2 Vectors

BRIEF REVIEW

Vector The physical quantities which have magnitude and direction and are added according to the triangle law of addition are called vectors, or, directed segment which follow triangle law of addition are called vectors.

Properties of Vectors In addition to magnitude and unit (a) it has specified direction, (b) it obeys triangle law of addition, (c) their addition is commutative i.e.

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

(d) Their addition is associative $\left(\vec{A}+\vec{B}\right)+\vec{C} = \vec{A}+\left(\vec{B}+\vec{C}\right)$

Representation of Vectors Vectors may be represented in two forms: polar and cartesian.

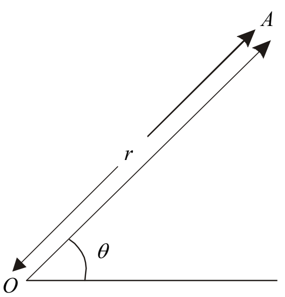


Fig. 2.1 Polar representation of a vector

Polar Form In this form $\overrightarrow{OA}=(r, \theta)$ where r is magnitude and θ is angle as shown in Fig 2.1

Cartesian Form In this form $\vec{A}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ where a_1, a_2 and a_3 are coefficients and $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit Vectors along x, y and z directions, respectively, as illustrated in Fig 2.2

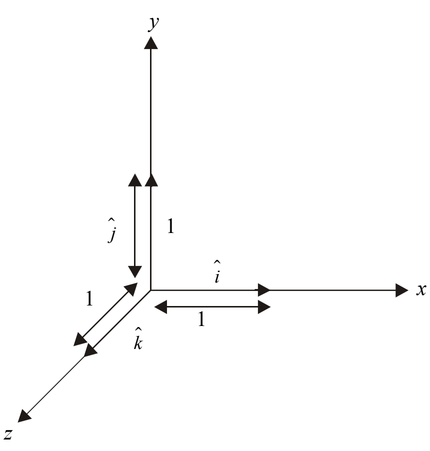


Fig. 2.2 Unit vector representation in rectangular coordinate system

Types of Vector In general vectors may be divided into three types:

1. Proper Vectors
2. Axial Vectors
3. Inertial or Pseudo Vectors.

Proper Vectors Displacement, force, momentum etc. are Proper Vectors.

Axial Vectors The vectors which act along axis of rotation are called axial vectors. For example, angular velocity, torque, angular momentum, angular acceleration are axial vectors.

Pseudo or Inertial Vectors The vectors used to make a non inertial frame of reference into inertial frame of reference are called pseudo or inertial. Vectors may further be subdivided as

(i) **Null Vector** It has zero magnitude and indeterminate direction.

(ii) **Unit Vector** Magnitude of unit vector is 1. It specifies direction only. Unit vector of a given vector is $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$

i.e. vector divided by its magnitude represents unit vector.

(iii) **Like Vector or Parallel Vectors** If two vectors have the same direction but different magnitude then they are said to be parallel or like vectors. Fig 2.3(a) shows like vectors.

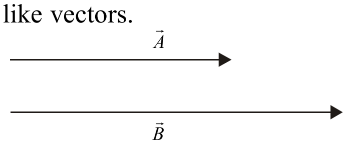


Fig. 2.3 (a) Like vectors

(iv) **Unlike Vectors** Two vectors having opposite directions and unequal magnitudes are called unlike vectors or parallel vectors in opposite sense. If their magnitudes are equal, they are called opposite vectors. Fig 2.3(b) shows unlike vectors.

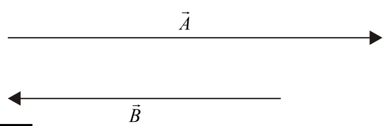


Fig. 2.3 (b) Unlike vectors

(v) **Equal Vectors** Two parallel vectors having equal magnitudes are called equal vectors.

(vi) **Co-initial Vectors** If vectors have a common initial point, they are known as co-initial vectors. In Fig. 2.4, $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are co-initial vectors.

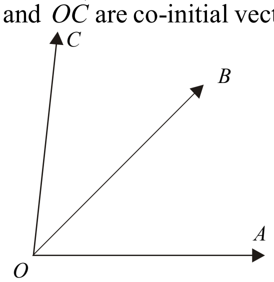


Fig. 2.4 Co-initial vectors

(vii) **Co-linear Vectors** Like, unlike, equal, opposite vectors may be grouped as co-linear vectors if they are either in the same line or parallel.

(viii) **Co-planar Vectors** Vectors lying in the same plane are termed as coplanar.

Resolution of a Vector Resolving a vector into its components is called resolution of a vector. Using triangle law one can write in Fig. 2.5 $\vec{A} = \vec{A}_x + \vec{A}_y$ or $\vec{A} = \vec{A}_x\hat{i} + \vec{A}_y\hat{j}$

or $\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$

$|\vec{A}| = \sqrt{A^2{}_x + A^2{}_y}$ and

$\tan\theta = \frac{A_y}{A_x}$ or $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

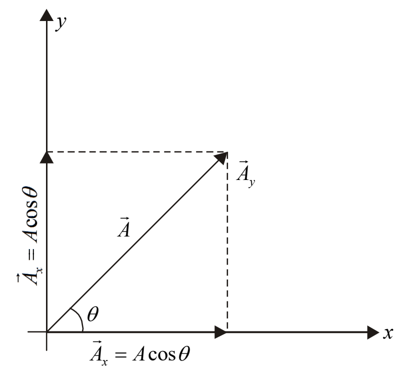


Fig. 2.5 Resolution of a vector

Laws of additions of vectors Vectors can be added using

(a) Triangle Law

(b) Parallelogram Law

(c) Polygon Law.

Triangle Law If two vectors acting on a body may be represented completely (in magnitude and direction) by two sides of a triangle taken in order, then their resultant is represented by third side of the triangle taken in opposite directions. In the fig 2.6 (a) $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$ or $\vec{A} + \vec{B} = \vec{R}$

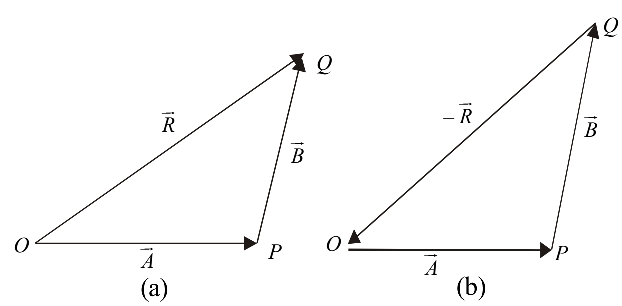


Fig. 2.6 Triangle law illustration

If three vectors acting on a body may completely be repesented by three sides of a triangle taken in order then the system is in equilibrium. In Fig. 2.6 (b)

$\overrightarrow{OP} + \overrightarrow{PQ} + \overrightarrow{QR} = \vec{A} + \vec{B} + (-\vec{R})$ or $\vec{R} - \vec{R} = 0$

Parallelogram Law If two vectors acting on a body may be represented completely by two adjacent sides of a parallelogram, then their resultant is represented by a diagonal passing through the common point. In Fig 2.7 from equal vector $\overrightarrow{PL} = \overrightarrow{OQ} = \vec{B}$ from Δ law $\vec{A} + \vec{B} = \vec{R}$

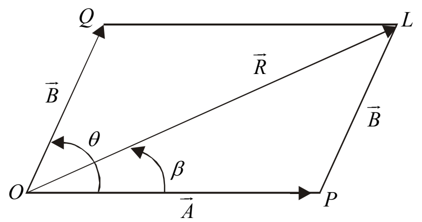


Fig. 2.7 Parallelogram law of vector illustration

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\sin\theta}$$

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

Note: $A - B \le R \le A + B$ i.e. $R_{min} = A - B$ when $\theta = 180°$ and $R_{max} = A + B$ when $\theta = 0°$ $0 \le \theta \le 180°$. Remember θ cannot exceed 180°.

Note: Minimum number of coplanar vectors whose sum can be zero (or required for equilibrium)

= 2 (if vectors are equal and opposite)

= 3 if vectors are unequal or not opposite, minimum number of noncoplanar vectors whose sum can be zero = 4

Note: Subtraction of a vector is equivalent to addition of a negative vector.

Multiplication of Vectors Two types of multiplication is defined (a) dot product or scalar product, (b) Cross product or vector product.

Dot product or Scalar Product If the product of two vectors is a scalar, then this rule is applied $\vec{A}.\vec{B} = AB\cos\theta$

Note: $\vec{A}.\vec{B} = \vec{B}.\vec{A}$, i.e., scalar product is commutative $\vec{A}.\left(\vec{B} + \vec{C}\right) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$, i.e., scalar product follows distributive law.

Rules: (i) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(ii) $\hat{i}.\hat{j} = \hat{j}.\hat{i} = \hat{j}.\hat{k} = \hat{k}.\hat{j} = \hat{i}.\hat{k} = \hat{k}.\hat{i} = 0$

Application of Dot Product

1. When the product of two vectors is a scalar. For example $W = \vec{F}.\vec{s}$, $P = \vec{F}.\vec{v}$ current $I = \int \vec{j}.\vec{ds}$,

magnetic flux $\phi = \int B.ds$ etc.

2. To find an angle between two vectors $\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{|A||B|}\right)$

3. If the dot product of two non zero vectors is zero, then they are perpendicular to one another.

4. Find the component of a vector along a given direction.

For instance, the component of $\vec{A}$ along $\vec{B}$ is $A\cos\theta = \frac{\vec{A}.\vec{B}}{AB}$

Cross product or Vector product This product is used when the product of two vectors is a vector, i.e.,

$\vec{A} \times \vec{B} = AB\sin\theta\ \hat{n}$ where $\hat{n}$ is a unit vector perpendicular to both A and B. Apply right-handed screw rule to find the direction of $\hat{n}$ or $\vec{A} \times \vec{B}$. Vector product is noncommutative i.e. $\vec{A} \times \vec{B} \ne \vec{B} \times \vec{A}$ (magnitude will be equal but direction will be opposite). Vector product is distributive, i.e., $\vec{A} \times \left(\vec{B} + \vec{C}\right) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

Rules: (i) $\vec{A} \times \vec{A} = 0 = \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$

(ii) $\hat{i} \times \hat{j} = \hat{k} = -\hat{j} \times \hat{i}$, $\hat{j} \times \hat{k} = \hat{i} = -\hat{k} \times \hat{j}$,

$\hat{k} \times \hat{i} = \hat{j} = -\hat{i} \times \hat{k}$

Application of Vector Product

1. Cross product is used in rotational motion or product of two vectors is a vector. For example, Torque $\vec{\tau} = \vec{r} \times \vec{F}$, Poynting vector $\vec{P} = \vec{E} \times \vec{H} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$,

Angular momentum $\vec{L} = \vec{r} \times \vec{p}$, $\vec{v} = \vec{\omega} \times \vec{r}$,

$F = q\left(\vec{v} \times \vec{B}\right)$

2. If the vector product of two nonzero vectors is zero, then they are parallel.

3. It can be used to find angle θ

$$\theta = \sin^{-1}\left[\frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}\right]$$

4. $|\vec{A} \times \vec{B}|$ represents area of a parallelogram whose sides are A and B. $\frac{1}{2}|\vec{D}_1 \times \vec{D}_2|$ represents area of a parallelogram where D_1 and D_2 are diagonals of the parallelogram.

Since $\overrightarrow{D_1} = \vec{A} + \vec{B}$ and $\overrightarrow{D_2} = \vec{A} - \vec{B}$

$\therefore$ Area of a ||gm $= \frac{1}{2}\left|\left(\vec{A} + \vec{B}\right) \times \left(\vec{A} - \vec{B}\right)\right|$

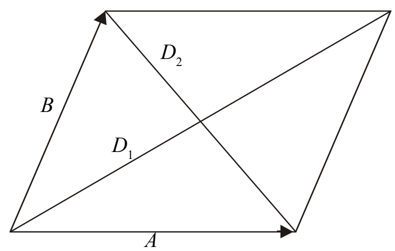


Fig. 2.8

5. $\frac{1}{2}\left|\vec{A}\times\vec{B}\right|$ represents area of a triangle when A and B are two sides of the triangle.

Relative Velocity Since absolute rest or absolute motion do not exist, therefore, every motion is a relative motion. Though, for convenience, we assume earth at rest and in common language measure the speed or velocity with respect to ground. But if two bodies A and B are moving with velocities V_A and V_B then relative velocity of A with respect B may be thought of velocity of A by bringing B to rest by applying equal and opposite velocity of B. Alternatively, Vector law may be applied from Fig. 2.9

$$\vec{V}_{AG} = \vec{V}_{AB} + \vec{V}_{BG} \quad \text{or}$$

$$\vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG} \quad \text{or}$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

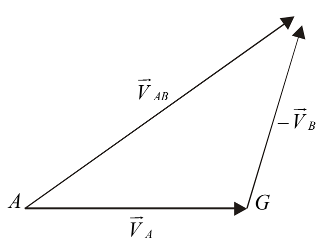


Fig. 2.9 **Relative velocity illustration**

The best way to solve the questions on relative velocity is to resolve it into x and y components, thus

$$\vec{V}_{AB} = (\vec{V}_{Ax}\hat{i} + \vec{V}_{Ay}\hat{j}) - (\vec{V}_{Bx}\hat{i} + \vec{V}_{By}\hat{j})$$

$$= \left(\vec{V}_{Ax} - \vec{V}_{Bx}\right)\hat{i} + \left(\vec{V}_{Ay} - \vec{V}_{By}\right)\hat{j}$$

Then $|V_{AB}| = \sqrt{\left(V_{Ax} - V_{Bx}\right)^2 + \left(V_{Ay} - V_{By}\right)^2}$ and

$\tan\beta = \dfrac{V_{Ay} - V_{By}}{V_{Ax} - V_{Bx}}$ with respect to x-axis or $\hat{i}$ direction

$\tan\beta' = \dfrac{V_{Ax} - V_{Bx}}{V_{Ay} - V_{By}}$ with respect to y-axis or $\hat{j}$ direction

SHORT CUTS AND POINTS TO NOTE

1. Laws of Addition are Triangle law, parallelogram law, Polygon law.

   Resultant $|R| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

   $\tan\beta = \dfrac{A\sin\theta}{B + A\cos\theta}$ (see fig 2.10 carefully).

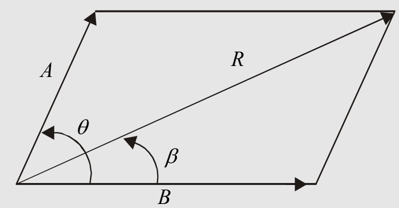


Fig. 2.10 **Parallelogram law**

2. Vector subtraction is identical to vector addition, i.e., $\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$ change θ to $180 - \theta$. in equation of resultant.

3. The vectors are represented in $\hat{i}$, $\hat{j}$ form then Resultant $\vec{R}$ is given by

$$\vec{R} = \left(A_x\hat{i} + A_y\hat{j}\right) + \left(B_x\hat{i} + B_y\hat{j}\right)$$

$$= \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

$$\left|\vec{R}\right| = \sqrt{\left(A_x + B_x\right)^2 + \left(A_y + B_y\right)^2} \quad \text{and}$$

$\tan\beta = \dfrac{A_y + B_y}{A_x + B_x}$ w.r.t x or $\hat{i}$ direction and

$\tan\beta' = \dfrac{A_x + B_x}{A_y + B_y}$ w.r.t y or $\hat{j}$ direction.

