13

Wave Motion and Wave in a String

BRIEF REVIEW

Waves are basically of three types — mechanical, electromagnetic and matter waves as illustrated in Fig. 13.1. In this chapter, a we will deal with mechanical waves.

A wave is a disturbance which propagates energy from one place to the other without transporting matter. It is spread over a region without clear-cut boundaries. It is not localized.

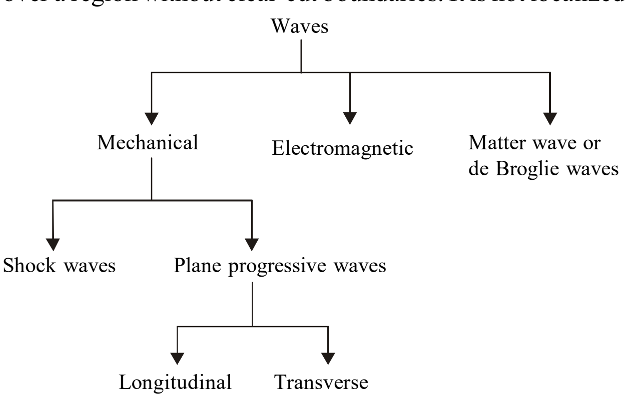


Fig. 13.1 **Waves description**

Diffraction differentiates between particle nature and wave nature. It is a convincing proof of wave nature.

Mechanical waves require a medium to propagate while other waves like electromagnetic and matter waves do not require any medium to propagate. Therefore, mechanical waves are also called **elastic waves.**

Shock waves are a kind of pulse propagation and are mathematically expressed as

$$y = \frac{a}{b+(x \pm vt)^2}.$$

Shock waves are produced during earthquakes, volcanic eruptions, bomb blasts and during a sonic boom.

$y = y_0\ (\omega t - kx)$ is the wave propagating along positive x direction..

Plane progressive wave is given by

$$y = y_0 \sin(\omega t - kx)$$

where k is called propagation constant or wave number, ω is called angular frequency, y_0 amplitude and y instantaneous displacement. Such a wave is called a displacement wave.

$K = \dfrac{2\pi}{\lambda}$ where λ is wavelength, $(\omega t - kx)$ is the phase at any instant. When path difference $\Delta x = \lambda$, then phase shift $\Delta\phi = 2\pi$. In general $k\Delta x = \Delta\phi$.

A wave can have two types of velocities.

Wave velocity or phase velocity and group velocity or particle velocity.

Wave velocity $v = \dfrac{dx}{dt} = \dfrac{\omega}{k} = f\lambda$

In a dispersive medium, wave travels with a group velocity

$$v_{\text{group}} = v - \lambda \frac{dv}{d\lambda}$$

This is the case for electromagnetic waves. For example, in water and glass and so on different wavelength travel with different velocities.

Particle velocity $v_{\text{particle}} = \dfrac{dy}{dt} = -\dfrac{dx}{dt} \times \dfrac{dy}{dx} = -v(\text{slope})$

= – wave velocity × solpe at that point

A plane progressive wave mechanical or electromagnetic may be expressed in one of the following forms

$$y = y_0 \sin(\omega t - kx)$$

$$y = y_0 \sin\omega \left(t - \frac{x}{v}\right)$$

$$y = y_0 \sin 2\pi \left(ft - \frac{x}{\lambda}\right)$$

$$y = y_0 \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$y = y_0 \sin k\,(vt - x)$$

- If the sign between t and x is negative then the wave propagates in positive x direction.
- When a wave passes from one medium to the other its frequency does not change.

Velocity of wave in a string $v = \sqrt{\dfrac{T}{\mu}}$

where T is tension in the string and π is mass/length of the string.

Frequency of Wave in a String

If vibrating in p-loops (transverse),

fundamental frequency $f = \dfrac{p}{2l}\sqrt{\dfrac{T}{\mu}}.$

Longitudinal waves in a string have p-loops

$$f = \frac{p}{l}\sqrt{\frac{T}{\mu}}.$$

Average power transmitted along the string

$$P_{\text{average}} = \frac{1}{2}\frac{\omega^2 x_0^2 F}{v} = 2\pi^2 \mu\, x_0^2 f^2 v$$

Average intensity

$$I_{\text{average}} = 2\pi^2 \rho x_0^2 f^2 v$$

Interference of Waves in the Same Direction

If $y_1 = y_{01}\sin(kx - \omega t)$ and

$y_2 = y_{02}\sin(\omega t - kx + \phi)$

then $y = y_1 + y_2$ and $y = y_0 \sin(\omega t - kx + \delta)$

Apply vector laws

$$y_0 = \sqrt{y_{01}^2 + y_{02}^2 + 2y_{01}y_{02}\cos\phi}$$

$$\tan\delta = \frac{y_{02}\sin\phi}{y_{01} + y_{02}\cos\phi}.$$

y_0 will be maximum when $\cos\phi = 1$ or $\phi = 0$ or $2n\pi$ where $n = 0, 1, 2 \ldots$

y_0 will be minimum when $\cos\phi = -1$

$\cos\phi = (2n+1)\pi$

when amplitude y_0 is maximum the **constructive interference** is said to take place as sound intensity will be maximum and the path difference is $n\lambda$.

$y_{0(\max)} = y_{01} + y_{02}$;

$y_{0\min} = y_{01} - y_{02}$

Amplitude is minimum when phase difference is an odd multiple of π or path difference is an odd multiple of half the wave length.

$$\frac{I_{\max}}{I_{\min}} = \frac{(y_{01} + y_{02})^2}{(y_{01} - y_{02})^2}$$

when intensity is minimum **destructive interference** occurs.

If the reflection occurs from a denser medium, it introduces a phase shift of π radian or 180^0 between incident and reflected wave as illustrated in Figure 13.2

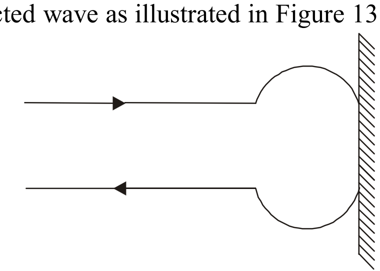
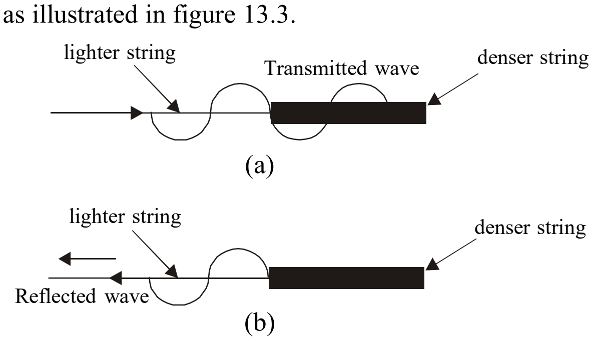


Fig. 13.2 **Reflection from the wall**

Note that if the wave is travelling in a string which is a combination of two mediums then, if the wave travels from the lighter to the denser string it is reflected out of phase (or 180^0) from the junction but is transmited in phase.

If a wave propagates from denser to lighter string phase is shifted neither for the reflected nor for the tranmitted wave as illustrated in figure 13.3.



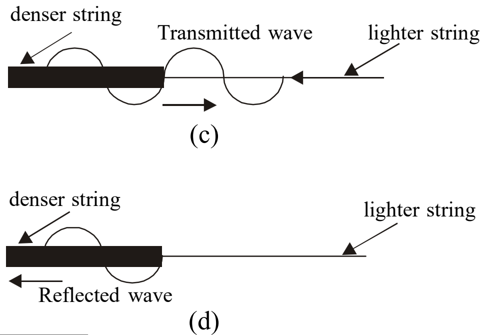


Fig. 13.3 **Reflection and transmission from denser and rarer medium.**

Standing waves or stationary waves are produced when two waves having the same amplitude and same frequency superpose while travelling in opposite directions.

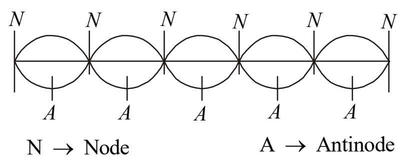


Fig. 13.4 **Standing wave illustration**

that is, $y_1 = y_0 \sin(kx - \omega t)$

and $y_2 = y_0 \sin(kx - \omega t)$ superpose.

Thus, $y = y_1 + y_2 = (2y_0 \sin kx) \cos \omega t$.

Normally a wave and its reflected wave superpose to produce standing waves. The points of maximum amplitude are called antinodes and minimum amplitude are called nodes. Separation between two consecutive nodes or antinodes is $\lambda/2$. Separation between a node and an antinode is $\lambda/4$.

Notes:

1. In a travelling wave the disturbance produced in a region propagates with a definite velocity and in a *standing wave* it is confined to the region where it is produced.
2. In a travelling wave the motion of all the particles is similar. In a *standing wave* different particles move with different amplitude.
3. In a *standing wave particles at node* always remain at *rest.* In a travelling wave there is no such particle which remains at rest always.
4. In a *standing wave* all the *particles cross* their *mean positions together.* In a travelling wave there is no instant when all the particles are at the mean position.
5. In a *standing wave* all the *particles* move in *phase.* In a travelling wave the phases of neighbouring particles are always different.
6. In a travelling wave energy from one region of space is transferred to the other region of space. In a *standing wave* the energy is always *confined* in that region.

Standing wave ratio (SWR) $\dfrac{y_{0\max}}{y_{0\min}} = \dfrac{y_{01} + y_{02}}{y_{01} - y_{02}}$

For a progressive wave, SWR = 1 (as $y_{02} = 0$). For standing wave SWR = ∞

In standing waves $\dfrac{d^2 y}{dx^2} = \dfrac{1}{v^2} \dfrac{d^2 y}{dt^2}$

The amplitude of the wave $y_0 = 2y_0 \sin kx$ is a periodic function of position (and not of time as in beats). If a loop vibrates in a single loop, the mode is fundamental. There are two nodes and one antinode and frequency is f. If there are n loops we say the string is vibrating with nth harmonic or $(n - 1)$th overtone and there will be n antinodes and $n + 1$ nodes. Frequency in this case will be nf.

That is, in a string fixed at both ends all integral multiples of fundamental frequency are allowed and

$$f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}.$$

Vibrations of Strings Fixed at One End

Note that at the open end an antinode will be formed and at the fixed end a node will be formed.

- Only odd multiple of frequencies are allowed.
- n_{th} harmonic = $(n - 1)$th overtone.
- Fundamental frequency is also called note or first harmonic.
- Octave is the tone whose frequency is double the fundamental frequency

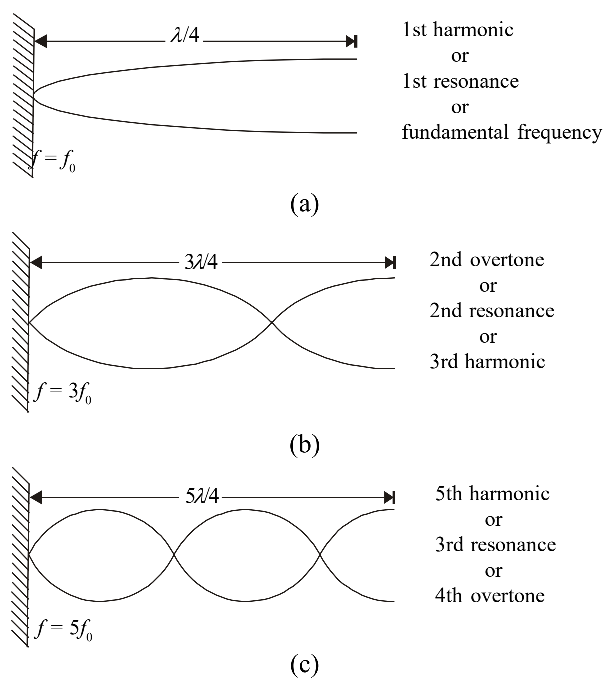


Fig. 13.5 **Standing wave in a string fixed at one end and open at other**

Melde's Experiment

$$f = \frac{p}{l}\sqrt{\frac{T}{\mu}} \; ; p = \text{number of loops}$$

Fig. 13.6 (a)

$$f = \frac{p}{2l}\sqrt{\frac{T}{\mu}} \text{ where } P = \text{number of loops}$$

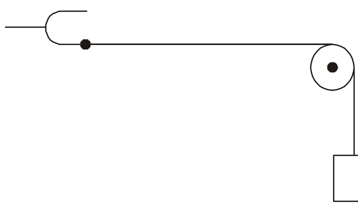


Fig. 13.6 (b) **Melde's experiment**

$$p_{\text{longitudinal}} = \frac{p_{transverse}}{2}$$

Velocity of a wave on the surface of a liquid is

$$v_s = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho}}$$

where T is surface tension and ρ is density of the liquid.

