12 Simple Harmonic Motion

BRIEF REVIEW

Harmonic or Periodic motion If a moving body repeats its motion after regular intervals of time, the motion is said to be **harmonic or periodic**. The time interval after which it repeats the motion is called **time period**. If the body moves **to and fro** on the same path, it is said to be **oscillation**. In simple harmonic motion the particle moves in a straight line or angle and the acceleration of the particle is always directed towards a fixed point on the line. This fixed point is called centre of oscillation. The acceleration in SHM is given by

$a = -\omega^2 x$ or $F = -m\omega^2 x$ or $F = -kx$

where $k = m\omega^2$ is called force constant or spring constant.

The force which brings the particle back towards the equilibrium or mean position is called restoring force. Such a motion is also called isochronous.

SHM may be thought of as a **projection of uniform circular motion along a diameter**

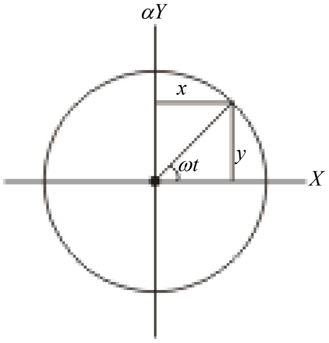


Fig. 12.1

$x = r\cos\omega t$; $y = r\sin\omega t$; $a = -\omega^2 x$

or $\dfrac{d^2x}{dt^2} = -\omega^2 x$ This differential equation gives the solution

$x = x_0 \sin\omega t$ (if the particle starts from mean position)

$x = x_0 \cos\omega t$ (if the particle starts from extreme position)

$x = x_0 \sin(\omega t \pm \phi)\}$ (if the particle starts in between mean and extreme position)

$x = x_0 \cos(\omega t \pm \phi)$.

The solution of differential equation in exponential form is $x = x_0\, e^{\pm(\omega t \pm \phi)}$.

Here x is instantaneous displacement, x_0 is amplitude (maximum displacement), ϕ is initial phase angle or epoch or angle of repose and, ω is angular frequency.

Linear frequency $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$; T being time period.

Velocity of the particle executing SHM

Assume $x = x_0 \sin\omega t$. then $v = \dfrac{dx}{dt} = x_0\omega\cos\omega t$

$v = x_0\omega\sqrt{1-\sin^2\omega t} = \omega\sqrt{x_0^2 - x^2}$

$\upsilon_{max} = x_0\omega$; $\upsilon_{min} = 0$ at extreme position

Fig. 12.2 (a) shows graph between velocity and displacement and Fig. 12.2 (b) shows the graph between velocity and time.

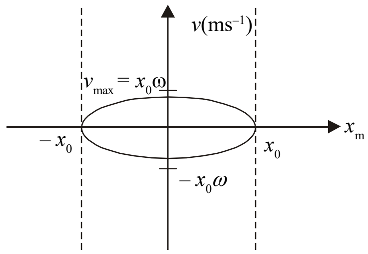


Fig. 12.2 (a) **Velocity — displacement graph**

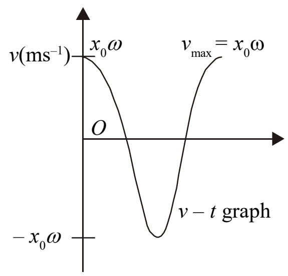


Fig. 12.2 (b) **Velocity — time graph**

Fig. 12.3 (a) and (b) shows graph between acceleration and displacement and acceleration and time.

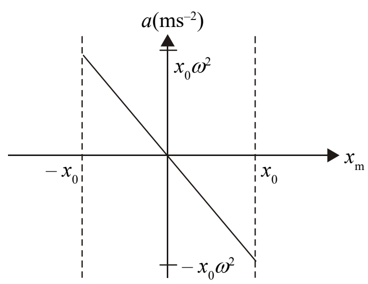


Fig. 12.3 (a) **Acceleration — displacement graph**

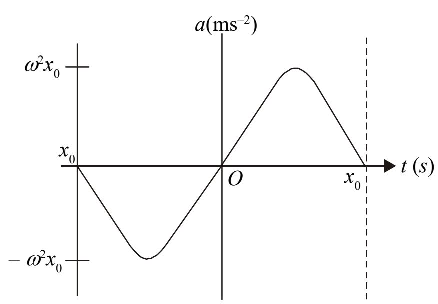


Fig. 12.3 (b) **Acceleration — time graph**

Note the graph between velocity and acceleration is an ellipse.

Note velocity leads the displacement by $\dfrac{\pi}{2}$ but velocity lags the acceleration by $\dfrac{\pi}{2}$

$$a_{max} = x_0\,\omega^2$$
$$\upsilon = x_0\omega\cos\,\omega t$$

$$\frac{dv}{dt} = -x_0\omega^2\sqrt{1-\cos^2\omega t} \text{ or } a = -\omega^2 x,\ a_{max} = \omega^2 x_0$$

$$a = -\,\omega\sqrt{(x_0\omega)^2 - (x_0\omega\cos\omega t)}$$

$$a = -\,\omega\sqrt{\upsilon_0^2 - \upsilon^2}$$

or $$\frac{a^2}{\omega^2\upsilon_0^2} + \frac{\upsilon^2}{\upsilon_0^2} = 1.$$

Note velocity is manimum at mean position and acceleration is zero at mean position. Velocity is zero at extreme position and acceleration is maximum at extreme position. Kinetic energy (*KE*) of a particle executing SHM = $\dfrac{1}{2}\,m\omega^2\,(x_0^{\,2} - x^2)$

Potential energy (*PE*) of a particle executing SHM = $\dfrac{1}{2}\,m\omega^2 x^2$

Total energy = *KE* + *PE* = $\dfrac{1}{2}\,m\omega^2 x^2$

Note *KE* is maximum at mean position and zero at extreme postion. *PE* is zero at mean position and maximum at extreme position. See Fig. 12.4.

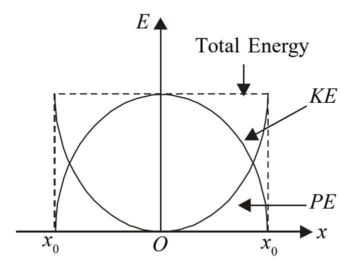


Fig. 12.4 **KE, PE and total energy depiction**

In SHM, velocity displacement curve is an ellipse. see Fig. 12.5 (a)

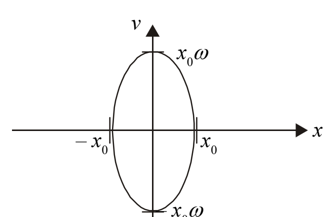


Fig. 12.5 (a) **Velocity displacement graph**

$$x = x_0\sin\,\omega t\,;$$
$$\upsilon = x_0\omega\cos\,\omega t$$

or $$\frac{x}{x_0} = \sin\,\omega t \text{ —(1);}$$

$$\frac{\upsilon}{x_0\omega} = \cos\,\omega t \text{ —(2)}$$

Square and add (1) and (2)

$$\frac{x^2}{x_0^2} + \frac{\upsilon^2}{x_0^2\omega^2} = 1$$

acceleration – velocity relationship in SHM is an ellispe

$$a = -\omega^2 x_0 \sin \omega t \quad ;$$
$$\upsilon = x_0\omega \cos \omega t$$

or $$\frac{a^2}{\omega^4 x_0^2} + \frac{\upsilon^2}{x_0^2\omega^2}$$ see Fig. 12.5 (b)

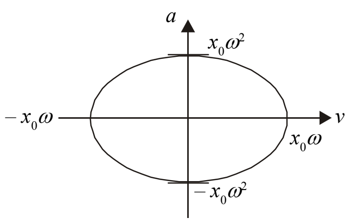


Fig. 12.5 (b) **Acceleration — velocity graph**

If a tunnel is dug in the earth diameterically or along a chord irrespective of its position or angle then $T = 2\pi\sqrt{\frac{R}{g}}$ = 84 min 36 s for a particle released in the tunnel. See Fig. 12.6

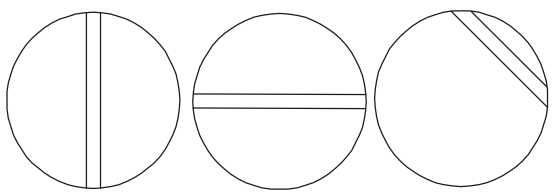


Fig. 12.6 **SHM in tunnel in the earth**

If a point charge q is tunnelled in a uniformly charged sphere having charge Q and radius R then

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 r^3 m}{Qq}}$$

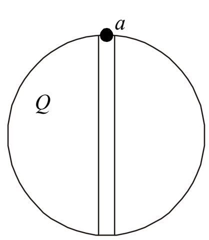


Fig. 12.7

Angular SHM A body free to rotate about a given axis can make angular oscillations when it is slightly pushed aside and released. The angular oscillations are called **angular SHM**.

(a) there is a mean position where the resultant torque on the body is zero ($\theta = 0$).

(b) the body is displaced through an angle from the mean position, a resultant torque $\propto \theta$ (angular displacement) acts.

(c) the nature of the torque (clockwise or anticlockwise) is to bring the body towards mean position.

$\tau = -k\,\theta$ *i.e.*

$$\alpha I = -k\,\theta \text{ or } \alpha = -\frac{k}{I}\,\theta$$

$$\alpha = -\omega^2\,\theta$$ or

$$\omega = \sqrt{\frac{k}{I}} \text{ or } T = 2\pi\sqrt{\frac{I}{k}}.$$

Solution of the equation $\alpha = -\omega^2\,\theta$ is

$\theta = \theta_0 \sin \omega t$ if the particle starts from mean position

$\theta = \theta_0 \cos \omega t$ if the particle starts from extreme position

$\theta = \theta_0 \sin(\omega t \pm \phi)$ if the particle starts from in between mean and extreme.

$$\theta = \theta_0 \cos(\omega t \pm \phi) \quad \Omega = \frac{d\theta}{dt} = \theta_0\,\omega \cos \omega t$$

or $$\frac{d\theta}{dt} = \omega\sqrt{\theta_0^2 - \theta^2}$$

Pendulums may be of 5 types: **simple pendulum**, **spring pendulum**, **conical pendulum**, **physical or compound** and **torsional pendulum**. Note the time period of each of them.

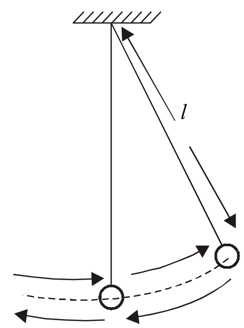


Fig. 12.8 (a) **Simple Pendulum**

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ if } \theta \text{ is small}$$

$$T = 2\pi\sqrt{\frac{I}{g}}\left[1 + \frac{\theta_0^2}{16}\right] \text{ if } \theta \text{ is finite and } \theta = \theta_0$$

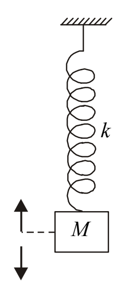


Fig. 12.8 (b) **Spring Pendulum**

$$T = 2\pi\sqrt{\frac{M}{k}}$$

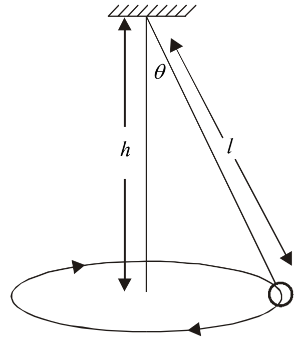


Fig. 12.8 (c) **Conical Pendulum**

$$T = 2\pi\sqrt{\frac{h}{g}}$$

or $$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

Note no effect of 'g' on spring pendulum.

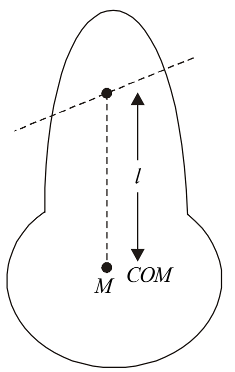


Fig. 12.8 (d) **Physical Pendulum**

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

or $$T = 2\pi\sqrt{\frac{k^2+l^2}{lg}} = 2\pi\sqrt{\frac{(k+l)^2+2kl}{lg}}$$

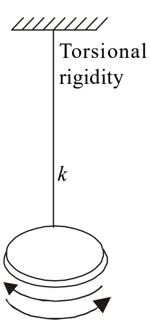


Fig. 12.8 (e) **Torsional Pendulum**

$$T = 2\pi\sqrt{\frac{I}{k}}$$

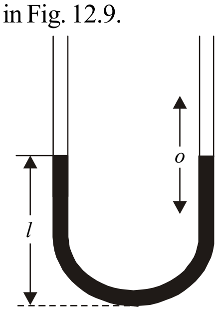
Note in physical pendulums T is maximum if $l = 0$ or $l = \infty$ and T is minimum if $k = l$.

Second's pendulum: If the time period of a simple pendulum is $2s$, it is called seconds.

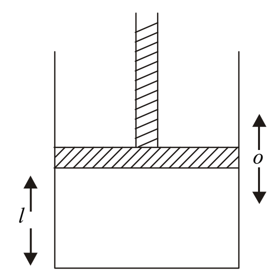
Longest time period (for $T = 2\pi\sqrt{\dfrac{1}{g\left(\dfrac{1}{l}+\dfrac{1}{R}\right)}}$ if $l \to \infty$ $T = 2\pi\sqrt{\dfrac{R}{g}}$ = 84 min. 36 s. for an infinitely long simple pendulum) where R is radius of the earth

If $l = R$, the radius of the earth then $T = 2\pi\sqrt{\dfrac{R}{2g}}$ = 60 min or 1h.

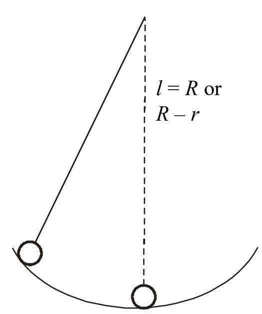
SHM under gravity If SHM occurs due to restoring force provided by weight or acceleration due to gravity then $T = 2\pi\sqrt{\dfrac{l}{g}}$. Some of the examples of this type are motion of a liquid in a U-tubevertical cylinder/piston. Motion of a ball in a concave mirror/ bowl and a floating cylinder as illustrated in Fig. 12.9.



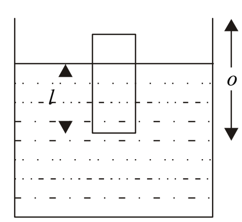
(a) $T = 2\pi\sqrt{\dfrac{l}{g}}$



(b) $T = 2\pi\sqrt{\dfrac{l}{g}}$



(c) $T = 2\pi\sqrt{\dfrac{R}{g}}$ if ball does not roll but slips. $T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}$ if the ball rolls.



(d) $T = 2\pi\sqrt{\dfrac{l}{g}}$

Fig. 12.9

Effect of temperature on time period of simple pendulum

$\frac{T}{T_0} = \left[1+\frac{\alpha\Delta\theta}{2}\right]$ where α is linear expansion coefficient and $\Delta\theta$ is rise in temperature. If temperature falls take $\Delta\theta$ negative.

or $\Delta T = T_0 \frac{\alpha\Delta\theta}{2}$

If the up thrust of the liquid is taken into account Then time period $T = 2\pi\sqrt{\frac{l}{g(1-\frac{\sigma}{\delta}}}$ and $a = g' = g\left(1-\frac{\sigma}{\delta}\right)$ where σ is density of liquid and δ is density of the body. Damping of liquid is assumed negligible.

If the supended wire stretches due to elasticity then time period $T' = 2\pi\sqrt{\frac{l}{g}}\left[1+\frac{Mg}{2\pi r^2 Y}\right]$ or $\Delta T = 2\pi\sqrt{\frac{l}{g}}\ \frac{Mg}{2\pi r^2 Y}$

or $\Delta T = T\ \frac{Mg}{2\pi r^2 Y}$ where $T = 2\pi\sqrt{\frac{l}{g}}$ and Y is young's modulus.