4. $R_{max} = A + B$ when $\theta = 0$, $R_{min} = A - B$ when $\theta = 180°$.

   Remember $0 \le \theta \le 180°$

5. If the system is in equilibrium, Lami's theorem may be applied. In Fig 2.11 $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$

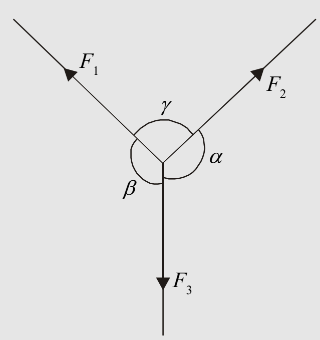


Fig. 2.11 **Lami's Theorem**

6. Equilibrium may be static or dynamic. However, in both cases $\sum \vec{F} = 0$ Linear equilibrium if $\sum \vec{F} = 0$. Rotational Equilibrium $\sum F = 0, \sum \tau = 0$ Equilibrium is stable if $\sum F = 0, \sum \tau = 0$ and PE is minimum. Unstable equilibrium means $\sum F = 0$, $\sum \tau = 0$ and PE is maximum. Neutral equilibrium means $\sum F = 0, \sum \tau = 0$ and PE is constant but not zero.

7. The best approach is to resolve the vectors into x and y components and then solve.

   Use $\vec{v}_R = \vec{v}_A + \vec{v}_B$ if resultant or net velocity is to be found.

   Use $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ if relative velocity is to be found.

8. Magnitude of a vectors $V = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

   $$|V| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

9. If two non zero vectors are perpendicular, their dot product is zero.

10. If two vectors $\vec{A}$ and $\vec{B}$ are parallel, then $\vec{A} = k\vec{B}$ where k is a positive or negative real number. Moreover $\vec{A} \times \vec{B} = 0$

11. To find $\vec{A}$ and $\vec{B}$ use determinant method i.e.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, \ \vec{B} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$\vec{A} \times \vec{B} = (a_y b_z - b_y a_z)\hat{i} - \hat{j}(a_x b_z - b_x a_z) + \hat{k}(a_x b_y - b_x a_y)$$

12. $\vec{A} .(\vec{A} \times \vec{B}) = \vec{B} .(\vec{A} \times \vec{B}) = 0$

13. $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} . \vec{C})\vec{B} - (\vec{A} . \vec{B})\vec{C}$

14. Vector division is not allowed.

15. Vector operator ∇ (nabla) is used to define

$$\nabla = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]$$

(a) ∇V represents electric field i.e.

$$E = \nabla V = \left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right]$$

(b) $\nabla . E$ will give divergence of E

(c) $\nabla \times E$ will represent curl of E

16. Unit vector of a vector is $\hat{a} = \dfrac{\vec{A}}{|\vec{A}|}$

17. To cross a river along the shortest path, the swimmer/boat shall head at an obtuse angle to the flow of river provided $V_{swimmer} > V_{river}$.

18. To cross the river in the shortest time, the swimmer/boat shall head at right angle to the flow of river provided $V_{swimmer} > V_{river}$.

19. If $V_{river} > V_{swimmer}$ them to reach opposite end in minimum time the swimmer shall swim at an obtuse angle.

CAUTION

1. Considering null vector has a specified direction.

⇒ Null vector has no specified direction.

2. Trying some other tools to prove two vectors are perpendicular.

⇒ For vectors most convenient method to prove two vectors perpendicular is $\vec{A}.\vec{B} = 0$

3. Not recognizing when relative velocity is to be found and when resultant velocity is to be determined.

⇒ If the word *appear* or *with respect to* has been used in problem, then find relative velocity.

If the word *actual* or *net* or *real* or *resultant* is used in problem, then find resultant velocity.

4. Not remembering vector laws of addition or not understanding its full meaning.

⇒ Parallelogram law or triangle law of addition leads to

$$|\vec{A} + \vec{B}| = |\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

$R_{max} = A + B$ if $\theta = 0$

$R_{min} = A - B$ if $\theta = 180°$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

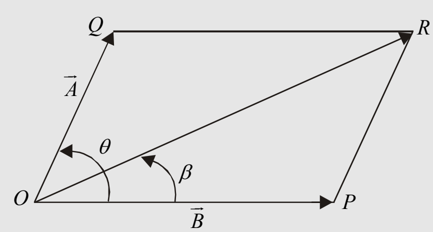


Fig. 2.12

5. Assuming $|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$ or $|\vec{A} - \vec{B}| = |\vec{A}| - |\vec{B}|$

⇒ This will be true only if vectors are like vectors/ parallel vectors. Otherwise apply triangle law.

6. Not resolving vectors (when vectors are more than two) and solving problem by conventional method.

⇒ Though problem can be solved using triangle law or parallelogram law but they make the problem unnecessarily lengthy and time consuming.

7. Not able to recognise direction in $\hat{i}$, $\hat{j}$ form.

⇒ If $\hat{j}$ is not vertical but on the earth's plane then right hand side is East (represented by $\hat{i}$); Left hand side is West (marked $-\hat{i}$); front of you is North (marked $\hat{j}$); your back represents South ($-\hat{j}$) as illustrated in Fig 2.13

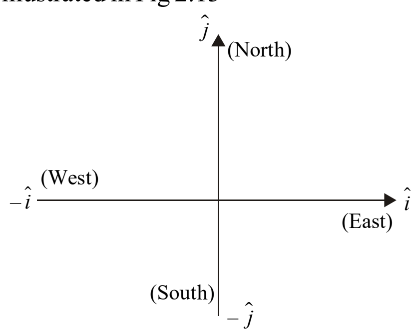


Fig. 2.13

$\hat{k}$ represents vertically up and ($-\hat{k}$) vertically down. Consider a specific problem:

$V_R = 3\hat{i} - 4\hat{j}$ lies in the 4th quadrant.

then $\tan\beta = \frac{4}{3}$ or 53° South of East.

It may also be written $\tan\beta' = \frac{3}{4}$ or 37° East of South.

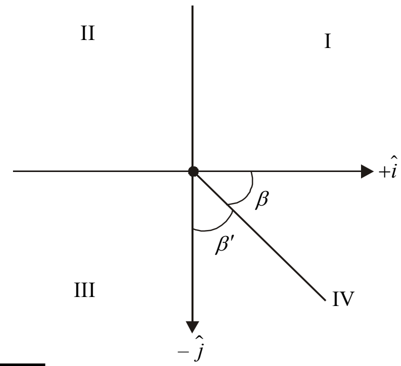


Fig. 2.14

8. Not recognizing the axes (in specific cases) about which the vectors be resolved.

⇒ If vectors are resolved about some specific axes in typical cases, problem becomes very simple. Recognition of such questions and axes is necessary.

9. Finding angle between vectors to prove vectors are parallel.

⇒ If $\vec{A} = k\vec{B}$ where k is a positive or negative number (integer or fraction) then $\vec{A}$ and $\vec{B}$ are parallel.

10. Assuming when a particle takes a u turn or 90° turn and continues to move with the same speed then ΔV (change in velocity) or average acceleration are zero.

⇒ Use vectors, i.e., write $\hat{i}$, $\hat{j}$, $\hat{k}$ etc. with initial and final velocities to find change in velocity or average acceleration.

SOLVED PROBLEMS

1. In CH_4 molecule, there are four $C - H$ bonds. If two adjacent bonds are in $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}-\hat{k}$ direction, then find the angle between these bonds.

(a) $\sin^{-1}\left(\frac{-1}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$

(c) $\sin^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{-1}{3}\right)$

Solution (d) $\cos\theta = \dfrac{(\hat{i}+\hat{j}+\hat{k}).(\hat{i}-\hat{j}-\hat{k})}{\sqrt{3}.\sqrt{3}} = \dfrac{-1}{3}$ or

$\theta = \cos^{-1}\left(\frac{-1}{3}\right)$

2. Two vectors $\vec{A}$ and $\vec{B}$ have magnitude 3 each. $\vec{A}\times\vec{B} = -5\hat{k}+2\hat{i}$. Find angle between A and B

(a) $\cos^{-1}\frac{\sqrt{29}}{9}$ (b) $\tan^{-1}\left(\frac{-5}{2}\right)$

(c) $\sin^{-1}\left(\frac{2}{5}\right)$ (d) $\sin^{-1}\left(\frac{\sqrt{29}}{9}\right)$

Solution (d) $\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|} = \dfrac{\sqrt{5^2+2^2}}{9}$

$= \dfrac{\sqrt{29}}{9}$ or $\sin^{-1}\left(\dfrac{\sqrt{29}}{9}\right)$

3. A particle moving eastwards with $5 ms^{-1}$. In 10 s the velocity changes to $5 ms^{-1}$ northwards. The average acceleration in this time is

[AIEEE 2005]

(a) $\frac{1}{\sqrt{2}}$ ms^{-2} towards North east

(b) $\frac{1}{2}$ ms^{-2} towards North

(c) $\frac{1}{\sqrt{2}}$ ms^{-2} towards North west

(d) Zero

Solution (c) $a_{av} = \frac{V_f - V_i}{t} = \frac{5\hat{j} - 5\hat{i}}{10}$ or $|a_{av}| = \frac{1}{\sqrt{2}}$ North west.

4. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$ then the value of α is

[CBSE PMT 2005]

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$

(c) 1 (d) –1

Solution (b) $\left(2\hat{i} + 3\hat{j} + 8\hat{k}\right).\left(4\hat{j} - 4\hat{i} + \alpha\hat{k}\right) = 0$ or

$$-8 + 12 + 8\alpha = 0 \quad \alpha = \frac{-1}{2}$$

5. If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $\left(\vec{B} \times \vec{A}\right).\vec{A}$ equals

[CBSE PMT 2005]

(a) $BA^2 \sin\theta$ (b) $BA^2 \cos\theta \sin\theta$

(c) $BA^2 \cos\theta$ (d) zero

Solution (d)

6. A river is flowing from W to E with a speed 5m/min. A man can swim in still waters at a velocity 10m/min. In which direction should a man swim to take the shortest path to reach the south bank?

[BHU 2005]

(a) 30° East of South (b) 60° East of North

(c) South (d) 30° West of North

Sol. (d) $v_s \sin\theta = v_{river}$ or

$$\sin\theta = \frac{1}{2} \quad \theta = 30°$$

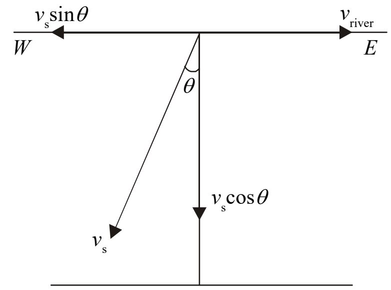


Fig. 2.15

7. Electrons in a TV tube move horizontally South to North. Vertical component of earth's magnetic field points down. The electron is deflected towards

(a) West (b) no deflection

(c) East (d) North to South

Solution (c) $F == q\left(\vec{v} \times \vec{B}\right) = -e\left(\hat{j} \times -\hat{k}\right) = \hat{i}\ e$

8. If $\vec{A}.\vec{B} = \vec{B}.\vec{C}$ then

(a) $\vec{A} = \vec{C}$ always (b) $\vec{A} \neq \vec{C}$ always

(c) $\vec{A}$ may not be equal to $\vec{C}$

(d) none of these

Solution (c)

9. $A = 3\hat{i} + 4\hat{j}$ Find a vector perpendicular to $\vec{A}$ in the plane of $\vec{A}$

(a) $\hat{k}$ (b) $-3\hat{i} - 4\hat{j}$

(c) $4\hat{j} - 3\hat{i}$ (d) $4\hat{i} - 3\hat{j}$

Solution (d), (a) is also perpendicualr to $\vec{A}$ but not in the same plane. Check using $\vec{A}.\vec{B} = 0$

10. Find a vector $\vec{x}$ which is perpendicular to both $\vec{A}$ and $\vec{B}$ but has magnitude equal to that of $\vec{B}$.

Rule: Inter change coeff. of $\hat{i}$ and $\hat{j}$ and change sign of one of the vectors.

$\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} - 2\hat{k}$

(a) $\frac{1}{\sqrt{10}}\left(\hat{i} + 10\hat{j} + 17\hat{k}\right)$ (b) $\frac{1}{\sqrt{10}}\left(\hat{i} - 10\hat{j} + 17\hat{k}\right)$

(c) $\sqrt{\frac{29}{390}}\left(\hat{i} - 10\hat{j} + 17\hat{k}\right)$ (d) $\sqrt{\frac{29}{390}}\left(\hat{i} + 10\hat{j} + 17\hat{k}\right)$

Solution (d) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix}$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{\hat{i}+10\hat{j}+17\hat{k}}{\sqrt{390}}$$

$$\vec{x} = |\vec{B}|\,\hat{n} = \frac{\sqrt{29}\left(\hat{i}+10\hat{j}+17\hat{k}\right)}{\sqrt{390}}$$

11. Rain is falling vertically with $3ms^{-1}$ and a man is moving due North with $4ms^{-1}$. In which direction he should hold the umbrella to protect himself from rains?

(a) 37° North of vertical (b) 37° South of vertical

(c) 53° North of vertical (d) 53° South of vertical

Solution (c) $V_{rm} = V_r - V_m = -3\hat{k} - 4\hat{j}$;

$$\tan\beta = \frac{4}{3} \Rightarrow \beta = 53° \text{ North of vertical}$$

12. A man is moving on his bike with 54 kmh^{-1}. He takes a u-turn in 10 *s* and continues to move with the same velocity. Find average acceleration during this time.

(a) 3.0 ms^{-2} (b) 1.5 ms^{-2}

(c) 0 (d) –1.5 ms^{-2}

Solution (a) $a_{av} = \dfrac{-15\hat{i} - \left(15\hat{i}\right)}{10} = -3\hat{i} = 3ms^{-2}$

13. A man starts from *O* moves 500 m turns by 60° and moves 500 m again turns by 60° and moves 500 m and so on. Find the displacement after (i) 5th turn, (ii) 3rd turn

(a) 500 m, 1000 m (b) 500 m, 500$\sqrt{3}$ m

(c) 1000 m, 500$\sqrt{3}$ m (d) none of these

Solution (a) A regular hexagon will be formed if we continue.

After 5th turn displacement = *OE* = 500m

After 3rd turn displacement = *OC* = 1000m (*OC* is diameter of the circle circumscribing regular hexagon).

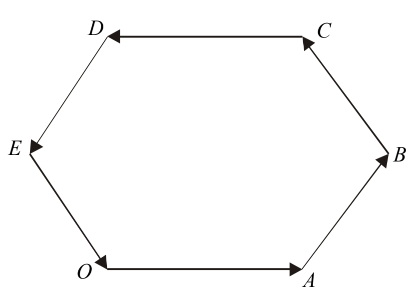


Fig. 2.16

14. If $\vec{B} = \lambda\,\vec{A}$ then $\dfrac{\vec{B}}{\vec{A}}$ = ...

(a) λ (b) $\dfrac{1}{\lambda}$

(c) $\dfrac{\lambda}{2}$ (d) Inderminate

Solution (d) $\because$ vector division is not allowed.

15. The acceleration of a particle as seen from two frames S_1 and S_2 has equal magnitude 5 ms^{-2} .

(a) The frames must be at rest with respect to each other.