Velocity of Torsional waves in a rod is

$$V_T = \sqrt{\frac{\eta}{\rho}}$$

SHORT CUTS AND POINTS TO NOTE

1. To produce longitudinal waves the medium should possess bulk modulus of elasticity.
2. To produce transverse wave the medium must possess shear modulus of elasticity.
3. $y = y_0\sin(\omega t - kx + \phi)$ is the equation of a plane progressive wave in positive x direction. ϕ is initial phase angle or epoch. Normally, we write a simplified equation, $y = y_0\sin(\omega t - kx)$. A more general equation of wave is $y = y_0 e^{\pm i(\omega t - kx)}$
4. $k = \dfrac{2\pi}{\lambda}$ is propagation vector or wave number.
5. Wave velocity is $\dfrac{dx}{dt} = v = \dfrac{\omega}{k} = f\lambda$ and may be called phase velocity. In a dispersive medium waves travel with group velocity v_g given by

   $v_g = v - \dfrac{\lambda dv}{d\lambda}$. In dispersive mediums waves of different wavelengths travel with different velocity.
6. Particle velocity = $\dfrac{dy}{dt} = \dfrac{-dx}{dt} \times \dfrac{dy}{dx} = -v_{\text{wave}}$ (slope) Maximum particle velocity = $y_0\omega$
7. Frequency of the wave does not vary when a wave passes from one medium to the other.
8. Power (average) transmitted along the string is

   $$p_{av} = \frac{\omega^2 x_0^2 F}{2v} = 2\pi^2 \mu x_0^2 f^2 v$$

   Intensity = $2\pi^2 \rho x_0^2 f^2 v$ where μ = mass/length and ρ = density of the medium.
9. Interference of waves travelling in the same direction is obtained using vector laws

   $$y_0 = \sqrt{y_{01}^2 + y_{02}^2 + 2y_{01}y_{02}\cos\phi} \quad \text{and}$$

   $$\tan\delta = \frac{y_{02}\sin\phi}{y_{01} + y_{02}\cos\phi}$$

   if $y_1 = y_{01}\sin(kx - \omega t)$ and
   $y_2 = y_{02}\sin(\omega t - kx + \phi)$ interfere.
10. $$\frac{I_{max}}{I_{min}} = \frac{(y_{01} + y_{02})^2}{(y_{01} - y_{02})^2}$$

    Maximum intensity is obtained when phase shift is zero or path difference is an integral multiple of wavelength. Minimum intensity or destructive interference occurs when phase shift is an odd integral multiple of π radian or odd integral multiple of half the wavelength.
11. Reflection from a denser medium causes a phase shift of 180^0 and reflection from rarer or lighter medium occurs without change of phase in the string.
12. Standing waves result when two waves having same amplitude and same frequency travelling in opposite directions superpose.
13. $y = 2y_0\sin kx \cos \omega t$ represents a stationary wave in strings. Note that $(2y_0\sin kx)$ shows amplitude and is a function of distance.

    At certain places. amplitude is maximum (antinodes) and at other places amplitude is zero (nodes). Separation between consecutive nodes or antinodes is $\lambda/2$. Distance between a node and consecutive antinode is $\lambda/4$.

14. $\dfrac{v^2 d^2 y}{dx^2} = \dfrac{d^2 y}{dt^2}$ satisfies the standing wave equation.

15. $y = 2y_0 \cos kx \sin \omega t$ represents stationary wave equation which fits in closed or open pipes.

16. $f = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$ is used for fundamental mode in strings vibrating with transverse stationary waves.

17. In a sonometer, transverse stationary waves are produced.

$$f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$ can be used for finding frequency if the string is vibrating in n loops.

18. In Melde's experiment if the transverse mode is used

$$f = \frac{p}{2l}\sqrt{\frac{T}{\mu}}$$ and

$$f = \frac{p}{l}\sqrt{\frac{T}{\mu}}$$ for longitudinal mode.

19. Only transverse waves and not longitudinal waves can be polarised.

20. Sound waves being pressure waves are longitudinal.

21. If the disturbance produced is always along a fixed direction. The wave is linearly polarised in that direction. For example, $y = y_0 \sin(\omega t - kx)$ is linearly polarised in y direction. Linearly polarised waves are also called plane polarised waves.

22. If each particle of a string moves in a small circle as the wave passes through it then the wave is circularly polarised. If each particle moves in an ellipse it is elliptically polarised and if each particle is randomly displaced, it is unpolarised.

23. A circularly polarised or unpolarised wave passing through a slit does not show change in intensity as the slit is rotated in its plane. But the transmitted wave becomes linearly polarised in the direction parallel to the slit.

24. Number of nodes and antinodes in a tuning fork when vibrating in fundamental mode as illustrated in figure. Antinodes are A_1 and A_2 and A_3. A_3 is longitudinal antinode while A_1 and A_2 are transverse antinodes. If the tuning fork vibrates in nth harmonic then number of nodes = $2n$ and number of antinodes = $(2n + 1)$

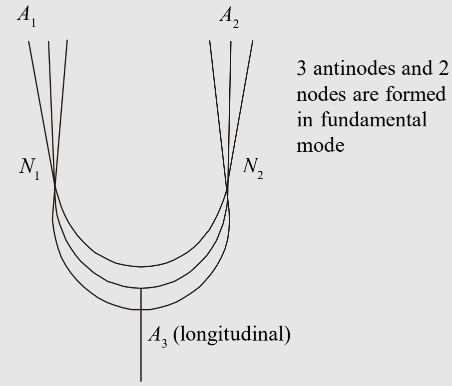


Fig. 13.7 **Nodes and Antinodes in a tuning fork**

CAUTION

1. Considering that all waves require a medium.

⇒ only mechanical waves require an elastic medium

2. Considering that both longitudinal and transverse waves can be produced in any medium.

⇒ Transverse waves require medium that shall possess shear modulus of elasticity. Therefore, transverse waves cannot be produced in gases.

3. Considering that waves could be only longitudinal or transverse.

⇒ Waves could be a combination of both. For example, ripples in water, seismic waves during earthquakes.

4. Considering that medium is also transported along with energy during propagation of a wave.

⇒ Only energy is transported and not the medium during propagation of the wave.

5. Considering only functions like $y_0 \sin(\omega t - kx)$ or $y_0 \cos(\omega t - kx)$ can represent a wave.

⇒ Functions like

$$y = y_0 \sin(\omega t - kx),\ y = y_0 \cos(\omega t - kx) \quad \text{and}$$

$$y = y_0 + \sum_{n=1}^{\infty} a_n \sin n\omega t + \sum_{n=1}^{\infty} b_n \cos n\omega t$$

also represent waves which are complex periodic waves. However, plane progressive harmonic waves can be represented as

$$y = y_{01} \sin(\omega t - kx) + y_{02} \sin(\omega t - kx)$$

$$y = y_0 \sin(\omega t - kx) \text{ or } y = y_0 \cos(\omega t - kx).$$

6. Confusion about the formula to be applied in strings to calculate frequency

⇒ use $f = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$ when vibrating in fundamental transverse mode and

$f = \dfrac{1}{l}\sqrt{\dfrac{T}{\mu}}$ when vibrating in fundamental longitudinal mode.

7. Confusion about the modes of production of transverse and longitudinal wave.

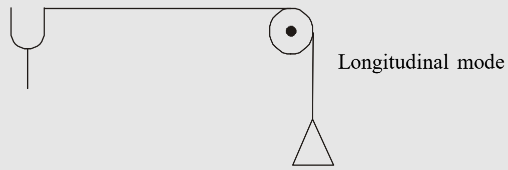


Fig. 13.8

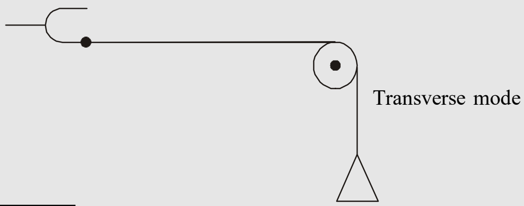


Fig. 13.9

8. Considering that velocity of a wave is same at every point in the string.

$\Rightarrow$ Velocity of wave in vertical strings is different at different points.

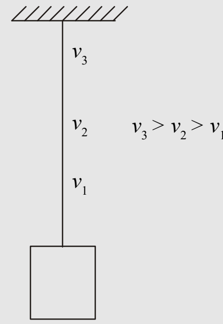


Fig. 13.10

SOLVED PROBLEMS

1. The displacement wave in a string is $y = (3\text{ cm})\sin 6.28(0.5x - 50t)$ where x is in centimetres and t in seconds. The wavelength and velocity of the wave is

(a) 2 cm, 100 cms^{-1} (b) 10 cm, 50 cm^{-1}
(c) 20 cm, 2 ms^{-1} (d) 2 m, 100 ms^{-1}

Solution (a) $k = \dfrac{2\pi}{\lambda}$

or $$\lambda = \frac{2\pi}{k} = \frac{6.28}{6.28(0.5)} = 2\text{ cm}$$

$$v = \frac{\omega}{k} = \frac{50\times 6.28}{0.5\times 6.28} = 100\text{ cms}^{-1}$$

2. The equation of a wave is $10\sin(6.28x - 314t)$ where x is in centimetres and t is in seconds. The maximum velocity of the particle is

(a) 62.8 cm s^{-1} (b) 3140 ms^{-1}
(c) 50 cms^{-1} (d) 31.4 m s^{-1}

Solution (d) $y_{max} = \omega y_0 = 314(10)$ cm/s or 31.4 ms^{-1}.

3. The speed of a transverse wave travelling on a wire having a length 50 cm and mass 50 g is 80 ms^{-1}. The area of cross-setion of the wire is 1 mm^2 and its Young's modulus is 16×10^{11} Nm^{-2}. Find the extension of the wire over natural length.

(a) 2 cm (b) 2 mm
(c) 0.2 mm (d) 0.02 mm

Solution (d) $v = \sqrt{\dfrac{T}{\mu}}$

or $$T = v^2\mu = (80)^2\left(\frac{5}{0.5}\times 10^{-3}\right) = 64\text{ N}$$

and $$Y = \frac{Fl}{A\,\Delta l}$$

or $$\Delta l = \frac{Fl}{AY} = \frac{64\times 0.5}{10^{-6}\times 16\times 10^{11}} = 2\times 10^{-5}\text{m.}$$

4. Which of the folowing waves is progressing in the y direction?

(a) $x = x_0\cos(\omega t - ky)$ (b) $y = y_0\cos(\omega t - ky)$
(c) $y = y_0\cos kx \sin \omega t$ (d) $y = y_0\sin kx \cos \omega t$

Solution (a) The wave $x = x_0\cos(\omega t - ky)$ travels along y direction.

