta} \right)^{1/6}$$

$$20(b) \text{ Using law of conservation of total momentum, we get}$$

$$(M+m)v = M\sqrt{2gh}$$

$$\text{or } v = \frac{M\sqrt{2gh}}{M+m}$$

Let opposition to penetration be F then

Total work done = change in K.E.

$$i.e. F = \frac{1}{2} (M+m) \frac{(M^2 2gh)}{d(M+m)^2} + (M+m)g$$

$$= \frac{M^2 gh}{(M+m)d} + (M+m)g$$

$$21(c) \text{ Loss in potential energy} = \text{Gain in kinetic energy}$$

$$mgh = \frac{1}{2} m v'^2$$

$$h = \frac{v'^2}{2g} \frac{(u+v)^2}{2g}$$

(\because here velocity is given as $(u+v)$)

$$22(b) \text{ Let } m \text{ be the mass of the bullet and } M \text{ be the mass of the gun. If } v \text{ is the velocity of bullet then}$$

$$MV = mv \text{ i.e., } V = \frac{mv}{M}$$

Thus, V is the velocity of recoil of gun when held loosely.

But when the gun is held tightly with the shoulder, the mass of the man supports the gun thus reducing the speed of gun. Hence, the gun experiences intensive recoil when fired with the butt held loosely with the shoulder.

$$23(d) \text{ Note } \frac{1}{2} kd^2 = mg(H+d)$$

$$\text{or } k = \frac{2mg(H+d)}{d^2}$$

$$24(a) \text{ Efficiency} = \frac{\text{output}}{\text{input}}$$

$$i.e. \text{ Output} = \text{efficiency} \times \text{input}$$

$$= \eta \times 100 \text{ calorie}$$

$$= \eta \times 100 \times 4.2$$

$$= 420 \eta \text{ joule}$$

$$\text{or } mgh = 420 \eta$$

$$\text{or } h = \frac{420 \eta}{mg}$$

$$25(c) \text{ Work done} = mgh$$

$$\frac{1}{2} \times \text{base} \times \text{height} = mgh$$

(area under the curve is work done)

$$\frac{1}{2} \times 10 \times 100 = 5 \times 10 \times h$$

$$\text{or } h = 10 \text{ m}$$

26(a) When plank is moved, the string is under tension. Work done by force F tries to neutralise the frictional force of plank and elongation of the string due to non sliding of the block due to its friction. Thus, work done by F is equivalent to option (a).

27(b) On ascending the plane, component of weight of tractor along the inclined plane will add to the given tension when the tractor and truck were moving on a level road i.e. new tension,

$$T' = T + mg \sin \theta$$

$$= 1000 + 1200 \times 9.8 \times \frac{1}{6}$$

$$= 1000 + 1960 = 2960 \text{ N}$$

28(c) Work done in one rotation = work done against gravity

$$\text{i.e. } F \times 2\pi \times \frac{1}{2} = 200 \times 10 \times 5 \times 10^{-3}$$

$$(F \times 2\pi = (mg) \times \text{pitch})$$

$$F = \frac{10}{3.14} = 3.18 \text{ N}$$

29. Using conservation of energy

$$\text{In first case, } \frac{1}{2} mu^2 + \frac{1}{2} m \left(\frac{u}{2} \right)^2$$

$$= mgL + mg \frac{L}{2}$$

$$\text{or } u = \left(\frac{12}{5} gL \right)$$

In second case

$$\frac{1}{2} (2) v^2 = 2 mgL \quad v = (2gL)^{1/2}$$

$$\therefore \frac{u}{v} = \frac{\left(\frac{12}{5} gL \right)^{1/2}}{[2gL]^{1/2}} = \left[\frac{6}{5} \right]^{1/2}$$

$$\text{or } v = \left[\frac{6}{5} \right]^{1/2}$$

30(a) Work done = Change in K.E.

$$\text{i.e. } W_g + W_a = \frac{1}{2} mv^2$$

where $W_g = mgs$ and W_a is the work done by air resistance

$$\therefore W_a = -mgs + \frac{1}{2} mv^2$$

$$= -\frac{m}{2} (-v^2 + 2gs)$$

$$= -\frac{m}{2} (2gs - v^2)$$

31(b) In case of vertical motion under gravity, when the body falls, force of gravity (a conservative force) is in the direction of motion so K.E. increases but when the body is projected upward, force of gravity is opposite to motion so K.E. of body decreases. If kinetic energy of a body increases the work is taken as positive and vice versa.