(b) The frames may be moving with respect to each other but neither should be accelerated with respect to the other.

(c) The acceleration of frame S_2 with respect to S_1 be 0 or 10 ms^{-2}.

(d) The acceleration of S_2 with respect to S_1 lies between 0 and 10 ms^{-2}.

Solution (d) use Parallelogram law.

16. A man running on a horizontal road at 8 ms^{-1} finds rain falling vertically. If he increases his speed to 12 ms^{-1}, he finds that drops make 30° angle with the vertical. Find velocity of rain with respect to the road.

(a) $4\sqrt{7}$ ms^{-1} (b) $8\sqrt{2}$ ms^{-1}

(c) $7\sqrt{3}$ ms^{-1} (d) $8ms^{-1}$

Solution (a) $V_{rm} = (V_{rx} - V_m)\,\hat{i} + V_{ry}\,\hat{j}$

case (i) $\tan 90 = \dfrac{V_{ry}}{V_{rx} - V_m} = \dfrac{V_{ry}}{V_{rx} - 8}$ or $V_{rx} = 8ms^{-1}$

case (ii) $\tan 30 = \dfrac{V_{rx} - V_m}{V_{ry}} = \dfrac{8-12}{V_{ry}}$ or $V_{ry} = -4\sqrt{3}\ ms^{-1}$

$$V_r = 8\,\hat{i} - 4\sqrt{3}\,\hat{j} = 4\sqrt{2^2+3} = 4\sqrt{7}\ ms^{-1}$$

$$\tan\theta = \frac{V_{ry}}{V_{rx}} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2} \quad \text{or}$$

$$\theta = \tan^{-1}\frac{\sqrt{3}}{2} \text{ with respect to road (horizontally).}$$

17. In Fig. 2.17 (a) shown find the velocity of block *m* if both the rope ends are pulled with *a* velocity *v*.

(a) $2v\cos\theta$ (b) $\dfrac{v}{\cos\theta}$

(c) $\dfrac{v}{2\cos\theta}$ (d) $\dfrac{2v}{\cos\theta}$

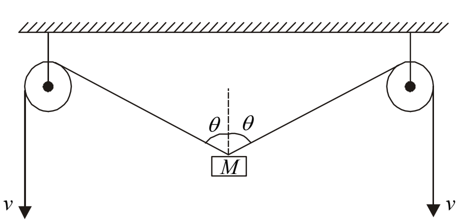


Fig. 2.17 (a)

Solution (b) $l^2 = x^2 + y^2$ or $2l\frac{dl}{dt} = 0 + 2y\frac{dy}{dt}$ ($\because x$ is constant its derivative is zero)

$$\frac{dy}{dt} = \frac{dl/dt}{y/l} = \frac{v}{\cos\theta}$$

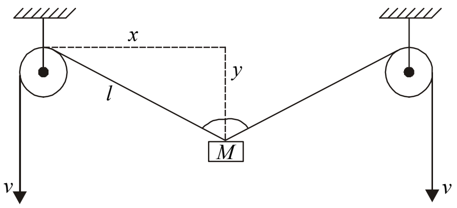


Fig. 2.17 (b)

18. Which of the following cannot be in equilibrium?

(a) 10N, 10N, 5N (b) 5N, 7N, 9N

(c) 8N, 4N, 13N (d) 9N, 6N, 5N

Solution (c) as 13N > 8 + 4 [$\because R_{\max} = A + B$]

19. $\vec{A} = 3\hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{B} = 6\hat{i} - \hat{j} + 3\hat{k}$. Find a vector parallel to $\vec{A}$ whose magnitude is equal to that of $\vec{B}$.

(a) $\sqrt{\frac{46}{29}}\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ (b) $\sqrt{\frac{46}{29}}\left(6\hat{i} - \hat{j} + 3\hat{k}\right)$

(c) $\sqrt{\frac{29}{46}}\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ (d) none

Solution (a)

$$\vec{x} = \hat{A}\,|B| = \frac{\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)\sqrt{36+1+9}}{\sqrt{9+16+4}}$$

$$= \sqrt{\frac{46}{29}}\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

20. $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors. Find the vector sum.
$\vec{a} = 4\hat{i} - \hat{j}$, $\vec{b} = -3\hat{i} + 2\hat{j}$, $\vec{c} = -3\hat{j}$

(a) $\sqrt{5}$, 297° (b) $\sqrt{5}$, 63°

(c) $\sqrt{3}$, 297° (d) $\sqrt{3}$, 63°

Solution (a) $\vec{R} = \vec{a} + \vec{b} + \vec{c} = \hat{i} - 2\hat{j}$

$$|R| = \sqrt{5} \text{ and } \tan\theta = -2 \text{ or } \theta = 297°$$

21. A block of mass m is connected to three springs, each of spring constant k as shown in Fig. 2.18. The block is pulled by x in the direction of C. Find resultant spring constant.

(a) k (b) $2k$

(c) $3k$ (d) $\frac{3k}{2}$

Solution (c) $F_{\text{net}} = -\left(kx + kx\cos 60 + kx\cos 60\right)$

$$= -2kx \therefore k_{eq} = 2k$$

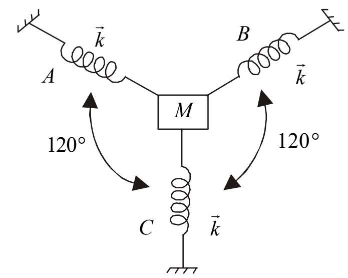


Fig. 2.18

22. A particle moves in the $x - y$ plane under the action of a force $\vec{F}$ such that the value of its linear momentum p at any instant is $p = 2(\cos t\,\hat{i} + \sin t\,\hat{j})$. The angle θ between F and p is

(a) 60° (b) 45°

(c) 30° (d) 90°

Solution (d) $F = \frac{dp}{dt} = 2\left(-\sin t\hat{i} + \cos t\hat{j}\right)$

$$\vec{F}.\vec{p} = 4\left(-\sin t\hat{i} + \cos t\hat{j}\right).\left(\cos t\hat{i} + \sin t\hat{j}\right) = 0 \therefore$$

$$\theta = 90°$$

23. Consider a collection of large number of particles, each moving with a speed v. The direction of velocity is randomly distributed in the collection. The magnitude of the relative velocity between a pair of particles averaged over all the pairs in the collection

(a) v (b) $\frac{2v}{\pi}$

(c) $\frac{\pi v}{4}$ (d) $\frac{4v}{\pi}$

Solution (d) $\vec{V}_{rel} = \vec{V}_A - \vec{V}_B$ or

$$|V_{AB}| = \sqrt{v^2 + v^2 - 2v^2\cos\theta}$$

$$= 2v\sin\frac{\theta}{2}$$

$$(v_{rel})\text{average} = \frac{\int_0^{2\pi} v_{rel}\,d\theta}{\int_0^{2\pi} d\theta} = \frac{1}{2\pi}\int_0^{2\pi} 2v\sin\frac{\theta}{2}\,d\theta$$

$$= \frac{2v}{\pi}\left|-\cos\left(\frac{\theta}{2}\right)\right|_0^{2\pi} = \frac{4v}{\pi}$$

24. A steamer is moving due east with 36 km/h. To a man in the steamer the wind appears to blow at 18 km/h due north. Find the velocity of the wind.

(a) $5\sqrt{5}$ ms^{-1} 30° North of east

(b) 5ms^{-1} 60° North of east

(c) $5\sqrt{5}$ ms^{-1} 60° North of east

(d) 5ms^{-1} 30° North of east

Solution (a) $V_{ws} = V_w - V_s \Rightarrow \left(V_{wx}\hat{i} + V_{wy}\hat{j}\right) - 10\,\hat{i} = 5\,\hat{j}$

or $\left(V_{wx}\hat{i} + V_{wy}\hat{j}\right) = 5\,\hat{j} + 10\,\hat{i}$ or

$|V_w| = 5\sqrt{5}$ $\tan\theta = \dfrac{1}{2}$ or

θ = 30° i.e. wind is blowing at $5\sqrt{5}$ ms^{-1} 30° North of east.

25. The position vector of a particle is $\vec{r} = a[\cos\omega t\,\hat{i} + \sin\omega t\;\hat{j}\,]$. The velocity of the particle is

(a) parallel to position vector

(b) directed towards origin

(c) directed away from origin

(d) perpendicular to position vector.

Solution (d) $\vec{v} = \dfrac{d\vec{r}}{dt} = a\omega[-\sin\omega t\,\hat{i} + \cos\omega t\;\hat{j}\,]$ and

$\vec{v}\,.\,\vec{r} = 0$

26. A force $6\hat{i} + 3\hat{j} + \hat{k}$ Newton displaces a particle from A(0,3,2) to B(5,1,6). Find the work done.

(a) 10 J (b) 22 J

(c) 32 J (d) 41J

Solution (c) $\vec{d} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

$$W = \vec{F}.\vec{d} = \left(6\hat{i} + 3\hat{j} + \hat{k}\right).\left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) = 32\text{ J}.$$

27. Wind is blowing NE with 18 $\sqrt{2}$ km h^{-1} and steamer is heading due west with 18 km h^{-1}. In which direction is the flag on the mast fluttering?

(a) North West (b) North

(c) South West (d) South.

Solution (d) $\vec{V}_{Res} = \vec{V}_{steamer} + \vec{V}_{wind} = -5\,\hat{i} + 5\,\hat{i} + 5\,\hat{j}$

$= 5\,\hat{j}$. The flag will flutter in a direction opposite to the direction of motion.

28. The resultant of two forces equal in magnitude is equal to either of two vectors in magnitude. Find the angle between the forces.

(a) 60° (b) 45°

(c) 90° (d) 120°

Solution (d) $F = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$ or

$\cos\theta = \dfrac{-1}{2}$; $\theta = 120°$

29. A man goes 100 m North then 100 m East and then 20 m North and then 100 $\sqrt{2}$ m South West. Find the displacement.

(a) 20 m West (b) 20 m East

(c) 20 m North (d) 20 m South

Solution (c) $d = 100\,\hat{j} + 100\,\hat{i} + 20\,\hat{j} + (-100\,\hat{i} - 100\,\hat{j}\,)$

$= 20\,\hat{j}$

TYPICAL PROBLEMS

30. A river flows 3 km h^{-1} and a man is capable of swimming 2 km h^{-1}. He wishes to cross it in minimum time. In which direction will he swim?

(a) $\sin^{-1}\left(\dfrac{2}{3}\right)$ (b) $\cos^{-1}\left(\dfrac{2}{3}\right)$

(c) $\tan^{-1}\left(\dfrac{2}{3}\right)$ (d) $\cot^{-1}\left(\dfrac{2}{3}\right)$

Solution (a) Let us assume he swims at an angle θ with the perpendicular as shown. If river is lm wide time taken to cross it

$t = \dfrac{l}{2\cos\theta}$, $v_x = 3 - 2\sin\theta$ horizontal distance covered along x direction during this period $x = v_x$.

$$t = (3 - 2\sin\theta)\frac{l}{2\cos\theta}$$

for t to be min $\dfrac{dx}{d\theta} = 0$, or

$$l\left[\frac{3}{2}\sec\theta\tan\theta - \sec^2\theta\right] = 0 \quad \text{or}$$

$$\sin\theta = \frac{2}{3}$$

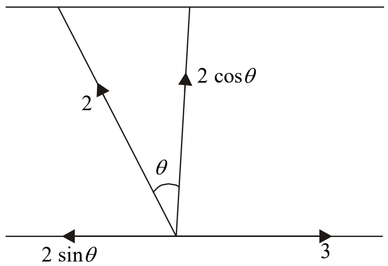


Fig. 2.19

31. A pilot is to flag an aircraft with velocity v due east. Wind is blowing due south with a velocity u. Find the time for a round journey A to B and back (A and B are l distance away).

(a) $\dfrac{l}{\sqrt{v^2-u^2}}$ (b) $\dfrac{2l}{\sqrt{v^2-u^2}}$

(c) $\dfrac{2l}{v}$ (d) $\dfrac{2l}{\sqrt{v^2+u^2}}$

Solution (b) See Fig. 2.20 the velocity in the direction A to B will be $\sqrt{v^2-u^2}$ $\therefore\ t=\dfrac{l}{\sqrt{v^2-u^2}}$ same time is needed to come back from B to A $\therefore\ t_{net}=\dfrac{2l}{\sqrt{v^2-u^2}}$

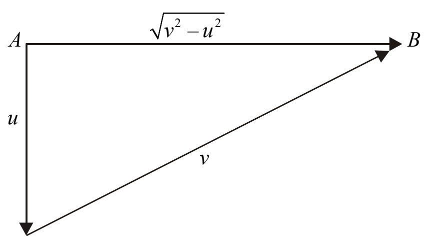


Fig. 2.20

32. When a mass m is rotated in a plane about a fixed point, its angular momentum is directed along

(a) the radius (b) tangent to the orbit

(c) the axis of rotation

(d) 45° to the axis of rotation

Solution (c) because angular momentum is an axial vector.

33. A pendulum hangs from the ceiling of a jeep moving with a speed v along a circle of radius R. Find the angle with the vertical made by the pendulum.

(a) 0 (b) $\tan^{-1}\left(\dfrac{v^2}{Rg}\right)$

(c) $\tan^{-1}\left(\dfrac{Rg}{v^2}\right)$ (d) none of these

Solution (b) $a_r=\dfrac{v^2}{R}$ $\tan\theta=\dfrac{\frac{v^2}{R}}{g}=\dfrac{v^2}{Rg}$

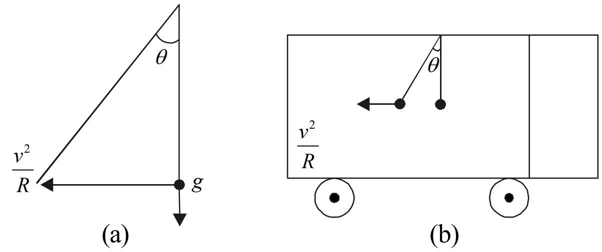


Fig. 2.21

34. If $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$ then angle between the vectors A and B is

(a) 0 (b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{4}$

Solution (c) See Fig. 2.22

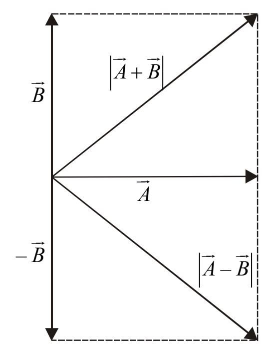


Fig. 2.22

35. Two identical pendulums are tied from the same rigid support. One is tied horizontally. The other is released when they are making the same angle θ with the vertical.

(a) $\tan^2\theta$ (b) $\cot^2\theta$

(c) 1 (d) $\sec^2\theta$

Solution (d) $T_1\cos\theta = mg$ or

$$T_1=\frac{mg}{\cos\theta}$$

$$T_2=mg\cos\theta$$

$$\therefore\ \frac{T_1}{T_2}=\frac{1}{\cos^2\theta}$$

$$=\sec^2\theta$$

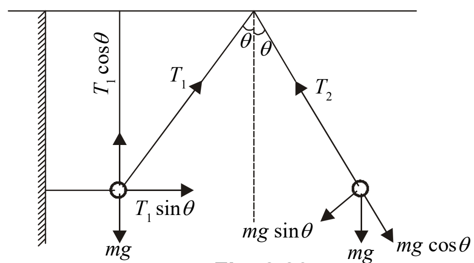


Fig. 2.23

36. Sixteen beads in a string are placed on a smooth incline as shown in equilibrium. The number of beads lying along the incline are

(a) 4 (b) 8

(c) 12 (d) none of these

Solution (c) Let n beads be hanging vertically. Then $(16-n)\,mg\sin\theta = n\,mg$. For $\sin\theta = \frac{1}{3}$, $n=4$ $\therefore$ $16-4=12$ beads lie along the plane.