If a carriage (lift) is moving up with an acceleration '*a*' carrying a pendulum then $T = 2\pi\sqrt{\frac{l}{g+a}}$

If the carriage (lift) moves down with an acceleration '*a*' carrying the pendulum then

$$T = 2\pi\sqrt{\frac{l}{(g-a)}}$$

If the carriage moves horizontally (e.g. a car) with an acceleration '*a*' then $T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

If the carriage is in circular motion of radius R with uniform speed υ then

$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{\upsilon^2}{r}\right)^2}}}$$

If the bob of a pendulum is charged and is placed in a uniform electric field [charge q on the bob is assumed + ve in Fig 12.10 (a) and 12.10 (b)]

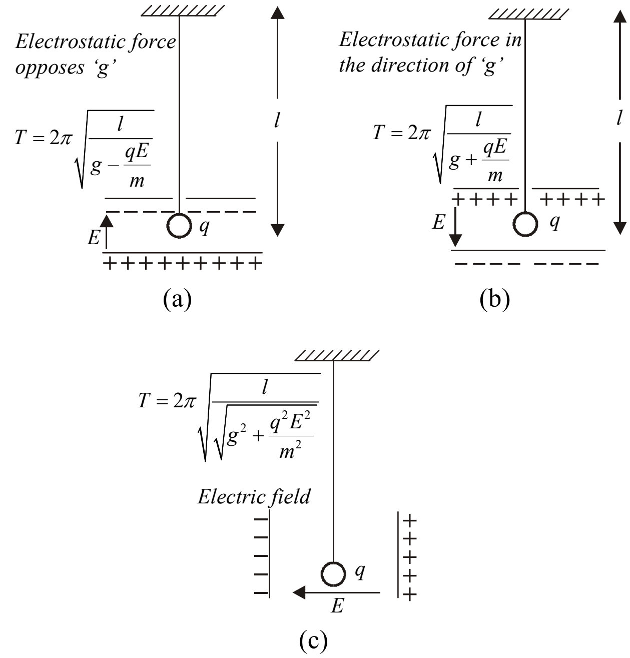


Fig. 12.10

For Spring System:

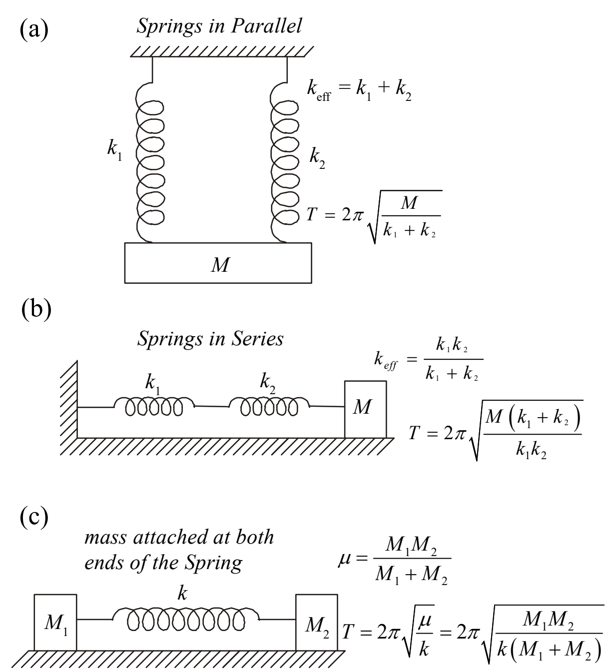
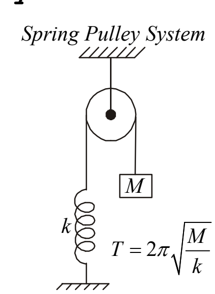


Fig. 12.11

Spring Pulley System:

(a)



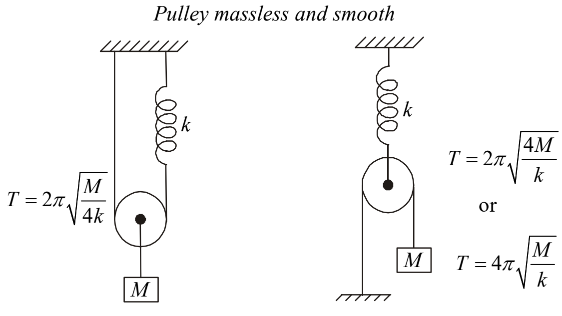


Fig. 12.12

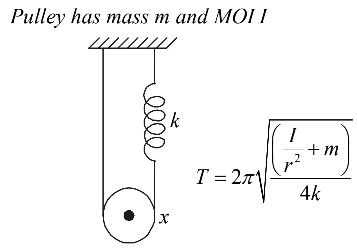


Fig. 12.13

Composition of two SHMs in same direction

$x_1 = x_{01} \sin \omega t$;

$x_2 = x_{02} \sin (\omega t + \theta)$

$x = x_0 \sin (\omega t + \phi) = x_1 + x_2 = x_{01} \sin \omega t + x_{02} \sin (\omega t + \theta)$

$x_0 = \sqrt{x_{01}^{\ 2} + x_{02}^2 + 2x_{01}x_{02}\cos\theta}$ and $\tan \phi$

$$= \frac{x_{02}\sin\theta}{x_{01} + x_{02}\cos\theta}$$

Note SHMs can be added like vectors. Result is same as parallelogram Law.

Composition of two perpendicular directions give rise to Lissajous figures.

$$x = x_0 \sin \omega t \text{ or } \sin \omega t = \frac{x}{x_0} \text{ and } \cos \omega t = \sqrt{1-\frac{x^2}{x_0^2}}$$

$y = y_0 \sin (\omega t + \phi) = y_0 \sin \omega t \cos \phi + y_0 \cos \omega t \sin \phi$

$$y = y_0 \frac{x}{x_0} \cos \phi + y_0 \sqrt{1-\frac{x^2}{x_0^2}} \sin \phi$$

or $$\left(\frac{y}{y_0} - \frac{x}{x_0}\cos\phi\right)^2 = \left(1-\frac{x^2}{x_0^2}\right) \sin^2 \phi$$

or $$\frac{y^2}{y_0^2} + \frac{x^2}{x_0^2} - \frac{2xy}{x_0 y_0} \cos \phi = \sin^2 \phi$$

If $\phi = 0$ $\left(\frac{y}{y_0} - \frac{x}{x_0}\right)^2 = 0$ or $y = \frac{y_0}{x_0} x$, see fig. 12.14 (a)

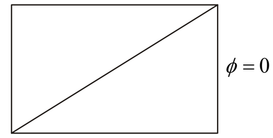


Fig. 12.14 (a)

If $0 < \phi < \frac{\pi}{2}$ for example $\phi = \frac{\pi}{4}$, oblique ellipse as shown in Fig. 12.14 (b) is obtained.

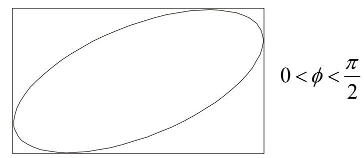


Fig. 12.14 (b)

If $\phi = \frac{\pi}{2}$, ellipse is obtained and if $x_0 = y_0$ the circle is obtained. See Fig. 12.15 (a) and (b)

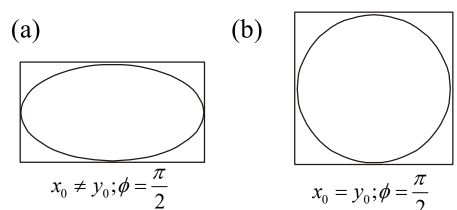


Fig. 12.15

If $\phi = 180°$ or π-radian then a straight line is obtained.

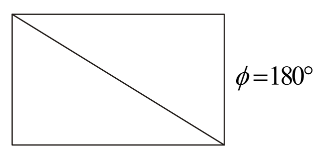


Fig. 12.16

Lissajous figures if the frequency of SHM in x-and y-direction are different then in Fig 12.17 (a)

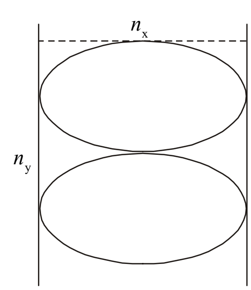


Fig. 12.17 (a)

$$\frac{\omega_x}{\omega_y} = \frac{n_y}{n_x}$$

$$= \frac{\text{number of times it touches y-axis}}{\text{number of times it touches x-axis}}$$

$= \frac{2}{1}$ and in Fig 12.17 (b)

$$\frac{\omega_x}{\omega_y} = \frac{2.5}{1}$$

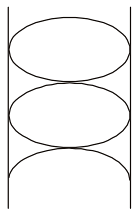


Fig. 12.17 (b)

Types of oscillations Oscillations may be of four types

(a) free or natural or fundamental frequency.

(b) forced.

(c) resonant.

(d) damped.

Free or natural oscillations depend upon dimensions and nature of the material (elastic constant).

If a periodic force of frequency other than the natural frequency of the material is applied then **forced oscillations result.**

For example if $y = y_0 \sin \omega t$ was the equation of SHM of a particle and a periodic force $p \sin \omega_1 t$ is applied $(\omega \neq \omega_1)$ then $y = y_0 \sin \omega t + p \sin \omega_1 t$. The resultant frequency is different from natural frequency of oscillation

Resonant oscillation are a special kind of forced oscillation in which frequency of the source = frequency of the applied force, i.e., $y = y_0 \sin \omega t + p \sin \omega t = (y_0 + p) \sin \omega t$. That is amplitude increases or intensity increases with resonance.

In damped oscillations amplitude of vibrations falls with time as shown in Fig. 12.18.

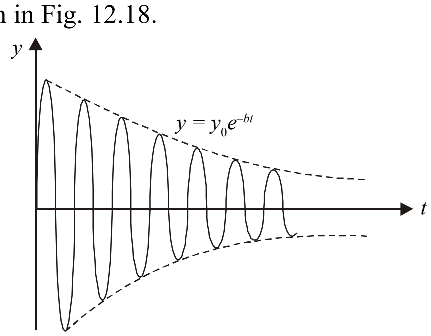


Fig. 12.18

Amplitude at any instant is given by $= y = y_0 e^{-bt}$ where y_0 is amplitude of first vibration and y is amplitude at time t and b is damping coefficient.

Damped harmonic motion

$$\frac{md^2x}{dt^2} + r\frac{dx}{dt} + kx = 0$$

or $$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

or $$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x = 0$$

where $b = \frac{r}{2m}$ is called damping coefficient.

$$x = \frac{x_0}{2}e^{-bt}\left[\left(1+\frac{b}{\sqrt{b^2-\omega^2}}\right)e^{t\sqrt{b^2-\omega^2}} + \left(1-\frac{b}{\sqrt{b^2-\omega^2}}\right)e^{-t\sqrt{b^2-\omega^2}}\right]$$

gives amplitude at any instant.

If $\frac{r}{2m} > \sqrt{\frac{k}{m}}$ or $b > \omega$ motion is over damped and non-oscillatory

If $\frac{r}{2m} = \sqrt{\frac{k}{m}}$ or $b = \omega$ motion is critically damped and $x = x_0 e^{-bt}$

If $\frac{r}{2m} < \sqrt{\frac{k}{m}}$ $b < \omega$ damped oscillatory motion with

time period $T = \frac{2\pi}{\sqrt{\omega^2 - b^2}} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}}$

If $r = 0$ motion is undamped and $T = 2\pi\sqrt{\frac{m}{k}}$.

SHORT CUTS AND POINTS TO NOTE

1. Periodic motion may also be termed as isochronous. Fourier theorem can be employed to express a complex perodic function as series of sine and cosine functions. That is, if $f(T)$ is a complex function of time then $f(T) = a_0 + \sum_{n=1}^{\infty} a_n \sin n\omega t + \sum_{n=1}^{\infty} b_n \cos n\omega t$.

2. SHM may be divided into two types

   (a) Linear (b) Angular.

   In Linear SHM

   $a = -\omega^2 x$ or $F = -kx$.

   In angular SHM

   $\alpha = -\omega^2\theta$

or $\tau = -C\theta$. Note in both cases acceleration is proportional to displacement.

3. Solution to equation $\frac{d^2x}{dt^2} = -\omega^2 x$ are (i) $x = x_0 \sin\omega t$ if motion starts form mean position at $t = 0$ (ii) $x = x_0 \cos\omega t$ if motion starts from extreme position at $t = 0$

$$\left.\begin{array}{l} x = x_0 \sin(\omega t \pm \phi) \\ x = x_0 \cos(\omega t \pm \phi) \end{array}\right\} \begin{array}{l}\text{if the motion starts in between} \\ \text{mean and extreme at } t = 0\end{array}$$

We may also represent $x = x_0\, e^{i\omega t \pm \phi}$ as SHM in exponential form.

4. If $x = x_0 \sin\omega t$ then $\upsilon = \frac{dx}{dt} = x_0\omega\cos\omega t$ use these equations when time is given in the problems.

If $x = x_0 \cos\omega t$ then $\upsilon = \frac{dx}{dt} = -x_0\omega \sin\omega t$

If displacement is given then $\upsilon = \omega\sqrt{x_0^2 - x^2}$.

Note $\upsilon_{max} = x_0\omega$ at mean position. $\upsilon = 0$ at extreme position. Velocity displacement graph is ellipse.

5. Use $a = -\omega^2 x$ if displacement is given. Acceleration displacement graph is a straight line with obtuse angle slope.

Use $a = -\omega^2 x_0 \sin\omega t$ if time is known and particle starts from mean position at $t = 0$

Use $a = -\omega^2 x_0 \cos\omega t$ if time is known and particle starts from extreme position at $t = 0$.

$a_{max} = -\omega^2 x_0 \cos\omega t$ at extreme position. $a_{min} = 0$ at mean position. acceleration – velocity graph is an ellipse.

6. $KE = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$ KE is maximum at mean position $KE_{max} = \frac{1}{2} m\omega^2 x_0^2$ KE is minimum at mean position $KE_{min} = 0$.

7. The frequency of KE or PE is twice the frequency of SHM.

8. PE $= \frac{1}{2} m\omega^2 x$. PE is maximum at extreme position when $x = x_0$. $PE_{max} = \frac{1}{2} m\omega^2 x_0$. PE is minimum at mean position. From this equation it appears $PE = 0$ if $x = 0$, i.e., at mean position. But it is not necessary that PE at mean position be zero. For example, if the bob of a pendulum is at a height h at $x = 0$, i.e, pendulum has some PE at mean position. Thus, general equation of $PE = \frac{1}{2} m\omega^2 x$ + positional PE.

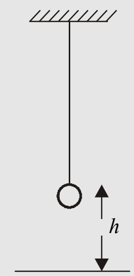


Fig. 12.19

9. If a tunnel is dug in earth diametrically or along a chord then $T = 2\pi\sqrt{\frac{R}{g}}$ along the tunnel.

However, if the ball is released from a height h along the tunnel as shown in Fig 12.20 Then

$$T = 4\sqrt{\frac{2h}{g}} + 4\sqrt{\frac{R}{g}} \sin^{-1}\left[\frac{h}{R+2h}\right].$$

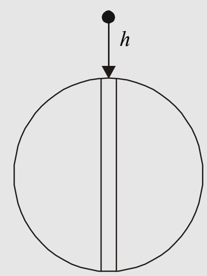


Fig. 12.20

10. If a charged paritcle having charge q is released in a tunnel in a charged solid sphere of charge Q and radius R then

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 R^3 m}{Qq}}.$$

11. If a dipole of moment p is suspended in a uniform electric field then time period for small oscillation is $T = 2\pi\sqrt{\frac{I}{pE}}$ where I is moment of inertia.

12. If a magnetic dipole of moment M is suspended in a magnetic field of induction B then time period

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

13. For a simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ if } \theta \text{ is small}$$

for a pendulum with finite angle $\theta = \theta_0$

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{\theta_0^2}{16}\right)$$

If the bob of the pendulum has radius r

$$T = 2\pi\sqrt{\frac{l^2+\frac{2}{5}r^2}{lg}}$$

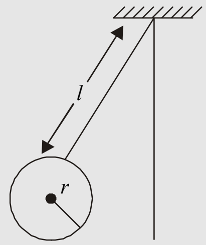


Fig. 12.21

Time period of a long pendulum

$$T = 2\pi\sqrt{\frac{1}{\left(\frac{1}{l}+\frac{1}{R}\right)g}}$$

if $l \to \infty$

$$T = 2\pi\sqrt{\frac{R}{g}} = 84\text{ min }36\text{s}.$$

14. For a physical or compound pendulum

$$T = 2\pi\sqrt{\frac{I}{mgl}} = \sqrt{\frac{k^2+l^2}{gl}}$$

where l is distance of axis rotation from COM.

where k is radius of gyration.

The plot timeperiod versus displacement from axis of rotation in a bar pendulum (or a compound pendulum) is shown in fig. 12.22.

Note T is maximum if $l = \infty$ or $l = 0$

$$T_{max} = \infty$$

T is minimum if $l = k$

$$T_{min} = 2\pi\sqrt{\frac{2kl}{lg}}.$$

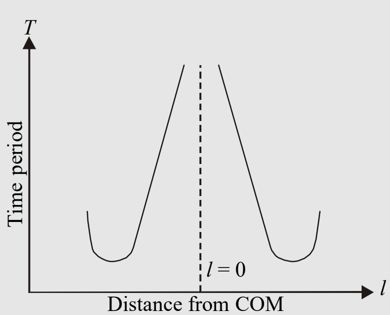


Fig. 12.22

15. For a conical pendulum

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}$$. See Fig. 12.23.

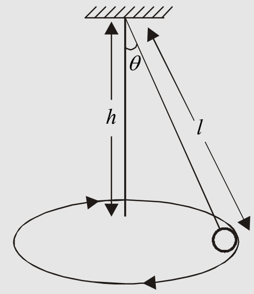


Fig. 12.23

16. For a torsional pendulum

$$T = 2\pi\sqrt{\frac{T}{K}}$$ where K is Torsional rigidity.