32(c) Work done = ΔKE ,

$$Mg \left(r - \frac{2r}{\pi} \right) = \frac{1}{2} Mv^2$$

(\therefore of c.g. of the chain)

$$\text{or } u = \sqrt{2gr \left(1 - \frac{2}{\pi} \right)}$$

33(d)

$$\frac{1}{2} mu^2 = -\mu mgS + \frac{1}{2} KS^2$$

$$\text{i.e. } S^2 + \frac{2\mu mgS}{K} - \frac{mu^2}{K} = 0$$

Solving for μ ,

$$S = \frac{1}{K} (\mu^2 m^2 g^2 - mKu^2 - \mu mg)$$

34. (b) For rigid cord,

$$mgl = \frac{1}{2} mv_1^2 \quad \dots\dots(i)$$

Equating (i) and (ii)

$$\frac{1}{2} mv_1^2 = \frac{1}{2} mv_2^2 + \frac{1}{2} k(\delta l)^2$$

Clearly $v_1 > v_2$

35(a) For equilibrium

$$Ma_{\max} \times h = Mg \times p$$

(q will not matter as per statement of the question because when the car is about to topple, the whole reaction is on the rear wheel as front wheel is no more in contact with the ground and here R is the reaction of the rear wheel)

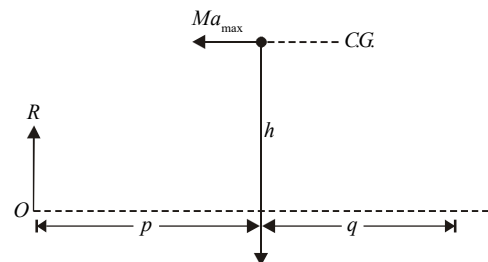


Fig 6.82

$$a_{\max} = \frac{qg}{h}$$

36(d) Force \times Time = Change in momentum

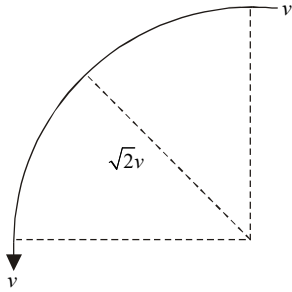


Fig 6.83

$$\text{or } F \times \Delta t = \Delta m v \sqrt{2} \text{ or } F = \frac{\Delta m}{\Delta t} v \sqrt{2}$$

$$= v \sqrt{2} \times \rho a \frac{\Delta l}{\Delta t}$$

$$(\because \Delta m = \rho a \Delta l)$$

$$\text{or } F = v \sqrt{2} \rho a v = v^2 \sqrt{2} \rho a$$

$$v = \left(\frac{F}{\sqrt{2} \rho a} \right)^{1/2}$$

37(a) Work done by man to stretch the spring

$$= \left(\frac{1}{2} kx \right) (S + x)$$

Work done by man on the floor

$$= - \left(\frac{1}{2} kx \right) (S)$$

$$\therefore \text{Total work done} = \frac{1}{2} kx^2$$

38(b) Let a small displacement be given to the system in vertical plane in frame such that ST remains horizontal then let vertical displacement of centres of rods UP and QR be y then vertical displacement of centres of UT and RS will be $3y$ and that of TS will be $4y$. Equating the total virtual work to zero we get