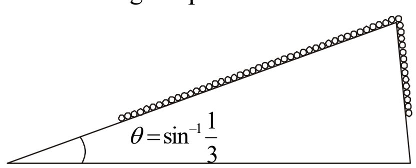


Fig. 2.24

37. Three particles A, B and C are situated at the verticles of an equilateral triangle of side l. Each of the particle starts moving with a constant velocity v such that A is always directed towards B, B towards C and C towards A. Find the time when they meet.

(a) $\frac{l}{\sqrt{3}v}$ (b) $\frac{2l}{v}$

(c) $\frac{2l}{3v}$ (d) none

Solution (c) We look into it as a problem of relative velocity and find v_{BA} in the direction of B.

$$t = \frac{l}{v_A - v_B\cos 120} = \frac{l}{v + v/2} = \frac{2l}{3v}$$

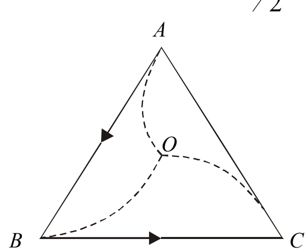


Fig. 2.25

38. Two particals are thrown horizontally in opposite directions from the same point from a height h with velocities 4 ms^{-1} and 3 ms^{-1}. Find the separation between them when their velocities are perpendicular.

(a) 0.15 s (b) 0.25 s

(c) 0.35 s (d) none of these

Solution (c) $\vec{V}_1 = -4\hat{i} - gt\hat{j}$ and $\vec{V}_2 = 3\hat{i} - gt\hat{j}$ are the velocities at any instant. For velocities to be perpendicular $\vec{V}_1 \cdot \vec{V}_2 = 0$

that is, $\vec{V}_1 \cdot \vec{V}_2 = 0$ or $t = \sqrt{\frac{12}{g^2}} = 0.35$ s

separation $= (u_1 + u_2)t = 7(0.35) = 2.45$ m.

39. A ball is thrown with a velocity $6\hat{j}$ with an acceleration $6\hat{i} + 2\hat{j}$. The velocity of the ball after 5 seconds is

(a) $30\hat{i} + 10\hat{j}$ (b) $30\hat{i} + 16\hat{j}$

(c) $10\hat{i} + 24\hat{j}$ (d) none of these

Solution (b) using $v = u + at$ $\quad v_x = 6(5) = 30$ ms^{-1}

$v_y = 6 + 2(5) = 16$ ms^{-1} $\quad v = 30\hat{i} + 16\hat{j}$

40. Ray AO in medium I emerges as OB in medium II then refractive index of medium II with respect to medium is

(a) $\frac{\overrightarrow{OA}.\overrightarrow{N_1O}}{\overrightarrow{OB}.\overrightarrow{N_2O}}$ (b) $\frac{|\overrightarrow{OA}\times\overrightarrow{N_1O}|}{|\overrightarrow{OB}\times\overrightarrow{N_2O}|}$

(c) $\frac{\overrightarrow{AO}.\overrightarrow{OP}}{\overrightarrow{OB}.\overrightarrow{OP}}$ (d) $\frac{\overrightarrow{OA}\times\overrightarrow{OP}}{\overrightarrow{OB}\times\overrightarrow{OP}}$

Assume OA, OB, N_1,O, N_2O and OP radius of a circle.

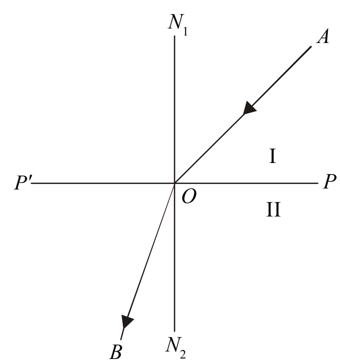


Fig. 2.26

Solution (b)

41. Vector Laws

(a) vary if scale is changed

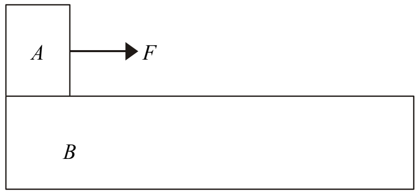
(b) vary if rotation of axes is performed

(c) vary if translation of coordinates is performed

(d) are invariant under translation and rotation of the coordinates.

Solution (d)

42. Block A is placed on B, whose mass is greater than that of A. Friction is present between the blocks while surface below B is smooth. Force F as shown increasing linearly with time, is applied at $t = 0$. The acceleration a_A and a_B of A and B, respectively, are plotted against time t. Choose the correct representation.



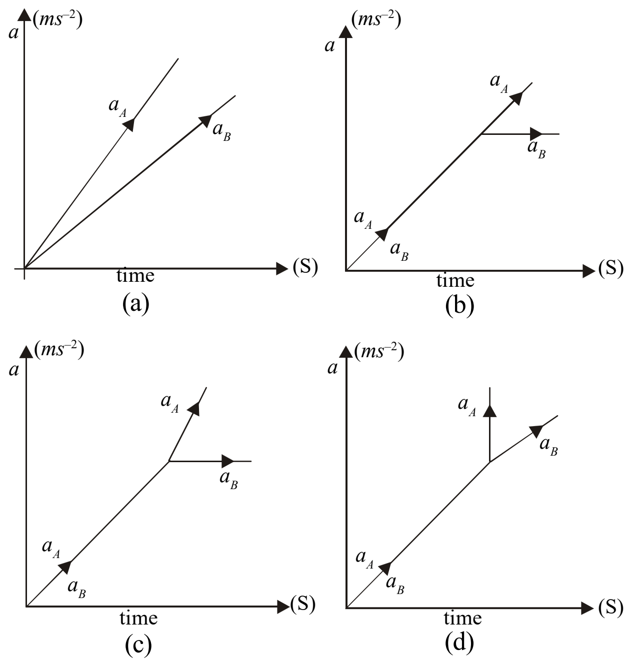


Fig. 2.27

Solution (c)

43. In Fig. 2.28 (a) mass of both the blocks are equal. v_A and v_B are instantaneous speed of A and B. Then

(a) B will never loose contact with the ground

(b) $|a_A| = |a_B|$

(c) $v_B = v_A\cos\theta$ (d) $v_B = \dfrac{v}{\cos\theta}$

Solution (a), (d) $BP + PA = B'P + PA'$ or

$l_1 + l_1 = (l_1 - x\cos\theta) + y$ .... (i)

differentiating (i) $0 = \dfrac{-dx}{dt}\cos\theta + \dfrac{dy}{dt} = 0$ or

$v_A = v_B\cos\theta$

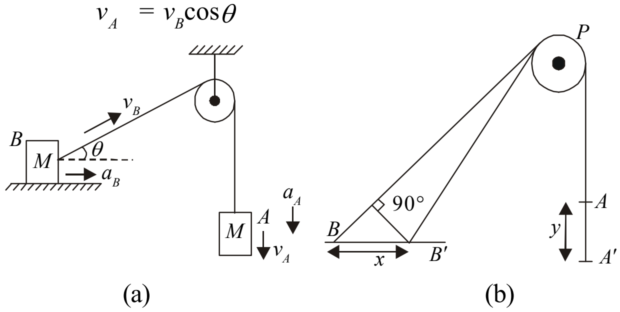


Fig. 2.28

44. The product of two vectors $\vec{A}$ and $\vec{B}$ may be

(a) $\geq AB$ (b) $\leq AB$

(c) $< AB$ (d) zero

Solution (b), (c), (d) $\because \vec{A}\,.\,\vec{B} = AB\cos\theta$ and $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

45. $\vec{X} = \vec{A}\,.\,\vec{B}$ and $\vec{X} = \vec{C}\,.\,\vec{B}$ then

(a) $\vec{A} = \vec{C}$ always

(b) $\vec{A}$ may not be equal to $\vec{C}$

(c) $\vec{A}$ and $\vec{C}$ are parallel

(d) $\vec{A}$ and $\vec{C}$ are antiparallel.

Solution (b)

46. $\vec{A} + \vec{B} = \vec{C}$ Vectors A and B if rotated by θ in the same sense to form $\vec{A}'$ and $\vec{B}'$ then

(a) $\vec{A}' + \vec{B}' = \vec{C}$ (b) $\vec{A}' + \vec{B}' \neq \vec{C}$

(c) $\vec{A}'\,.\,\vec{B}' = \vec{A}\,.\,\vec{B}$ (d) $\left|\vec{A}' + \vec{B}'\right| = \left|\vec{C}\right|$

Solution (b), (c), (d)

47. $\vec{A}$ and $\vec{B}$ are two vectors such that $\vec{A} + \vec{B} = \vec{C}$ in a given coordinate sustem. The axes are rotated by θ. Then in new coordinate system

(a) $\vec{A} + \vec{B} = \vec{C}$ (b) $\vec{A} + \vec{B} \neq \vec{C}$

(c) $\vec{A}\times\vec{B}$ (old system) = $\vec{A}\times\vec{B}$ (new system)

(d) $\vec{A}$ and $\vec{B}$ (interchange in new system)

Solution (b)

48. A point moves according to the law $x = at$, $y = at\,(1 - \alpha\, t)$ where a and α are positive constants and t is time. Find the moment at which angle between velocity vector and acceleration vector is $\dfrac{\pi}{4}$.

Solution $\vec{v} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} = a\,\hat{i} + (a - 2a\,\alpha\, t)\,\hat{j}$

$f = \dfrac{dv}{dt} = -2a\,\alpha\,\hat{j}$ or

$$\cos\frac{\pi}{4} = \frac{-2a\alpha\hat{j}.\left[a\hat{i} + \left(a - 2a\alpha t\right)\hat{j}\right]}{2a\alpha\sqrt{a^2 + \left(a - 2a\alpha t\right)^2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{-[a-2a\alpha t]}{\sqrt{a^2+(a-2a\alpha t)^2}} \text{ or}$$

$2(1-2\alpha t)^2 = 1 + (1-2\alpha t)^2$

or $(1-2\alpha t)^2 = 1$ or $t = \dfrac{1}{\alpha}$

PASSAGE 1

Read the following passage and answer the questions given at the end.

The advantage of the method of breaking up vectors into its components rather than adding directly with the use of suitable trigonometric relations, is that we always deal with right-angled triangles and simplify the calculations.

In adding vectors by the analytical method, the choice of coordinate axes determines how simple the process will be. Sometimes the components of the vectors with respect to a particular set of axes are known to begin with, so that choice of axes is obvious. At other times the choice of axes can greatly simplify the job of resolution of the vectors into components. For example, the axes can be oriented so that at least one of the vectors lies parallel to an axis.

1. Using the approach mentioned in the lines 'The advantage .... simplify the calculations.' Solve the problem shown in Fig. 2.29

Solution $\vec{R} = \overrightarrow{OA}\cos45\,\hat{i} + \overrightarrow{OA}\sin45\,\hat{j} + \text{AB}\,\hat{i} - \text{BC}\cos60\,\hat{i} + \text{BC}\sin60\,\hat{j}$

$$\vec{R} = 6\hat{i} + 6\hat{j} + 4\hat{i} - 2\hat{i} + 2\sqrt{3}\,\hat{j}$$

$$|\vec{R}| = 8\hat{i} + (6+2\sqrt{3})\hat{j}\ ;$$

$$|\vec{R}| = \sqrt{8^2+(6+2\sqrt{3})^2}$$

$$= 12.38\ N$$

$$\tan\beta = \frac{9.464}{8} \text{ with respect to } x\text{-axis.}$$

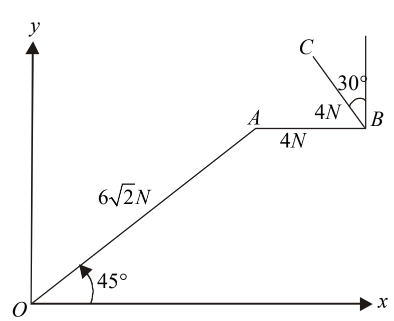


Fig. 2.29

2. Using the procedure listed in the paragraph, find the resultant of vectors $\vec{A}$ and $\vec{B}$ in Fig. 2.30 (a)

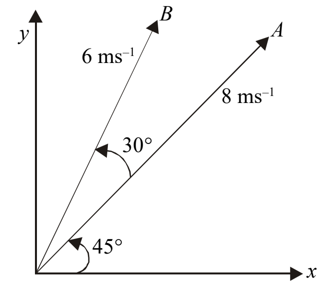


Fig. 2.30 (a)

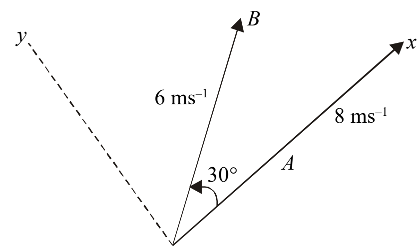


Fig. 2.30 (b)

Solution See Fig 2.30 (b) $\vec{R} = 8\hat{i} + 6\cos30\,\hat{i} + 6\sin30\,\hat{j}$

$$\vec{R} = (8+3\sqrt{3})\hat{i} + 3\hat{j}\ ;\ |\vec{R}| = 13.54\text{ms}^{-1}$$

$$\tan\beta = \frac{3}{8+3\sqrt{3}} = \frac{3}{13.196} \text{ with respect to } 8\text{ms}^{-1}$$

vector.

3. How $\vec{A} - \vec{B}$ can be accomplished?

Solution Apply $\vec{A} + (-\vec{B})$ i.e. Adding $-\vec{B}$ to $\vec{A}$.

$$|R| = \sqrt{A^2+B^2+2AB\cos(180-\theta)}$$

PASSAGE 2

Read the following passage and answer the questions given at the end.

Three kinds of multiplication operation for vectors can be defined (i) multiplication of a vector by a scalar, (ii) multiplication of two vectors in such a way as to yield a scalar, and (iii) multiplication of two vectors in such a way as to yield another vector.

The multiplication of a vector by a scalar simply means $\vec{X} = k\vec{A}$ where k is a scalar. Such a product only changes the magnitude of the vector. The directions will be the same if k is positive and direction will be opposite if k is negative.

Dividing the vector by a scalar would only mean $\vec{X} = \dfrac{1}{k}\vec{A}$ i.e. multiplication by reciprocal of k.

When we multiply a vector quantity by another vector quantity, we must distinguish between the scalar (or dot) product and the vector (or cross) product. The scalar product of two vectors $\vec{A}$ and $\vec{B}$ written as $\vec{A}\cdot\vec{B} = AB\cos\theta$ where θ is angle between the vectors $\vec{A}$ and $\vec{B}$.

The vector product of two vector implies $\vec{X} = \vec{A} \times \vec{B} = AB\sin\theta\, \hat{n}$ where $\hat{n}$ is a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$. The direction of $\hat{n}$ or $\vec{X}$ is given by the direction of right-handed screw or right hand thumb rule.

1. The flow of electromagnetic energy can be accomplished by
   (a) dot product (b) cross product
   (c) product of vector with a scalar
   (d) by division of one vector with another.