5. Velocity of sound in air is 332 m/s. Its velocity in vacuum is

(a) > 332 ms^{-1} (b) 3×10^8ms^{-1}
(c) 332 ms^{-1} (d) none of these

Solution (d) None of these as velocity is zero as sound waves require medium.

6. A cork floating in a calm lake is executing SHM of frequency f. When a boat passes close to the cork then the

(a) frequency becomes greater than f.
(b) frequency becomes less than f.
(c) frequency remains constant.
(d) none of these.

Solution (c) Frequency remains constant and velocity will vary, that is, wavelength will vary.

7. Two waves of equal amplitude x_0 and equal frequency travel in the same direction in a medium. The amplitude of the resultant wave is

(a) 0 (b) x_0

(c) $2x_0$ (d) between 0 and $2x_0$

Solution (d) use $x_0' = \sqrt{x_{01}^2 + x_{02}^2 + 2x_{01}x_{02}\cos\theta}$

$\because$ $\theta = 0$ and $x_{01} = x_{02} = x_0$

$x_0' = 2x_0$.

8. The fundamental frequency of a string is proportional to

(a) inverse of the length. (b) the diameter.

(c) tension. (d) density.

Solution (a) $f \propto \frac{1}{l}$

9. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope.

(a) 0.06 m (b) 0.12 m

(c) 0.09 m (d) none of these

[IIT 1984]

Solution (d) $v = \frac{T}{\mu}$

$$\frac{v_{top}}{v_{bottom}} = \sqrt{\frac{T_T}{T_B}} = \sqrt{\frac{(6+2)g}{2g}} = 2$$

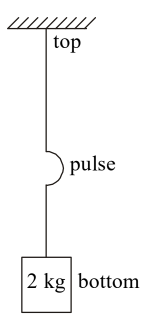


Fig. 13.11

$$\frac{f\lambda_{\text{Top}}}{f\lambda_{\text{Bottom}}} = 2 \text{ as frequency does not change}$$

$\therefore$ $\lambda_{\text{top}} = \lambda_{\text{bott}} \times 2 = 0.12$ m.

10. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling. The speed of transverse waves in the rope at a point 0.5 m from the lower end is

(a) 2.21 ms^{-1} (b) 4.21 ms^{-1}

(c) 7.21 ms^{-1} (d) 3.31 ms^{-1}

[Roorkee 1991]

Solution (a) $T = \frac{M}{L}(x)g$

and $v = \sqrt{\frac{\frac{M}{L}(x)g}{M/L}} = \sqrt{gx} = \sqrt{9.8 \times 0.5}$

$= 2.21$ m/s

11. The equations of motion of two waves propagating in the same direction is given by

$y_1 = A\sin(\omega t - kx)$

and $y_2 = A\sin(\omega t - kx - \theta)$.

The amplitude of the medium particle will be

(a) $\sqrt{2}\,A\cos\theta$ (b) $2A\cos\theta$

(c) $\sqrt{2}\,A\cos\theta/2$ (d) $2A\cos\theta/2$

[BHU 2003]

Solution (d) $y_0 = \sqrt{A^2 + A^2 + 2A(\cos\theta)}$

$= A\sqrt{2(1+\cos\theta)} = 2A\cos\theta/2$.

12. The displacement y of a wave travelling in x direction is given by

$$y = 10^{-1}\sin\left(600t - 2x + \frac{\pi}{3}\right)\text{m}$$

Where x is expressed in metres and t in seconds. The speed of the wavemotion in metre per second is

(a) 600 (b) 1200

(c) 200 (d) 300

Solution (d) $v = \frac{\omega}{k} = \frac{600}{2} = 300\ ms^{-1}$.

13. A steel wire of linear mass density 9.8 g/m is stretched with a tension of 10 kg. It is kept between poles of an electromagnet and it vibrates in resonance when carrying an arc of frequency n. The frequency n is

(a) 100 Hz (b) 200 Hz

(c) 25 Hz (d) 50 Hz

Solution (d) $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2}\sqrt{\frac{10 \times 9.8}{9.8 \times 10^{-3}}}$

$$= \frac{10^2}{2} = 50 \text{ Hz}.$$

14. The equation of a progressive wave is

$y = 8\sin\left[\pi\left(\frac{t}{10} - \frac{x}{4}\right) + \frac{\pi}{3}\right]$. The wavelength of the wave is

(a) 8 m (b) 4 m

(c) 2 m (d) 10 m

[CET Maharashtra 2002]

Solution (a) $\frac{2\pi}{\lambda} = k$ or $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/4} = 8$

15. The equation of a stationary wave is $y = \sin\frac{\pi x}{3}\cos 10\pi t$ where x and y are in centimetres and t in seconds. The separation between two consecutive nodes is

(a) 1.5 cm (b) 6.0 cm

(c) 3.0 cm (d) 18 cm

[DPMT 2002]

Solution (c) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/3} = 6$ cm.

Separation between two consecutive nodes

$= \lambda/2 = 3$ cm.

16. If the amplitude of velocity of a particle acted by a force $F = F_0 \cos \omega t$ along x-axis is given by

$$v_0 = \frac{1}{(a\omega^2 - b\omega + c)^{1/2}} \text{ where } b^2 > 4ac.$$

The frequency of resonance is:

(a) $\omega = b/a$ (b) $b/2a$

(c) a/b (d) $a/2b$

Solution (b) For resonance $v_0 \to \infty$(max)

$\therefore$ $(a\omega^2 - b\omega + c)^{1/2}$ should be minimum

or $\frac{d}{d\omega}(a\omega^2 - b\omega + c) = 0$

or $2a\omega - b = 0$ or $\omega = \frac{b}{2a}$

17. An observer on the sea shore observes 54 waves reaching the coast per minute. If the wavelength is 10 m. The velocity is

(a) 9 ms^{-1} (b) 54 ms^{-1}

(c) 18 ms^{-1} (d) 36 ms^{-1}

Solution (a) $f = \frac{54}{60} = \frac{9}{10}$ Hz

$$v = f\lambda = \frac{9}{10} \times 10 = 9 \text{ ms}^{-1}$$

18. A light pointer fixed to one prong of a tuning fork touches a vertical smoked plate. The fork is set to vibration and the plate is allowed to fall freely. Eight complete waves are counted when the plate falls through 10 cm. The frequency of the tuning fork is

(a) 112 Hz (b) 14 Hz

(c) 28 Hz (d) 56 Hz

[IIT 1996]

Solution (d) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 0.1}{9.8}} = \frac{1}{7}\, s.$

$$f = \frac{\text{number of waves}}{\text{time}} = \frac{8}{1/7} = 56 \text{ Hz.}$$

19. A progressive wave of frequency 500 Hz is travelling with a velocity 360 ms^{-1}. How far are two points 60^0 out of phase?

(a) 0.06 m (b) 0.12 m

(c) 0.18 m (d) 0.24 m

Solution (b) $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

or $$\Delta x = \frac{\Delta\phi\lambda}{2\pi} = \frac{\pi/3(0.72)}{2\pi} = 0.12 \text{ m}$$

20. Two blocks each having a mass 3.2 kg are connected by a wire CD and the system is suspended from the ceiling by another wire AB as shown in figure. The linear mass density of AB is 10 gm^{-1} and that of the CD is 8 gm^{-1}. The speed of the transverse wave pulse produced in AB and CD is

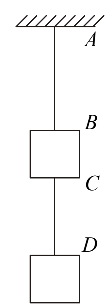


Fig. 13.12

(a) 80 ms^{-1}, 40 ms^{-1} (b) 40 ms^{-1}, 80 ms^{-1}

(b) 80 ms^{-1}, 63 ms^{-1} (d) none of these

Solution (c) $v = \sqrt{\frac{T}{\mu}} \Rightarrow v_{AB}$

$$= \sqrt{\frac{6.4\times 10}{10\times 10^{-3}}} = 80 \text{ ms}^{-1}$$

$$v_{CD} = \sqrt{\frac{3.2\times 10}{8\times 10^{-3}}} = 63 \text{ ms}^{-1}$$

21. A transverse wave described by $y = 0.02 \sin(x + 30t)$ propagates on a stretehed string of linear density 12 gm^{-1}. The tension in the string is

(a) 2.16 N (b) 1.08 N
(c) 0.108 N (d) 0.0108 N

Solution (c) $v = \dfrac{\omega}{k} = \dfrac{30}{1} = 30 \text{ ms}^{-1}$

$$T = v^2\mu = (30)^2 \times 12 \times 10^{-3} = 0.108 \text{ N}.$$

22. A circular loop of the string rotates about its axis on a frictionless horizontal plane at a uniform rate so that the tangential speed of any particle of the string is v. If a small transverse disturbance is produced at a point of the loop. The speed (relative to the string) at which the disturbance will travel is

(a) v (b) $\dfrac{v}{2}$
(c) $2v$ (d) $\dfrac{v}{4}$

Solution (a) v

TYPICAL PROBLEMS

23. In the figure the string has a mass 4.5 g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley? (Take g = 10 ms^{-2})

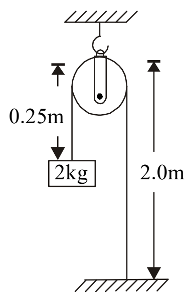


Fig. 13.13

(a) 0.04 s (b) 0.2 s
(c) 0.4 s (d) 0.02 s

Solution (d) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{20}{2 \times 10^{-3}}} = 100 \text{ ms}^{-1}$

$$\mu = \frac{4.5}{2.25} = 2 \text{ gm}^{-1}$$

$$s = ut - \frac{1}{2} gt^2$$

$$2 = 100t - 5t^2$$

$$t^2 - 20t + 0.4 = 0$$

$$t = \frac{+20 \pm \sqrt{400 - 1.6}}{2} = 0.02 \text{ s}$$

24. Two wires are kept tight between the same pair of supports. The tensions in the wires are in the ratio 2:1. The radii are in the ratio 3 : 1 and the densities are in the ratio 1 : 2. The ratio of their fundamental frequencies are

(a) 1/2 (b) 2/3
(c) 3/4 (d) 4/9

Solution (d) $\dfrac{f_1}{f_2} = \dfrac{\dfrac{1}{2l}\sqrt{\dfrac{T_1}{\mu_1}}}{\dfrac{1}{2l}\sqrt{\dfrac{T_2}{\mu_2}}} = \sqrt{\dfrac{T_1\rho_2 A_2}{T_2\rho_1 A_1}}$

$$= \sqrt{\frac{T_1\rho_2\pi r_2^2}{T_2\rho_1\pi r_1^2}} = \frac{2}{3}$$

25. The equation of a standing wave produced on a string fixed at both the ends is $y = 0.4 \sin(0.314x) \cos(600\pi t)$. The smallest length of the string would be—— (where x is in cm, t is in seconds)

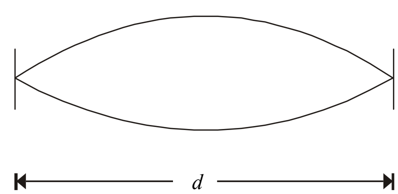


Fig. 13.14

(a) 20 cm (b) 40 cm
(c) 10 cm (d) none of these

Solution (c) $\lambda = \dfrac{2\pi}{k}$

$$\lambda = \frac{2\pi}{0.314} = 20 \text{ cm}$$

Smallest length $d = \dfrac{\lambda}{2} = 10$ cm.

26. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling. The time taken by the transverse wave to travel through the full length of the rope is

(a) 2.0 s (b) 1.2 s
(c) 1.0 s (d) 2.2 s

Solution (c)$T = \left(\frac{M}{L}x\right)$ g and $v = \sqrt{\frac{\frac{Mx}{L}g}{\mu / L}} = \sqrt{gx}$

$$v = \frac{dx}{dt} = \sqrt{gx} \text{ or } dt = \frac{dx}{\sqrt{gx}}$$

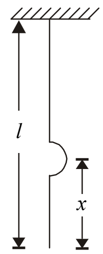


Fig. 13.15

or $t = \frac{1}{\sqrt{g}}\int_0^L x^{-1/2}dx$

or $t = 2\sqrt{\frac{L}{g}} = 2\sqrt{\frac{2.45}{9.8}} = 1$ s.

27. An aluminium wire of length 60 cm is joined to a steel wire of length 80 cm and stretched between two fixed supports. The tension produced is 40 N. Cross-sectional area is 1 mm^2 (steel) and 3 mm^2 (aluminium). Minimum frequency of the tuning fork which can produce standing waves with the joint as a node is

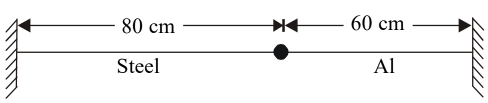


Fig. 13.16

(density of A1 = 2.6 g cc^{-1} and density of steel = 7.8 g cc^{-1})

(a) 90 Hz (b) 145 Hz
(c) 180 Hz (d) 250 Hz

Solution (c) $f = \frac{n}{2l}\sqrt{\frac{T}{\mu}} = \frac{n}{2l}\sqrt{\frac{T}{A\rho}}$

since frequency will remain same

$$\therefore \quad \frac{n}{2l_1}\sqrt{\frac{T}{A_1\rho_1}} = \frac{p}{2l_2}\sqrt{\frac{T}{A_2\rho_2}}$$

or $\frac{n}{p} = \frac{l_1}{l_2}$ that is, $\frac{n}{p} = \frac{4}{3}$

$$f = \frac{3}{2(0.6)}\sqrt{\frac{40}{10^{-6}\times 2.6\times 10^3\times 3}}$$

$$= \frac{1}{0.4}\sqrt{\frac{40\times 10^3}{2.6\times 3}}$$

$$= \frac{100}{0.4}\sqrt{\frac{2}{3.9}} = 180\,\text{Hz}.$$

28. A 200 Hz wave with amplitude 1 mm travels on a long string of linear mass density 6 gm^{-1} kept under a tension of 60 N. The average power transmitted across a given point in the string is

(a) 0.53 W (b) 0.83 W
(c) 0.47 W (d) 0.89 W

Solution $p_{\text{average}} = 2\pi^2\mu\, x_0^2 f^2 v$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{60}{6\times 10^{-3}}} = 100 \text{ ms}^{-1}$$

$= 2 \times \pi^2(6 \times 10^{-3})\,(10^{-6})\,(2 \times 10^2)^2 \times 100$
$= 0.47$ W.

29. The ends of a stretched string of length L are fixed at $x = 0$ amd $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin \frac{\pi x}{L} \cos \omega t$ and energy E_1 and in another experiment, its displacement is $y_2 = A \sin \frac{2\pi x}{L} \sin 2\omega t$ and energy E_2 then

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$
(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

[IIT screening 2002]

Solution (c) sin $E\ \alpha\ y_0^2$ f2 and $y_0 = A$ in each case but $f_1 = f; f_2 = 2f$, therefore, $E_2 = 4E_1$.

30. A sonometer wire resonates. with a given tuning fork forms a standing wave with 5 antinodes between two bridges when a 9 kg weight is suspended from the wire. When the mass is replaced by a mass M, the wire resonates with the same tuning fork forming 3 antinodes for the same position of wedges. The value of M is

(a) 2.25 kg (b) 5 kg
(c) 12.5 kg (d) $\frac{1}{25}$ kg

[IIT screening 2002]

Solution (a) $f = \frac{n_1}{2l}\sqrt{\frac{T_1}{\mu}} = \frac{n_2}{2l}\sqrt{\frac{T_2}{\mu}}\ \sqrt{\frac{T_1}{T_2}} = \frac{n_2}{n_1}$

or $\frac{T_1}{T_2} = \frac{n_2^2}{n_1^1} = \left(\frac{4}{2}\right)^2 = 4\ M = \frac{9}{4} = 2.25$ kg.

PASSAGE 1

Read the following passage and answer the questions given at the end.

You have designed a new musical instrument of very simple construction. Your design consists of a metal tube with length l and diameter $\frac{l}{10}$. You have stretched a string of mass per unit length μ across the open end of the tube. The other end of the tube is closed. To produce the musical effect you are looking for, you want the frequency of the third harmonic standing wave on the string to be same as fundamental frequency for sound waves in the air column in tube. Velocity of sound in air column is v

1. What must be the tension in the string to obtain the desired effect.

(a) $\frac{v^2\mu}{4}$ (b) $\frac{v^2\mu}{12}$

(c) $\frac{v^2\mu}{36}$ (d) $\frac{v^2}{36\mu}$

2. What other harmonics of the string, if any, are in resonance with standing waves in air column?

(a) 9th harmonic of string with 6th harmonic of air column.

(b) 9th harmonic of string with 3rd harmonic of air column.

(c) 6th harmonic of string with 2nd harmonic of air column.

(d) None.

3. What will happen to the sound produced by instrument if the tension is changed to twice the value in Question 1?

(a) Sound becomes shrill as frequency increases.

(b) Sound becomes hoarse.

(c) Sound intensity in creases.

(d) Sound intensity falls.

Solution 1. (c) $f = \frac{v}{4l} = \frac{3}{2l}\sqrt{\frac{T}{\mu}}$

or $T = \frac{v^2\mu}{36}$

Solution 2. (b) In closed pipe only odd harmonics f_0, $3f_0$, $5f_0$, ____ occur.

$\therefore$ 3rd harmonic of air column will be in resonance with 9th harmonic of string.

Solution 3. (a, d) If tension is doubled then $f_{new} = 3\sqrt{2}\, f_0$

Resonance will break therefore intensity will fall. However, the sound becomes shrill.

PASSAGE 2

Read the following passage and answer the questions given at the end.

Dolphins emit high frequency sound waves typically 10^5 Hz and use the echos for guidance and for hunting. The corresponding wave length in water is 1.48 cm with this sonar system they can sense objects as small as (roughly) the wavelength. Ultrasonic imaging is a medical technique that uses exactly the same physical principle, sound waves of very short wavelength (or high frequency) called ultrasound are scanned over the human body the echos from the interior organs are used to create image. Ultrasound is more sensitive than x-rays in distinguishing various kinds of tissues and does not have the radiation hazards associated with the x-rays. Ultra sound is used for the study of heart valve action, detection of tumors, and prenatal tests.

1. Find the size of object which can be featured with 5 MHz in water.

(a) 0.148 mm (b) 0.3 mm

(c) 0.5 mm (d) 0.1 mm

2. A submarine is 200 m deep. What time the wave will take to detect it?

(a) 0.25 s (b) 0.12 s

(c) 0.35 s (d) 1.2 s

3. Find the incorrect statement.

(a) Ultrasound can be used to determine the sex of the unborn baby (embryo).

(b) Ultrasound can be used to keep away rodents in cold storage.

(c) Ultrasound can be used to detect even bone fractures.

(d) Ultrasound travels faster than sound.

Solution 1. (b) $v = 1.48 \times 10^5 \times 10^{-2} = 1480\ ms^{-1}$

$$\lambda = \frac{v}{f} = \frac{1480}{5\times10^6} = 0.3 \text{ mm}$$

Solution 2. (a) $t = \frac{2x}{v} = \frac{2\times200}{1480} = 0.25$ s

Solution 3. (c,d)

PASSAGE 3

Read the following passage and answer the questions given at the end.

A deep-sea diver is suspended beneath the surface of Loch Ness by a 100 m long cable that is attached to a boat on the surface. The diver and his suit have a total mass of 120 kg and a volume 0.08 m^3. The cable has a diameter of 2 cm and a linear mass density 1.10 kg/m. The diver thinks he sees something moving in the murky depths and jerks the end of the cable to send signal, by means of transverse waves up the cable, to his companions in the boat. Density of water is 10^3 kg m^{-3}.

1. Find the tension in the cable at its lower end, where it is attached to the diver.

(a) 230 N (b) 200 N
(c) 120 N (d) 150 N
(e) 400 N

2. Calculate the tension in the cable at a distance x from the diver.

(a) $(40 + .1x)g$ (b) $(40 + 0.786x)g$
(c) $(120 + 1.1x)g$ (d) $(120 - 1.1x)g$

3. The time required for the signal to reach the companions for the first time.

(a) 1.92 s (b) 1.61 s
(c) 2.83 s (d) none of these

Solution **1.** (e) $T = F - B = (120 - .08 \times 10^3)g = 40\,gN$
where B is Buoyant force

Solution **2.** (b) $T(x) = Mg + F(x) - B(x) = (120 + 1.1x - (.08 \times 10^3) - (\pi \times 10^{-4})x \times 10^3)g$
$= [(120 - 80) + (1.1x - .314x)]g = (40 + .786x)g$

Solution **3.** $v = \dfrac{dx}{dt} = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{(40 + 0.786x)g}{1.1}}$

or $$dt = \frac{dx}{\sqrt{\dfrac{40}{1.1} + 0.714gx}}$$

or $$t = \int_0^{100} \left(\frac{40}{1.1} + 0.714gx\right)^{-1/2}$$

$$dx = \left.\frac{\frac{1}{2}\sqrt{\dfrac{40}{1.1} + .714gx}}{.714g}\right|_0^{100}$$

$$= \frac{\sqrt{751}}{14.2} = \frac{27.5}{14.2} = 1.92 \text{ s}$$

PASSAGE 4

Read the following passage and answer the questions givenat the end.

A guitar string of length L is plucked in such a way that the total wave produced is sum of the fundamental and the second harmonic. That is standing wave is given by
$y(x,t) = y_1(x,t) + y_2(x,t)$
$= c \sin\omega_1 t \sin k_1 x + c \sin\omega_2 t \sin k_2 x$
where $\omega_1 = vk_1$ and $\omega_2 = vk_2$. Nodes and antinodes are observed

1. At what values of x nodes of $y_1(x,t)$ are observed?

(a) 0, L (b) 0, $L/2$, L
(c) $\dfrac{L}{2}$ (d) none of these

2. At what values of x antinodes of $y_2(x,t)$ are observed?

(a) $\dfrac{L}{2}$ (b) $\dfrac{L}{4}, 3\dfrac{L}{4}$
(c) $0, \dfrac{L}{2}, L$ (d) none

3. Will the sum of two standing waves produce a new standing wave?

(a) yes.
(b) no.
(c) sometimes it can produce.
(d) in some conditions it may not produce.

Solution **1.** (a) $(0, L)$ see Fig X

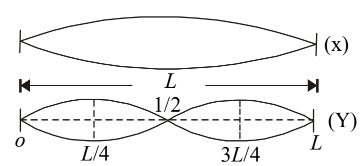


Fig. 13.17

Solution **2.** (b) $\left(\dfrac{L}{4}, \dfrac{3L}{4}\right)$ see Fig. Y

Solution **3.** (b) Sum of stationary waves of different frequency will not result into stationary waves.

QUESTIONS FOR PRACTICE

1. What happens when a sound wave is reflected from the boundary of a denser medium? The compression of the incident wave is returned as a

(a) rarefaction. (b) crest.
(c) trough. (d) compression.