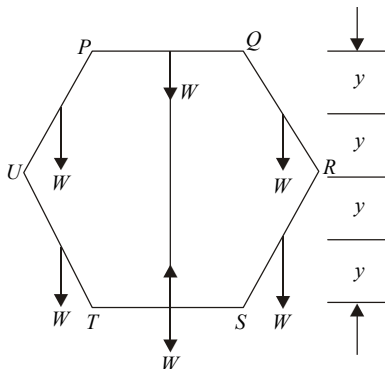


Fig 6.84

$$(W + W) \delta y + (W + W) 3 \delta y + W (4 \delta y) - T (4 \delta y) = 0$$

(where T is the tension in thread)

$$\text{or } 2W + 6W + 4W = 4T$$

$$\text{or } T = 3W$$

$$39(c) \text{C.G. of first slab} = \frac{b}{2}$$

Weight of each slab

$$= \text{volume} \times \text{density} \times g$$

$$= b^3 \rho g$$

C.G. of column of slabs

$$= \frac{\text{Total height of } N \text{ slabs}}{2}$$

$$= \frac{Nb}{2}$$

Height of displacement of force

$$= \left(\frac{Nb}{2} - \frac{b}{2} \right) = (N-1) \frac{b}{2}$$

$$\text{Work done} = N \times b^3 \rho g \times (N-1) \frac{b}{2}$$

$$= \frac{1}{2} (N^2 - N) b^4 \rho g$$

$$40(b) mg + F \cos \theta = R$$

Where m is mass of wiper, F is applied force towards centre and R is normal reaction of ground upward.

If motion is allowed on the floor,

$$F \sin \theta - \mu N \geq 0$$

$$\text{i.e. } F \sin \theta - \mu (mg + F \cos \theta) \geq 0$$

$$\text{i.e. } \sin \theta - \mu \cos \theta \geq \frac{\mu mg}{F}$$

$$\text{or } \tan \theta = \mu$$

and in limiting case $\tan \theta_0 = \mu_s$

$$\text{or } \theta_0 = \tan^{-1} \mu_s$$

41(d) Let $AB = BC = CD = l$ unit and end A moves slightly towards right such that l is increased to $(l + \delta l)$.

Let the total work be zero, we get

$$F_1 (3 \delta l) - F_2 (\delta l) = 0$$

$$(\because \text{displacement of } A = 3 \delta l$$

and that of C is δl)

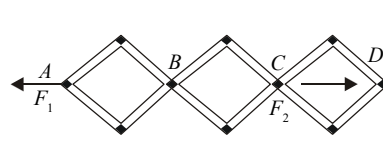


Fig 6.85

$$\text{or } F_1 = \frac{F_2}{3}$$

- 42(d) Let the extension in cord be δL . Using conservation of energy, we get

$$Mg(L + \delta L) = \frac{1}{2} K (\delta L)^2$$

$$K(\delta L) + 2 - 2MgL - 2Mg\delta L = 0$$

or $K(\delta L)^2 - 2Mg\delta L - 2MgL = 0$

or $\delta L = \frac{2Mg \pm \sqrt{4M^2g^2 + 4K \times 2MgL}}{2K}$

(By solving the quadratic equation)

$$i.e. \delta L = \frac{Mg}{K} \left(1 + \sqrt{1 + \frac{2KL}{Mg}} \right)$$

(\therefore -ve sign is meaning less)

- 43(a) Let the particle be at a distance x at any instant and T be the tension in the string then

$$T = K(l'^2 - l^2)^{1/2} = Kx$$

Net force tending to make the particle move further through dx ,

$$\text{Work done} = \left[\frac{Kx^2}{l} - \mu(Mg - Kx) \right] dx$$

$$\frac{1}{2} Mv^2 = \int_0^x \left[\frac{Kx^2}{l} - \mu(Mg - Kx) \right] dx$$

$$= \frac{Mg}{3l} \left(3 - \frac{3}{2} \mu l \right)$$

or $v = \left[\frac{gS}{3l} (2S - 3\mu l) \right]^{1/2}$

- 44(a) Let the pulley be given a small rotation θ in anticlockwise direction, the pulley and load will be lifted by $r_2\theta$ and P moves down by $(r_1\theta - r_2\theta)$.