17. In a spring pendulum

$$T = 2\pi\sqrt{\frac{M}{R}}$$ if spring has mass M_s then

$$T = 2\pi\sqrt{\frac{M+\frac{M_s}{3}}{k}}$$

If springs are in parallel, use $k_{eff} = k_1 + k_2 + ...$

If springs are in series, use $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + ...$

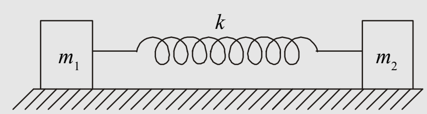


Fig. 12.24

If masses M_1 and M_2 are attached to two ends of a spring of spring constant k. The spring is compressed by x and released to oscillate. Use reduced mass in such cases.

$$\frac{1}{\mu} = \frac{1}{M_1} + \frac{1}{M_2}$$

or $$\mu = \frac{M_1M_2}{M_1+M_2}$$

$\therefore$ $$T = 2\pi\sqrt{\frac{\mu}{k}}$$

$$= 2\pi\sqrt{\frac{M_1M_2}{(M_1+M_2)k}}.$$

18. In a spring pulley system:

Fig. 12.25 and Fig. 12.26

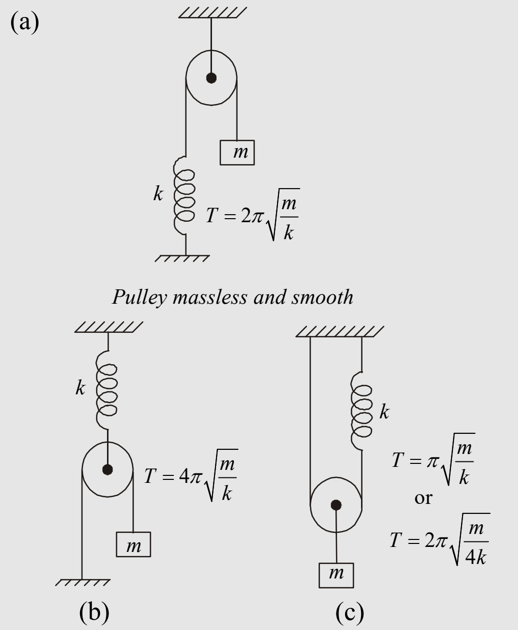


Fig. 12.25

If pulley has mass m and MOI I then

using conservation of energy and differentiating energy to find force and relating $F = -kx$ we find ω or T

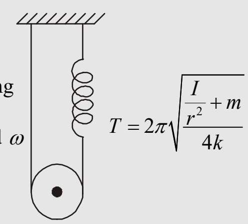


Fig. 12.26

19. SHM of liquid in U-tube:

In Fig. 12.27 (a)

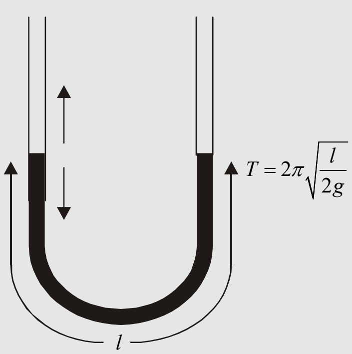


Fig. 12.27 (a)

If one side of liquid has length as shown in Fig 12.27 (b) then

$$T = 2\pi\sqrt{\frac{l}{g}}$$

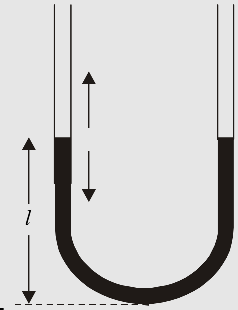


Fig. 12.27 (b)

20. SHM in cylinder – piston In a vertical cylinder

$$T = 2\pi\sqrt{\frac{l}{g}}$$ as shown in Fig. 12.28.

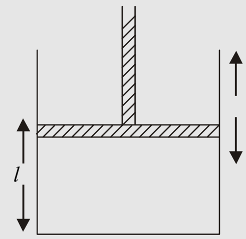


Fig. 12.28 (a)

In a horizontal cylinder/piston system having a gas of bulk modulus B or pressure P. Volume V_0

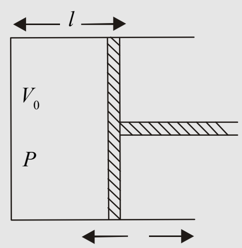


Fig. 12.28 (b

$$T = 2\pi\sqrt{\frac{MV_0}{A^2B}} = 2\pi\sqrt{\frac{MV_0}{A^2\gamma P}}$$

In (adiabatic conditions)

$$\gamma = \frac{C_P}{C_V}$$

$$T = 2\pi\sqrt{\frac{MV_0}{A^2P}} = 2\pi\sqrt{\frac{Ml}{AP}}$$

In (isothermal conditions).

21. If pendulum is in a carraige moving vertically up or down with an acceleration a, then $T = 2\pi\sqrt{\frac{l}{g \pm a}}$

use $+ve$ sign for upward motion and use $-ve$ sign for downword motion.

22. If the carriage is accelerated horizontally with an acceleration 'a' than $T = 2\pi\sqrt{\dfrac{l}{(g^2+a^2)^{1/2}}}$

23. If the carriage moves in a circle of radius r with velocity v, i.e., it is in a merry-go-round then

$$T = 2\pi\sqrt{\frac{l}{\left(g^2+\left(\frac{v^2}{r}\right)^2\right)^{1/2}}}$$

24. If the sphere of radius r slides on a concave mirror of radius R then

$$T = 2\pi\sqrt{\frac{(R-r)}{g}}$$

25. If the sphere of radius r rolls on a concave mirror of radius R then

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$. See Fig. 12.29.

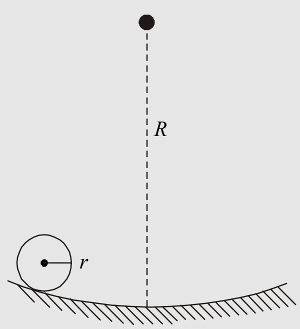


Fig. 12.29

26. If the bob of a pendulum is immersed in a liquid (non viscous) $T = 2\pi\sqrt{\dfrac{l}{g\left(1-\dfrac{\sigma}{\rho}\right)}}$

27. SHM in the same direction but with phase difference are added like vectors. Thus, if $x_1 = x_{01}\sin\omega t$ and $x_2 = x_{02}\sin(\omega t+\theta)$ are added then resultant is an SHM

$x = x_0\sin(\omega t+\phi)$. So that

$$x_0 = \sqrt{x_{01}^2+x_{02}^2+2x_{01}x_{02}\cos\theta} \text{ and}$$

$$\tan\phi = \frac{x_{02}\sin\theta}{x_{01}+x_{02}\cos\theta}.$$

28. If two SHMs are at right angles i.e. along x and y directions and a phase shift of 0° or 180° exists between them they form a straight line otherwise an ellipse is formed. If ϕ= 90° or $\frac{\pi}{2}$ radian and $x_0 = y_0$ (amplitudes are equal) then a circle results.

29. If the frequency of SHMs in x and y directions are different, Lissajous figures are formed

$$\frac{\omega_x}{\omega_y} = \frac{n_y}{n_x} = \frac{\text{number of times it touches y-axis}}{\text{number of times it touches x-axis}}$$

phase difference ϕ can be found from oblique ellipse as shown in Fig. 12.30 in Fig.

$$\phi = \sin^{-1}\frac{a}{b}$$

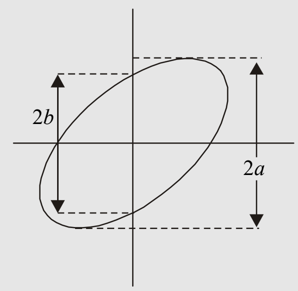


Fig. 12.30

30. Oscillation are of four types: free, forced, resonant and damped. Resonant oscillations are a special kind of forced oscillations in which frequency of force = frequency of source. In damped oscillation amplitude at any instant is obtained using $x_0' = x_0 e^{-bt}$ where x_0 is amplitude of first oscillation.

31. Quality factor $Q = 2\pi\dfrac{\text{Average energy stored}}{\text{energy loss in one cycle}}$ $= \omega_0\tau$ where τ is relaxation time. Relaxation time $= \dfrac{m}{b}$ (for energy). Relaxation time for velocity = $\dfrac{2m}{b}$.

32. If $b = \dfrac{r}{2m} > \omega\left[=\sqrt{\dfrac{k}{m}}\right]$ motion is non oscillatory and overdamped

If $b = \omega$ motion is critically damped.

If $b < \omega$ damped oscillatory motion occurs.

If $r = 0$ undamped oscillations result.

CAUTION

1. Considering every vibratory motion as SHM.

⇒ Only those vibratory motions in which $a = -\omega^2 x$ or $\alpha = -\omega^2\theta$ are SHMs. Note in SHMs amplitude x_0 or θ_0 are extremely small. Force is directed towards mean or equilibrium position.

2. Assuming that decreasing amplitude with time in simple pendulum also decreases time.

⇒ Time period remains unchanged.

3. Considering amplitude synonym of span of SHM.

⇒ Span of SHM = twice the amplitude.

4. Considering ϕ as net phase in $x = x_0 \sin(\omega t + \phi)$

⇒ ϕ is initial phase at $t = 0$. It is also called angle of repose or epoch. Note ϕ should be small if motion is to be SHM.

5. Considering that the time periods in case of spring pendulums on different planes like the one on inclined plane, other on vertical plane and yet another on horizontal plane are different.

⇒ Time period in spring pendulum is independent of '*g*'.

6. Considering total length as length of pendulum in a compound or physical pendulum.

⇒ length from COM to point of suspension is to be used.

7. Assuming at half the amplitude time will be $\frac{1}{8}$ th of the total time period.

⇒ $t = \frac{T}{12}$ if the particle starts from mean position,

and, $t = \frac{T}{6}$ if the particle starts from extreme position.

8. Assuming spring constant remains invariant when spring is cut.

⇒ Spring constant $k \propto \frac{1}{l}$.

9. Considering total energy = *KE* + *PE*.

⇒ Total energy = *KE* + *PE* + resting energy (positional *PE*)

10. Assuming Average *PE* = Average *KE* always.

⇒ Average *KE* = Average *PE* = $\frac{1}{4} m \omega^2 x_0^2$ (with respect to time averaged over time period).

Average *KE* = 2 Average *PE* with respect to position. Average *KE* with respect to position = $\frac{1}{3} m\omega^2 x_0^2$ and Average *PE* with respect to position = $\frac{1}{6} m\omega^2 x_0^2$.

11. Considering motion in *V*-tube is alike motion in *U*-tube.

⇒ In *V*-tube shown in Fig.. 12.31

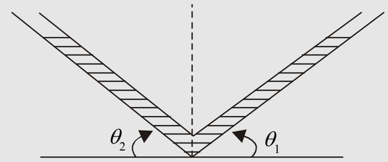


Fig. 12.31

$$T = 2\pi\sqrt{\frac{m}{A\rho g(\sin\theta_1 + \sin\theta_2)}}$$ where ρ is density of the liquid.

12. Considering that time period of a simple pendulum depends upon mass or amplitude as time period of a cylindrical bob having a hole at the base varies when the sand/water leaks through it.

⇒ $T = 2\pi\sqrt{\frac{l}{g}}$ time period varies with length *l* or *g*.

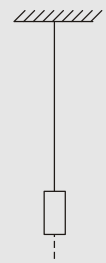


Fig. 12.32

When the sand/water is being vacated distance from COM (length) varies hence, *T* varies.

13. Considering mass of the spring does not affect time period.

⇒ If mass of the spring is M_S then net mass is $m + \frac{M_S}{3}$ and $T = 2\pi\sqrt{\frac{\frac{M_S}{3} + m}{k}}$.

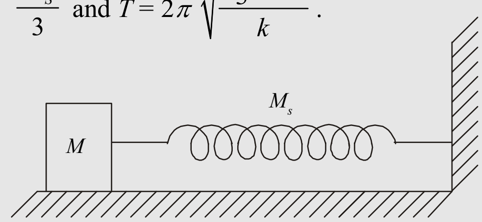


Fig. 12.33

14. Considering we cannot find frequency of oscillation using energy conservation.

⇒ In SHM total energy is conserved. Therefore, $\frac{dE}{dt} = 0$.

15. Considering that only one restoring force can act in SHM.

⇒ There can be more than one restoring force.

16. Not remembering trignometric relations.

⇒ Remember trignometric relations for better understanding of SHM and waves.

SOLVED PROBLEMS

1. The circular motion of a particle with constant speed is

(a) periodic but not SHM.

(b) SHM but not periodic.

(c) periodic and SHM.

(d) neither periodic nor SHM.

[CBSE PMT 2005]

Solution (a)

2. Two SHMs are represented by $y_1 = 0.1 \sin\left(100\,\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the particle 2 is

(a) $\frac{-\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{-\pi}{3}$ (d) $\frac{+\pi}{6}$

[AIEEE 2005]

Solution (a) $\phi = \frac{\pi}{3} - \frac{\pi}{2} = \frac{-\pi}{6}$

$\because \quad \cos \pi t = \sin\left(\frac{\pi}{2} + \pi t\right).$

3. Which of the following functions represent a simple harmonic motion?

(a) $\sin \omega t - \cos \omega t$ (b) $\sin^2 \omega t$

(c) $\sin \omega t + \sin^2 \omega t$ (d) $\sin \omega t - \sin^2 \omega t$

[AIIMS 2005]

Solution (a) $\because \ \frac{d^2 y}{dt^2} \propto y.$

4. A small body attached to one end of a vertically hanging spring is performing SHM about its mean position with angular frequency ω and amplitude a. If at height y from the mean position the body gets detatched from the spring. Calculate the value of y so that height obtained by the mass is maximum. The body does not interact with the spring during its subsequent motion after detachment.

[IIT Mains 2005]

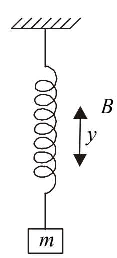


Fig. 12.34

Solution The point B should be such that $PE = 0$ at B so that KE is maximum and it can rise to a maximum height.

$$\therefore \quad y = \frac{mg}{k} = \frac{g}{\omega^2} < a \text{ (amplitude)}$$

5. From the displacement time graph of an oscillating particle. Find the maximum velocity of the particle.

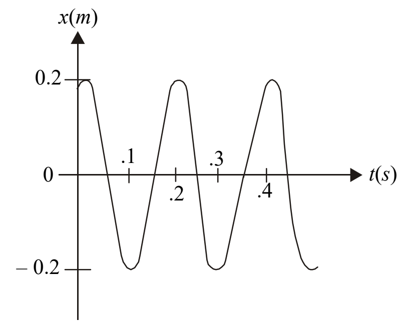


Fig. 12.35

(a) 2 ms^{-1} (b) π ms^{-1}

(c) 2π ms^{-1} (d) $\frac{\pi}{2}$ ms^{-1}

Solution (c) $\upsilon_{\max} = x_0\,\omega = 0.2\left(\frac{2\pi}{0.2}\right) = 2\pi\,\text{ms}^{-1}$

6. A machine part is undergoing SHM with a frequency of 5 Hz and amplitude 1.8 cm. How long does it take the part to go from $x = 0$ to $x = -1.8$ cm?

(a) $\frac{1}{20}$ s (b) $\frac{1}{15}$ s

(c) $\frac{1}{10}$ s (d) $\frac{1}{30}$ s

Solution (a) $t = \frac{T}{4} = \frac{1}{5 \times 4} = \frac{1}{20}$ s.

7. A 42.5 kg chair is attached to a spring. It takes 1.3 s to make one complete oscillation when the chair is empty. When a lady is sitting on the chair with her feet off the floor, the chair now takes 1.84 s for one cycle. The mass of the lady is

(a) 35.5 kg (b) 40.5 kg

(c) 42.5 kg (d) 45 kg

Solution (b) $T = 2\pi\sqrt{\frac{M}{k}}$

$$\text{or} \quad \left(\frac{T_2}{T_1}\right)^2 = \frac{M + M_{\text{Lady}}}{M}$$

$$\Rightarrow \left(\frac{1.84}{1.3}\right)^2 = (1.40)^2 = 1.96 = 1 + \frac{M_{\text{lady}}}{42.5}.$$

$$M_{\text{lady}} = 42.5 \times 0.96 = 40.5 \text{ kg}$$

8. Find the velocity when $KE = PE$ of the body undergoing SHM. Amplitude $= x_0$ and angular frequency is ω. How many times in a cycle $KE = PE$.

(a) $\frac{\omega x_0}{\sqrt{2}}$, 2 (b) ωx_0, 2

(c) $\frac{\omega x_0}{\sqrt{2}}$, 4 (d) ωx_0, 4

Solution (c) $\frac{1}{2} m \omega^2 (x_0^2 - x^2) = \frac{1}{2} m \omega^2 x^2$

or $x = \frac{x_0}{\sqrt{2}}$

$$V = \omega \sqrt{x_0^2 - x^2} = \frac{\omega x_0}{\sqrt{2}}.$$

This will be achieved 4 times in a cycle.