2. The velocity of sound in vacuum is

(a) 332 ms^{-1} (b) 288 ms^{-1}
(c) 330 ms^{-1} (d) zero

3. The frequency of a man's voice is 300 Hz. If velocity of sound waves is 336 ms^{-1}, the wavelength of the sound is

(a) 1.12 m (b) 300 × 336 m
(c) 330/336 m (d) none of these

4. The angle between particle velocity and wave velocity in transverse waves is

(a) π (b) $\pi/2$
(c) $\pi/4$ (d) zero

5. Longitudinal waves cannot travel through
(a) liquids. (b) gases.
(c) vacuum. (d) solid.

6. In case of the electromagnetic waves the angle between the electric and magnetic field vectors is
(a) π (b) $\pi/2$
(c) zero (d) none of these

7. A wave of length 2 m is superposed on its reflected wave to form a stationary wave. A node is located at x = 3 m, the next node will be located at x =
(a) 4 m (b) 3.75 m
(c) 3.50 m (d) 3.25 m

8. Velocity of sound in the atmosphere of a planet is 500 ms^{-1}. The minimum distance between the source of sound and the obstacle for an echo heard is
(a) 17 m (b) 50 m
(c) 25 m (d) 20 m

9. If F is restoring force, k is force constant and y is displacement, which of the following expressions represent the equation of simple harmonic motion?
(a) $F = -ky$ (b) $F = \sqrt{ky}$
(c) $F = ky$ (d) none of these

10. The number of beats produced per second by two tuning forks when sounded together is 4. One of them has a frequency of 250 Hz. The frequency of the other cannot be less than
(a) 254 Hz (b) 252 Hz
(c) 248 Hz (d) 246 Hz

11. A bomb explodes on the moon. How long will it take for the sound to reach the earth?
(a) 1 day (b) 1000 s
(c) 10 s (d)none of these

12. Two simple harmonic waves having the same amplitude and frequency with zero phase difference superimpose at right angles to each other. The resultant motion will be
(a) linear. (b) eliptical.
(c) circular. (d) none of these.

13. A property of the progressive wave that does not depend upon other characteristics mentioned below is
(a) wavelength. (b) amplitude.
(c) frequency. (d) wave velocity.

14. Two waves of same frequency but amplitudes equal to a and $2a$ travelling in the same direction superimpose out of phase. The resultant amplitude will be
(a) $\sqrt{a^2 + 2a^2}$ (b) $3a$
(c) $2a$ (d) a

15. The oscillators that can be described in terms of sine or cosine functions are called
(a) simple harmonic. (b) natural.
(c) sympathetic. (d) free.

16. The distance between two consecutive antinodes is 0.5 m. The distance travelled by the wave in half the time period is
(a) 2 m (b) 1 m
(c) 0.5 m (d) 0.25 m

17. Which of the following expressions is that of a simple harmonic progressive wave?
(a) $A \sin(\omega t - kx)$ (b) $A \sin \omega t$
(c) $A \sin \omega t \cos kx$ (d) $A \cos kx$

18. A wave of frequency 400 Hz has a velocity of 320 ms^{-1}. The distance between the particles differing in phase by 90^0 is
(a) 80 cm (b) 60 cm
(c) 40 cm (d) 20 cm

19. The ratio of intensities of two waves is 1 : 16. The ratio of their amplitudes is
(a) 16/17 (b) 1/16
(c) 1/4 (d) 1/2

20. Two waves each of loudness L superimpose to produce beats. The maximum loudness of the beats will be
(a) $4L$ (b) $2L$
(c) L (d) none of these

21. Two waves of intensities I and $4I$ superimpose. The minimum and maximum intensities will respectively be
(a) I, $9I$ (b) $3I$, $5I$
(c) I, $5I$ (d) none of these

22. The velocity of sound in oxygen at NTP is v. The velocity of sound in hydrogen at NTP will be
(a) $2\sqrt{2}\,v$ (b) $2v$
(c) $4v$ (d) none of these

23. The isothermal elasticity of a medium is E_i and the adiabatic elasticity in E_a. The velocity of the sound in the medium is proportional to
(a) $\sqrt{E_i}$ (b) E_a
(c) $\sqrt{E_a}$ (d) E_i

24. The velocity of sound in air is v and the root mean square velocity of the molecules is c. Then v/c =
(a) $\gamma/3$ (b) $\gamma/\sqrt{3}$
(c) $\frac{\sqrt{\gamma}}{3}$ (d) $\sqrt{\frac{\gamma}{3}}$

25. The velocity of sound at 0^0C is 332 ms^{-1}. At what temperature will it be 664 ms^{-1}?

(a) 273^0C (b) 546^0C

(c) 819^0C (d) 1092^0C

26. Velocity of hydrogen at NTP is v. What will be the velocity of sound in a mixture of hydrogen and oxygen in the ratio 4 : 1 at NTP is

(a) v (b) $2v$

(c) $v/2$ (d) $v/4$

27. A sound wave is travelling in a medium in which the velocity is v. It is incident on the second medium in which the velocity of the wave is $2v$. What should be the minimum angle of incidence on the first medium, so that the wave fails to cross the surface of separation of the two media?

(a) 60^0 (b) 45^0

(c) 30^0 (d) 15^0

28. Beats are produced because of the superposition of two progressive notes. Maximum loudness at the waxing is n times the loudness of either notes. What is the value of n?

(a) 4 (b) 2

(c) $\sqrt{2}$ (d) 1

29. The first resonance length in a closed organ pipe is 50 cm. Then the second resonance length will be

(a) 50 cm (b) 100 cm

(c) 150 cm (d) 200 cm

30. Which type of oscillations give rise to resonance?

(a)damped (b) free

(c)forced (d) all of these

31. At what temperature the speed of sound in air will be 1.5 times its value at 27^0 C in air?

(a) 102^0C (b) 204^0K

(c) 204^0C (d) 402^0C

32. The ratio of speeds of sound in hydrogen gas and oxyen gas at same temperature will be

(a) 8 : 1 (b) 4 : 1

(c) 1 : 8 (d) 1 : 4

33. The distance between a node and an anti-node is

(a) 2λ (b) λ

(c) $\lambda/2$ (d) $\lambda/4$

34. The speed of a supersonic wave, as compared to that of sound is

(a) less. (b) more.

(c) equal. (d) 1/10.

35. The increase in the speed of sound, on increasing the temperature of the medium by 10^0C, will be

(a) 600 ms^{-1} (b) 6 ms^{-1}

(c) 0.61 ms^{-1} (d) 60 ms^{-1}

36. The velocity of sound in air is 350 ms^{-1}. The fundamental frequency of an open pipe of length 50 cm will be

(a) 700 s^{-1} (b)350 s^{-1}

(c) 175 s^{-1} (d)50 s^{-1}

37. The fundamental frequency of an open organ pipe is n. The pipe is vertically immersed in water such that half of its length is submerged. The fundamental frequency of air column in this position will be

(a) $n/3$ (b) $n/2$

(c) n (d) $2n$

38. If the ratio of amplitudes of two waves at any point in the medium is 1 : 3, then the ratio of maximum and minimum intensities because of their superposition will be

(a) 2 : 1 (b) 3 : 1

(c) 4 : 1 (d) 2 : 3

39. The phase difference between the particles vibrating between two consecutive nodes is

(a) zero (b) $\pi/2$

(c) π (d) 2π

40. The frequency of an open organ pipe is n. If one end is closed then its fundamental frequency will be

(a) $n/2$ (b) $3n/4$

(c) n (d) $2n$

41. Two sound waves of equal intensity I superimpose at point P in 90^0 out of phase. The resultant intensity at point P will be

(a) $4I$ (b) $\sqrt{2}\,I$

(c) $2I$ (d) I

42. The equation of a wave propagating in a string is $y = 2\cos\pi(100t - x)$. Its wave length will be

(a) 2 cm (b) 5 cm

(c) 2 m (d) 50 cm

43. On vibrating a tuning fork of frequency 256 Hz with another fork A, six beats per second are heard. On loading A, again six beats per second are heard. The frequency of A will be

(a) 244 Hz (b) 250 Hz

(c) 262 Hz (c) 268 Hz

44. The ratio of frequencies in a stretched string is

(a) 1 : 2 : 3 (b) 1 : 3 : 5

(c) 2 : 4 : 6 (d) 3 : 2 : 1

45. The property of a medium necessary for wave propagation is its

(a) elasticity. (b) low resistance.
(c) inertia. (d) all of above.

46. The ratio (v) of velocities of sound in dry air and humid air is
(a) $v < 1$ (b) $v > 1$
(c) $v = 1$ (d) zero

47. The waves propagating on water surface are
(a) ultrasonic. (b) longitudinal.
(c) unaudible. (d) transverse.

48. A tuning fork produces four beats per second with 49 cm and 50 cm lengths of a stretched wire of a sonometer. The frequency of the fork is
(a) 196 Hz (b) 296 Hz
(c) 396 Hz (d) 693 Hz

49. In Melde's experiment, eight loops are formed with a tension of 0.75 N. If the tension is increassed to four times then the number of loops produced will be
(a) 2 (b) 4
(c) 8 (d) 16

50. The third harmonic in an open organ pipe is known as
(a) fundamental frequency.
(b) second overtone.
(c) third overtone.
(d) first overtone.

51. The correct graph between the frequency n and square root of density (ρ) of a wire, keeping its length, radius and tension constant, is

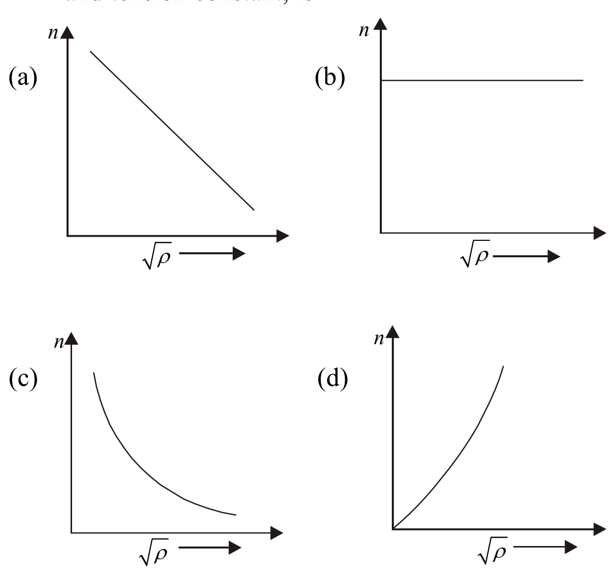


Fig. 13.18

52. P is the junction of two wires A and B. B is made of steel and is thicker while A is made of aluminium and is thinner as shown. If a wave pulse as shown in the figure approaches P, the reflected and transmitted waves from P are respectively

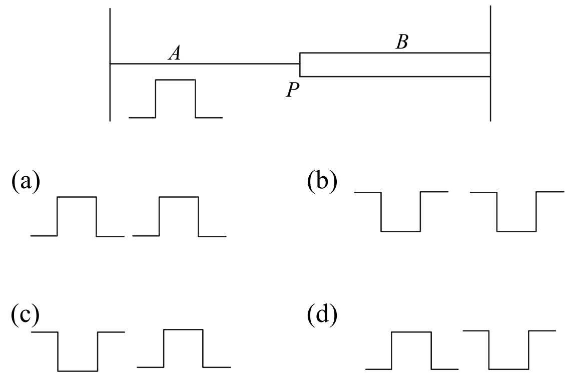


Fig. 13.19

53. In Kundt's tube, when waves of frequency 10^3 Hz are produced the distance between five consecutive nodes is 82.5 cm. The speed of sound in gas filled in the tube will be
(a) 660 ms^{-1} (b) 330 ms^{-1}
(c) 230 ms^{-1} (d) 100 ms^{-1}

54. A resonance tube is resonated with tuning fork of frequency 256 Hz. If the length of resonating air columms are 32 cm and 100 cm, then end correction will be
(a) 1 cm (b) 2 cm
(c) 4 cm (d) 6 cm

55. On decreasing the temperature the frequency of an organ pipe becomes
(a) less. (b) more.
(c) equal. (d) infinity.

56. When a sound wave is reflected from a rigid support then its path changes by
(a) $\lambda/2$ (b) λ
(c) π (d) 2π

57. The vibration in the stem of tuning fork are
(a) transverse. (b) longitudinal.
(c) both. (d) none of these.

58. The ratio of the velocityof body and velocity of sound is known as
(a) laplace number. (b) positive integer.
(c) stable number. (d) mach number.