Equating virtual work done to zero

$$P(r_1\theta - r_2\theta) - Wr_2\theta = 0$$

or $\frac{P}{W} = \frac{r_2}{r_1 - r_2}$

- 45(a) Here Brake power = $\frac{(T - S)(2\pi RN)}{60,000}$ kW

or $(T - S) = \frac{6000 \times \text{Brake power}}{2\pi RN}$

$$= \frac{6000 \times 5.25}{2\pi \times 0.5 \times 220}$$

$$= 455.8 \text{ N} \approx 456 \text{ N}$$

- 46(a) Normal reaction,

$$R = mg = 2 \times 9.8 \text{ N}$$

Frictional force

$$F = \mu R = 0.2 \times 2 \times 9.8$$

$$= 3.92 \text{ N}$$

Distance travelled

$$5s = S \text{ i.e. } 2 \times 5 = 10 \text{ m}$$

$$\therefore \text{Work done} = F \times S = 3.92 \times 10 = 39.2 \text{ J}$$

- 47(b) Here $mg y - \frac{1}{2} m v_F^2 = f_2 s_1 + f_2 s_2$

$$i.e. \frac{1}{2} m v_F^2 = mg y - \mu mg \cos \beta s_1 - \mu mg \cos \gamma s_2$$

or $\frac{1}{2} v_F^2 = g \left(y - \frac{\mu x_1 s_1}{s_1} - \frac{\mu x_2 s_2}{s_2} \right)$

or $v_F = \sqrt{2g(y - \mu x)}$

- 48(c) $P = Fv = c$

$$\therefore \frac{mdv}{v} \quad v = c$$

$$v dv = \frac{cdt}{m} \text{ or } \frac{v^2}{2} = \frac{ct}{m}$$

(after integration)

$$\left(\frac{ds}{dt} \right)^2 = \frac{2ct}{m} \text{ or } \left(\frac{ds}{dt} \right) = \sqrt{\frac{2c}{m}} t^{1/2}$$

or $s \propto t^{3/2}$

- 49(a) Here momentum of third fragment is

$$p^3 = \sqrt{p_1^2 + p_2^2}$$

or $p^3 = \sqrt{(mv)^2 + (mv)^2}$

$$= \sqrt{2} mv$$

Final K.E. of the system

$$= \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{p_3^2}{2(2m)}$$

$$= \frac{3}{2} mv^2 + \frac{3}{2} mv^2 + \frac{3}{2} mv^2$$

$$= \frac{3}{2} mv^2$$

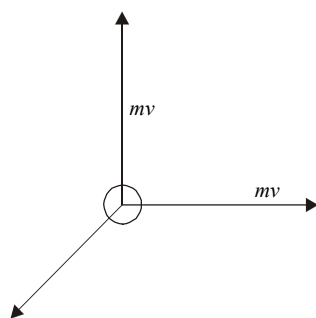


Fig 6.86

Since initial K.E. = 0 therefore energy released = $\frac{3}{2} mv^2$.

50(c) Force exerted by engine = $mg \sin \theta + \mu mg \cos \theta = mg (\sin \theta + \mu \cos \theta)$