9. In the Vander Waals interaction

$$U = U_0 \left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^6\right].$$

A small displacement x is given from equilibrium position $r = R_0$. Find the approximate PE function.

(a) $\frac{36U_0}{R_0^2} x^2 - U_0$ (b) $\frac{24U_0}{R_0} x - U_0$

(c) $\frac{96U_0}{R_0^2} - U_0$ (d) none of these

Solution (a) $U = U_0 \left[\left(\frac{R_0}{R_0 + x}\right)^{12} - 2\left[\frac{R_0}{R_0 + x}\right]^6\right]$

$$= U_0 \left[\frac{R_0^{12}}{R_0^{12}}\left[\frac{1}{1+\frac{x}{R_0}}\right]^{12}\right] - 2\left[\frac{1}{1+\frac{x}{R_0}}\right]^6$$

$$= U_0 \left[\left(1+\frac{x}{R_0}\right)^{-12} - 2\left[1+\frac{x}{R_0}\right]^{-6}\right]$$

$$= U_0 \left[\left[1 - \frac{12x}{R_0} + \frac{66x^2}{R_0^2}\right] - 2\left(1 - \frac{6x}{R_0} + \frac{15x^2}{R_0^2}\right)\right]$$

$$= \frac{36U_0}{R_0^2} x^2 - U_0$$

10. A hydrogen atom has mass 1.68×10^{-27} kg. When attached to a certain massive molecule it oscillates with a frequency 10^{14} Hz and with an amplitude 10^{-9} cm. Find the force acting on the hydrogen atom.

(a) 2.21×10^{-9} N (b) 3.31×10^{-9} N

(c) 4.42×10^{-9} N (d) 6.63×10^{-9} N

Solution (d) $\omega^2 = \frac{k}{m}$ or $4\pi^2 f^2 m = k$

or $F = kx_0 = 4\pi^2 f^2 m x_0$

or $F = 4 \times \pi^2 \times 10^{28} \times 1.68 \times 10^{-27} \times 10^{-11}$

$= 6.63 \times 10^{-9}$ N.

11. An unhappy mouse of mass m_0, moving on the end of a spring of spring constant p is acted upon by a damping force $F_x = -b\, \upsilon_x$. For what value of b the motion is critically damped.

(a) $b = \sqrt{\frac{p}{m_0}}$ (b) $b = 2\sqrt{pm_0}$

(c) $b = \sqrt{\frac{p^2}{2m_0}}$ (d) $b = \sqrt{\frac{p}{2m_0}}$

Solution (b) $\frac{b}{2m_0} = \sqrt{\frac{p}{m_0}}$

or $b = 2\sqrt{pm_0}$.

12. Using equation $x = A\, e^{\frac{-bt}{2m}}\, t \cos(\omega' t + \phi)$ and assuming $\phi = 0$ at $t = 0$, find the expression for acceleration at $t = 0$

(a) $A\left[\frac{k}{m} - \frac{b^2}{4m^2}\right]$ (b) $A\left[\frac{k}{m} + \frac{b^2}{4m^2}\right]$

(c) $A\left[\frac{b^2}{4m^2} - \frac{k}{m}\right]$ (d) $A\left[\frac{b}{2m} - \frac{k}{m}\right]$

Solution (c) $\frac{dx}{dt} = \frac{-b}{2m} A\, e^{\frac{-bt}{2m}} \cos(\omega' t) - A\, e^{\frac{-bt}{2m}}$

$\omega' \sin(\omega' t)$.

$$\frac{d^2x}{dt^2} = -\frac{-Ab}{2m}\left[\frac{-b}{2m}e^{\frac{-bt}{2m}}\cos(\omega' t) - e^{\frac{-bt}{2m}}\omega'\sin(\omega' t)\right]$$

$$+\frac{bA}{2m}\,\omega' e^{\frac{bt}{2m}}\sin(\omega' t+\phi) - A\,\omega'^2\,e^{\frac{-bt}{2m}}$$

$$\cos(\omega' t+\phi)\quad \left.\frac{d^2x}{dt^2}\right|_{t=0}$$

$$= -A\,\omega'^2 + A\,\frac{b^2}{4m^2} = A\left[\frac{b^2}{4m^2} - \frac{k}{m}\right].$$

13. A glider is oscillating in SHM on an air track with an amplitude A. You slow it so that its amplitude becomes half. Find the total mechanical energy in terms of previous value.

(a) $1/2$ (b) $1/3$

(c) $1/\sqrt{5}$ (d) $1/4$

Solution (d) initial total energy $= \frac{1}{2}\, m\,\omega^2 A^2$;

final total energy $= \frac{1}{2}\, m\,\omega^2 \left(A/2\right)^2$.

$\therefore$ Total mechanical energy becomes ¼ th.

14. A block of mass M is connected to one end of a spring of spring constant k. The other end is connected to the wall. Another block of mass m is placed on M. The coefficient of static friction is μ. Find the maximum amplitude of oscillation so that block of mass m does not slip on the lower block.

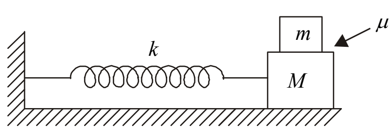


Fig. 12.36

(a) $\frac{\mu m g}{k}$ (b) $\frac{\mu M g}{k}$

(c) $\frac{\mu(M+m)g}{k}$ (d) $\mu\left(\frac{M}{M+m}\right)g\Big/k$

Solution (c) $\mu\,(M+m)\,g = k\,x_0$

or $x_0 = \frac{\mu(M+m)g}{k}$.

15. A hanging wire is 185 cm long having a bob of 1.25 kg. It shows a time period of 1.42 s on a Planet Newtonia. If the circuimference of Newtonia is 51400 km, find the mass of the planet.

(a) 3.5×10^{25} kg (b) 9.08×10^{24} kg

(c) 2.6×10^{25} kg (d) 3.14×10^{24} kg

Solution (a) $T = 2\pi\sqrt{\frac{l}{g}}$ or $g = \frac{4\pi^2 l}{T^2}$ and

$$\frac{GM}{R^2} = \frac{4\pi^2 l}{T^2}$$

or $$M = \frac{4\pi^2 l R^2}{GT^2}.$$

$$M = \frac{4\times10\times1.85\times(8185)^2\times10^6}{6.67\times10^{-11}\times(1.42)^2}$$

$$= 3.5\times10^{25}\text{ kg}$$

16. A uniform rod of length l mass m is fixed at the centre. A spring of spring constant k is connected to rod and wall as shown in Fig. 12.37. The rod is displaced by small angle θ and released. Find time period of oscillation.

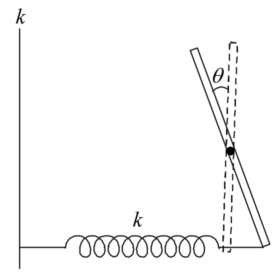


Fig. 12.37

(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{m}{2k}}$

(c) $2\pi\sqrt{\frac{3m}{k}}$ (d) $2\pi\sqrt{\frac{m}{3k}}$.

Solution (d) $\tau = I\,\alpha = -(kx)\,\frac{l}{2}$

or $$\frac{ml^2}{12}\,\alpha = -\left(k\frac{l}{2}\theta\right)\frac{l}{2}.$$

or $$\alpha = -\frac{3k}{m}\,\theta$$

or $$\omega = \sqrt{\frac{3k}{m}}$$

or $$T = 2\pi\sqrt{\frac{m}{3k}}$$

17. A uniform rod of length L oscillates through small angles about a point x from its centre for what value of L its angular frequency will be maximum.

(a) $\frac{L}{12}$ (b) $\frac{L}{3}$

(c) $\frac{L}{\sqrt{8}}$ (d) $\frac{L}{\sqrt{12}}$

Solution (d) ω will be maximum if T is minimum. T is minimum if $x = k$ the radius of gyration

or $$x = \frac{L}{\sqrt{12}}.$$

18. Two solid cylinders connected with a short light rod about common axis have radius R and total mass M rest on a horizontal table top connected to a spring of spring constant k as shown. The cylinders are pulled to the left by x and released. There is sufficient friction for the cylinders to roll. Find time period of oscillation.

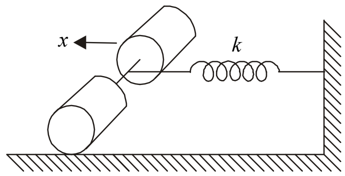


Fig. 12.38

(a) $2\pi\sqrt{\frac{M}{k}}$ (b) $2\pi\sqrt{\frac{M}{2k}}$

(c) $2\pi\sqrt{\frac{3M}{2k}}$ (d) $2\pi\sqrt{\frac{M}{3k}}$

Solution (c) $\tau = \left(\frac{MR^2}{2} + MR^2\right)\alpha = -kx \,.\, R$

or $$a = R\,\alpha = -\frac{2kx}{3M}$$

or $$\omega = \sqrt{\frac{2k}{3M}}$$

or $$T = 2\pi\sqrt{\frac{3M}{2k}}$$

19. A particle moves according to the equation $x = a\sin^2(\omega t - \frac{\pi}{4})$. Find the amplitude and frequency of oscillations.

(a) a, ω (b) $\frac{a}{2}, \omega$

(c) $\frac{a}{2}, \frac{\omega}{2}$ (d) $\frac{a}{2}, 2\omega$

Solution (d) $x = a\sin^2(\omega t - \frac{\pi}{4})$

$$= \frac{a}{2}\left[1 - \cos 2\left(\omega t - \frac{\pi}{4}\right)\right]$$

20. The superposition of two SHMs of the same direction results in the oscillation of a point according to the law $x = x_0\cos(2.1\,t)\cos 50\,t$. Find the angular frequencies of the constituent oscillations and period with which they beat.

(a) 52.1 s^{-1}, 47.9 s^{-1}, 0.2 s (b) 50 s^{-1}, 2.1 s^{-1}, 0.22 s

(c) 52.1 s^{-1}, 47.9 s^{-1}, 1.5 s (d) none

Solution (c) $x_0\cos(2.1\,t)\cos 50\,t$

$$= \frac{x_0}{2}[\cos 52.1\,t + \cos 47.9\,t].$$

$$f = \frac{\omega_1 - \omega_2}{2\pi}$$

or $$T = \frac{2\pi}{\omega_2 - \omega_1} = \frac{2\times\pi}{4.2} \sqcup 1.5\text{ s}$$

21. A point moves in the plane xy according to the law $x = a\sin\omega t$, $y = b\cos\omega t$. The particle has a trajectory ____.

(a) hyperbolic (b) elliptic

(c) circular (d) straight line

Solution (b) $\frac{x}{a} = \sin\omega t$ .... (1)

$\frac{y}{b} = \cos\omega t$ .... (2)

square & add (1) and (2)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ which shows ellipse.

22. PE of a particle is $U(x) = \frac{a}{x^2} - \frac{b}{x}$. Find the time period of small oscillation.

(a) $2\pi\sqrt{\frac{8a^3m}{b^4}}$ (b) $2\pi\sqrt{\frac{8b^3m}{b^4}}$

(c) $2\pi\sqrt{\frac{8a^4m}{b^3}}$ (d) $2\pi\sqrt{\frac{8b^4m}{a^3}}$

Solution (a) The equilibrium position is $\frac{du}{dx} = 0$

$$\Rightarrow \frac{-2a}{x_0^3} + \frac{b}{x_0^2} = 0 \text{ or } x_0 = \frac{2a}{b}.$$

$$u(x) = \frac{a}{x_0^2} - \frac{b}{x_0} + (x - x_0)\left.\frac{dU}{dx}\right|_{x=x_0} + \frac{1}{2}(x - x_0)^2 \left.\frac{d^2u}{dx^2}\right|_{x=x_0}.$$

$$\left.\frac{d^2u}{dx^2}\right|_{x=x_0} = \frac{6a}{x_0^4} - \frac{2b}{x_0^3} = \frac{6a}{\left(\frac{2a}{b}\right)^4} - \frac{2b}{\left(\frac{2a}{b}\right)^3} = \frac{b^4}{8a^3}$$

Thus, $u(x) = U(x_0) + \frac{1}{2}\left(\frac{b^4}{8a^3}\right)y^2$

Comparing with $\frac{1}{2}m\omega^2 y^2 = \frac{1}{2}\frac{b^4}{8a^3}y^2$

$$\omega = \sqrt{\frac{b^4}{8a^3 m}}$$

or $$T = 2\pi\sqrt{\frac{8a^3 m}{b^4}}.$$

23. Determine the period of oscillation of 200 g of Hg into a bent tube whose right arm forms an angle 30° with the vertical as shown. The cross-sectional area of the tube is 0.5 cm^2. Neglect viscosity.

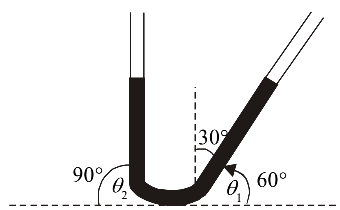


Fig. 12.39

(a) 0.68 s (b) 0.74 s
(c) 0.8 s (d) 0.88 s

Solution (c) $T = 2\pi\sqrt{\frac{M}{A\rho g(\sin\theta_1 + \sin\theta_2)}}$

$$= 2\pi\sqrt{\frac{.2}{.5\times10^{-4}\times13.6\times10^3\times10\left(1+\frac{\sqrt{3}}{2}\right)}}.$$

$= 0.8$s

24. In the system shown, a long uniform rod is attached at one end of a spring constant k and the other end is hinged. It is displaced slightly and allowed to oscillate. The time period of oscillation is

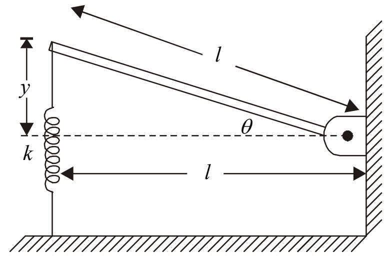


Fig. 12.40

(a) $2\pi\sqrt{\frac{M}{k}}$ (b) $2\pi\sqrt{\frac{M}{2k}}$

(c) $2\pi\sqrt{\frac{M}{3k}}$ (d) none of these

Solution (c) $\tau = kyl = kl^2\theta$

$$\tau = \frac{Ml^2}{3}\alpha \quad \text{therefore,}$$

$$\frac{Ml^2}{3}\alpha = -kl^2\theta$$

or $$\alpha = \frac{-3kl^2}{Ml^2}\theta \quad \omega^2 = \frac{3k}{M}$$

$$T = 2\pi\sqrt{\frac{M}{3k}}$$

25. A solid ball of mass m is allowed to fall from a height h to a pan suspended with a spring of spring constant k. Assume the ball does not rebound and pan is massless, then amplitude of the oscillation is

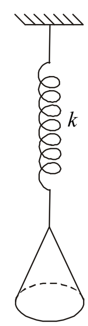


Fig. 12.41

(a) $\frac{mg}{k}$ (b) $\frac{mg}{k} + \left(\frac{2hk}{mg}\right)^{1/2}$

(c) $mg\sqrt{1+\frac{1+2hk}{mg}}$ (d) $\frac{mg}{k}\left[1+\sqrt{1+\frac{2hk}{mg}}\right]$

Solution (d) mg $(h + x) = \frac{1}{2}kx^2$

or $\quad x^2 - \left(\frac{2mg}{k}\right)x - \frac{2mgh}{k} = 0$

or $\quad x = \frac{mg}{k} + \frac{mg}{k}\sqrt{1+\frac{2hk}{mg}}$

26. A partical executes SHM of frequency f. The frequency of its kinetic energy is

(a) f (b) $\frac{f}{2}$

(c) $2f$ (d) zero

Solution (c) KE $= \frac{1}{2}m\omega^2\left(x_0^2 - x^2\right)$

$$= \frac{1}{2}m\omega^2\left(x_0^2 - x_0^2\sin^2\omega\right)$$

$$= \frac{1}{2}m\omega^2 x_0^2\left(\frac{1+\cos^2\omega}{2}\right)$$

Note the frequency is 2ω or $2f$

27. For the particle executing SHM the displacement x is given by $x = A$ cos ωt. Identify which represents variation of potential energy as a function of time t and displacement x

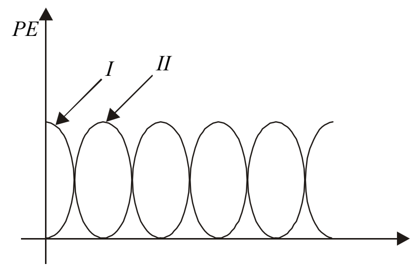


Fig. 12.42 (a)

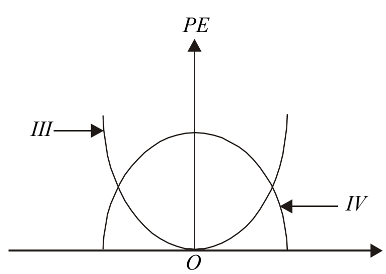


Fig. 12.42 (b)

(a) I, III (b) II, IV

(c) II, III (d) I, IV

[IIT Screening 2003]

Solution (a) $x = A\cos\omega t$

$$PE = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2\cos^2\omega t$$

that is, at t = 0 potential energy is maximum and potnetial energy $\alpha\, x^2$ So choice is I and III.