Solution (b) $\vec{P} = \frac{1}{\mu_0}\left[\vec{E} \times \vec{B}\right]$

2. Choose correct statement on the basis of the paragraph.
   (a) Dot product means multiplying the magnitude of vector A by the projection of B on A.
   (b) Dot product means multiplying the projection of two vectors along x-axis.
   (c) Dot product means multiplying a vector by the perpendicular projection of the other vector on the first vector.
   (d) Dot product makes the result a scalar.

Solution (a) and (d)

3. Choose the correct statement on the basis of the paragraph.
   (a) Cross product is product of projection of one vector on the other and the other vector.
   (b) Cross product is the product of one vector with perpendicular projection of other on the first vector.
   (c) $\vec{X} = \vec{A} \times \vec{B}$ means $\vec{X}$, $\vec{A}$ and $\vec{B}$ are mutually perpendicular.
   (d) $\vec{X} = \vec{A} \times \vec{B}$ means $\vec{X} \perp \vec{A}$ and $\vec{X} \perp \vec{B}$ only.

Solution (b) and (d)

PASSAGE 3

Read the following passage and answer the questions given at the end.

It was thought until about 1956 that all laws of physics were invariant under another kind of transformation of coordinates, the substitution of right-handed coordinates system with a left handed system. In that year, however, some experiments involving the decay of certain elementary particles were studied in which the result of the experiment did turn out to depend on the handedness of coordinates system used to express the results. C. N. Yang and T. D. Lee were awarded Nobel prize in 1957 for their theoretical prediction that it would be the case. In other words, the experiment and its image in a mirror would yield different results. This surprising result led to re-examination of the whole question of the symmetry of physical laws.

1. Are the laws of physics invariant under another kind of transformation of coordinates?

Solution Experimentally, decay of elementary particles proved dependence on the coordinates system. Theoretically Yang and Lee predicted the results should depend upon the coordinate system.

2. Do the vector laws vary in mirror image coordinate system?

Solution This is a matter of debate yet. It has not been confirmed as such.

QUESTION FOR PRACTICE

1. A large number of particles are moving with same magnitude of velocity v but having random directions. The average relative velocity between any two particles averaged over all the pairs is
   (a) $\frac{\pi}{4}v$ (b) $\frac{\pi}{2}v$
   (c) $\frac{3}{\pi}v$ (d) $\frac{4}{\pi}v$

2. A boy swims in a straight line to reach the other side of a river. His velocity is 5 ms^{-1} and the angle of swim with shore is 30°. Flow of river opposes his movement at 2 ms^{-1}. If width of river is 200 m, where does he reach the other bank?

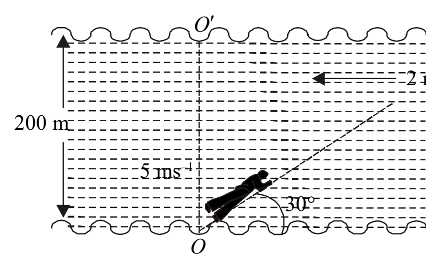


Fig. 2.31

   (a) 106 m from O' downstream
   (b) 186 m from O' downstream
   (c) 186 m from O' upstream
   (d) 106 m from O' upstream.

3. Two cars are moving with same velocity of 30 kmh^{-1} maintaining a distance of 5 km between them. Speed

of third car moving in the opposite direction and meeting the two cars at an interval of 240 s is

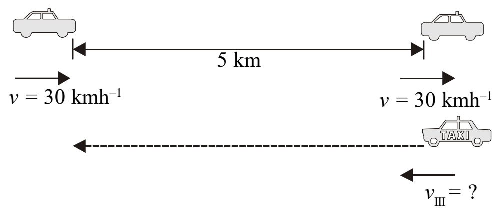


Fig. 2.32

(a) 45 kmh^{-1} (b) 30 kmh^{-1}

(c) 55 kmh^{-1} (d) 35 kmh^{-1}

4. Given $\vec{F} = \left(4\hat{i} - 10\hat{j}\right)$ and $\vec{r} = \left(5\hat{i} - 3\hat{j}\right)$, compute torque.

(a) $-62\,\hat{j}$ unit (b) $62\,\hat{k}$ unit

(c) $48\,\hat{i}$ unit (d) $-48\,\hat{k}$ unit.

5. Two swimmers start from point P on one bank of the river to reach Q on the opposite bank. Velocity of each swimmer in still waters is 2.5 kmh^{-1}. One of the swimmers crosses the river along the straight route PQ and the other swims right angles to the stream and then walks the distance which he has been carried away by the river to get to point Q. Stream velocity is 2 kmh^{-1}. If both the swimmers reach point Q simultaneously, the velocity of walking of second swimmer is

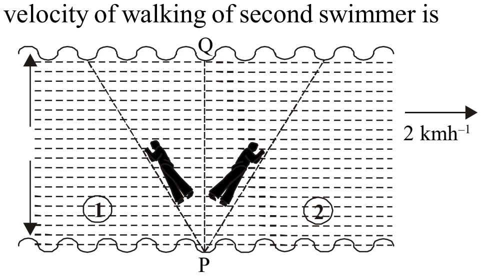


Fig. 2.33

(a) 3 kmh^{-1} (b) 4 kmh^{-1}

(c) 2 kmh^{-1} (d) 3.5 kmh^{-1}.

6. A boat sails 2 km east, then 4 km northeast and then in an unknown direction. Final position of the boat is 5 km east from starting point. Unknown displacement is

(a) 2.8 km, 3°26' with north towards east

(b) 3 km, 2°26' with east

(c) 3.5 km, 2°30' with south towards west

(d) 1.8 km, 2°36' with north towards east.

7. A parallelogram has diagonals expressed as $\vec{A} = 5\hat{i} - 4\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} + 2\hat{j} - \hat{k}$. Area of parallelogram is

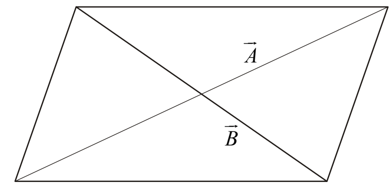


Fig. 2.34

(a) $\sqrt{117}$ units (b) $\sqrt{171}$ units

(c) $\sqrt{711}$ units (d) $\sqrt{107}$ units.

8. Two particles are projected simultaneously in a vertical plane from the same point. These particles have different velocities at different angles with the horizontal. The path seen by each other is

(a) parabola (b) hyperbola

(c) elliptical (d) straight line.

9. Rain appears to fall vertically or a man walking at 3 kmh^{-1} but if he doubles his speed to the rain appears to fall at 45°. The real velocity of rain is

(a) $3\sqrt{2}$ kmh^{-1}, 45° (b) $2\sqrt{3}$ kmh^{-1}, 45°

(c) $3\sqrt{2}$ kmh^{-1}, 30° (d) $2\sqrt{3}$ kmh^{-1}, 60°.

10. A helicopter is to reach a point 200,000 m east of his existing place. Its velocity relative to wind blowing at 30 kmh^{-1} from northwest taking scheduled arrival time duration as 40 minute is

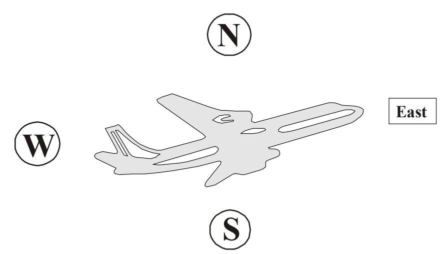


Fig. 2.35

(a) $21\,\hat{i} + 279\,\hat{j}$ (b) $279\,\hat{i} + 21\,\hat{j}$

(c) $729\,\hat{i} + 12\,\hat{j}$ (d) $12\,\hat{i} + 729\,\hat{j}$

11. Two bodies move uniformly towards each other. They become 4 m nearer in every 1 second, and get 4 m closer every 10 second. If they move in the same direction with their previous speeds, the speeds of the bodies are

(a) 1.8 ms^{-1}, 1.8 ms^{-1} (b) 2.2 ms^{-1}, 2.0 ms^{-1}

(c) 2.2 ms^{-1}, 1.8 ms^{-1} (d) 1.5 ms^{-1}, 2.5 ms^{-1}

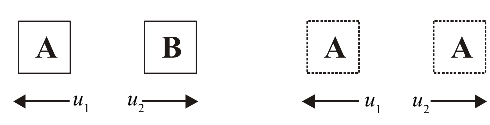


Fig. 2.36

12. A man holds an umbrella at 30° vertically to keep himself dry. He, then, runs at a speed of 10 ms^{-1} and finds the raindrops to be hitting vertically. Speed of the raindrops w.r.t. earth and w.r.t. man are

Fig. 2.37

(a) 20 ms^{-1}, 10 ms^{-1} (b) 10 ms^{-1}, $20\sqrt{3}$ ms^{-1}

(c) $10\sqrt{3}$ ms^{-1}, 20 ms^{-1} (d) 20 ms^{-1}, $10\sqrt{3}$ ms^{-1}.

13. A nut is screwed onto a bolt with 12 turns per cm and diameter 1.18 cm. The bolt is lying in horizontal direction. The nut spins at 216 r.p.m. Time taken by the nut to cover 1.5 cm along the bolt is

(a) 2 s (b) 3 s

(c) 4 s (d) 5 s.

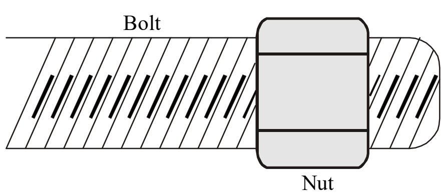


Fig. 2.38

14. A particle moves from one end of a strip to the other end in time t with uniform velocity. The particle then flies off vertically making the strip move on the smooth horizontal table. Further distance travelled by the strip in time T is equal to the length of the strip L. Then

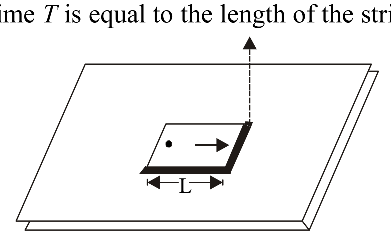


Fig. 2.39

(a) T > t (b) T = t

(c) T < t (d) data is insufficient.

15. An open umbrella is held upright and rotated about the handle at a uniform rate of 21 revolution in 44 s. If the rim of the umbrella is a circle of 100 cm diameter and the height of the rim above ground is 150 cm, then the drops of water spinning off the rim will hit the ground at

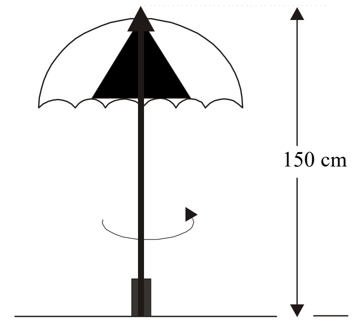


Fig. 2.40

(a) 200 cm (b) 52 cm

(c) 172 cm (d) 97 cm.

16. Two motorcycles M_1 and M_2 are heading towards each other with a speed of 30 kmh^{-1} each. A bird flies off M_1 at 60 kmh^{-1} when distance between the motorcycles is 60 km. It heads towards M_2 and then back to M_1 and so on. The total distance the bird moves till the motorcycles meet is

Fig. 2.41

(a) 60 km (b) 40 km

(c) 50 km (d) 30 km

(e) none

17. Two towns T_1 and T_2 are connected by regular bus service. A scooterist moving from T_1 to T_2 with speed of 20 kmh^{-1} notices that a bus goes past it every 21 minute in the direction of his motion and every 7 minute in the opposite direction. If a bus leaves in either direction every t minute, the period t is

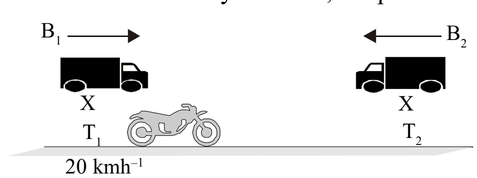


Fig. 2.42

(a) 1.5 minute (b) 9.5 minute

(c) 6 minute (d) 10.5 minute.

18. If $\vec{P}+\vec{Q}+\vec{R} = 0$ and out of these, two vectors are equal in magnitude and the third vector has magnitude $\sqrt{2}$ times that of any of these two vectors, then angles among the three vectors are

(a) 45°, 75°, 75° (b) 45°, 90°, 135°

(c) 90°, 135°, 180° (d) 90°, 135°, 135°.

19. A hunter is at (4, – 1, 5) units. He observes two preys at P_1 (– 1, 2, 0) units and P_2 (1, 1, 4), respectively. At zero instant he starts moving in the plane of their positions with uniform speed of 5 units s^{-1} in a direction perpendicular to line P_1P_2 till he sees P_1 and P_2 collinear at time T. Time T is

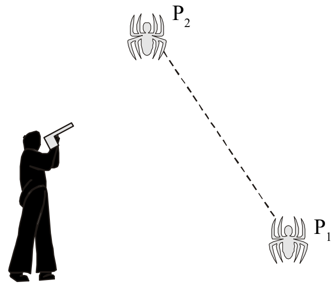


Fig. 2.43

(a) 0.53 s (b) 0.73 s

(c) 0.35 s (d) 0.92 s.

20. An aeroplane flies from P and Q with speed v and then from Q and P with the same speed. If wind blows normal to straight line PQ with the speed V, the total time for to and fro motion is

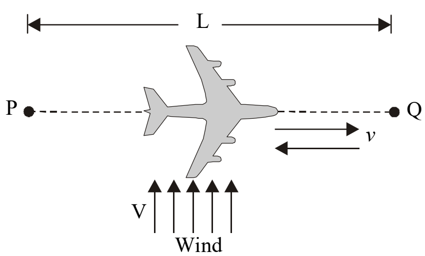


Fig. 2.44

(a) $\dfrac{L}{(v^2-V^2)^{1/2}}$ (b) $\dfrac{2L}{(v^2-V^2)^{1/2}}$

(c) $\dfrac{2L}{V-v}$ (d) $\dfrac{2L}{(v^2-V^2)}$

21. If $\vec{a}$ and $\vec{b}$ are the intersecting face diagonals of a cube of side x in plane XOY and YOZ, respectively, with respect to reference frame at the point of intersection of the vectors and sides of cube as the axes, the components of vector $\vec{r} = \vec{a}\times\vec{b}$ are

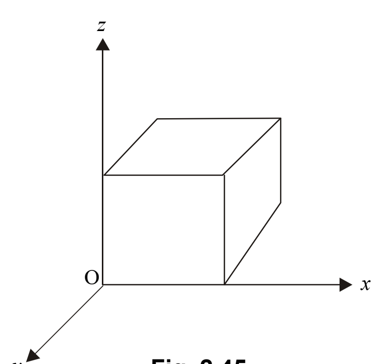


Fig. 2.45

(a) $x, -x, x$ (b) $-x^2, -x^2, x^2$

(c) $x^2, -x^2, x^2$ (d) $x, x^2, -x$.