59. For constructive interference, the path difference between two waves must be
(a) $(2n + 1)\ \lambda/2$ (b) $(2n + 1)\lambda$
(c) $n\dfrac{\lambda}{2}$ (d) $n\lambda$

60. The frequencies of two sound sources are 256 Hz and 260 Hz respectively. The beat frequencies produced by them will be

(a) $0.025\ s^{-1}$ (b) $2.5\ s^{-1}$
(c) $4.00\ s^{-1}$ (d) $25\ s^{-1}$

61. On sounding a string and a tuning fork simultaneously, six beats per second are produced if the length of the string is 95 cm or 100 cm. The frequency of the fork is

(a) 117 Hz (b) 234 Hz
(c) 432 Hz (d) 702 Hz

62. The resultant amplitude, when two waves of same frequency but with amplitudes a_1 and a_2 superimpose with a phase difference of $\pi/2$ will be

(a) $a_1^2 + a_2^2$ (b) $\sqrt{a_1^2 + a_2^2}$
(c) $a_1 - a_2$ (d) $a_1 + a_2$

63. In a stationary wave the distance between consecutive antinodes is 25 cm. If the wave velocity is 300 ms^{-1} then the frequency of wave will be

(a) 150 Hz (b) 300 Hz
(c) 600 Hz (d) 750 Hz

64. The intensity of sound after passing through a slab decreases by 20%. On passing through two such slabs, the intensity will decrease by

(a) 30% (b) 36%
(c) 40% (d) 50%

65. Two waves travel in the mutually opposite directions in a medium. When superimposed, the phenomenon observed is

(a) stationary waves. (b) harmonic nodes.
(c) beats. (d) resonance.

66. When two plane progressive waves travelling in same direction superpose over each other, the velocity of resultant wave will

(a) increase. (b) remain unchanged.
(c) be zero. (d) decrease.

67. The fundamental frequency in a stretched string is 100 Hz. To double the frequency, the tension in it must be changed to

(a) $T_2 = 2T_1$ (b) $T_2 = 4T_1$
(c) $T_2 = T_1$ (d) $T_2 = \frac{T_1}{4}$

68. The frequency of an open pipe is 300 Hz. The first overtone of this pipe is the same as the second overtone of a closed pipe. The length of the closed organ pipe is

(a) 11 cm (b) 21 cm
(c) 42 cm (d) 84 cm

69. In the equation of the motion of a particle $y = 0.5 \sin(0.3t + 0.1)$, the initial phase of motion is

(a) $(0.3t + 0.1)$ (b) 0.3
(c) $0.3t$ (d) 0.1

70. Two waves of same amplitude and same frequency reach a point in a medium simultaneously. The phase difference between them for resultant amplitude to be zero, will be

(a) 4π (b) 2π
(c) π (d) 0^0

71. Under similar conditions of temperature and pressure, the velocity of sound is maximum in

(a) CO_2 (b) H_2
(c) N_2 (d) O_2

72. The sound box of a sonometer increases

(a) speed of sound.
(b) sound intensity.
(c) wavelength of sound.
(d) sound frequency.

73. The loudness of sound depends on

(a) amplitude. (b) wavelength.
(c) pitch. (d) speed.

74. The ratio of intensities of two sound waves is 4 : 9 The ratio of their amplitudes will

(a) 9 : 4 (b) 4 : 9
(c) 2 : 3 (d) 3 : 2

75. The speed of sound in air is 320 ms^{-1}. The length of a closed pipe is 1 m. Neglecting the end correction, the resonant frequency for the pipe will be

(a) 80 Hz (b) 240 Hz
(c) 320 Hz (d) 400 Hz

76. On increasing the distance between the source of sound and observer thrice, the intensity of sound becomes

(a) 3 (b) 1/3
(c) 9 (d) 1/9

77. The velocity of sound in air at room temperature is 340 m/s and density of air is 1.2 kgm^{-1}. The value of atmospheric pressure in terms of the height of mercury column in metre will be

(a) 0.75 (b) 7.5
(c) 75 (d) 750

78. The best source of sound in order to obtain a pure note is

(a) harmonium. (b) tuning fork.
(c) flute. (d) sonometer.

79. Sound waves are propagating in a medium. The moduli of isothermal and adiabatic elasticity of the medium are E_T and E_s respectively. The velocity of sound wave is proportional to

(a) $\sqrt{E_T}$ (b) $\sqrt{E_s}$

(c) E_T (d) $\frac{E_S}{E_T}$

80. For a pulse moving in a heavy string the junctions of the string behaves as a

(a) perfectly rigid end (b) free end

(c) partially rigid end (d) rigid end

81. The intensity of a soundwave in an elastic medium falls by 10% on travelling a distance of 1 m. If the initial intensity of the wave is 100% then on travelling a distance of 3 m in that medium the intensity will become

(a) 81% (b) 70%

(c) 72.9% (d) 60%

82. A 75 cm long wire is reduced by 0.5 cm then it produces three beats per second with the fork. The frequency of the fork is

(a) 0.47 Hz (b) 4.47 Hz

(c) 47.7 Hz (d) 447.0 Hz

83. When two identical wires on a sonometre are kept under same tension their fundamental frequency is 500 Hz. In order to produce five beats per second the percentage change in the tension of one of the wires will be

(a) 2% (b) 4%

(c) 6% (d)8%

84. The minimum distance between the sound and the reflecting surface, in order to hear an echo, must be

(a) 0.65 m (b) 1.65 m

(c) 16.5 m (d) 165 m

85. The total mass of a sonometer wire remains constant. On increasing the distance between two bridges to four times, its frequency will become

(a) 0.25 times (b) 0.5 times

(c) 4 times (d) 2 times

86. The lowest pitch out of the following sources is that of a

(a) man. (b) boy.

(c) lion. (d) mosquito.

87. If the frequency of a sound wave is doubled then the velocity of sound will be

(a) zero. (b) half.

(c) double. (d) unchanged.

88. If the wavelength of a wave is decreased by 20% then its frequency will become

(a) 20% less. (b) 25% more.

(c) 20% more. (d) 25% less.

89. If the energy density and velocity of a wave are u and c respectively then the energy propagating per second per unit area will be

(a) u/c. (b) c^2u.

(c) uc. (d) c/u.

90. In which of the following is the energy loss maximum?

(a) sonometer (b) tuning fork

(c) thin tube (d)broader pipe

91. What is produced at a rigid reflecting plane for a displacement wave?

(a) beats. (b) node and antinode.

(c) antinode. (d) node.

92. If the equations of two sound waves are $y_1 = 5 \sin 252 \pi t$ and $y_2 = 5 \sin 280 \pi t$ respectively. Then the number of beats heard per second will be

(a) beats will not be heared. (b) 6.

(c) 12. (d) 34.

93. If the density of materials of two strings of same length, tension and area of cross-section are 2 kgm^{-3} and 4 kgm^{-3} respectively then the ratio of their frequencies will be

(a) $1 : \sqrt{2}$ (b) 2 : 1

(c) 1 : 2 (d) $\sqrt{2} : 1$

94. The time taken by a particle to travel between a trough and a crest in a transverse wave is

(a) T (b) $3T/4$

(c) $T/2$ (d) $T/4$

95. The second overtone of a closed organ pipe P_1 and the third overtone of an open pipe P_2 are in unison with a tuning fork. The ratio of the length of P_1 and P_2 will be

(a) 3 : 8 (b) 1 : 3

(c) 8 : 3 (d) 3 : 1

96. Two open pipes, whose lengths are 50 cm and 51 cm respectively, produce five beat per second when sounded together. The fundamental frequencies in Hertz will be

(a) 65 and 60 (b) 90 and 95

(c) 95 and 90 (d) 255 and 250

97. The mass of a 4m long string is 0.01 kg. It is stretched by a force of 400 N. The velocity of the transverse wave propagating in the string is

(a) 100 ms^{-1} (b) 200 ms^{-1}

(c) 300 ms^{-1} (d) 400 ms^{-1}

98. The length, mass and tension of a string are 1000 cm, 0.01 kdg and 10 N respectively, the speed of transverse waves in the string will be

(a) 10^2 ms^{-1} (b) 10^4 ms^{-1}

(c) 10^6 ms^{-1} (d) 10^8 ms^{-1}

99. In Melde's experiment, the string vibrates in seven segments under tension of 9 gram weight. If string is to be vibrated in three segmets then the tension required will be

(a) 1.4 gm-wt (b) 13 gm-wt

(c) 49 gm-wt (d) 61 gm-wt

100. The length of strings of a cello is 0.8 m. In order to change the pitch in frequency ratio 5/4, their length should be decreased by

(a) 0.08 m (b) 0.2 m

(c) 0.13 m (d) 0.16 m

101. Two waves of same frequency but of amplitudes a and $2a$ respectively superimpose over each other. The intensity at a point where the phase difference is $3\pi/2$, will be

(a) a (b) $3a^2$

(c) $5a^2$ (d) $9a^1$

102. A resonance tube of diameter 2 cm is vibrating in unison with a tuning fork of frequency 512 Hz When the length of air column above water level is 13.5 cm, the wavelength of the note will be

(a) 31.0 cm (b) 32.2 cm

(c) 54 cm (d) 62.0 cm

103. If the slope is s, wave velocity is v_w and particle velocity is v_p then

(a) $v_p = sv_w$ (b) $v_p = \frac{v_w}{s}$

(c) $v_p = sv_w$ (d) $v_p = \frac{-v_w}{s}$

104. A wave $y = 10 \sin (200\pi t - 0.5x)$ travels in space. The separation between two rarefactions is

(a) 10 cm (b) 2π cm

(c) 2 cm (d) 4π cm

105. The density of air at NTP is 1.293 kgm^{-3} and density of mercury at 0^0C is 13.6×10^3 kgm^{-3}. If $C_p = 0.2417$ calkg^{-10}C^{-1} and $C_V = 0.1715$, the speed of sound in air at 100^OC will be (g = 9.8 Nkg^{-1})

(a) 260 ms^{-1} (b) 332 ms^{-1}

(c) 350.2 ms^{-1} (d) 388.4 ms^{-1}

106. Two organ pipes are of same size. Hydrogen and oxygen gases are filled in them. Taking the elasticity of two gases to be same, the ratio of their fundamental frequencies will be

(a) 1 : 4 (b) 4 : 1

(c) 8 : 1 (d) 16 : 1

107. A sitar wire vibrates with frequency of 330 vibrations per second. If its length is increased three times and tension is increased four times then the frequency of the wire will be

(a) 110 Hz (b) 220 Hz

(c) 330 Hz (d) 440 Hz

108. A u-tube of uniform cross-section is kept in a vertical position. A liquid of mass m and density d is filled in one of its limbs. This liquid will oscillate in this tube with time period T given by

(a) $T = 2\pi\sqrt{\frac{m}{gda}}$ (b) $T = 2\pi\sqrt{\frac{ma}{gd}}$

(c) $T = 2\pi\sqrt{\frac{m}{g}}$ (d) $T = 2\pi\sqrt{\frac{m}{2agd}}$

109. The ends of two wires of radii r and $2r$ are mutually joined. The compound wire is used in a sonometer. The joint is kept at the centre between two bridges and tension T is produced into it. If the joint is a node, then the ratio of the number of loops produced in two wires will be