28. The earing of a lady is 5 cm long. She sits in a merry-go-round moving at 4 ms^{-1} in a circle of radius 2 m. The time period of oscillations is nearly

(a) 0.6 s (b) 0.4 s

(c) 0.8 s (d) none of these

Solution (b) $T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{v^2}{R}\right)^2}}}$

$$= 2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{v^2}{R}\right)^2}}} \cong 0.4 \text{ s (nearly)}$$

29. A rod of length l and mass m is hanged from one edge. The time period of small oscillations is

(a) $2\pi\sqrt{\frac{l}{3g}}$ (b) $2\pi\sqrt{\frac{l}{g}}$

(c) $2\pi\sqrt{\frac{2l}{3g}}$ (d) none of these

Solution (c) $T = 2\pi\sqrt{\frac{I}{mgl}}$

$$= 2\pi\sqrt{\frac{\frac{Ml^2}{12}+\frac{Ml^2}{4}}{Mgl/2}}$$

$$= 2\pi\sqrt{\frac{2l}{3g}}$$

30. A, B, C are identical springs each of spring constant k as shown in Fig. Mass M is displaced slightly along C and released. Find the time period of small oscillation.

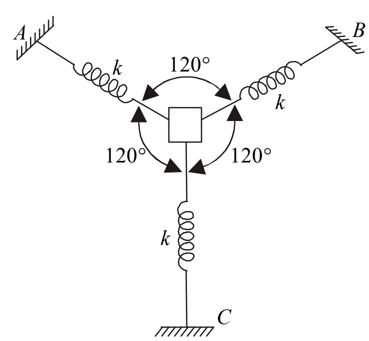


Fig. 12.43 (a)

(a) $T=2\pi\sqrt{\frac{m}{k}}$ (b) $T=2\pi\sqrt{\frac{m}{2k}}$

(c) $T=2\pi\sqrt{\frac{m}{2k}}$ (d) $T=2\pi\sqrt{\frac{3m}{2k}}$

Solution (c) Net force as shown in Fig. 12.43 (b) is

$$F = -(kx + kx\cos 60 + kx\cos 60) = -2kx$$

$$ma = -2kx$$

or $$a = \frac{-2k}{m}x$$

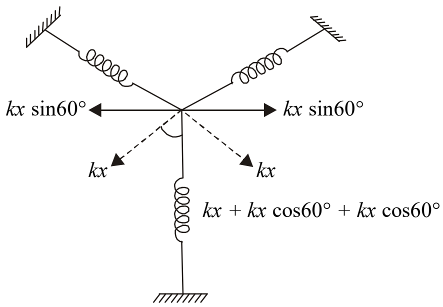


Fig. 12.43 (b)

$$\omega = \sqrt{\frac{2k}{m}}$$

or $$T = 2\pi\sqrt{\frac{m}{2k}}$$

31. The pulley shown in figure has a MOI I about its axis and mass m. Find the time period of vertical oscillations of its COM. The spring has a spring constant k and the string does not slip over the pulley.

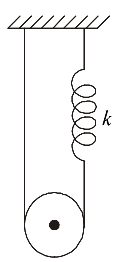


Fig. 12.44

Solution $2T$ = mg or $2ky$ = mg or $y = \frac{Mg}{2k}$

Writing energy

$$U = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 - mgx + \frac{1}{2}k\left[\frac{mg}{2k} + 2x\right]^2$$

$$= \frac{1}{2}\left[\frac{I}{r^2} + m\right]v^2 + \frac{m^2g^2}{8k} + 2kx^2$$

Put $\frac{du}{dt}$ = 0 as the system is conservative, giving

$$0 = \left[\frac{I}{r^2} + m\right]\frac{vdv}{dt} + 4kxv$$

or $$\frac{dv}{dt} = a = -\frac{4kx}{\left(\frac{I}{r^2} + m\right)}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{I/r^2 + m}{4k}}.$$

32. A ball is suspended by a thread of length l at the point O on the wall PQ which is inclined to the vertical by α. The thread with the ball is displaced by a small angle β away from the vertical and also away from the wall. If the ball is released, find the period of oscillation of the pendulum when (a) $\beta < \alpha$ (b) $\beta > \alpha$. Assume the collision to be perfectly elastic.

[Olympiad 1992 Roorkee 91]

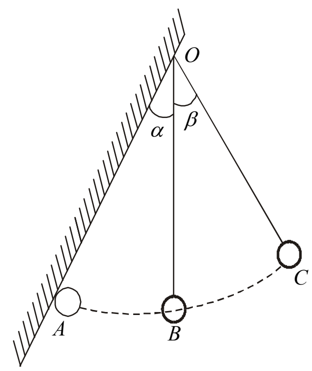


Fig. 12.45

Solution $\theta = \theta_0 \sin \omega t$ where $\omega = \sqrt{\frac{g}{L}}$

(a) When $\beta < \alpha$ $\quad T = 2\pi\sqrt{\frac{L}{g}}$

(b) when $\beta > \alpha$ time taken by pendulum from B to C and C to B

$$t_1 = \frac{T}{2} = \frac{1}{2} \times 2\pi\sqrt{\frac{L}{g}} = \pi\sqrt{\frac{L}{g}}$$

Time for B to A and A to B

$$t_2 = 2t = \frac{2}{\omega}\sin^{-1}\left(\frac{\alpha}{\beta}\right)$$

Using $\theta = \theta_0 \sin \omega t$

$$\alpha = \beta \sin \omega t \text{ or } t = \frac{1}{\omega}\sin^{-1}\left(\frac{\alpha}{\beta}\right)$$

$\therefore$ Time period of motion

$$T = t_1 + t_2 = \sqrt{\frac{L}{g}}\left[\pi + 2\sin^{-1}\frac{\alpha}{\beta}\right]$$

33. A uniform rod is placed on two spinning wheels as shown in the Fig. 12.46 (a). The axes of the wheels are separated by l. The coefficient of friction between the rod and the wheel is μ. Show that rod performs SHM and find the period of small oscillations.

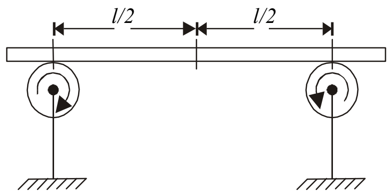


Fig. 12.46 (a)

Solution $N_1 + N_2 = mg$ .... (1)

$\mu N_1 - \mu N_2 = ma$ .... (2)

$$N_1\left(\frac{l}{2}+x\right) = N_2\left(\frac{l}{2}-x\right) \quad \text{....... (3)}$$

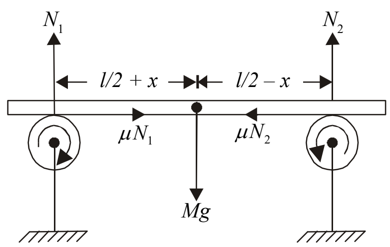


Fig. 12.46 (b)

Solving (1) (2) and (3), we get

$$a = -\frac{\mu 2gx}{l}$$

or $$T = 2\pi\sqrt{\frac{l}{2g\mu}}$$

34. A particle is executing SHM $x = 3\cos\omega t + 4\sin\omega t$. Find the phase shift and amplitude.

(a) 50°, 5 units (b) 37°, 3.5 units

(c) 53°, 3.5 units (d) 37°, 5 units

Solution (d) $x = x_0\sin(\omega t + \phi) = x_0\sin\omega t\cos\phi + x_0\cos\omega t\sin\phi = 3\cos\omega t + 4\sin\omega t$

Comparing we get $x_0\cos\phi = 4$ .... (1)

and $x_0\sin\phi = 3$ .... (2)

dividing (2) by (1) $\tan\phi = ¾$ or $\phi = 37°$. Squaring and adding (1) and (2) we get $x_0 = 5$.

35. When two sinusoids oscillating in different frequencies are fed to x and y plates of a cathode ray oscilloscope the pattern shown in Fig. Q 35. is observed.

$$\frac{\omega_x}{\omega_y} = \_\_\_\_\_\_$$

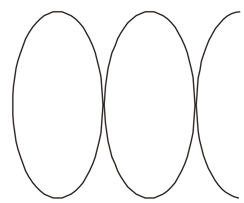


Fig. 12.47

(a) $\frac{5}{2}$ (b) $\frac{2}{5}$

(c) $\frac{1}{3}$ (d) 3

Solution (b) $\frac{\omega_x}{\omega_y} = \frac{n_y}{n_x} = \frac{1}{2.5}$

TYPICAL PROBLEMS

36. Two particles are moving in uniform circular motion in opposite direction with same angular velocity. Radius of the circle is R. Their resultant motion is equivalent to

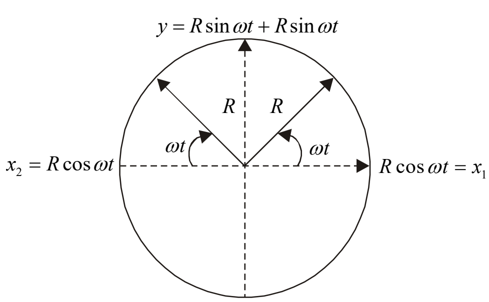


Fig. 12.48

(a) angular SHM of angular frequency 2ω.

(b) linear SHM of angular frequency 2ω.

(c) linear SHM of amplitude $2R$.

(d) angular SHM of amplitude 2 Radian.

Solution (c) Resolving $x_1 = R\cos\omega t$; $x_2 = -R\cos\omega t$

$y_1 = R\sin\omega t$; $y_2 = R\sin\omega t$.

$x = x_1 + x_2 = 0$

$y = y_1 + y_2 = 2R\sin\omega t$

37. A common hydrometer has 1 and 0.8 specific gravity marks 4 cm apart. Calculate the time period of vertical oscillations when it floats in water. Neglect resistance of water.

(a) 0.6 s (b) 0.7 s

(c) 0.65 s (d) 0.8 s

Solution (d) $(l + 4)\,0.8\,g = l\,(1)\,y$

or $0.2\,l = 0.8 \times 4$

or $\quad l = 16\,\text{cm}.$

$$T = 2\pi\sqrt{\frac{.16}{9.8}} = 0.8\text{ s}$$

38. A planck with a body of mass m placed on to it starts moving straight up with the law $y = a(1 - \cos\omega t)$ where ω is displacement. Find the time dependent force

(a) $-ma\,\omega^2\cos\omega t$ (b) $ma\,\omega^2\cos\omega t$

(c) $ma\,\omega^2\sin\omega t$ (d) $mg + ma\,\omega^2\cos\omega t$

Solution (d) $\frac{dy}{dt} = +a\,\omega\sin\omega t$ and

$$\frac{d^2y}{dt^2} = a\,\omega^2\cos\omega t$$

$$F = mg + ma\,\omega^2\cos\omega t.$$

39. A particle of mass m moves according to the equation $F = -\alpha\, m\, r$ where α is a positive constant, r is radius vector. $r = r_0\,\hat{i}$ and $\upsilon = \upsilon_0\,\hat{j}$ at $t = 0$. Describe the trajectory.

(a) $\left(\frac{x}{r_0}\right)^2 + \alpha\left(\frac{y}{\upsilon_0}\right)^2 = 1$

(b) $\left(\frac{x}{r_0}\right)^2 + \alpha\left(\frac{y}{\upsilon_0}\right)^2 = 0$

(c) $\left(\frac{x}{r_0}\right)^2 + \left(\frac{y}{\upsilon_0}\right)^2 = \frac{1}{\alpha}$

(d) none of these

Solution (a) $m\frac{d^2x}{dt^2}\,\hat{i} + m\frac{d^2y}{dt^2}\,\hat{j}$

$$= -\alpha m\,[x\,\hat{i} + y\,\hat{j}]$$

or $\quad \frac{d^2x}{dt^2} = -\alpha x$ and

$$\frac{d^2y}{dt^2} = -\alpha y$$

Thus $x = x_0\cos(\omega t + \theta)$ and $\omega^2 = \alpha$

$$\frac{dx}{dt} = -x_0\omega\sin(\omega t + \theta) = 0\ (\text{given} \therefore \theta = 0)$$

$$y = y_0\sin(\omega t + \beta)$$

$\because \quad y = 0$ at $t = 0 \qquad \therefore \beta = 0$

$\left|x = x_0\cos\omega t\right|_{t=0} = r_0 \qquad \left|\therefore x_0 = r_0\right|$

hence $x = r_0\cos\omega t.$

$$\frac{dy}{dt} = y_0\,\omega\cos\omega t = \upsilon_0 \text{ or } y_0 = \frac{\upsilon_0}{\omega}$$

$$\left(\frac{x}{r_0}\right)^2 + \left(\frac{y}{\upsilon_0/\omega}\right)^2 = 1 \text{ or } \left(\frac{x}{r_0}\right)^2 + \alpha\frac{y^2}{\upsilon_0^2} = 1$$

40. A pendulum is constructed as a thin walled sphere of radius R filled up with water and suspended from a point O. Centre of sphere from O is l apart. How many times will the time period of small oscillation change when the water freezes?

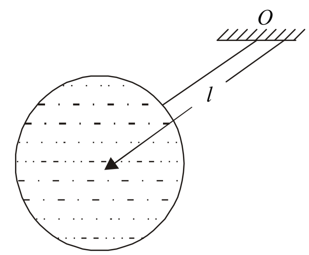


Fig. 12.49

(a) $\frac{2}{5}\left(\frac{R}{l}\right)^2$ (b) $1 + \frac{2}{5}\left(\frac{R}{l}\right)^2$

(c) $\sqrt{1 + \frac{2}{5}\left(\frac{R}{l}\right)^2}$ (d) $\sqrt{\frac{1}{1 + \frac{2}{5}\left(\frac{R}{l}\right)^2}}$

Solution (c) **Case (i)** When water is present

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

Case (ii) When water freezes

$$T_1 = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{ml^2 + \frac{2}{5}mR^2}{mgl}}.$$

$$T_1 = 2\pi\sqrt{\frac{l}{g}}\sqrt{1 + \frac{2}{5}\left(\frac{R}{l}\right)^2}.$$

41. A coin is placed on a horizontal platform, which undergoes vertical SHM of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

(a) at the highest position of the platform.

(b) at the mean position of the platform.

(c) for an amplitude g/ω^2.

(d) for an amplitude of $\sqrt{g/\omega}$.

Solution (a), (c) $g = x_0\,\omega^2$ or $x_0 = g/\omega^2$.

42. Which of the following quantities is always negative in SHM?

(a) $\vec{F}\,.\,\vec{a}$ (b) $\vec{F}\,.\,\vec{r}$

(c) $\vec{\upsilon}\,.\,\vec{r}$ (d) $\vec{a}\,.\,\vec{r}$

Solution (b) and (d) $\because\ a = -\omega^2 r$

43. Which of the following quantities are always zero in SHM?

(a) $\vec{F}\times\vec{a}$ (b) $\vec{\upsilon}\times\vec{r}$

(c) $\vec{a}\times\vec{r}$ (d) $\vec{F}\times\vec{r}$

Solution (All)

44. Average energy in one time period in an SHM is

(a) $\frac{1}{2} m\,\omega^2 y_0^2$ (b) $m\,\omega^2 y_0^2$

(c) $\dfrac{m\omega^2 y_0^2}{3}$ (d) $\dfrac{m\omega^2 y_0^2}{4}$

Solution (a) Average energy = $KE_{aV} + PE_{aV}$

$$= \frac{m\omega^2 y_0^2}{2}.$$

45. Average KE in one complete oscillation with respect to position is

(a) $\dfrac{m\omega^2 x_0^2}{2}$ (b) $\dfrac{m\omega^2 x_0^2}{3}$

(c) $\dfrac{m\omega^2 x_0^2}{4}$ (d) $\dfrac{m\omega^2 x_0^2}{6}$

Solution (b) $KE = \frac{1}{2}\,m\,\omega^2\,(x_0^2 - x^2)$;

$$KE_{aV} = \frac{1}{2}\,m\,\omega^2\,\frac{\int_{-0}^{x_0}\left(x_0^2 - x^2\right)dx}{\int_0^x dx}.$$

$$= \frac{1}{2}\,\frac{m\omega^2\left[x_0^3 - \frac{x_0^3}{3}\right]}{x_0} = \frac{m\omega^2 x_0^2}{3}.$$

46. A particle is moving in a circular path with continuously increasing speed. Its motion is

(a) periodic but not oscillatory.

(b) periodic but not SHM.

(c) oscillatory.

(d) neither periodic nor SHM.

(e) none of these.