22. Two particles are originally placed at P and Q distant d apart. At zero instant, they start moving such that velocity $\vec{v}$ of P is aimed towards Q and velocity $\vec{u}$ of Q is perpendicular to $\vec{v}$. The two projectiles meet at time T =

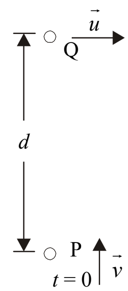


Fig. 2.46

(a) $\dfrac{(v^2-u^2)d^2}{v^3 d}$ (b) $\dfrac{(v+u)d}{v^2}$

(c) $\dfrac{v(v-u)}{d}$ (d) $\dfrac{vd}{(v^2-u^2)}$

23. A particle is moving in a circle of radius R in such a way that at any instant the a_r and a_t are equal. If the speed at $t = 0$ is v_0, the time taken to complete the first revolution is

(a) $\dfrac{R}{v_0}e^{-2\pi}$ (b) v_0R

(c) $\dfrac{R}{v_0}$ (d) $\dfrac{R}{v_0}(1 - e^{-2\pi})$.

24. A boat which has a speed of 5 kmh^{-1} in still waters crosses a river of width 1 km along the shortest possible path in 15 minutes. The speed of the river in kmh^{-1} is

(a) 1 (b) 3

(c) 4 (d) $\sqrt{41}$

[Based on I.I.T.]

25. A particle is moving in a plane with velocity given by $\vec{v} = \hat{i}u_0 + \hat{j}\,a\omega\,\cos\omega t$ if the particle is at origin at t = 0. Distance from origin at time $3\pi/2\omega$ is

(a) $\sqrt{a^2+(3\pi u_0/2\omega)^2}$ (b) $\sqrt{a^2+(2\pi u_0/\omega)^2}$

(c) $\left(\dfrac{\pi u_0}{\omega}\right)^2 + a^2$ (d) $\sqrt{a^2+\left(\dfrac{2\pi u_0}{3\omega}\right)^2}$

[Based on Roorkee]

26. Mass A is released from the top of a frictionless inclined plane 18 m long and reaches the bottom 3 s later. At the instant when A is released, a second mass B is projected upwards along the plate from the bottom with a certain initial velocity.

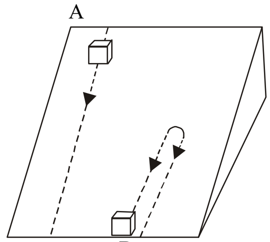


Fig. 2.46

Mass B travels a distance up the plane, stops and returns to the bottom so that it arrives simultaneously with A. The two masses do not collide. Initial velocity of A is

(a) 4 ms^{-1} (b) 5 ms^{-1}

(c) 6 ms^{-1} (d) 7ms^{-1}

[Based on I.I.T.]

27. A particle is moving eastward with a velocity of 5 ms^{-1}. If in 10 s the velocity changes by 5 ms^{-1} northwards, what is the average acceleration in this time?

(a) $\frac{1}{\sqrt{2}}$ ms^{-1} NW (b) $\frac{1}{2}$ ms^{-1} EN

(c) $\sqrt{2}$ ms^{-1} NW (d) $2\sqrt{2}$ ms^{-1} NW.

[Based on I.I.T.]

28. The driver of a truck travelling with a velocity v suddenly notices a brick wall in front of him at a distance d. To avoid crashing into the wall

(a) he should apply brakes

(b) he should take a circular turn without applying brakes

(c) both (a) and (b) alternately

(d) data is insufficient.

[Based on Roorkee]

29. To get a resultant displacement of 10 cm, two displacement vectors, one of magnitude 6 cm and another of 8 m, should be combined

(a) at an angle 60° (b) perpendicular to each

(c) parallel (d) anti-parallel

30. When mass is rotating in a plane about a fixed point, its angular momentum is directed along

(a) the axis of rotation

(b) line at an angle of 45° to the axis of rotation

(c) the radius

(d) the tangent to the orbit

31. Which of the following is a vector?

(a) force (b) mass

(c) energy (d) power

32. When two vectors $\vec{A}$ and $\vec{B}$ of magnitude a and b are added, the magnitude of the resultant vector is always

(a) greater than $(a + b)$

(b) not greater than $(a + b)$

(c) equal to $(a + b)$

(d) less than $(a + b)$

33. Identity the vector quantity.

(a) heat (b) angular momentum

(c) time (d) work

34. Which of the following quantities is a scalar?

(a) magnetic moment

(b) acceleration due to gravity

(c) electric field

(d) electrostatic potential

35. Which of the following quantities is a vector?

(a) volume (b) temperature

(c) displacement (d) density

36. The rectangular components of force 5 dyne are

(a) 3 and 4 dyne (b) 2.5 and 25 dyne

(c) 1 and 2 dyne (d) 2 and 3 dyne

37. Identify the scalar quantity.

(a) work (b) impulse

(c) force (d) acceleration

38. Moment of inertia is

(a) vector (b) scalar

(c) phasor (d) tensor

39. If the magnitude of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units, respectively, and $\vec{A} + \vec{B} = \vec{C}$, the angle between vectors $\vec{A}$ and $\vec{B}$ is

(a) $\pi/4$ (b) $\pi/2$

(c) π (d) 0

40. Angular displacement is

(a) a scalar (b) a vector

(c) neither (a) nor (b) (d) either (a) or (b)

41. A mosquito flies from the hole in a mosquito net top corner diametrically opposite. If the net is 3 × 2 × 2 m then the displacement of the mosquito is

(a) $\sqrt{13}\ m$ (b) $\sqrt{17}\ m$

(c) $\sqrt{11}\ m$ (d) none of these

42. A man travels 1 mile due east, 5 mile due south, 2 mile due east and finally 9 miles due north. How far is the starting point?

(a) 3 miles (b) 5 miles

(c) 4 miles (d) between 5 and 9 miles

43. Two forces of magnitude 7 N and 5 N act on a p article at an angle θ to each other, θ can have any value. The minimum magnitude of the resultant force is

(a) 12 N (b) 8 N

(c) 2 N (d) 5 N

44. I started walking down a road to day-break facing the sun. After walking for some time, I turned to my left then I turned to the right once again. In which direction was I going?

(a) northeast (b) south

(c) east (d) northwest

45. If $\vec{A} = \vec{B} + \vec{C}$ and the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 5, 4 and 3 units, respectively. then the angle between $\vec{A}$ and $\vec{C}$ is

(a) $\pi/2$ (b) sin–1 (3/4

(c) $\cos^{-1}(3/5)$ (d) $\cos^{-1}(4/5)$

46. A boat which has a speed of 5 $kmhr^{-1}$ in still waters crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the water in $kmhr^{-1}$ is

(a) 4 (b) $\sqrt{41}$

(c) 1 (d) 3

47. If two waves of same frequency and same amplitude on superimposition produce a resultant wave of the same amplitude, the wave differs in phase by

(a) $\pi/5$ (b) $2\pi/3$

(c) $\pi/4$ (d) zero

48. If $\vec{n}$ is a unit vector in the direction of the vector $\vec{A}$, then

(a) $\vec{n} = |\vec{A}|/\vec{A}$ (b) $\vec{n} = \vec{n} \times \vec{n}$

(c) $\vec{n} = \vec{A}/|\vec{A}|$ (d) $\vec{n} = \vec{A}\,|\vec{A}|$

49. Two forces of 4 dyne and 3 dyne act upon a body. The resultant force on the body can only be

(a) between 3 and 4 dynes

(b) between 1 and 7 dynes

(c) more than 3 dynes

(d) more than 4 dynes

50. A river is flowing from west to east at a speed of 3 m/ minute. A man on the south bank of the river, capable of swimming at 10 m in still waters wants to swim the river in the shortest time. He should swim in a direction

(a) 30° west of north

(b) 60° east of north

(c) 30° east of north

(d) due north

51. The resultant of two equal forces is double of either of the force. The angle between them is

(a) 0° (b) 60°

(c) 90° (d) 120°

52. An aeroplane is moving on a circular path with a speed 250 $kmhr^{-1}$. What is the change in velocity in half revolution?

(a) 0 (b) 125 $kmhr^{-1}$

(c) 250 $kmhr^{-1}$ (d) 500 $kmhr^{-1}$

53. A body constrained to move in y direction is subject to force given by $\vec{F} = (-2\vec{i} + 15\vec{j} + 6\vec{k})$ N. What is the work done by this force, in moving the body through a distance of 10m along y-axis?

(a) 20 J (b) 150 J

(c) 160 J (d) 190 J

54. I walked 4 miles turned to my left and walked 6 miles then turned to my right again and walked 4 mile. Which of the choice mentions the distance from the straight point to the place where *I* stopped?

(a) 10 mile (b) 14 mile

(c) 15 mile (d) 20 mile

55. A force $\vec{F} = 6\vec{i} - 8\vec{j} + 10\vec{k}$ newton produces an acceleration of 1 ms^{-2} in a body, the body would be

(a) 10 $\sqrt{2}$ kg (b) 6 $\sqrt{2}$ kg

(c) 20 kg (d) 200 kg

56. Maximum and minimum magnitudes of the resultant of two vectors of magnitudes P and Q are in the ratio 3 : 1. Which of the following relations is true?

(a) $PQ = 1$ (b) $P = 2Q$

(c) $P = Q$ (d) none of these

57. What is the projection of $\vec{P}$ on $\vec{Q}$?

(a) $\vec{Q}\cdot\vec{P}$ (b) $\hat{P}\cdot\hat{Q}$

(c) $\vec{P}\cdot\vec{Q}$ (d) $\vec{P}\cdot\hat{Q}$

58. Rain is falling vertically 4 ms^{-1}. A man is moving due east with 3 ms^{-1}. The direction in which he shall hold the umbrella with the vertical is

(a) 53° east of vertical

(b) 37° east of vertical

(c) 53° west of vertical

(d) 37° west of vertical

59. There are *N* co-planar vectors each of magnitude *V*. Each vector is inclined to the proceeding vector at angle 2 π/N. What is the magnitude of their resultant?

(a) zero (b) V/N

(c) V (d) NV

60. Which of the following operations between the two vectors can yield a vector perpendicular to either of them?

(a) subtraction (b) division

(c) addition (d) multiplication

61. Three vectors *A*, *B* and *C* satisfy the relation $\vec{A}.\vec{B}=0$ and $A.\ C=0$. The vector *A* is parallel to

(a) $\vec{B}.\vec{C}$ (b) $\vec{B}$

(c) $\vec{C}$ (d) $\vec{B}\times\vec{C}$

62. Angle between the vectors $\left(\vec{i}+\vec{j}\right)$ and $\left(\vec{j}+\vec{k}\right)$ is

(a) 60° (b) 90°

(c) 180° (d) 0°

63. Resultant of two vectors $\vec{P}$ and $\vec{Q}$ is inclined at 45° to either of them. What is the magnitude of the resultant?

(a) $\sqrt{P^2+Q^2}$ (b) $\sqrt{P^2-Q^2}$

(c) P + Q (d) P – Q

64. A steamer is heading due North with 20 ms^{-1}. The wind is blowing 10 ms^{-1}. The wind is blowing 10 ms^{-1} due west. In which direction the flag on the mast flutters?

(a) $\tan^{-1}\frac{1}{2}$ west of north

(b) $\tan^{-1}\frac{1}{2}$ east of north

(c) $\tan^{-1}\frac{1}{2}$ north of east

(d) $\tan^{-1}\frac{1}{2}$ north of west

65. What is the angle between $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}$?

(a) $\pi/3$ (b) $\pi/4$

(c) $\pi/6$ (d) none of these

66. What is the maximum number of components into which a vector can be split?

(a) 2 (b) 3

(c) 4 (d) more than 4

67. What is the maximum number of a rectangular components into which a vector can be split in its own plane?

(a) two (b) three

(c) four (d) more than 4

68. A force of 6 kg and 8 kg can be applied together to produce the effect of a single force of

(a) 20 kg (b) 15 kg

(c) 11 kg (d) 1 kg

69. To a person going east in a car with a velocity of 25 $kmhr^{-1}$, a train appears to move towards north with a velocity of 25 $\sqrt{3}$ km/hr. The actual velocity

(a) 5 $kmhr^{-1}$ (b) 25 $kmhr^{-1}$

(c) 50 $kmhr^{-1}$ (d) 53 $kmhr^{-1}$

70. The area of a Δ formed with sides $5i+3j-k$ and $3i+2j-k$ is

(a) $\sqrt{6}$ (b) $\sqrt{3}$

(c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{5}{2}}$

71. At what angle should be the two forces $2p$ and $\sqrt{2}\,p$ act so that the resultant force is $p\sqrt{10}$

(a) 45° (b) 60°

(c) 90° (d) 120°

72. Two cars are moving. A along east with 10 ms^{-1}. At any instant it is 1500 m away from the crossing. *B* at the same instant is 1800 m away from the crossing and is moving towards the crossing with 15 ms^{-1}. When do they come closest?

(a) 109.3 s (b) 129.2 s

(c) 119.3 s (d) 99.3 s

73. What is the angle between $\vec{P}$ and the resultant of $\left(\vec{P}+\vec{Q}\right)$ and $\left(\vec{P}-\vec{Q}\right)$?

(a) $\frac{\tan^{-1}\left|\left(\vec{P}-\vec{Q}\right)\right|}{|P+Q|}$ (b) $\tan^{-1}(Q/P)$

(c) tan–1 (P/Q) (d) zero

74. The length of seconds hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is

(a) zero (b) $\left(\frac{\pi}{15\sqrt{2}}\right)$ cms^{-1}

(c) $\left(\frac{\pi}{15}\right)$ cms^{-1} (d) $\left(\frac{\pi\sqrt{2}}{15}\right)$ cms^{-1}

75. Rain falling vertically downwards with a velocity of 3 kmh^{-1}. A person moves on a straight road with a velocity of 4 kmh^{-1}. Then the apparent velocity of the rain with respect to the person is

(a) 1 kmh^{-1} (b) 5 kmh^{-1}

(c) 4 kmh^{-1} (d) 3 kmh^{-1}

76. A large number of particles are moving towards each other with velocity v having directions of motion randomly distributed. What is the average relative velocity between any two particles averaged over all the pairs?

(a) $4v/\pi$ (b) $4\pi v$

(c) v (d) $\pi v/4$

77. The magnitudes of the X and Y components $\vec{P}$ are 7 and 6. The magnitudes of the X and Y components of $\vec{P}$ + $\vec{Q}$ are 11 and 9, respectively. What is the magnitude of Q?

(a) 9 (b) 8

(c) 6 (d) 5

78. A swimmer can swim in still waters with speed v and the river flowing with velocity $v/2$. To cross the river in shortest time, he should swim making angle θ with the upstream. What is the ratio of the time taken to swim across in the shortest time to that in swimming across over shortest distance?

(a) sin θ (b) cos θ

(c) tan θ (d) cot θ

79. A vector of magnitude a is rotated through angle θ. What is the magnitude of the change in the vector?

(a) $2\alpha \sin\theta$ (b) $2\alpha\cos\theta$

(c) $2\alpha\sin(\theta/2)$ (d) $2\alpha\cos(\theta/2)$

80. Consider a vector $\vec{F} = 4\hat{i} - 3\hat{j}$. Another vector that is perpendicular to $\vec{F}$ is

(a) $4i + 3j$ (b) $7k$

(c) $3i - 4j$ (d) $6i$

81. A helicopter is flying south with a speed of 50 km h^{-1}. A train is moving with the same speed towards east. The relative velocity of the helicopter as seen by the passengers in the train will be 50 $\sqrt{2}$ km h^{-1} towards

(a) northwest (b) southwest

(c) northeast (d) southeast

82. A man is walking due east at the rate of 4 kmhr^{-1} and the rain is falling at an angle of 30° east of vertical with a velocity of 6 kmhr^{-1}. The velocity of the rain relative to the man will be

(a) 5 km hr^{-1} (b) 7.118 km hr^{-1}

(c) 8.718 km hr^{-1} (d) 10 km hr^{-1}

83. A truck travelling due north at 20 ms^{-1} turns west and travels at the same speed. Then the change in velocity is

(a) 40 ms^{-1} northwest

(b) 20 $\sqrt{2}$ ms^{-1} northwest

(c) 20 $\sqrt{2}$ ms^{-1} southwest

(d) 40 ms^{-1} southwest

84. Given that P is a point on a wheel rolling on a horizontal ground. The radius of the wheel is R. Initially if the point P is in contact with the ground the wheel rolls through half the revolution. What is the displacement of point P?