Power exerted by engine = Fv

Work done by engine = Fvt

Efficiency of engine $\eta = \frac{\text{output}}{\text{input}}$

$$= \frac{Fvt}{\text{input}}$$

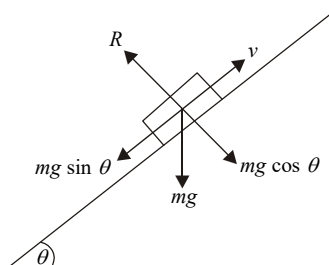


Fig 6.87

\therefore Input of engine

$$= \frac{Fvt}{\eta} = \frac{mg(\sin \theta + \mu \cos \theta)vt}{\eta}$$

51(c) Power = $P = \frac{dW}{dt} = \frac{d}{dt} (mgh)$

$$= \frac{dV \rho gh}{dt}$$

$$\text{or } P = \frac{dV \rho gh}{dt} = \rho dh \frac{dV}{dt}$$

$$\text{Since } \eta = \frac{P_{\text{output}}}{P_{\text{input}}}$$

$$\therefore P_{\text{output}} = \eta P_{\text{input}} = \eta \rho gh \frac{dV}{dt}$$

$$\text{or } \frac{dV}{dt} \eta = \frac{P_{\text{output}}}{\eta \rho gh^2}$$

$$= \frac{40}{0.9} \rho gh = \frac{44.44}{\rho gh}$$

52(d) Momentum of hydrogen atom = momentum of emitted photon

$$\text{i.e. } mv = \frac{E}{c}$$

$$\text{or } v = \frac{E}{mc}$$

$$= \frac{(0.73 + 1.82)1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^8} \\ = 0.81 \text{ ms}^{-1}$$

53(b) In stretched position of spring,

$$\text{system P.E.} = 2 \times \frac{1}{2} k \times 2x^2 = 4kx^2$$

In mean position, both balls have kinetic energy only;

$$K.E. = 2 \left[\frac{1}{2} mv^2 \right] = mv^2$$

but P.E. = K.E.

$$\therefore 4Kx^2 = mv^2$$

$$\text{or } v = 2x \sqrt{\frac{K}{m}} = 2R\theta \sqrt{\frac{K}{m}}$$

$$54(b) a_c = k^2 r t^2$$

$$\text{i.e. } \frac{v^2}{r} = k^2 r t^2 \quad \text{or } v = krt$$

$$\text{Also } a_t = \frac{dv}{dt} = kr$$

$$\therefore F = ma_t = mkr$$

$$\text{Then, power} = Fv = mkr(krt) = mk^2 r^2 t$$

$$55(a) \vec{V}_{\text{cart}} = 4 \hat{i} \quad \dots\dots(i)$$

$$\vec{V}_{\text{stone + cart}} = (6 \sin 30) \hat{j} + (6 \cos 30) \hat{k} \quad \dots\dots(ii)$$

$$= (3 \hat{j} + 3\sqrt{3} \hat{k})$$

$$\text{Then } V_{\text{stone}} = (ii) + (i) = 4 \hat{i} + 3 \hat{j} + 3\sqrt{3} \hat{k}$$

Velocity of stone at highest point

$$\vec{V}_{\text{stone + height}} = 4 \hat{i} + 3 \hat{j}$$

[At highest point vertical component (*i.e.* z component) is zero]

or speed of stone at highest point

$$V = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = 5 \text{ ms}^{-1}$$

Using conservation of momentum

$$mV = 2mV_{\text{combined}}$$

$$\text{or } V_{\text{combined}} = \frac{V}{2} = \frac{5}{2} = 2.5 \text{ ms}^{-1}$$

56(a) At horizontal position, tension is zero *i.e.* velocity of combined mass is zero.

$$V_{\text{combined}}^2 - 2gl = 0$$

$$\text{or } l = \frac{V_{\text{combined}}^2}{2g} = \frac{2.5^2}{2 \times 9.8}$$

$$= 0.32 \text{ m}$$

57(a) Using conservation of energy

$$\frac{1}{2} mv^2 + mgL = \frac{1}{2} mu^2$$

$$\text{or } v = \sqrt{u^2 - 2gL}$$

58(c) Work done = $\int F dt$

$$\text{Then } W_1 = \int_0^a -(Kx \hat{j}), dx \hat{i} = 0$$

$$\text{and } W_2 = \int_0^a -K(y \hat{i} + a \hat{j}) \cdot dy \hat{j}$$

$$= -Ka \int_0^a dy = -Ka^2$$

$$\therefore W = W_1 + W_2 = -Ka^2$$

59(d) The graph of $U(x)$ with x is as shown in figure

Potential energy is zero at $x = 0$ and maximum at

As mechanical energy has fixed value *i.e.* $k/2$, the kinetic energy has to be maximum at $x = 0$ and maximum at $x = \pm \alpha$.