Solution (e)

47. The block A in fig shown moves at speed υ towards B placed in equilibrium. All collisions to take place are elastic and surfaces are frictionless. Find the time period of the periodic motions of the two blocks.

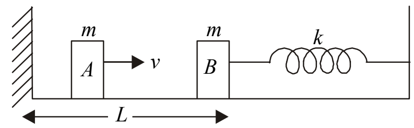


Fig. 12.50

Solution for block B $T = 2\pi\sqrt{\dfrac{m}{k}}$

for block A $T = \dfrac{2L}{v} + \dfrac{2\pi\sqrt{\dfrac{m}{k}}}{2}$

or $$T = \frac{2L}{\upsilon} + \pi\sqrt{\frac{m}{k}}.$$

48. A closed circular wire hung on a nail in a wall undergoes small oscillations of amplitude 2° and time period 2 s. Find the radius of the wire and velocity at the mean position.

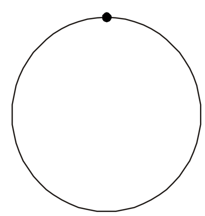


Fig. 12.51

(a) 50 cm, 11 cm (b) 1 m, 22 cm

(c) 33 cm, 8 cm (d) none

Solution (a) $T = 2\pi\sqrt{\dfrac{I}{mgl}}$

or $$2 = 2\pi\sqrt{\frac{2Mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$$

or $$r = \frac{1}{2}\text{ m.}$$

$$\upsilon_{\max} = (2r\,\theta)\,\frac{2\pi}{T} = \frac{2\times 50\times 2\times\pi\times}{180}\times\frac{2\pi}{2}$$

$= 11$ cm.

49. 3 SHMs of equal amplitude A and equal time periods in the same direction combine. The phase of 2nd motion is 60° ahead of the first and phase of 3rd is 60° ahead of the second. Find the amplitude of the resultant.

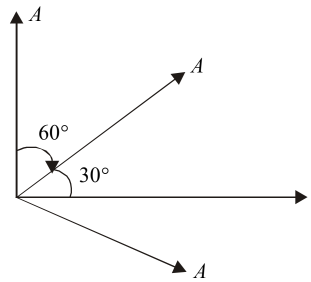


Fig. 12.52

(a) A (b) $\frac{3}{2}A$

(c) $2A$ (d) $3A$

Solution (c) 2nd and 3rd combine to give amplitude $A' = \sqrt{A^2 + A^2 + 2A \times A\cos 60} = \sqrt{3}A$

$$\tan\phi = \frac{A\sin 60}{A + A\cos 60} = \frac{1}{\sqrt{3}} \quad \phi = 30°$$

$$A_{\text{net}} = \sqrt{A^2 + (\sqrt{3}A)^2} = 2A.$$

50. A particle is subjected to two SHMs one along the x-axis and the other on a line making an angle 45° with the x-axis. The two motions are $x = x_0 \sin \omega t$ and $p = p_0 \sin \omega t$. Find their resultant.

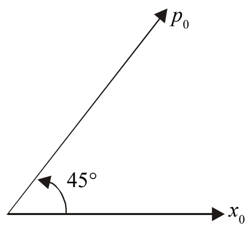


Fig. 12.53

(a) $x_0 + p_0$ (b) $\sqrt{x_0^2 + p_0^2 + \sqrt{2}x_0 p_0}$

(c) $\sqrt{x_0^2 + p_0^2 - \sqrt{2}x_0 p_0}$ (d) $\sqrt{x_0^2 + p_0^2}$

Solution (b) $A_{\text{res}} = \sqrt{x_0^2 + p_0^2 + 2x_0 p_0 \cos 45}$

$$= \sqrt{x_0^2 + p_0^2 + \sqrt{2}x_0 p_0}\ .$$

51. Consider the situation shown in Fig. 12.54. A horizontal disc of mass m, radius R suspended from the centre through a wire of torsional rigidity k. If at $t = 0$, the disc is deviated by ϕ_0, find the energy of small torsional vibrations.

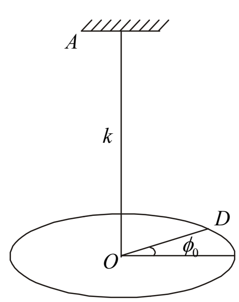


Fig. 12.54

(a) $\frac{1}{2}k\phi_0^2$ (b) $\frac{1}{4}k\phi_0^2$

(c) $k\phi_0^2$ (d) $2k\phi_0^2$

Solution (a) $\frac{1}{2}k\phi_0^2$

52. Two physical pendulums perform small oscillations about the same horizontal axis with frequencies ω_1 and ω_2. Their moments of inertia are I_1 and I_2 respectively. In a state of stable equilibrium pendulums were fastened rigidly together. The frequency of small oscillations of the compound pendulum will be

(a) $\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$ (b) $\frac{\omega_1 + \omega_2}{2}$

(c) $\frac{I_1\omega_1 + I_2\omega_2}{2(I_1 + I_2)}$ (d) $\frac{I_1\omega_1 - I_2\omega_2}{I_1 - I_2}$

Solution (a) $(I_1 + I_2)\,\omega = I_1\,\omega_1 + I_2\,\omega_2$ (conserve angular momentum)

$$\therefore \quad \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

53. Two identical rods A and B are connected by a wire of length l in the middle as shown in Fig. 12.55. The wire is bent into an arc of a circle of radius R and released. If Y is Young's modulus of wire. Find time period of small oscillations. MOI of each rod is I_b.

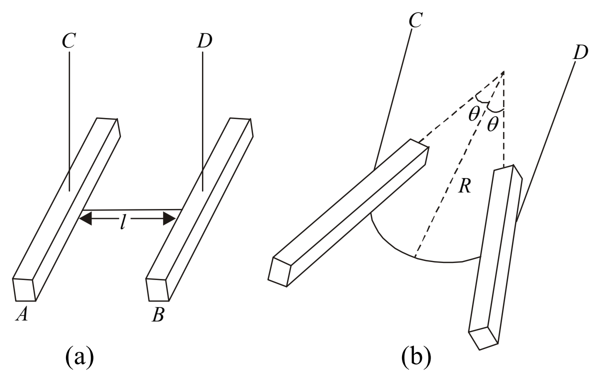


Fig. 12.55

(a) $\sqrt{\frac{2l}{Yr^4}}$ (b) $\sqrt{\frac{2lI_b}{Yr^4}}$

(c) $\sqrt{\frac{2I_b}{Yr^4}}$ (d) $\sqrt{\frac{2lI_b}{Y\pi r^4}}$

Solution (d) $\frac{Yi}{R} = -I_{\text{bar}}\frac{d^2\theta}{dt^2}$

$\frac{l}{R} = 2\theta,\ i = \frac{\pi r^4}{4}$ and is called geometric moment of inertia.

$$\frac{Y\pi r^4}{4l}(2\theta) = -I_{\text{bar}}\frac{d^2\theta}{dt^2}$$

or $$\frac{d^2\theta}{dt^2} = -\frac{Y\pi r^4}{2lI_b}\theta$$

comparing with $\alpha = \omega^2\theta$

$$\omega = \sqrt{\frac{Y\pi r^4}{2lI_b}}$$

or $$T = \sqrt{\frac{2lI_b}{Y\pi r^4}}.$$

PASSAGE 1

Read the following passage and answer the questions given at the end.

Consider a spherical ball of radius r, rolling on a concave surface of radius R. A student measured the time period T for small oscillation. Her teacher told her that this arrangement is called dynamic spherometer. The teacher insisted that she shall use it like a spherometer. However, student was interested to find g. One of her friends was interested to find G by using the same experiment using radius of the earth and mass of the earth.

1. What do you mean by dynamic spherometer?

(a) An instrument used to measure radius of curvature directly.

(b) An instrument used to measure radius of curvature more accurately than usual spherometer.

(c) An instrument used to measure radius of curvature indirectly.

(d) None of these.

Solution (b)

2. If T is the time period measured. What will be the radius of curvature R of the concave surface?

(a) $\frac{T^2g}{4\pi^2}+r$ (b) $\frac{1.4T^2g}{4\pi^2}+r$

(c) $\frac{T^2g}{1.4(4\pi^2)}+r$ (d) $\frac{T^2g}{2.8\pi^2}+r$

Solution (c)

$$T = 2p\sqrt{\frac{(R-r)\left(1+\frac{k^2}{r^2}\right)}{g}}$$

$$= 2\pi\sqrt{\frac{7(R-r)}{5g}}.$$

$$R-r = \frac{5T^2g}{7(4\pi^2)}$$

or $$R = \frac{T^2g}{1.4(4\pi^2)}+r.$$

3. How her friend will measure G?

(a) $G = \frac{5.6\pi R\ R_E^2}{T^2M_E}$

(b) $G = \left(\frac{5.6\pi^2R}{T^2}-r\right)\frac{R_E^2}{M_E}$

(c) $G = \frac{(R-r)5.6\pi^2R_E^2}{T^2M_E}$

(d) none

Solution (c)

$$T = 2\pi\sqrt{\frac{1.4(R-r)}{g}}$$

or $$g = \frac{(4\pi^2)(1.4)(R-r)}{T^2}$$

$$\frac{GM_E}{R_E^2} = \frac{5.6\pi^2(R-r)}{T^2}$$

or $$G = \frac{5.6\pi^2(R-r)R_E^2}{T^2M_E}$$

PASSAGE 2

Read the following passage and answer the questions given at the end.

Charge q is placed exactly in the center of two charges Q each as shown in Fig. 12.57. A demonstrator in a TV show slightly displaces q along perpendicular bisector the charge q executes SHM. He then displaces it axially. Charge q again executes SHM with a different time period.

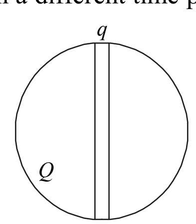


Fig. 12.56

He then took out a sphere in which a tunnel was carved out along the diameter. He charged the sphere and took a small particle (charged) q. He allowed the charge particle to fall in tunnel. The charged particle executed SHM. He posed the following questions.

1. What is the nature of charge on q with respect to Q?

(a) like charge (b) unlike

(c) either

(d) insufficient information to reply.

Solution (b)

2. What is the time period of oscillation in each case?

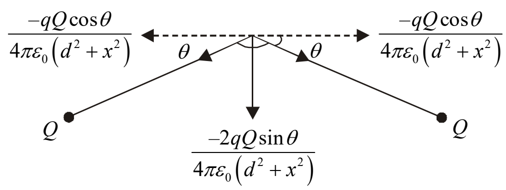


Fig. 12.57

Solution **Case (i)**

$$F_{net} = \frac{-2qQ}{4\pi\in_0(d^2+x^2)}\sin\theta$$

$$= \frac{-2qQx}{4\pi\in_0(d^2+x^2)^{3/2}}.$$

since $x << d$ $\therefore$ $ma = \frac{-2qQx}{4\pi\in_0 d^3}$

or $a = \frac{-2qQx}{4\pi\in_0 md^3}.$

$a = -\omega^2 x$ comparing with we get

$$\omega = \sqrt{\frac{2qQ}{4\pi\in_0 md^3}}$$

or $T = 2\pi\sqrt{\frac{4\pi\in_0 md^3}{2qQ}}$

Case (ii)

$$F_{net} = \frac{-qQ}{4\pi\in_0}\left[\frac{1}{(d-x)^2}-\frac{1}{(d+x)^2}\right]$$

$$ma = \frac{-qQ(4dx)}{4\pi\in_0(d^2-x^2)^2}$$

or $a = \frac{-4qQ}{4\pi\in_0 d^3m}x$ as $d>>x$,

or $\omega = \sqrt{\frac{qQ}{\in_0\pi d^3m}}$

or $T = 2\pi\sqrt{\frac{\pi\in_0 d^3m}{qQ}}$

Case (iii)

$$E_{in} = \frac{Qx}{4\pi\in_0 R^3} \quad F = ma = \frac{-qQx}{4\pi\in_0 R^3},$$

$$a = \frac{-qQx}{4\pi\in_0 mR^3}$$

$$\omega = \sqrt{\frac{Qq}{4\pi\in_0 mR^3}}$$

or $T = 2\pi\sqrt{\frac{4\pi\in_0 mR^3}{Qq}}$

PASSAGE 3

Read the following passage and answer the questions given at the end.

Many diatomic (two – atom) molecules are bound together by covalent bonds that are much stronger than Vander waals interaction, Their examples include H_2, N_2, O_2 etc. Experiments indicate that for many such molecules, the interaction can be described by a force of the kind

$$F(r) = A\,[e^{-2b(r-R_0)} - e^{-b(r-R_0)}]$$

where A and b are positive constants, r is center, to center separation of the atoms, and R_0 is the equilibrium separation. For the hydrogen molecule (H_2) $A = 2.97\times10^{-8}$ N, $b = 1.95\times10^{10}$ m^{-1} and $R_0 = 7.4\times10^{-11}$ m

1. Find force constant for small oscillation around equilibrium.

(a) 580 Nm^{-1} (b) 58 Nm^{-1}

(c) 400 Nm^{-1} (d) none of these

2. Find the frequency of small oscillation.

(a) 6.1×10^{12} Hz (b) 8.2×10^{13} Hz

(c) 7.3×10^{12} Hz (d) 5.3×10^{11} Hz

Solution **1.** (a)

$$F(r) = A\,[e^{-2b(r-R0)} - e^{-b(r-R0)}]$$

$$= A\,[1-2b(r-R_0)-1-b(r-R_0)]$$

$$= -A\,b(r-R_0)$$

comparing with $F = -kr$

$k = Ab$

$k = 2.97\times10^{-8}\times1.95\times10^{10} = 580$ Nm^{-1}

Solution **2.** (b) $w = \sqrt{\frac{k}{m}}$

or $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$= \frac{1}{6.28}\sqrt{\frac{580}{1.08\times2\times10^{-27}}}$$

$$= 8.2\times10^{13}\text{ Hz}$$

PASSAGE 4

Read the following passage and answer the questions given at the end.

All walking animals, including humans, have natural walking pace, a number of steps per minute that is more comfortable than a faster or slower pace. Assume this natural pace is equal to the period of the leg, viewed as a uniform rod pivoted at the hip joint. Fossil evidence shows that Tyrannosourus rex, a two-legged dinossaur that lived 65 million years ago at the end of cretacens period, had a leg length of 3.1m and a stride length (the distance from one footprint to next print of the same foot) 4 m.

1. How does the natural walking pace depend on the length L of the length?

(a) $T = 2\pi\sqrt{\dfrac{2l}{3g}}$ (b) $T = 2\pi\sqrt{\dfrac{l}{g}}$

(c) $T = 2\pi\sqrt{\dfrac{3l}{2g}}$ (d) none of these

2. Estimate the speed of Tyrannosourus rex.

(a) 3 km h^{-1} (b) 4 km h^{-1}

(c) 5 km h^{-1} (d) none of these

Solution **1.** (a)

$$T = 2\pi\sqrt{\frac{I}{Mgl_{com}}}$$

$$= 2\pi\sqrt{\frac{2l}{3g}}$$

$$= 2\pi\sqrt{\frac{2\times 3.1}{3\times 9.8}} = 2.9 \text{ s}$$

Solution **2.** (c)

$$v = \frac{\text{Stride length}}{T}$$

$$= \frac{4}{2.9} = 1.4 \text{ ms}^{-1}$$

or $= 5 \text{ km h}^{-1}$

QUESTIONS FOR PRACTICE

1. Two simple pendulums of lengths 1 m and 16 m respectively are both given small displacements in the same direction at the same instant. They will again be in phase after the shorter pendulum has completed n oscillations where n is

(a) 1/3 (b) 1/4

(c) 4 (d) 5

2. A simple spring has length l and force constant k. It is cut into two springs of length l_1 and l_2 such that $l_1 = nl_2$ (n = an integer) the force constant of spring of length l_1 is

(a) $k(1 + n)$ (b) $k/(n + 1)$

(c) $k(1 + n)$ (d) k

3. The kinetic energy and potential energy of a particle executing SHM will be equal when displacement (amplitude = a) is

(a) $a\sqrt{2/3}$ (b) $a/2$

(c) $a/\sqrt{2}$ (d) $a\sqrt{2}$

4. If a particle performs SHM with a frequecy v, then its K.E. will oscillate with a frequency

(a) zero (b) $v/2$

(c) v (d) $2v$

5. The dispalacement y in centimeters is given in terms of time t s by the equation: $y = 3\sin 314\, t + 4\cos 314\, t$, then the amplitude of SHM is

(a) 3 cm (b) 4 cm

(c) 5 cm (d) 7 cm

6. A body of mass m is suspended from rubber and with force constant k. The maximum distance over which the body can be pulled down for the body's oscillation to remain harmonic is

(a) $mg/2k$ (b) $2k/mg$

(c) $2\ mg/k$ (d) mg/k

7. Length of second's pendulums is decreased by 1% then the gain or loss in time per day will be nearly

(a) 0.44 s (b) 4.4 s

(c) 44 s (d) 440 s

8. Two pendulums oscillate with a constant phase difference of 90°, and same amplitude. The maximum velocity of one is v. The maximum velocity of other will be

(a) $\sqrt{2v}$ (b) $v\sqrt{2}$

(c) v (d) $2v$

9. What is the length of second's pendulum where g is 980 cms^{-2}?

(a) 102.4 cm (b) 99.2 cm
(c) 88 cm (d) 78 cm

10. The displacement of particle in SHM in one time period is
(a) zero (b) a
(c) $2a$ (d) $4a$

11. For a particle executing SHM having amplitude 'a' the speed of the particle is one half of its maximum speed when its displacement from the mean position is
(a) $a/2$ (b) a
(c) $a\sqrt{3/2}$ (d) $2a$

12. A spring pendulum is suspended from the top of a car. If the car accelerates on a horizontal road, the frequency of oscillation will
(a) be zero. (b) remain same.
(c) increase. (d) decrease.