(a) $R\sqrt{\pi^2+1}$ (b) $R\sqrt{\pi^2+4}$

(c) πR (d) $2\pi R$

85. A vector $\vec{F}_1$ is along x axis. If $\vec{F}_1 \cdot \vec{F}_2$ is zero $\vec{F}_2$ could be

(a) $(j + k)$ (b) $-(i + j)$

(c) $4(i + k)$ (d) $-4i$

86. A parallelogram is formed with $\vec{a}$ and $\vec{b}$ as the sides. Let $\vec{d}_1$ and $\vec{d}_2$ be the diagonals of the parallelogram then $a^2 + b^2 =$

(a) $\left(d_1^2+d_2^2\right)/2$ (b) $\left(d_1^2-d_2^2\right)/2$

(c) $d_1^2+d_2^2$ (d) $d_1^2-d_2^2$

87. If $|\vec{A}| = |\vec{B}|$, then what is the angle between $\vec{A}$ + $\vec{B}$ and $\vec{A}$ – $\vec{B}$

(a) 90° (b) 60°

(c) 30° (d) 0°

PASSAGE 1

Read the following passage and answer the questions given at the end.

A graphic artist is creating a new logo for her company's Website. In the graphics program she is using, each pixel in an image file has coordinates (x, y) where the origin $(0, 0)$ is at the upper left corner of the image, the positive x axis points to the right and positive y axis points down. Distances are measured in pixels. The artist draws a line from pixel location A (10,20) to the location B (210, 200). She wishes to draw a socond line that starts at A (10, 20) and is 250 pixel long. The line is at 30° angle measured clockwise from the first line. Call this point C. The artist now joins BC.

1. Find the coordinates of C.

(a) (97, 248) (b) (87, 238)

(c) (87, 258) (d) (97, 268)

2. Find the length of BC

(a) 137 pixel, 35° above right left

(b) 137 pixel, 25° below stright left

(c) 157 pixel, 45° below straight left

(d) 137 pixel 35° below skright left

Solution **1.** (c)

$$l = \{(x-10)\,\hat{i} + (y-20) \quad \text{...(1)}$$

$$250 = \sqrt{(x-10)^2 + (y-20)^2}$$

$$\cos 30 = \frac{\sqrt{3}}{2} = \frac{(x-100)200 + (y-20)180}{250 \times 269} \quad \text{...(2)}$$

Solving (1) and (2) we get C (87, 258)

Solution **2.** (b)

$$|BC| = \sqrt{(210-87)^2 + (200-258)^2} = 137$$

$$\cos\theta = \frac{(210-87)\hat{i}\cdot(-\hat{i})}{137} = \frac{123}{137} \text{ or } \theta = 25^\circ$$

PASSAGE 2

Read the following passage and answer the questions given at the end.

At Enormous State University (ESU), the football team records its plays using vector displacements with the origin taken to be the position of the ball before the play starts. In a certain pass play, the receiver starts at (10, –5) where the unit are yards and $\hat{i}$, $\hat{j}$ are unit vectors along the right and the downfield, $6\hat{i} + 4\hat{j}$ (zigs) and $12\hat{i} + 18\hat{j}$ (zags). Meanwhile quarterback has dropped straight back at $-7\hat{j}$.

1. How far must the quarter back throw the ball?

(a) 35 yards (b) 33.9 yards

(c) 37.2 yards (d) 31.1 yards

2. The direction in which he throws the ball is

(a) 31°36′ right of down field

(b) 31°36′ left of down field

(c) 28.3° right of down field

(d) none

Solution **1.** (d)

$$R = x\hat{i} + y\hat{j} = 10\hat{i} - 5\hat{j} + (-6\hat{i} + 4\hat{j})$$

$$+ (12\hat{i} - 18\hat{j}) - 7\hat{j} = 16\hat{i} - 26\hat{j}$$

$$|R| = \sqrt{16^2 + 26^2} = 31.1$$

Solution **2.** (a) $\tan\beta = \frac{16}{26}$ or $\beta = 31°36'$ right of down field.

PASSAGE 3

Read the following passage and answer the questions given at the end.

All the stars of the Big Dipper (part of the constellation Ursa Major) may appear to be the same distance from the Earth. But they are in fact very far away from each other. The Fig shows the distance from the earth to each of these stars. The distances are given by (light years), the distance that light travels in years.

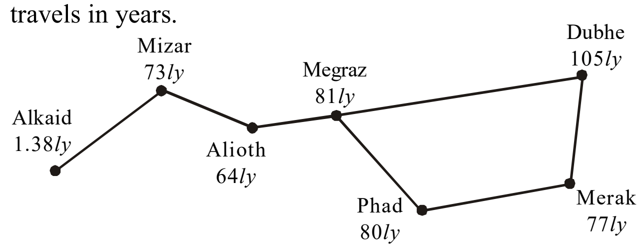


Fig. 2.47

One light year = 9.461′ 10^{15} m. Alkaid and Merak are 25.6° apart in the earth's sky.

1. Find the distance in light years from Alkaid to merak.

(a) 76 ly (b) 56 ly

(c) 66.3 ly (d) 69.9 ly

2. To an inhabitant of a planet orbiting Merak, how many degrees apart in the sky would Alkaid and sun be?

(a) 129° (b) 108°

(c) 100° (d) 27°

Solution **1.** (a) 77 – 1.38 = 75.62 or 76 ly

Solution **2.** (a) Angle between SA and SM = 26.4, given angle between A_m and A_s (with nearly same dimensions) will be around 26°

\< ANS = 180 – 52 = 128°

Answers to Question for Practice

1.	(d)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(a)	**6.**	(a)	**7.**	(b)
8.	(d)	**9.**	(a)	**10.**	(b)	**11.**	(c)	**12.**	(c)	**13.**	(d)	**14.**	(a)
15.	(d)	**16.**	(a)	**17.**	(d)	**18.**	(d)	**19.**	(a)	**20.**	(b)	**21**	(c)
22.	(d)	**23.**	(d)	**24.**	(b)	**25.**	(a)	**26.**	(c)	**27.**	(a)	**28.**	(a)
29.	(b)	**30.**	(a)	**31.**	(a)	**32.**	(b)	**33.**	(b)	**34.**	(d)	**35.**	(c)
36.	(a)	**37.**	(a)	**38.**	(d)	**39.**	(b)	**40.**	(d)	**41.**	(b)	**42.**	(b)
43.	(c)	**44.**	(c)	**45.**	(c)	**46.**	(d)	**47.**	(b)	**48.**	(c)	**49.**	(b)
50.	(d)	**51.**	(a)	**52.**	(d)	**53.**	(b)	**54.**	(a)	**55.**	(a)	**56.**	(b)
57.	(d)	**58.**	(b)	**59.**	(a)	**60.**	(d)	**61.**	(d)	**62.**	(a)	**63.**	(a)
64.	(b)	**65.**	(d)	**66.**	(d)	**67.**	(a)	**68.**	(c)	**69.**	(c)	**70.**	(c)
71.	(a)	**72.**	(b)	**73.**	(d)	**74.**	(b)	**75.**	(b)	**76.**	(d)	**77.**	(d)
78.	(a)	**79.**	(c)	**80.**	(b)	**81.**	(a)	**82.**	(c)	**83.**	(c)	**84.**	(b)
85.	(a)	**86.**	(a)	**87.**	(a)								

EXPLANATION

1(d) Let α be the angle between velocities of a pair of particles, then relative velocity is given by

$$v_r = \sqrt{v^2 + v^2 - 2v \times v \times \cos\alpha}$$

$$= 2v \sin\frac{\alpha}{2} \quad \left(\because 1 - \cos\alpha = 2\sin^2\frac{\alpha}{2}\right)$$

Average relative velocity is given by

$$\text{average } v_r = \frac{\int_0^{2\pi} 2v(\sin\alpha/2)\,d\alpha}{\int_0^{2\pi} d\alpha} = \frac{4}{\pi}v$$

2(c) Velocity of river $\vec{v}_r = -2\,\hat{i}$

Velocity of swimmer w.r.t. river is

$$\vec{v}_s = 5\cos 30\,\hat{i} + 5\sin 30\,\hat{j}$$

$$= 4.33\,\hat{i} + 2.5\,\hat{j}$$

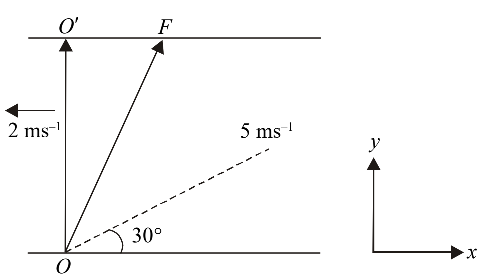


Fig. 2.48

Using $\vec{v}_R = \vec{v}_s + \vec{v}_r$ we get

$$\vec{v}_R = 2.33\,\hat{i} + 2.5\,\hat{j}$$

$$\text{Time taken by swimmer} = \frac{\text{Distance along } y\text{-axis}}{y \text{ component of velocity}}$$

$$= \frac{200}{2.5} = 80 \text{ s}$$

Distance moved along x-axis,

$O'F$ = x component of relative velocity × time

$= 2.33 \times 80 = 186.4$ m

$\therefore$ 186 m upstream

3(a) Time interval,

$$t = \frac{\text{distance between cars}}{\text{relative velocity}}$$

$$= \frac{d}{v_{III} + 30}$$

where v_{III} is velocity of 3rd car and velocity of I or II is 30 kmh^{-1} or

$$\frac{240}{3600} = \frac{5}{v_{III} + 30} \qquad \text{or}$$

$$v_{III} = (15 \times 5 - 30)$$

$$= 45 \text{ kmh}^{-1}$$

4(b) Here $\vec{r} = -5\,\hat{i} - 3\,\hat{j} + 0\,\hat{k}$ and

$$\vec{F} = 4\,\hat{i} - 10\,\hat{j} + 0\,\hat{k}$$

$$\therefore \quad \tau = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -3 & 0 \\ 4 & -10 & 0 \end{vmatrix}$$

$$= \hat{k}\,(50 + 12) = 62\,\hat{k} \text{ unit}$$

5(a) Resultant velocity of swimmer 1 is

$$v_{PQ} = \sqrt{2.5^2 - 2^2} = \sqrt{2.25}$$

$$= 1.5 \text{ kmh}^{-1}$$

Let width of river be W then time taken,

$t_1 = \dfrac{W}{1.5}$ h. Time taken by swimmer 2 is

$$t_2 = \frac{W}{2.5}$$

$\therefore$ Distance Q Q′ = velocity × time = $2 \times \dfrac{W}{2.5}$

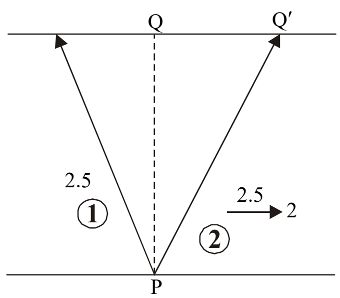


Fig. 2.49

Time taken by swimmer 2 to move to distance Q Q′ is

$$t = t_1 - t_2$$

$$= \frac{W}{1.5} - \frac{W}{2.5} = \frac{W}{1.5 \times 2.5}$$

Desired velocity

$$= \frac{QQ'}{t} = \frac{2W}{2.5} \times \frac{W}{1.5 \times 2.5}$$

$$= 3 \text{ kmh}^{-1}$$

6(a) $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$

$$5\hat{i} = 2\hat{i} + [(4\cos 45°)\hat{i} - (4\sin 45°)\hat{j}] + \vec{S}_3$$

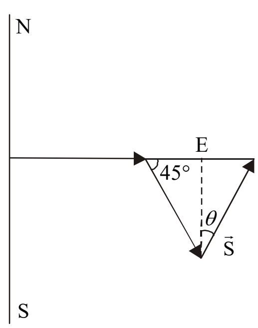


Fig. 2.50

i.e., $\vec{S}_3 = (3\hat{i} - 2\sqrt{2})\hat{i} + 2\sqrt{2}\,\hat{j}$

$$= 0.172\,\hat{i} + 2.828\,\hat{j}$$

i.e., $S_3 = \sqrt{0.172^2 + 2.828^2}$

$$= 2.8 \text{ km}$$

and $\tan\theta = \dfrac{0.172}{2.828} = 0.0608$

or $\theta = \tan^{-1} 0.0608$

or $\theta = 3° 26'$

7(b) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -4 & 3 \\ 3 & 2 & -1 \end{vmatrix}$

$$\vec{A} \times \vec{B} = -2\hat{i} + 14\hat{j} + 22\hat{k}$$

$$\text{Area} = \frac{1}{2}\left|\vec{A} \times \vec{B}\right| = \sqrt{1^2 + 7^2 + 11^2} = \sqrt{171} \text{ unit}$$

8(d) Here, $\vec{v}_1 = (u_1 \cos\alpha_1)\hat{i} + (u_1 \sin\alpha_1 - gt)\hat{j}$

and $\vec{v}_2 = (u_1 \cos\alpha_2)\hat{i} + (u_2 \sin\alpha_2 - gt)\hat{j}$

Relative velocity $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$ has horizontal as well as vertical components and it is constant also, so that path seen by each other is straight line with angle not equal to 90°.

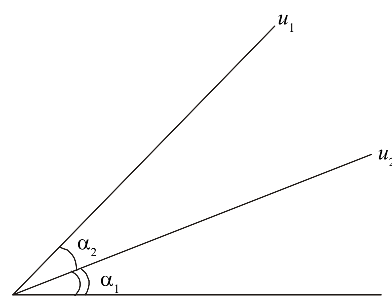


Fig. 2.51

9. (a) Here $\overrightarrow{OP}$ equal to $3\hat{i}$ is the velocity of man $\overrightarrow{OQ}$ is the velocity of rain.

$\therefore$ PQ is velocity of rain relative to man. In the second case, $\overrightarrow{OR}$ equal to $6\hat{i}$ is the new velocity of man and $\overrightarrow{RQ}$ = new velocity of rain relative to man.