(a) 1 : 4 (b) 1 : 5

(c) 3 : 1 (d) 1 : 2

110. The length of a string is 1 m, tension in it is 40 N and mass of the string is 0.1 kg. Then the velocity of transverse waves produced in the string will be

(a) 400 ms^{-1} (b) 180 ms^{-1}

(c) 80 ms^{-1} (d) 20 ms^{-1}

111. The equation $y = 0.15 \sin 5x \cos 300t$ represents a stationary wave. The wavelength of this stationary wave will be

(a) 0.628 m (b) 2.512 m

(c) 1.256 m (d) zero

112. A resonance tube of length 1 m is resonated with a tuning fork of frequency 700 Hz. If the velocity of sound in air is 330 ms^{-1} then the number of harmonics produced in the tube will be

(a) 1 (b) 2

(c) 3 (d) 4

113. A man is standing between two cliffs. If he claps his hands once, a series of echoes at the intervals of one second are heard. If the speed of sound is 340 ms^{-1} the distance between the cliffs is

(a) 170 m (b) 680 m

(c) 340 m (d) 510 m

114. A wire of mass 4 kg, length 4 m is hanged vertically. A weight of 2 kg is attached at the bottom. The ratio of velocity top to bottom is

(a) $\sqrt{3}$ (b) $\sqrt{2}$

(c) 3 (d) 2

115. A transverse wave is given by $y = a \sin 2\pi (ft - x/\lambda)$. The maximum particle velocity is four times the wave velocity, when

(a) $\lambda = 2\pi a$ (b) $\lambda = \pi a$

(c) $\lambda = \pi a/4$ (d) $\lambda = \pi a/2$

116. A student sees a jet plane flying from east to west. When the jet is seen just above his head the sound of Jet appears to reach him making angle 60° with the horizontal from east. If the velocity of sound is C, then that of the jet plane is

(a) $\frac{2}{\sqrt{3}} C$ (b) $\frac{C}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $2C$

117. A ship sends a longitudinal wave towards the bottom of the sea. The wave returns from the bottom of the sea after 2.0 s. The bulk modulus of the sea vater is $2.2 \times 10^9 Nm^{-2}$ and the density of water is 1.1 g cm^{-3}. The depth of the sea is about

(a) 2200 m (b) 2000 m

(c) 1400 m (d) 1100 m

118. A jet aeroplane is flying at the speed of sound. When the sound of the jet appears to be coming verically downwards, the angle of sight of the aeroplane with the horizontal cannot be

(a) 60° (b) 40°

(c) 30° (d) 25°

119. A vibrating tuning fork is placed close to another of equal frequency. The other also starts vibrating. This happens because of

(a) interference.

(b) resonance.

(c) superposition.

(d) formation of stationary waves.

120. The distance between two consecutive nodes on a stretched string is 10 cm. It is in resonance with tuning fork of frequency 256 Hz.What is the velocity of the progressive wave in the string?

(a) 6. 40 ms^{-1} (b) 51.20 ms^{-1}

(c) 25.60 ms^{-1} (d) 12.50 ms^{-1}

121. An organ pipe P_1 closed at one end vibrating in its first harmonic and pipe P_2 open at both ends vibrating in its third harmonic are in resonance with the same tuning fork. The ratio of their lengths is

(a) 3/8 (b) 3/4

(c) 1/8 (d) 1/6

122. In an open organ pipe the harmonics that are missing are

(a) depends upon length of the pipe.

(b) none.

(c) even.

(d) odd.

123. The string of a sonometer is divided into two parts with the help of wedge. The total length of the string is 1 m and the two parts differ in length by 2 mm. When sounded together, they produce two beats. The frequencies of the notes emitted by the two parts are

(a) 499 and 497 (b) 501 and 503

(c) 501 and 499 (d) none of these

124. The frequency of the first overtone of an open organ pipe is f, then that of the closed organ pipe will be

(a) $3f/4$ (b) f

(c) $f/4$ (d) $f/2$

125. A tube closed at one end and containing air produces fundamental note of frequency of 256 Hz. If the tube is open at both ends, the fundamental frequency will be

(a) 512 Hz (b) 384 Hz

(c) 128 Hz (d) 64 Hz

126. A sonometer wire is to be divided in to three segments having fundamental frequencies in the ratio 1 : 2 : 3 What should be the ratio of lengths?

(a) 4 : 2 : 1 (b) 4 : 3 : 1

(c) 6 : 3 : 1 (d) 3 : 2 : 1

127. If we add 8 kg load to the hanger of a sonometer. the fundamental frequency becomes three times of its initial value. The initial load in the hanger was

(a) 4 kg-wt (b) 2 kg-wt

(c) 1 kg-wt (d) 0.5 kg-wt

128. Two identical organ pipes are producing fundamental notes of frequency 200 Hz at 15°C. If the temperature of one pipe is raised to 27°C, the number of beats produced will be

(a) 8 (b) 6

(c) 4 (d) 2

129. A rod 70 cm long is clamped from middle. The velocity of sound in the material of the rod is 3500 ms^{-1}. The frequency of fundamental note produced by it is

(a) 3500 Hz (b) 2500 Hz

(c) 1250 Hz (d) 700 Hz

130. The tension in a wire is decreased by 19% . The percentage decrease in frequency will be

(a) 10% (b) 19%

(c) 0.19% (d) none of these

131. An open organ pipe of length 50 cm vibrates in unison with a tuning fork of frequency f. The diameter of the pipe is 3 cm. What is the wavelength of the note produced?

(a) 204 cm (b) 102 cm
(c) 96 cm (d) 52 cm

132. Two waves of same amplitude superimpose to produce two beats per second. What is the ratio of maximum loudness to that of one of the waves?
(a) 2 (b) 4
(c) 8 (d) ∞

133. What is the phase difference between the particles of a string on the two sides of an antinode?
(a) 180° (b) 90°
(c) 45° (d) 0°

134. A progressive wave of frequency 500 Hz is travelling at a speed of 360 ms^{-1}. How far are the two points on it having a phase difference of 60°?
(a) 0.12 m (b) 0.36 m
(c) 0.24 m (d) 0.18 m

135. The standing waves set upon a string are given by

$$y = 4 \sin\left(\frac{\pi x}{12}\right) \cos(52\pi t).$$

If x and y are in centimetres and t is in seconds, what is the amplitude of the particle at $x = 2$ cm?
(a) 12 cm (b) 4 cm
(c) 2 cm (d) 1 cm

136. A boat at anchor is rocked by a wave whose crests are 100 m apart and whose velocity is 0.25 ms^{-1}. These waves reach the boat once every
(a) 2500 s (b) 1500 s
(c) 4.00 s (d) 0.25 s

137. If the amplitude of waves at a distance 10 cm from a point source is A, the amplitude at a distance 40 cm will be
(a) $A/2$ (b) $A/4$
(c) A (d) $2A$

138. A wave has amplitude of velocity

$v_0 = \dfrac{1}{4\omega^2 - 11\omega + 6}$. The resonant frequency is

(a) $\omega = 2s^{-1}$ (b) $\omega = \dfrac{3}{4}\ s^{-1}$
(c) zero (d) $\omega = \dfrac{11}{8}\ s^{-1}$

139. The distance between two consecutive crests in a wave train produced in a string is 5 cm. If two complete waves pass through any point per second. The velocity of the wave is
(a) 15 cms^{-1}. (b) 10 cms^{-1}.
(c) 2.5 cms^{-1}. (d) 5 cms^{-1}.

140. A tuning fork of frequency 290 Hz appear to be vibrating with a frequency of 10 Hz when observed in a flashing illumination. The frequency of the flashing light is
(a) 10 Hz (b) 270 Hz
(c) 280 Hz (d) 290 Hz

141. For a resonance tube the air columns for the first and the second resonance differ in length by 31.5 cm. The wavelength of the sound wave is (in cm)
(a) 126.0 (b) 63.0
(b) 31.5 (d) 252.0

142. Transverse waves are being produced in a stretched string whose equation is $y = 0.021 \sin(x + 30t)$. If the linear density of string is 1.3×10^{-4} kgm^{-1} then the tension produced in string will be
(a) 10 N (b) 0.5 N
(c) 0.117 N (d) 0.107 N

143. Two waves of frequencies 1 Hz and 3 Hz are moving in a medium with same velocity. The ratio of intensities at a point where amplitudes of two waves are equal will be
(a) 1 : 1 (b) 1 : 4
(c) 1 : 2 (d) 1 : 9

144. Power of 10 W is emitted by a loudspeaker. The sound intensity radiated by it at a distance of 3 m is 2 W/m^2. If the intensity of loudspeaker is doubled the intensity at 6 m will be
(a) 1 W/m^2 (b) 4 W/m^2
(c) 0.5 W/m^2 (d) 2 W/m^2

145. A wave represented by $y = 100 \sin(ax + bt)$ is reflected from a dense plane at the origin. If 36% of energy is lost and rest of the energy is reflected then the equation of the reflected wave will be
(a) $y = -80 \sin(ax + bt)$ (b) $y = -8.1 \sin(ax + bt)$
(c) $y = -10 \sin(ax + bt)$ (d) $y = -8.1 \sin(ax + bt)$

146. If one of the arms of a tuning fork is broken then its frequency of vibration is
(a) not effected. (b) less than before.
(c) as before. (d) more than before.

147. If the frequency of the wave is 100 Hz then the particles of the medium cross the mean position in 1 s will be
(a) 50 times. (b) 100 times.
(c) 200 times. (d) 400 times.

148. Which of the following laws of strings is not correct?
(a) $n \propto \dfrac{1}{\sqrt{m}}$ (b) $n \propto l$
(c) $n \propto \sqrt{T}$ (d) $n \propto \dfrac{1}{l}$

149. The wavelength of a wave in a medium is 0.5 m. Due to this wave the phase difference between two particles of the medium is $\pi/5$. The minimum distance between these points is
(a) 5 km. (b) 5 m.
(c) 5 mm. (d) 5 cm.

150. In strings, the position of antinodes are obtained at

(a) $\lambda, 2\lambda, 3\lambda$ (b) $0, \lambda/2, \lambda$

(c) $2\lambda, 4, 6\lambda$ (d) $\lambda/4, 3\lambda/4, 5\lambda/4$

PASSAGE 1

Read the following passage and answer the questions given at the end.

Your roommates have lost the TV remote control and no amount of searching can find it. Rather than buy a new one, you build a low cost replacement. You attach one end of a small liver mechanism to the TV channel changing button. You plan to attach the other end of the liver to a 3 m long string that will run from TV to the couch. When you pull the string tight and pluck your end of the string a wave will travel down the string and trigger the fever, changing the channel. Your design assumes you will disturb the string vertically by 5 mm when you pluck it and that your wave will take only 0.2 s to travel horizontally along the string from your end to the lever. Unfortunately you could not find a single string 3 m long. You could only find two 1.5 m long strings one weighing 90 g and the other weighing 10 g You tie the two pieces to make a 3 m long string and attach one end of the combined string to lever mechanism. You then take the other end in hand and head for the couch.

1. How hard do you have to pull to make it stretch taut?

(a) 7.5 N (b) 75 N

(c) 1.5 N (d) 13.5 N

2. How many loops will be seen in each string?

(a) 1 in thinner wire and three in thicker wire.

(b) 3 in thinner wire and 1 in thicker wire.

(c) 1 each.

(d) 3 each.