13. The length of the seconds pendulum on the surface of earth is 1m. Its length on the surface of earth is
(a) 1/36 m (b) 1/6 m
(c) 6m (d) 36 m

14. The phase angle between the projections of uniform circular motion on two mutually perpendicular diameter is
(a) π (b) $3\pi/4$
(c) $\pi/2$ (d) zero

15. When particle oscillates simple harmonically its potential energy varies periodically. If frequency of the particle is n, the frequency of the potential energy is
(a) $n/2$ (b) n
(c) $2n$ (d) $4n$

16. A simple pendulum performs SHM about $x = 0$ with an amplitude A and time period T. The speed of the pendulum at $x = A/2$ will be
(a) $3\pi 2A/T$ (b) $\pi A\frac{\sqrt{3}}{T}$
(c) $\pi\frac{A}{T}$ (d) $\pi A\frac{\sqrt{3}}{2T}$

17. The potential energy of a particle with displacement x is $U(x)$. The motion is simple harmonic. If k is a positive constant then
(a) $U = kx$ (b) $U = k$
(c) $U = -kx^2/2$ (d) $U = kx^2$

18. A bob is suspended by a string of length l. The minimum horizontal velocity imparted to the ball for reaching it to the height of suspension is
(a) $\sqrt{l/g}$ (b) $\sqrt{2gl}$
(c) $\sqrt{g/l}$ (d) $2\sqrt{gl}$

19. The angle between the instantaneous velocity and the acceleration of a particle executing SHM is
(a) zero or π (b) $\pi/2$
(c)zero (d) π

20. The frequency of SHM is 100 Hz. Its time period is
(a) 0.01 s (b) 0.1 s
(c) 1s (d) 100 s

21. The displacement y of a particle executing periodic motion is given by

$$y = 4\cos 2(t/2)\sin(100t).$$

This expression may be considered to be a result of the super position of
(a) 2 (b) 3
(c) 4 (d) 5

[IIT 92]

22. Two bodies M and N of equal masses are suspended from two separate massless spring of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal the ratio of the amplitude of M to that of N is
(a) k_2/k_1 (b) $\sqrt{k_2/k_1}$
(c) k_1/k_2 (d) $\sqrt{k_1/k_2}$

[IIT 88]

23. If E is the total energy of a particle executing SHM and 'A' is the amplitude of the vibratory motion, the E and 'A' are related as
(a) $E \propto A^2$ (b) $E \propto 1/A^2$
(c) $E \propto A$ (d) $E \propto 1/A$

24. A particle excites SHM with a time period of 2 s and amplitude 5 cm. Maximum magnitude of its velocity is
(a) $10\,\pi$ cm s^{-1} (b) $20\,\pi$ cm s^{-1}
(c) $2.5\,\pi$ cm s^{-1} (d) $5\,\pi$ cm s^{-1}

25. The dimensional formula for amplitude of SHM is
(a) MLT (b) $M^\circ L^\circ T^\circ$
(c) $M^\circ LT^\circ$ (d) MLT°

26. A simple pendulum is attached to the roof of a lift. Its time period of oscillation, when the lift is stationary is 0.5 s. Its frequency of oscillation when the lift falls freely will be
(a) infinite. (b) zero.
(c) 2 Hz. (d) 20 Hz.

27. A spring of force constant k is cut into two equal parts, which are then joined parallel to each other. The force constant of the combination will be

(a) $4k$ (b) $2k$

(c) k (d) $k/2$

28. A particle is placed on a plank undergoing SHM of frequency $3/\pi$ Hz. The maximum amplitude of the plank so that the particle does not leave the plank will be

(a) $\frac{5}{18}$ m (b) $\frac{5}{8}$ m

(c) $\frac{2}{9}$ m (d) none of these

29. The intensities of two notes are equal. If frequency of one note is one-fourth that of the other then the ratio of their amplitudes is

(a) 16 (b) 4

(c) 2 (d) 1

30. A person wearing a wrist watch that keeps correct time at the equator goes to N-pole. His watch will

(a) keep correct time. (b) gain time.

(c) loose time. (d) cannot say.

31. Which of the following is not essential for the free oscillation of a mass attached to a spring?

(a) Elasticity (b) Gravity

(c) Inertia (d) Restoring force

32. A pendulum suspended from the ceiling of the train beats seconds when the train is at rest. What will be the time period of the pendulum if the train accelerates at 10 ms^{-2}? Take $g = 10$ ms^{-2}.

(a) $(2/\sqrt{2})$ s (b) 2 s

(c) $2\sqrt{2}$ s (d) none of these

33. Which of the following quantities connected with SHM do not vary periodically?

(a) Total energy (b) Velocity

(c) Displacement (d) Acceleration

34. A mass m is suspended from a spring of force constant k. The angular frequency of oscillation of the spring will be

(a) k/m (b) $\sqrt{m/k}$

(c) m/k (d) $\sqrt{k/m}$

35. What is the number of degrees of freedom of an oscillating simple pendulum?

(a) more than three (b) 3

(c) 2 (d) 1

36. The graph between restoring force and time in case of SHM is a

(a) parabola. (b) sine curve.

(c) straight line. (d) circle.

37. In SHM, which of the following quantities does not vary as per nature of the sine curve?

(a) acceleration (b) time period

(c) displacement (d) velocity

38. Two particles P and Q describe SHM of same amplitude a and frequency v along the same straight line. The maximum distance between the two particles is $a\sqrt{2}$. The initial phase difference between the particle is

(a) $\pi/3$ (b) $\pi/2$

(c) $\pi/6$ (d) zero

39. A particle is moving on a circle with uniform speed. Its motion is

(a) aperiodic motion.

(b) periodic and SHM.

(c) periodic but not SHM.

(d) none of these.

40. The mass and radius of a planet are double that of the earth. The time period of a pendulum on that planet which is a seconds pendulum on earth, will be

(a) $\sqrt{\frac{1}{\sqrt{2}}}$ s (b) 0.5 s

(c) $2\sqrt{2}$ s (d) 2 s

41. The work done by a simple pendulum in one completed oscillation is

(a) equal to Ek. (b) equal to U.

(c) zero. (d) equal to $U + Ek$.

42. A particle is moving such that its acceleration is represented by the equation $a = -bx$, where x is its displacement from mean position and b is a constant. Its time period will be

(a) $2\pi/\sqrt{b}$ (b) $2\pi/b$

(c) $2\pi\sqrt{b}$ (d) $2\sqrt{\frac{\pi}{b}}$

43. The displacement of a particle executing SHM is half its amplitude. The fraction of its kinetic energy will be

(a) 2/3 (b) 3/4

(c) 1/3 (d) 1/2

44. The phase difference between the velocity and displacement of a particle executing SHM is

(a) $\pi/2$ radian (b) π radian
(c) 2π radian (d) zero

45. The ratio of the maximum velocity and maximum displacement of a particle executing SHM is equal to

(a) n (b) g
(c) T (d) ω

46. The physical quantity conserved in simple harmonic motion is

(a) time period. (b) total energy.
(c) potential energy. (d) kinetic energy.

47. The bob of a simple pendulum consists of a sphere filled with mercury. If a small quantity of mercury is taken out, then the period of pendulum will

(a) become erroneous. (b) decrease.
(c) increase. (d) remain unchanged.

48. The time period of a second's pendulum is 2 s. The mass of the spherical bob is 50 g and is empty. If it is replaced by another solid bob of same radius but mass 100 g then its time period will be

(a) 8 s (b) 4 s
(c) 2 s (d) 1 s

49. The amplitude and time period of simple harmonic oscillator are a and T respectively. The time taken by it in displacing from $x = 0$ to $x = a/2$ will be

(a) T (b) $T/2$
(c) $T/4$ (d) $T/6$

50. The time period of the hour hand of a watch is

(a) 24 h (b) 12 h
(c) 1 h (d) 1 min.

51. A simple pendulum is released when $\theta = \pi/6$. The time period of oscillation is

(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{l}{g}\left(\frac{293}{288}\right)}$

(c) $2\pi\sqrt{\frac{l}{g}\left(\frac{288}{293}\right)}$ (d) none of these

52. A mass m is suspended from a spring. Its frequency of oscillation is f. The spring is cut into two equal halves and the same mass is suspended from one of the two pieces of the spring. The frequency of oscillation of the mass will be

(a) $\sqrt{2}\,f$ (b) $2f$
(c) $f/2$ (d) f

53. Which of the following characteristics must remain constant for undamped oscillations of the particle?

(a) acceleration (b) phase
(c) amplitude (d) velocity

54. Identical springs of spring constant K are connected in series and parallel combinations. A mass m is suspended from them. The ratio of their frequencies of vertical oscillations will be

(a) 1 : 4 (b) 1: 2
(c) 4 : 1 (d) 2 : 1

55. The maximum displacement of a particle exectuting SHM from its mean position is 2 cm and its time period is 1 s. The equation of its displacement will be

(a) $x = 2\sin 4\pi t$ (b) $x = 2\sin 2\pi t$
(c) $x = \sin 2\pi t$ (d) $x = 4\sin 2\pi t$

56. The maximum value of the time period of a simple pendulum is

(a) 84.6 min (b) 1 year
(c) 1 day (d) 12 h

57. If a simple pendulum of length l has maximum angular displacement θ, then the maximum kinetic energy of the bob of mass m is

(a) $\frac{1}{2}\,m\,(l/g)$ (b) $mgl\,(1 - \cos\theta)$

(c) $(mgl\sin\theta)/2$ (d) $mg/2l$

58. A simple pendulum is carried at a depth of 1 km below sea level. It becomes

(a) slow. (b) unchanged.
(c) fast. (d) none of these.

59. The ratio of the kinetic energy E_k and potential energy of a particle executing SHM, at a distance x from mean position will be

(a) $\frac{x^2}{x^2 - a^2}$ (b) $\frac{x^2 - a^2}{x^2}$

(c) $\frac{x^2}{a^2 - x^2}$ (d) $\frac{a^2 - x^2}{x^2}$

60. The curve between the acceleration and velocity of a particle executing SHM is a an

(a) ellipse. (b) circle.
(c) parabola. (d) straight line.

61. On increasing the lingth of a second's pendulum by Z%, its time period will

(a) increase by 2 Z%. (b) decrease by Z%.
(c) decrease by 2 Z%. (d) increase by 0.5 Z%.

62. A clock purchased in 1942 loses 1 min in 1 day. Its time period must have become

(a) extremely small. (b) extremely large.
(c) shorter. (d) longer.

63. The displacement of a particle executing SHM at any instant t is $x = 0.01 \sin 100(t + 0.05)$ then its time period will be

(a) 0.06 s (b) 0.2 s

(c) 0.1 s (d) 0.02 s

64. The equation of displacement of a particle executing SHM is $x = 0.40 \cos(2000\,t + 18)$. The frequency of the particle is

(a) 10^3 Hz (b) 20 Hz

(c) 2×10^3 Hz (d) $\frac{10^3}{\pi}$ Hz

65. The displacement of a particle executing SHM at any instant t is $x = a \sin \omega t$. The acceleration of the particle at $t = \frac{T}{4}$ will be

(a) $a\omega$ (b) $a\omega^2$

(c) $-a\omega$ (d) $-a\omega^2$

66. Which of the following quantities is doubled on doubling the amplitude of a harmonic oscillator

(a) E_t (b) E_k

(c) v_{max} (d) U

67. The amplitude of a particle executing SHM is 2 cm and its frequency is 50 cycles/s. Its maximum acceleration will be

(a) 1972 ms^{-2} (b) 19.72 ms^{-2}

(c) 1.972 ms^{-2} (d) zero

68. The work done in stretching a spring of force constant k from y_1 to y_2 from mean position will be

(a) $\frac{k}{2}\left(y_2^2 - y_1^2\right)$ (b) $\frac{k}{2}\left(y_1^2 - y_2^2\right)$

(c) $k(y_2 - y_1)$ (d) zero

69. A watch becomes fast by 5 minutes in a day. In the watch makers shop it keeps correct time. It is due to

(a) natural vibrations. (b) forced vibrations.

(c) damped vibrations. (d) none of these.

70. The potential energy of a simple pendulum in its resting position is 10 J and its mean kinetic energy is 5 J. Its total energy at any instant will be

(a) 5 J (b) 10 J

(c) 15 J (d) 20 J

71. A simple pendulum is hanging from the ceiling of a train. If the train gets accelerated with a uniform acceleraion a, then its direction from the vertical wall be

(a) $\tan^{-1}(a/g)$ (b) $\sin^{-1}(a/g)$

(c) $\cot^{-1}(a/g)$ (d) $\cos^{-1}(a/g)$

72. The ratio of the kinetic energy of a particle executing SHM at its mean position to its potential energy at extreme position is

(a) = 1 (b) = $1/g$

(c) > 1 (d) < 1

73. The time period of a torsional pendulum is

(a) $T = \pi\sqrt{C/I}$ (b) $T = 2\pi\sqrt{I/C}$

(c) $T = 2\pi\sqrt{C/I}$ (d) $T = \pi\sqrt{I/C}$

74. The average kinetic energy of a simple harmonic oscillator with respect to position will be

(a) $\frac{ka^2}{6}$ (b) $\frac{ka^2}{4}$

(c) $\frac{ka^2}{3}$ (d) $\frac{ka^2}{2}$

75. The time period of a simple pendulum of infinite length is

(a) $T = 2\pi\sqrt{\frac{R_e}{g}}$ (b) infinity

(c) $T = \pi\sqrt{\frac{R_e}{g}}$ (d) zero

76. If the amplitude of oscillation of a simple pendulum is increased by 30%, then the percentage change in its time period will be

(a) 90% (b) 60%

(c) 30% (d) zero

77. If at any instant of time the displacement of a harmonic oscillator is 0.02 m and its acceleration is 2 ms^{-2}, its angular frequency at that instant will be

(a) 0.1 $rads^{-1}$ (b) 1 $rads^{-1}$

(c) 10 $rads^{-1}$ (d) 100 $rads^{-1}$

78. The potential energy of a simple harmonic oscillator is given by $U(x) = \alpha + \beta x + \gamma x^2 + \delta x^3$ the term representing the SHM will be

(a) α (b) βx

(c) γx^2 (d) δx^3

79. The phase shift of the resultant $y = 3 \sin \omega t + 4 \cos \omega t$ is

(a) 37° (b) 53°

(c) 90° (d) none of these

80. Which physical quantity oscillates with the same frequency as that of SHM for a harmonic oscillator?

(a) acceleration (b) E_{total}

(c) KE (d) PE

81. Two particles execute SHM along a straight line with same amplitude and same frequency. Everytime they meet in opposite directions at a displacement equal to half the amplitude. The difference between them is

(a) $2\pi/3$ (b) π
(c) $\pi/3$ (d) $\pi/2$

82. The work done in displacing a harmonic oscillator through an angle ϕ from its mean position is

(a) $mgl\,(1 - \cos\phi)$ (b) mgl
(c) $mgl \cos\phi$ (d) zero

83. The time period of a second's pendulum on the surface of moon will be

(a) $\frac{2}{\sqrt{6}}$ s (b) $6\sqrt{2}$ s
(c) $2\sqrt{6}$ s (d) 2 s

84. The equation of simple harmonic motion of a particle is

$\frac{d^2x}{dt^2} + 0.2\frac{dx}{dt} + 36x = 0$. Its time period is approximately

(a) $\pi/6$ s (b) $\pi/4$ s
(c) $\pi/3$ s (d) $\pi/2$ s

85. A spring of spring constant k is cut into two pieces such that their lengths are in the ratio 1 : 2, then the force constant of the bigger piece will be

(a) $3/2\,k$ (b) $k/2$
(c) $2k$ (d) $3k$

86. A body of mass 100 gm is suspended from a spring of force constant 50 N/m. The maximum acceleration produced in the spring is

(a) $g/2$ (b) g
(c) $g/3$ (d) $g/4$

87. A second's pendulum is kept in a satellite revolving at a height 3 R_E from earth's surface. Its time period will be

(a) $2\sqrt{3}$ s (b) 1 s
(c) zero (d) infinity

88. The time period of a particle executing SHM is $\frac{2\pi}{\omega}$ and its velocity at a distance b from mean position is $\sqrt{3}\,b\omega$. Its amplitude is

(a) b (b) $2b$
(c) $3b$ (d) $4b$

89. The time period of a second's pendulum on the surface of moon is found to be 5 s. The acceleration due to gravity on the surface of moon is

(a) 3.2 ms^{-2} (b) 1.6 ms^{-2}
(c) 0.8 ms^{-2} (d) 0.6 ms^{-2}

90. The displacement of a particle executing SHM at any instant t is $x = 7 \cos 0.5\,\pi t$. The time taken by the particle in reaching from mean position to extreme position will be

(a) 4 s (b) 2 s
(c) 1 s (d) 1/2 s

91. A body of mass M and charge q is suspended from a spring. When slightly displaced, it oscillates with period T. If a uniform electric field acts vertically downwards, then the new time period will be

(a) $T' = T$ (b) $T' < T$
(c) $T' > T$ (d) cannot be predicted

92. A body of mass M is situated in a potential field $U(x) = U_0(1 - \cos dx)$, where U_0 and d are constants. The time period of small oscillations will be

(a) $2\pi\sqrt{MU_0d^2}$ (b) $2\pi\sqrt{\frac{M}{U_0d^2}}$
(c) $2\pi\sqrt{\frac{U_0d^2}{M}}$ (d) $2\pi\sqrt{\frac{U_0}{Md^2}}$

93. Restoring force in the SHM is

(a) conservative. (b) nonconservative.
(c) frictional. (d) centripetal.