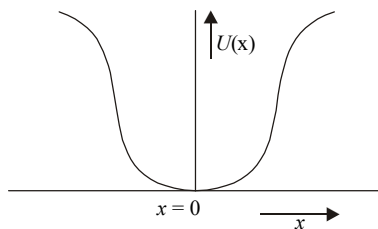


Fig 6.88

60(d) Using $F = -\frac{dU}{dx}$ we get

$$U = \frac{kx^2}{2} - \frac{ax^4}{4} + C$$

$$U = 0 \text{ for } x = 0$$

$$\text{and } x = \left(\frac{2k}{a} \right)^{1/2}$$

which is satisfied by graph (d).

61(a,c) When the string breaks, the particle starts from the point $(2, 0, 5)$ with speed $u_y = 3 \text{ m/s}$, and moves as a projectile. It will reach the xy plane in 1 second.

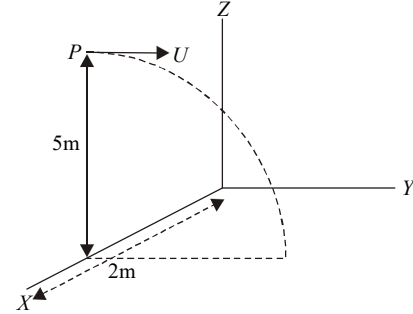


Fig 6.89

62(b,c,d) The horizontal momentum of the system is conserved ($= 0$) till the collision, as there are no horizontal forces acting on the system. At the collision, an additional impulse $2mv$ is given by the wall to the system.

63(a,c) There are no horizontal forces acting on the 'R plus B' system. Hence, its centre of mass will move down vertically, and horizontal momentum will be conserved.

64(a,c) $mv_1 = Mv_2$, where v_1 and v_2 are speeds of m and M , as seen from ground.

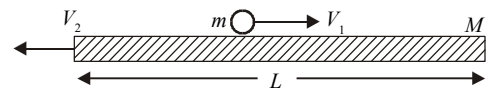


Fig 6.90

The velocity of m relative to M is

$$v_{12} = v_1 - (-v_2)$$

$$\text{Hence, } t = \frac{l}{v_{12}} = \frac{l}{v_1 + v_2}$$

$$\text{or } v_1 + v_2 = l/t.$$

65(a,c) If the system is isolated (no external forces), p and E are conserved. Electrostatic forces are internal forces. To fix Y , external forces must act on the system, *viz.*, on Y . In that case, p is not conserved. However, these external forces do no work, as there is no displacement of Y . E is conserved.

67(b,c,d) There is an exchange of linear velocities. However, the two spheres cannot exert torques on each other, as their surfaces are frictionless, and the angular velocities of the spheres do not change.

69(b,c) Let u = speed of A before impact. Thus, $p = mu$.

Let v_1, v_2 = speeds of A and B after impact.

$$u = v_1 + v_2 \text{ and } v_1 - v_2 = -eu$$

$$\therefore v_1 = \frac{1}{2} u (1 - e) \text{ and } v_2 = \frac{1}{2} u (1 + e)$$

$$J = mv_2 = m \left[\frac{1}{2} u (1 + e) \right] = \frac{1}{2} p (1 + e)$$

74(a,b,c,d) Draw the free-body diagrams for B and C . Balance horizontal and vertical forces separately for both.

75(a,b) The impulse given to the particle is equal to the area under the $F-t$ graph $= 0.07 \text{ kg m/s} = 2 \times \text{initial momentum}$ of the particle. Hence, the particle will reverse in direction and move with its initial speed.