3. What is the frequency of the wave?

(a) 21.5 Hz (b) 20 Hz

(c) 14.3 Hz (d) 11.1 Hz

Solution **1.** (a) $v = \sqrt{\dfrac{T}{\mu}} \Rightarrow \dfrac{3}{0.2} = \sqrt{\dfrac{T}{\frac{.1}{3}}}$

or $3\,T = 22.5\,N$

or $T = 7.5\,N$

Solution **2.** (a) $f = \dfrac{n_1}{2l}\sqrt{\dfrac{T}{\frac{.01}{1.5}}} = \dfrac{n_2}{2l}\sqrt{\dfrac{T}{\frac{.09}{1.5}}}$

or $\dfrac{n_1}{n_2} = \dfrac{1}{3}$

Solution **3.** (d) $f = \dfrac{1}{3}\sqrt{\dfrac{7.5\times 1.5}{.01}} = \sqrt{125} = 11.1$ Hz.

PASSAGE 2

Read the following passage and answer the questions given at the end.

A boy of 5th standard is playing with the cloths line. He unties one end, holds it taut and wiggles the end up and down sinusoidlly with frequency 2 Hz and amplitude 0.075 m. The wave speed is 12 ms^{-1}. At $t = 0$ the end has maximum displacement and is instantaneously at rest. Assume no wave bounces back from the far end to muddle up the pattern.

1. What is wave number?

(a) 1.05 m^{-1} (b) 1.32 m^{-1}

(c) 0.78 m^{-1} (d) 2.34 m^{-1}

2. Write a wave function describing the wave.

(a) $y = 0.075 \cos (1.05x - 4\pi t)$

(b) $y = 0.075 \cos (1.05x - 2t)$

(c) $y = 0.075 \sin (1.05x - 4\pi t)$

(d) $y = 0.075 \sin (1.05x - 2t)$

3. Write equations for the displacement as a function of time 3 m of the boy's end of the clothesline

(a) $y = 0.075 \cos 4\pi$ (b) $y = (-\,0.075 \cos 4\pi)$

(c) $y = 0.075 \sin 4\pi$ (d) none

Solution **1.** (a) $v = \dfrac{\omega}{k}$

or $k = \dfrac{\omega}{v} = \dfrac{2\pi\times 2}{12} = \dfrac{\pi}{3} = 1.05$ m^{-1}.

Solution **2.** (a)

Solution **3.** (b) $y = y_0 \cos 2\pi\left(\dfrac{x}{\lambda} - \dfrac{t}{T}\right) = 0.075 \cos 2\pi \left(\dfrac{3}{6} - 2t\right)$

$\because \quad \lambda = \dfrac{v}{f} = \dfrac{12}{2} = 6$ m.

$= 0.075 \cos (\pi - 4\pi t)$

$= -\,0.075 \cos 4\pi t$

PASSAGE 3

Read the following passage and answer the questions given at the end.

One of the strings of a Guitar lies along the x-axis when in equilibrium. The end of the string at x = 0 (the bridge of the guitar) is tied down. An incident sinusoidal wave travels the string in the $-x$ direction at 143 ms^{-1} with an amplitude 0.75 mm and a frequency of 440 Hz. This wave is reflected from the $x = 0$ end (fixed end) and the super position of incident and reflected travelling waves forms a standing wave.

1. The equation of the wave representing stationary wave is

(a) $0.75 \sin 19.3x \cos 880\pi t$.

(b) $(0.75 \times 10^{-3}) \sin 19.3x \cos 880\pi t$.

(c) $1.5 \times 10^{-3} \sin 19.3x \cos 880\pi t$.

(d) $1.50 \times 10^{-3} \sin 19.3x \cos 440\pi t$.

2. The separation between the two nearest points on the string that donot move at all is

(a) 0.163 m (b) 0.325 m

(c) 0.202 m (d) 0.244 m

(e) none

3. The maximum transverse velocity and maximum transverse acceleration at point of maximum oscillation is

(a) $4.15\ \text{ms}^{-1}, 1.15 \times 10^4\ \text{ms}^{-2}$

(b) $4.15\ \text{ms}^{-1}, -1.15 \times 10^4\ \text{ms}^{-2}$

(c) $1.15\ \text{ms}^{-1}, 4.15 \times 10^4\ \text{ms}^{-2}$

(d) $3.98\ \text{ms}^{-1}, 1.35 \times 10^4\ \text{ms}^{-2}$

Solution **1.** (c) $y = 2y_0 \sin kx \cos \omega t = 1.5 \times 10^{-3} \sin \left(\dfrac{440 \times 2\pi}{143} x\right) \cos (440 \times 2\pi)t$

Solution **2.** (a) $l = \dfrac{\lambda}{2} = \dfrac{143}{2 \times 440} = 0.163$ m.

Solution **3.** (a) $v_{\max} = \left.\dfrac{\partial y}{\partial t}\right|_{\max} = 1.5 \times 10^{-3} \times 2760 \sin 19.3x$

$\cos 880\pi t|_{\max} = 4.15\ \text{ms}^{-1}$

$a_{\max} = \left.\dfrac{\partial^2 y}{dt^2}\right|_{\max} = 4.15 \times 2760 = 1.15 \times 10^4\ \text{ms}^{-2}$

Answers to Questions for Practice

1.	(d)	**2.**	(d)	**3.**	(a)	**4.**	(b)	**5.**	(c)	**6.**	(b)	**7.**	(a)
8.	(c)	**9.**	(a)	**10.**	(d)	**11.**	(d)	**12.**	(a)	**13.**	(b)	**14.**	(d)
15.	(a)	**16.**	(c)	**17.**	(a)	**18.**	(d)	**19.**	(c)	**20.**	(a)	**21.**	(a)
22.	(c)	**23.**	(c)	**24.**	(d)	**25.**	(c)	**26.**	(c)	**27.**	(c)	**28.**	(a)
29.	(c)	**30.**	(c)	**31.**	(d)	**32.**	(b)	**33.**	(d)	**34.**	(b)	**35.**	(b)
36.	(b)	**37.**	(c)	**38.**	0(c)	**39.**	(a)	**40.**	(a)	**41.**	(b)	**42.**	(c)
43.	(c)	**44.**	(a)	**45.**	(d)	**46.**	(a)	**47.**	(d)	**48.**	(c)	**49.**	(b)
50.	(b)	**51.**	(c)	**52.**	(c)	**53.**	(b)	**54.**	(b)	**55.**	(a)	**56.**	(a)
57.	(b)	**58.**	(d)	**59.**	(d)	**60.**	(c)	**61.**	(b)	**62.**	(b)	**63.**	(c)
64.	(b)	**65.**	(a)	**66.**	(b)	**67.**	(b)	**68.**	(c)	**69.**	(d)	**70.**	(c)
71.	(b)	**72.**	(b)	**73.**	(a)	**74.**	(c)	**75.**	(a)	**76.**	(d)	**77.**	(a)
78.	(b)	**79.**	(b)	**80.**	(a)	**81.**	(c)	**82.**	(d)	**83.**	(a)	**84.**	(c)
85.	(a)	**86.**	(c)	**87.**	(d)	**88.**	(b)	**89.**	(c)	**90.**	(d)	**91.**	(d)
92.	(a)	**93.**	(d)	**94.**	(c)	**95.**	(a)	**96.**	(d)	**97.**	(d)	**98.**	(a)
99.	(c)	**100.**	(d)	**101.**	(c)	**102.**	(c)	**103.**	(c)	**104.**	(d)	**105.**	(d)
106.	(b)	**107.**	(b)	**108.**	(c)	**109.**	(d)	**110.**	(d)	**111.**	(c)	**112.**	(b)
113.	(c)	**114.**	(a)	**115.**	(d)	**116.**	(b)	**117.**	(c)	**118.**	(a)	**119.**	(b)
120.	(b)	**121.**	(d)	**122.**	(b)	**123.**	(c)	**124.**	(a)	**125.**	(a)	**126.**	(c)
127.	(c)	**128.**	(c)	**129.**	(b)	**130.**	(a)	**131.**	(b)	**132.**	(b)	**133.**	(a)
134.	(a)	**135.**	(c)	**136.**	(c)	**137.**	(b)	**138.**	(d)	**139.**	(b)	**140.**	(c)
141.	(b)	**142.**	(c)	**143.**	(d)	**144.**	(a)	**145.**	(a)	**146.**	(a)	**147.**	(c)
148.	(b)	**149.**	(d)	**150.**	(d)								

EXPLANATION

27. (c) $\dfrac{\sin i}{\sin r} = \dfrac{\text{velocity in medium 2}}{\text{velocity in medium 1}} = \dfrac{v}{2v}$

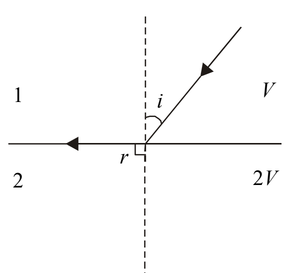


Fig. 13.20

$\dfrac{\sin i}{\sin 90^\circ} = \dfrac{1}{2}$ or $i = 30^\circ$.

48. (c) $\dfrac{v}{98} - \dfrac{v}{100} = 8$ or $v = 4 \times 98$ m/s$f_2 = 400$ Hz, $f_1 = 392$ Hz

$\therefore$ Using fork which produces 4 beats with these wires has frequency 396 Hz.

53. (b) $\dfrac{5\lambda}{2} = 82.5$ cm

$\lambda = 33$ cm and $v = f\lambda = 330\ \text{ms}^{-1}$.

54. (b) $\dfrac{\lambda}{4} = 32 + 0.3d$

$\dfrac{3\lambda}{4} = 100 + 0.3d$

$$\frac{\lambda}{2} = 68 \text{ cm}$$

$$\frac{\lambda}{4} = 34 \text{ cm or } 0.3d = 2 \text{ cm.}$$

109. (d) $\left.\dfrac{dy}{dt}\right|_{\max} = A\pi f$ and wave velocity $= f\lambda$;

$$A\pi f = 4f\lambda \text{ or } \lambda = \frac{A\pi}{4}$$

116. (b) $v = c \cos 60° = \dfrac{c}{2}$

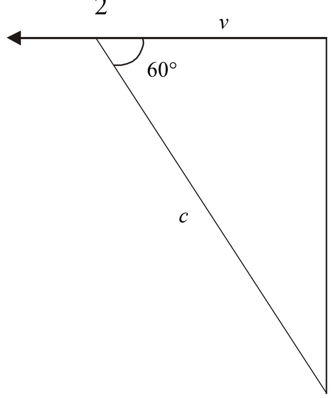


Fig. 13.21

117. (c) $v = \sqrt{\dfrac{\beta}{\rho}} = \sqrt{\dfrac{2.2\times10^9}{1.1\times10^3}} = 10^3\sqrt{2}\ \text{ms}^{-1}$

$$x = \frac{vt}{2} = \frac{10^3\sqrt{2}\times2}{2} = 1400 \text{ m.}$$

123. (c) $\dfrac{v}{49.9} - \dfrac{v}{5.1} = 2$ or $.2v = 2 \times 50 \times 49.9$

or $v = 50.1 \times 499$

$$f_1 = \frac{50.1\times499}{50.1} = 499 \text{ Hz } f_2 = 501 \text{ Hz}$$

128. (c) $\Delta f = \dfrac{\square v}{\lambda}$ $\Delta f = 0.6 \times 12 = 7.2\ \text{ms}^{-1}$

$$\lambda = \frac{340}{200} = 1.7 \therefore \Delta f = 4 \text{ (nearly.)}$$

130. (a) $f \propto \sqrt{2}\ f_{new} = \sqrt{0.81T} = 0.9\sqrt{T}$, that is,

or decrease of 10%.

144. (a) $\dfrac{k\dfrac{10}{3^2}}{k\dfrac{20}{6^2}} = \dfrac{2}{x}$

or $x = 1\ \text{Wm}^{-2}$.