94. What will be the percentage change in the time period of a simple pendulum if its length is increased by 5%?

(a) 1/9% (b) 2.5%
(c) 5.2% (d) 9%

95. Which of the following quantities is non-zero at the mean position for a particle executing SHM?

(a) force (b) acceleration
(c) velocity (d) displacement

96. A bar magnet is oscillating in the earth's magnetic field with time period T. If its mass is increased four times then its time period will be

(a) $T/2$ (b) T
(c) $2T$ (d) $4T$

97. A 5 kg. weight is suspended from a spring. The spring stretches by 2 cm/kg. If the spring is stretched and released, its time period will be

(a) 0.0628 s (b) 0.628 s
(c) 6.28 s (d) 62.8 s

98. The ratio of total energy of a harmonic oscillator to its average potential energy with respect to position will be

(a) 1 : 2 (b) 3 : 1
(c) 1 : 3 (d) 2 : 11

99. The height of liquid column in a U-tube is 0.3 m. If the liquid in one of the limbs is depressed and then released, then the time period of liquid column will be

(a) 0.11 s (b) 1.1 s

(c) 2 s (d) 19 s

100. The displacement of a particle executing simple harmonic motion at any instant t is $x = 3\cos 2\pi t$. The acceleration produced in the particle will be

(a) $-12\pi^2 \sin 2\pi t$ (b) $-12\pi^2\cos 2\pi t$

(c) $12\pi\cos 2\pi t$ (d) zero

101. A simple pendulum of mass m and length l is oscillating along a circular arc of angle ϕ. Another bob of mass m is lying at the extreme position of the arc. The momentum imparted by the moving bob to the stationary bob will be

(a) $mlv \cos \phi$ (b) $mv \sin \phi$

(c) mlv/ϕ (d) zero

102. A block of 4 kg produces an extension of 0.16 m in a spring. The block is replaced by a body of mass 0.50 kg. If the spring is stretched and then released the time period of motion will be

(a) 28.3 s (b) 2.83 s

(c) 0.283 s (d) 0.0283 s

103. The resultant of $y = y_0 \sin \omega t$ and $x = x_0 \sin \omega t$ will be a/an

(a) ellipse. (b) circle.

(c) figure of 8. (d) straight line.

104. Two particles are executing SHM. Their displacements at any instant of time t are $x = a \sin(\omega t - \phi)$ and $y = b \cos(\omega t - \phi)$. The phase difference between them will be

(a) 0° (b) ϕ

(c) 90° (d) 180°

105. The equation of displacement of a harmonic oscillator is $x = 3 \sin \omega t + 4 \cos \omega t$. The amplitude of the particles will be

(a) 1 (b) 5

(c) 7 (d) 12

106. The maximum velocity of a harmonic oscillator is d and its maximum acceleration is β. Its time period will be

(a) $\frac{\pi\beta}{d}$ (b) $2\pi d\beta$

(c) $\frac{2\pi d}{\beta}$ (d) $\frac{2\pi\beta}{d}$

107. A particle of mass 0.1 kg is executing simple harmonic motion of amplitude 1 m and time period 0.2 s. The maximum force on the particle will be

(a) 0.99 N (b) 9.9 N

(c) 9 N (d) 99 N

108. The bob of a simple pendulum of mass m is oscillating with angular amplitude 40°. The tension in the string at the instant when its angular displacement is 20° will be

(a) $mg \cos 20°$ (b) $> mg \cos 20°$

(c) $< mg \cos 20°$ (d) mg

109. A simple pendulum is suspended from the celling of a lift. When the lift is at rest, its time period is T. With what acceleration should lift be accelerated upwards in order to reduce its time period to $T/2$?

(a) $-3g$ (b) g

(c) $2g$ (d) $3g$

110. The kinetic energy of a harmonic oscillator is $K = K_0 \cos 2\ \omega t$. The maximum potential energy of the particle is

(a) zero (b) $\frac{K_0}{2}$

(c) K_0 (d) $2K$

111. A body is lying on a piston which is executing vertical SHM. Its time period is 2 s. For what value of amplitude, the body will leave the piston

(a) = 1 m (b) 0.248 m

(c) 0.428 m (d) 0.842 m

112. The time period of a simple pendulum as seen by an astronaut in a spaceship is

(a) 84.6 min (b) 2 s

(c) ∞ (d) 0

113. Infinite springs with force constant k, 2k, 4k 8k ____ respectively are connected in series. The effective force constant of the spring will be

(a) $k/2$ (b) k

(c) $2k$ (d) $2048k$

114. A particle is executing SHM along a straight line 8 cm long. While passing through mean position its velocity is 16 cms^{-1}. Its time period will be

(a) 0.0157 s (b) 0.157 s

(c) 1.57 s (d) 15.7 s

115. The time period of the second's hand of watch is

(a) 1 s (b) 1 min

(c) 1 h (d) 12 h

PASSAGE 1

Read the following passage and answer the questions given at the end.

While on a visit to Minnesota (Land of 10,000 lakes), a man signs up to take an excursion around one of the larger lakes. When you go to the dock where the 1500 kg boat is tied, he finds that the boat is bobbing up and down in the waves, executing simple harmonic motion of amplitude 20 cm. The boat takes 3.5 *s* to make one complete up down motion cycle when the boat is at its highest point, its dock is at the same point as for stationary dock. As the man watches the boat bob up and down, the man (mass 60 kg) begins to feel a bit woozy, due in part to previous night's dinner of lutefish. The man, as a result, refuses to board the boat unless the level of the boat deck's is with in 10 cm of the dock level.

1. How much time the man has to board the boat comfortably during each cycle of up and down motion?

(a) 0.585 s (b) 1.17 s
(c) 2.33 s (d) 0.293 s

2. What is the maximum velocity of oscillation of the boat?

(a) $0.36\ ms^{-1}$ (b) $0.18\ ms^{-1}$
(c) $0.54\ ms^{-1}$ (d) none

3. What is the velocity of the boat when the level of deck is 10cm of the dock?

(a) $0.18\ ms^{-1}$ (b) $0.26\ ms^{-1}$
(c) $0.32\ ms^{-1}$ (d) $0.43\ ms^{-1}$

Solution **1.** (b) $10 = 20 \cos wt$

or $$wt = \frac{\pi}{3} \text{ or } \frac{2\pi t}{3.5} = \frac{\pi}{3}$$

$$t_{tot} = \frac{3.5}{6} + \frac{3.5}{6}$$

$$= 1.17\, s \left(\frac{3.5}{6}\, s \text{ on each side of vibration}\right)$$

Solution **2.** (a) $v = x_0 w$

$$= 0.2\left(\frac{2\pi}{3.5}\right) = \frac{1.256}{3.5} = 0.36\ ms^{-1}.$$

Solution **3.** (c) $v = w\sqrt{x_0^2 - x^2} = \frac{2\pi}{3.5}\sqrt{.2^2 - .1^2}$

$$= \frac{2\pi}{3.5}\sqrt{.03} = 0.18\sqrt{3}\ ms^{-1}$$

PASSAGE 2

Read the following passage and answer the questions given at the end.

While visiting friends at cal state chico, you pay a visit to the Crazy Horse Saloon. This fine establishment features a 200 kg mechanical bucking bull, that has a mechanism that makes it move vertically in SHM. Whether the bull has a rider or not it moves with same amplitude (0.25m) and frequency (1.5 Hz). After watching other saloon patrons hold on the bull while riding, you (mass 75 kg) decide it to ride the macho way by not holding on. No one is terribly surprised when you come out of the saddle. Later while waiting for your bruises and pride to heal, you pass the time by calculating

1. If you leave the saddle when it is moving upward. The magnitude of down ward acceleration of the saddle is ----- when you lose contact with it.

(a) $6\ ms^{-2}$ (b) $7.2\ ms^{-2}$
(c) $9.8\ ms^{-2}$ (d) $12.2\ ms^{-2}$

2. How high is the saddle surface above the equilibrium position when you first become air borne ?

(a) 0.25 m (b) 0.21 m
(c) 0.16 m (d) 0.11 m

3. To what time you remain air borne (free fall) until you return to the saddle.

(a) 0.582 s (b) 0.528 s
(c) 0.483 s (d) none

Solution **1.** (c)

Solution **2.** (a) when at the highest point saddle just begins to come down you become air borne

Solution **3.** (b) The motion of the man is upward with velocity $v = 0.25\ (2p \times 1.5)$

$v = 0.75\ \pi ms^{-1}$ and falls with acceleration g. Time taken to return to leaving point

$$= \frac{2v}{g} = \frac{2 \times (.75\pi)}{g} = \frac{1.5}{\pi}$$

= 0.48 s. In that time saddle has not reached to him. As

$T = \frac{1}{1.5} = 0.666$ s. Therefore, man will settle down on, saddle again when the two are at same position. When man is at extreme position during return (.25m) saddle is at $y = .25 \cos 3\pi(.48) = -.047$ m from mean position Therefore, the saddle reaches to him when $a = g$

$$g = y\,(2\pi f)^2$$

or $$y = \frac{1}{9}\text{ m};\ y = y_0 \sin \omega t$$

$\Rightarrow$ $t = 0.048$; total time = 0.48 + .048 = .528 s

PASSAGE 3

Read the following passage and answer the questions given at the end.

The equation of a particle moving in S.H.M. is

$$\frac{d^2X}{dt^2} = -\omega^2 x$$

where ω is a constant, being equal to $\dfrac{2\pi}{\text{Time period}}$. The velocity of such a particle is maximum when it passes through its mean position while it is subjected to maximum acceleration at the extreme positions. The solution to the above equation is

$$x = A \sin(\omega t + \theta)$$

Where θ is a constant called the initial phase of the motion.

1. The time period of a simple pendulum is given by the equation $T = 2\pi\sqrt{\dfrac{l}{g}}$. The equation of motion of the pendulum will be $\dfrac{d^2x}{dt^2} =$

(a) $-\dfrac{l}{g}x$ (b) $\dfrac{g}{l}x$

(c) $\dfrac{l}{g}x$ (d) $-\dfrac{g}{l}x$

Solution **1.** (b)

2. A particle in S.H.M. has an amplitude of 20 cm and time period of 2 sec. Its maximum velocity will be........m/s.

(a) $0.04\ \pi^2$ (b) $0.2\ \pi$

(c) $0.2\ \pi^2$ (d) $0.04\ \pi$

Solution **2.** (b)

3. The time period of a particle in S.H.M. does not depend upon its

(a) frequency. (b) acceleration.

(c) amplitude.

(d) it depends on all the above.

Solution **3.** (b)

4. If a system is displaced from its equilibrium position and released, it moves according to the equation

$$\ddot{\theta} = -\frac{I^2}{kl}\theta$$

where I, k and l are constants. It will oscillate with a frequency

(a) $\sqrt{\dfrac{I^2}{kl}}$ (b) $2\pi\sqrt{\dfrac{kl}{I^2}}$

(c) $\dfrac{1}{2\pi}\sqrt{\dfrac{I^2}{kl}}$ (d) $\dfrac{1}{2\pi}\sqrt{\dfrac{kl}{I^2}}$

Solution **4.** (c)

Answers to Questions for Practice

1.	(c)	**2.**	(c)	**3.**	(c)	**4.**	(d)	**5.**	(c)	**6.**	(d)	**7.**	(d)
8.	(c)	**9.**	(b)	**10.**	(a)	**11.**	(c)	**12.**	(b)	**13.**	(b)	**14.**	(c)
15.	(c)	**16.**	(b)	**17.**	(d)	**18.**	(b)	**19.**	(b)	**20.**	(a)	**21.**	(b)
22.	(b)	**23.**	(a)	**24.**	(d)	**25.**	(c)	**26.**	(b)	**27.**	(a)	**28.**	(a)
29.	(b)	**30.**	(a)	**31.**	(b)	**32.**	(d)	**33.**	(a)	**34.**	(d)	**35.**	(c)
36.	(b)	**37.**	(b)	**38.**	(b)	**39.**	(c)	**40.**	(c)	**41.**	(c)	**42.**	(a)
43.	(b)	**44.**	(a)	**45.**	(d)	**46.**	(b)	**47.**	(c)	**48.**	(c)	**49.**	(d)
50.	(b)	**51.**	(b)	**52.**	(a)	**53.**	(c)	**54.**	(b)	**55.**	(b)	**56.**	(a)
57.	(b)	**58.**	(a)	**59.**	(d)	**60.**	(a)	**61.**	(d)	**62.**	(c)	**63.**	(a)
64.	(d)	**65.**	(d)	**66.**	(c)	**67.**	(a)	**68.**	(a)	**69.**	(b)	**70.**	(b)
71.	(a)	**72.**	(a)	**73.**	(b)	**74.**	(c)	**75.**	(a)	**76.**	(d)	**77.**	(c)
78.	(c)	**79.**	(b)	**80.**	(a)	**81.**	(a)	**82.**	(a)	**83.**	(c)	**84.**	(c)
85.	(a)	**86.**	(b)	**87.**	(d)	**88.**	(b)	**89.**	(b)	**90.**	(c)	**91.**	(b)
92.	(b)	**93.**	(a)	**94.**	(b)	**95.**	(c)	**96.**	(c)	**97.**	(c)	**98.**	(b)
99.	(b)	**100.**	(b)	**101.**	(d)	**102.**	(c)	**103.**	(d)	**104.**	(c)	**105.**	(b)
106.	(c)	**107.**	(d)	**108.**	(b)	**109.**	(d)	**110.**	(c)	**111.**	(a)	**112.**	(c)
113.	(a)	**114.**	(c)	**115.**	(b)								

EXPLANATION

8. (c) $\Delta T = \dfrac{T\Delta l}{2l} = \dfrac{86400}{2\times 100}$

28. (a) $x_0\omega^2 = g$ or $x_0 = \dfrac{10}{\left(2\pi\times\dfrac{3}{\pi}\right)^2} = \dfrac{5}{18}$.

54. (b) $T = 2\pi\sqrt{\dfrac{l}{g}}\left(1+\dfrac{\theta_0^2}{16}\right) = 2\pi\sqrt{\dfrac{l}{g}}\left(1+\dfrac{(\pi/6)^2}{16}\right)$

$$= 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{10}{36\times 16}\right)$$

58. (a) $T = \sqrt{\dfrac{1}{g\left(1 - \dfrac{\sigma}{\rho}\right)}}$

71. (a) Total energy = Resting energy + KE + PE

$$= 10\text{ J} + 5\text{J} + 5\text{J}$$

$$= 201.$$

84. (c) $\omega^2 = 36$ or $\omega = 6$

or $\quad \dfrac{2\pi}{T} = 6$

or $\quad T = \dfrac{\pi}{3}.$

92. (b) $U = U_0(1 - \cos dx)$

$$F = -\frac{dU}{dx}$$

$$= -U_0 d \sin dx$$

or $\quad a = \dfrac{-U_0 d^2}{M}\, x$

or $\quad \omega = \sqrt{\dfrac{U_0 d^2}{M}}.$

106. (c) $v_{\max} = x_0\omega \;\; a_{\max} = x_0\omega^2$

$\therefore \quad \omega = \dfrac{a_{\max}}{v_{\max}} = \dfrac{\beta}{d}$

or $\quad T = \dfrac{2\pi}{\beta} = \dfrac{\beta}{d}$

or $\quad T = \dfrac{2\pi d}{\beta}.$