76(a,b,d) The force exerted by the rope and the monkey on each other = the force of friction between the rope and the monkey = tension in the rope = reading of the spring balance.

77(a,b,d) The readings of the spring balance and the weighing machine are equal to the forces exerted by them on the body.

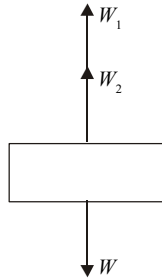


Fig 6.91

79(a,c) $Mg - T = Ma$ $T - mg = ma$

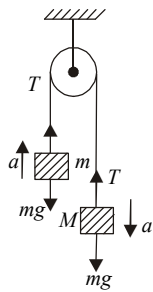


Fig 6.92

Solving, $a = \left(\frac{M - m}{M + m} \right) g$ and $T = \frac{2Mmg}{M + m}$

$$a_{CM} = \frac{Ma - ma}{M + m} = \left(\frac{M - m}{M + m} \right) \left(\frac{M - m}{M + m} \right) g.$$

82(a,d) Force of friction acts opposite to the direction of motion or tendency of motion.

83(a,b,d) As there are no external forces acting on the ' $A + B$ ' system, its total momentum is conserved. If the masses of A and B are $2m$ and 2 respectively, and v is the final common velocity, $mu = (m + 2m)v$ or $v = \frac{u}{3}$.

Work done against friction = loss in KE $= \frac{1}{2} mu^2 - \frac{1}{2} (3m) v^2$

$$= \frac{1}{2} mu^2 - \frac{1}{2} (3m) \frac{u^2}{9} = \frac{1}{2} mu^2 \left[1 - \frac{1}{3} \right] = \frac{2}{3} \times \frac{1}{2} mu^2.$$

The force of friction between the blocks is μmg .

Acceleration of A (to the right) $= a_1 = \frac{\mu mg}{2m} = \frac{\mu g}{2}$.

Acceleration of B (to the left) $= a_2 = \frac{\mu mg}{m} = \mu g$.

Acceleration of A relative to $B = a_1 - (-a_2) = \frac{3}{2} \mu g$.

84(a,b,d) R is the resultant of the normal reaction, $N = mg$, and the force of friction, $F \leq \mu mg$. As P is increased, F ($= P$) increases, while N is constant.

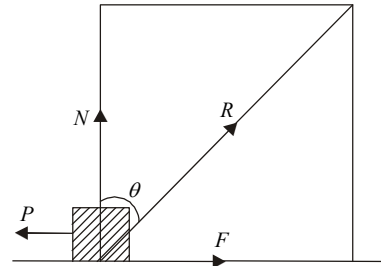


Fig 6.93

85(a,b,c,d) Let T = tension in the rope. Force T acts on both. The limiting force of friction is larger for the block than for the man. Each body will move when T exceeds the force of friction.

86(a,b,c) Let T = tension in the string,

F = force of friction between C and P .

If the string is under tension, the acceleration of C to the right = acceleration of P to the left $= a$.

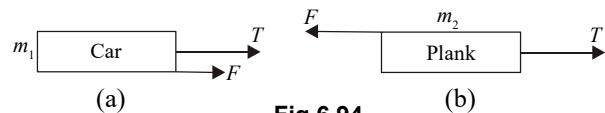


Fig 6.94

$$T + F = m_1 a \quad F - T = m_2 a$$

$$\therefore T = \frac{1}{2} (m_1 - m_2) a \quad \text{or } T > 0 \text{ if } m_1 > m_2.$$

If $m_1 < m_2$, T becomes < 0 , i.e., it becomes slack.

If $m_1 = m_2$, $T = 0$

87(a,b,c,d) The horizontal forces on the man must balance, i.e., the forces exerted by the two walls on him must be equal.

The vertical forces can balance even if the forces of friction on the two walls are unequal. The torques due to the forces of friction about his centre of mass must balance. This requires friction on both walls.