Now $OQ^2 = OP^2 + PQ^2$

or $OQ^2 = 3^2 + 3^2$ ($\because$ PQ = PR = 6 – 3 = 3)

or $OQ = 3\sqrt{2}$ kmh^{-1}

$\tan\theta = \dfrac{PQ}{OQ} = \dfrac{3}{3} = 1$ or $\theta = 45°$

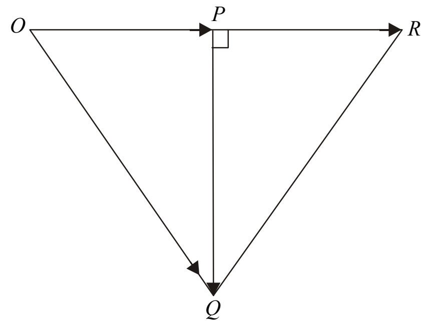


Fig. 2.52

10(b) Velocity of wind w.r.t. earth is,

$v_{WE} = 30$ kmh^{-1} towards NS *i.e.,* SE Velocity of helicopter w.r.t. earth is

$$v_{HE} = \frac{200000}{1000 \times \left(\frac{40}{60}\right)}$$

$= 300$ km towards East

In vector form,

$$\vec{v}_{HE} = 300\,\hat{i} \text{ and}$$

$$\vec{v}_{WE} = (30\cos 45°)\,\hat{i} - (30\sin 45°)\,\hat{j}$$

$$= 15\sqrt{2}\,\hat{i} - 15\sqrt{2}\,\hat{j}$$

Then resultant velocity is also given by, $\vec{v}_{NH} = \vec{v}_{WE} + \vec{v}_{HW}$

or $\vec{v}_{HW} = \vec{v}_{HE} - \vec{v}_{WE}$

$$= 300\,\hat{i} - (15\sqrt{2}\,\hat{i} - 15\sqrt{2}\,\hat{j})$$

$$= 279\,\hat{i} + 21\,\hat{j}.$$

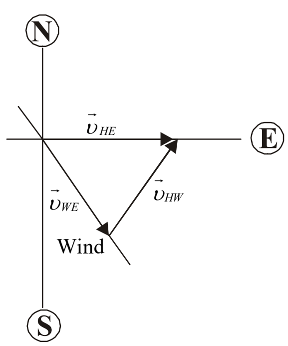


Fig. 2.53

11(c) $u_1 + u_2 = 4$ ...(i)

$u_1 - u_2 = 0.4$ ...(ii)

Adding (i) and (ii)

$2u_1 = 4.4$ or $u_1 = 2.2$ ms^{-1}

and $u_2 = 1.8$ ms^{-1}

12(c) Velocity of man $|\vec{v}m| = 10$ ms^{-1} Using $\sin 30° = \frac{v_m}{v_{re}}$

vm = velocity of man v_{re} = velocity of rain w.r.t. earth

v_{rm} = velocity of rain w.r.t. man

or $v_{re} = \frac{v_m}{\sin 30} = \frac{10}{1/2} = 20$ ms^{-1}

Agiain $\cos 30° = \frac{v_{rm}}{v_{re}}$

or $v_{rm} = v_{re} \cos 30$

$= 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$ ms^{-1}

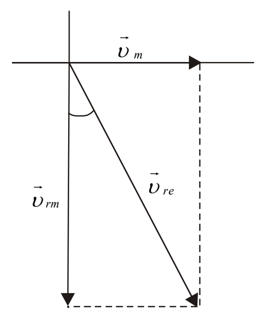


Fig. 2.54

13(d) Here, number of revolutions to cover 1.5 cm 'n'

$= \frac{1.5}{1/12} = 18$

Angular speed $= \omega\ 2\pi\nu = 2\pi \times \frac{216}{60} = 7.2\pi$ rd s^{-1}

But $\omega = \frac{\theta}{\omega}$ or $t = \frac{\theta}{t} = \frac{2\pi n}{\omega} = \frac{2\pi \times 18}{7.2\pi} = 5$s

14(a) The particile and the strips have to move in oppositic direction. Their relative velocity is given by $v_r = \frac{L}{t}$

The strip continues to move at its previous speed which is less than the relative velocity. It will now cover the same distance rather slowly.

$\therefore$ T > t

15(d) Velocity of water drop.

$v = r\omega = 0.5 \times \frac{(2\pi \times 21)}{44} = 1.5$ ms^{-1}

Time taken to reach ground,

$t = \sqrt{\frac{2h}{8}} = \sqrt{\frac{2 \times 1.5}{9.8}} = 0.55$ s

Range of drop, x = $vt = 1.5 \times 0.55 = 0.83$ m

distance R = $\sqrt{r^2 + x^2}$ = $\sqrt{0.5^2 + 0.83^2}$ = 0.97 m
= 97 cm

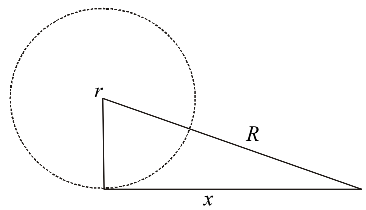


Fig. 2.55

16(a)v_{ret} = 30 + 30 = 60 kmh^{-1}

Time in which motorcycles will meet

$= \dfrac{x}{v_{ret}} = \dfrac{60 \text{ km}}{60 \text{ kmh}^{-1}}$

= 1 h

distance moved by bird in 1 hr = 60 × 1 = 60 km

17(d)Bus (say B1) in the same direction relative speed of bus and cyclist,

$\nu_r = \nu_b - \nu_c$ Distance travelled by B1 relative of cyclist in 21 minutes = $\nu_r \times 21$

Distance covered by bus in T minute

= νb T

Then 21 $\nu_r = \nu_b + T$

Bus in the opposite direction (say B_2)

Relative speed between B_2 and cyclist,

$\nu_r = \nu_b + \nu_c$

Distance travelled by B2 relative to cyclist in 7 minutes = νr × 7

Then 7 $\nu_r = \nu_b$ T

frp, (i) and (ii)

21 ν_r = 7 ν_r or 3 $\nu_r = \nu_r$ or 3 $(\nu_b - \nu_c) = \nu_b + \nu_c$ or ν_b = 2ν_c = 2 × 20 = 40 kmh^{-1}

Thus T = $\dfrac{7v_r}{v_b} = \dfrac{7(v_b + v_c)}{v_b} = \dfrac{7(40+20)}{40} = \dfrac{7\times 3}{2}$

$= \dfrac{21}{2}$ = 10.5 minute

18. (d) Let $P = Q = x$ and R = $\sqrt{2}x$ Using $\vec{P} + \vec{Q} + \vec{R}$ = 0, we get $\vec{P} + \vec{Q} = -\vec{C}$

$(\vec{P} + \vec{Q}).(\vec{P} + \vec{Q}) = (\vec{R}).(-\vec{R})$ Then P^2 + Q^2 + 2PQ cos θ = R^2 i,e., x^2 + x^2 + 2x^2 cos θ = 2x^2 i,e., cos θ = 90^o Again $\vec{Q} + \vec{R} = -\vec{P}$

$(\vec{Q} + \vec{R}).(\vec{Q} + \vec{R}) = (-\vec{P}).(-\vec{P})$ Then Q^2 + R^2 + 2QR cos α = P^2 i,e., cos α = –1/ 2$\sqrt{2}$ or φ = 135^o Third angle = 360 – (135 + 90) = 360 – 225 = 135^o

19(a)Here $\vec{A}$ = (–1–4) $\hat{i}$ + (2+1) $\hat{j}$ + (0–5) $\hat{k}$ = – 5 $\hat{i}$ + 3 $\hat{j}$ – 5 $\hat{k}$ and $\vec{B}$ = (1+1) $\hat{i}$ + (1–2) $\hat{j}$ + (4–0) $\hat{k}$ = 2 $\hat{i}$ – $\hat{j}$ + 4 $\hat{k}$

Now R = | $\vec{A}$ | sin θ and | $\vec{A} + \vec{B}$ | = AB sin θ or sin θ

$= \dfrac{|\vec{A}\times\vec{B}|}{AB}$ i,e., R = A $\dfrac{|\vec{A}\times\vec{B}|}{AB} = \dfrac{|\vec{A}\times\vec{B}|}{B}$

Now $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 3 & -5 \\ 2 & -1 & 4 \end{vmatrix}$

= (12 –5) $\hat{i}$ + (–10 + 20) $\hat{j}$ + (5 – 6) $\hat{k}$ = 7 $\hat{i}$ + 10 $\hat{j}$ – $\hat{k}$

| $\vec{A} \times \vec{B}$ | = (7^2 + 10^2 + 1)$^{1/2}$ = 12.25 | $\vec{B}$ |
= (2^2 + 1^2 + 4^2)$^{1/2}$ = 4.58

∴ R = $\dfrac{12.25}{4.58}$ = 2.67m

Time taken to reach destination

$= \dfrac{R}{v} = \dfrac{2.67}{5}$ = 0.53 s

20. The aeroplane has to follow a path such that resultant of ν and V should be in line with PQ

Thus R = $\sqrt{v^2 - V^2}$ time taken to cover distance PQ

$= \dfrac{L}{\sqrt{v^2 - V^2}}$

Since velocity is the same during return also \ time taken to cover distance QP = $\dfrac{L}{\sqrt{v^2 - V^2}}$. Thus, total time

$= \dfrac{L}{\sqrt{v^2 - V^2}}$

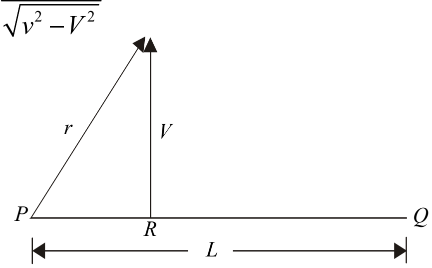


Fig. 2.56

21(c)Here $\vec{a} = x\hat{i} + x\hat{j}$ and $\vec{b} = x\hat{j} + x\hat{k}$ Since $\vec{R}$

$$= \vec{a} \times \vec{b} \text{ we get } \vec{R} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & x & 0 \\ 0 & x & x \end{vmatrix} = x^2\hat{i} - x^2\hat{j} + x^2\hat{k}$$

Clearly, the components are $R_x = x^2$, $R_y = -x^2$, $R_z = x^2$

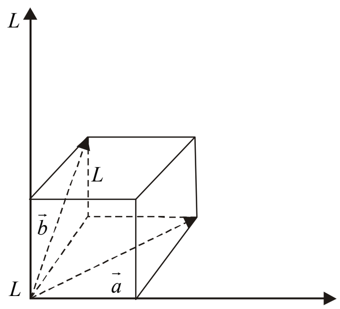


Fig. 2.57

22(d)Relative velocity of P and Q is ($v - u\cos\theta$). The particles will meet when

$$\int_0^T (v - u\cos\theta)\,dt = d \text{ and } \int_0^T v\cos\theta\,dt = d$$

and $\int_0^T \cos\theta\,dt = \dfrac{uT}{v}$ or $vT - u\dfrac{uT}{v} = d$ or $(v^2 - u^2)\,T = vd$

or $T = \dfrac{vd}{v^2 - u^2}$

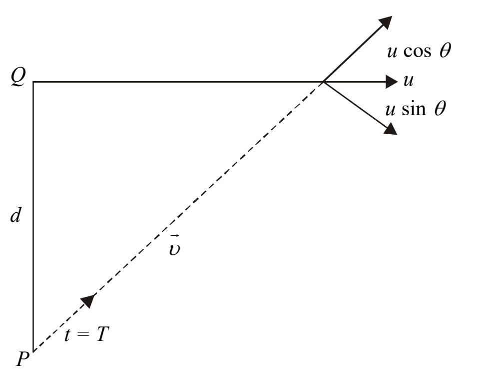


Fig. 2.58

23(d)Given, $a_r = a_t$ i,e., $R\omega^2 = R\alpha$ i,e., $R\omega^2 = R\dfrac{d\omega}{dt}$ or $\dfrac{d\omega}{dt} = \omega^2$ or $\dfrac{d\omega}{\omega^2} = dt$ i,e., $\int_{\omega_o}^{\omega} \dfrac{d\omega}{\omega^2} = \int_0^t dt$ i,e., ω

$$= \frac{\omega_o}{1 - \omega_{0'}}$$

i,e., $\dfrac{d\theta}{dt} = \dfrac{\omega_0}{1 - \cdot\omega_{0'}}$

i,e., $d\theta = \left(\dfrac{\omega_0}{1 - \omega_{0'}}\right)dt$ again, $\int_0^{2\pi} d\theta = \int_0^T \left(\dfrac{\omega_0}{1 - \omega_{0'}}\right)$

or $(1 - \omega_0 T) = e^{-2\pi}$ or $T = \dfrac{1}{\omega_0}(1 - e^{-2\pi})$

or $T = \dfrac{R}{v_0}(1 - e^{-2\pi})$

24(b) Speed of boat, $v_b = 5$ kmh^{-1}

Speed of boat when water flows, $vr = \dfrac{1}{1/4} = 4$ kmh^{-1}

Resultant speed $v = \sqrt{vb^2 - vr^2} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$

$= \sqrt{9}$

25(a)Comparing the given equation with $\vec{v} = \hat{i}\,v_x + \hat{j}\,v_y$,

we get $v_x = u_0$ and $\dfrac{dy}{dt} = a\omega\cos\omega t$ or $\dfrac{dx}{dt} = u_0$ and $\dfrac{dy}{dt} = a\omega\cos\omega t$. Integrating $x = \int u_0 dt$ and $y = \int a\omega \cos\omega dt$ or $x = u_0 t + c_1$ and $y = a\sin\omega t + c_2$ At $t = 0$, $x = 0$ and $y = 0$ we get c_1 and c_2 as zero $\therefore$ $x = u_0 t$ and $y = a\sin\omega t$ but $t = 3\pi/2\omega$ $\therefore$ $x = u_0(3\pi/2\omega)$ and $y = -a$. Then distance from origin, $d = \sqrt{x^2 + y^2}$

$= \sqrt{a^2 + (3\pi u_0/2\omega)^2}$

26(c)Here for A, $18 = 0 \times 3 + \dfrac{1}{2}a \times 3^2$ or $a = 4$ ms^{-2} for B, time taken to move up is given by, $t1 = u/a$ ($\therefore$ the relation $v = u + at$ here becomes $0 = u - at_1$). Distance moved up is given by the relation $0 = u^2 - 2aS$ i.e., $S = u^2/2a$ $S = \dfrac{1}{2}at2^2$ and $t_2 = \sqrt{\dfrac{2S}{a}}$ or $t_2 = \sqrt{\dfrac{2}{a}\dfrac{u^2}{2a}} = \dfrac{u}{a}$

But $\dfrac{u}{a} = t1$ hen $t1 + t2 = 3$ or $\dfrac{2u}{a} = 3$ or $u = \dfrac{3a}{2}$ or $u = \dfrac{3}{2} \times 4 = 6$ ms^{-1}

27(a)Change in velocity,

$\Delta\vec{v} = \vec{v}_2 + (-\vec{v}_1) = (5^2 + 5^2)^{1/2}$ NW $= 5\sqrt{2}$ ms^{-1} NW

Then $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{5\sqrt{2}}{10}$ NW $= \dfrac{1}{\sqrt{2}}$ ms^{-2} NW

28(a)For taking a circular turn to avoid. accident, the acceleration acquired will be $\dfrac{v^2}{d}$ Thus additional effort is required to meet this acceleration.

72(b) $x^2 = (1500 - 10t)^2 + (1800 - 15t)^2$, for x to be minimum,

its first derivative should be zero. Thus $\frac{dx}{dt} = 0$

$= -20(1500 - 10t) - 30(1800 - 15t)$

or $t = 129.23$ s

74(b) $\Delta v = 2v \sin \theta/2 = 2 \times \frac{2\pi}{60} \times \frac{1}{\sqrt{2}}$

84(b)Displacement of $= \pi R\,\hat{i} + 2R\,\hat{j}$

Displacement of $P = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4}$