8 Rotational Motion

BRIEF REVIEW

Moment of Inertia (MOI) Moment of inertia plays the same role in rotational motion as mass in the linear motion. Moment of inertia $I = \sum m_i r_i^2$. MOI is a tensor quantity.

$I = \int r^2 dm$ if mass is uniformly distributed; $I = M k^2$ where M is total mass of the body and k is radius of gyration.

Radius of Gyration (k) is the root mean square perpendicular distance of the body from axis of rotation.

$$k = \sqrt{\frac{m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2}{m_1 + m_2 + ... + m_n}}$$

$$= \frac{1}{M} \int r^2 dm$$

MOI of the bodies about an axis passing through their COM (centre of mass) and perpendicular to the plane of the body are given as

MOI of a Ring $I_{\text{ring}} = M R^2$

MOI of a disc (solid) $I_{\text{disc}} = \frac{M R^2}{2}$

MOI of an annular disc

$$I_{\text{Annular disc}} = \frac{M}{2} \left(R_1^2 + R_2^2\right) \text{ (fig. 8.1).}$$

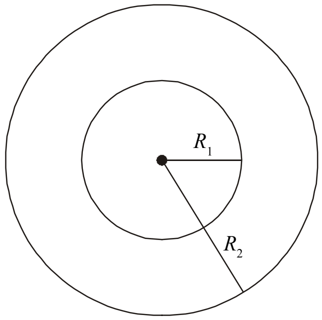

Fig. 8.1 **Annular disc**

MOI of a solid cylinder $I_{\text{cylinder}} = \frac{M R^2}{2}$

MOI of a hollow cylinder = $M R^2$ (if shell type, i.e., extremely thin walls).

$$= \frac{M}{2} \left(R_1^2 + R_2^2\right) \text{ (fig. 8.2).}$$

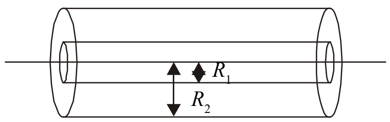

Fig. 8.2 **Hollow cylinder**

MOI of a spherical shell = $\frac{2}{3} M R^2$

MOI of a solid sphere = $\frac{2}{5} M R^2$

MOI of a hollow sphere $= \frac{2}{5} M \left(R_1^2 + R_2^2\right)$

MOI of a rod (cylindrical) $= \frac{M\,l^2}{12}$ (fig. 8.3)

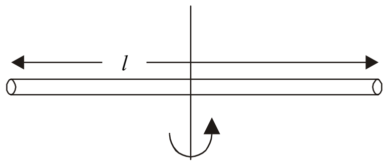

Fig. 8.3 **Rod**

MOI of a rod (rectangular) $= \frac{M(l^2 + b^2)}{12}$

MOI of a Lamina (rectangular) $= \frac{M}{12}\,(l^2 + b^2)$

MOI of a parallelopiped $= \frac{M}{12}\,(l^2 + b^2)$ (fig. 8.4)

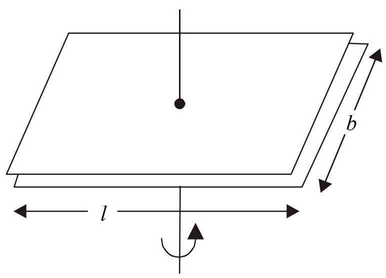

Fig. 8.4 **Rectangular lamina**

MOI of an elliptical disc $= \frac{M}{4}\,(a^2 + b^2)$ (fig. 8.5)

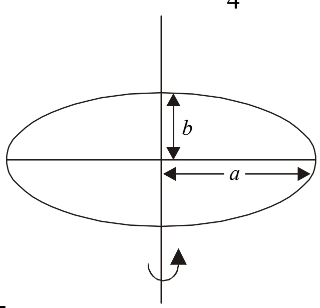

Fig. 8.5 **Elliptical disc**

MOI of a cone (Right circular cone) $= \frac{3}{10}\,M R^2$ (fig. 8.6)

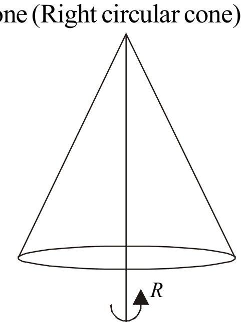

Fig. 8.6 **MOI of prism**

MOI of a prism or equilateral triangle $= \frac{M\,l^2}{6}$

MOI of a triangular lamina (about base)

$= \frac{M\,b^2}{6}$ (see fig. 8.7)

MOI of a triangular lamina about perpendicular

$I_p = \frac{M\,p^2}{6}$.

MOI of a triangular lamina about hypotenuse

$I_h = \frac{mb^2 p^2}{6(p^2 + b^2)}$.

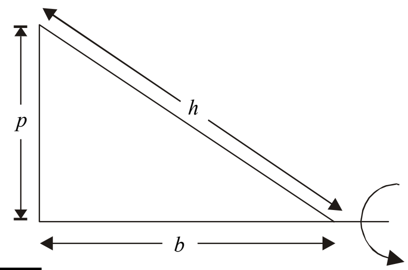

Fig. 8.7 **MOI of a triangular lamina**

MOI of a cone about XOX' $= \frac{3}{5} M \left(\frac{R^2}{4} + h^2\right)$ (fig. 8.8)

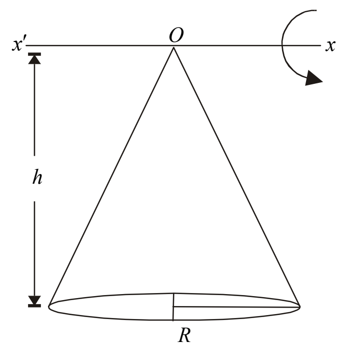

Fig. 8.8 **MOI of a cone**

MOI of a rod about one end $= \frac{Ml^2}{3}$ [fig. 8.9]

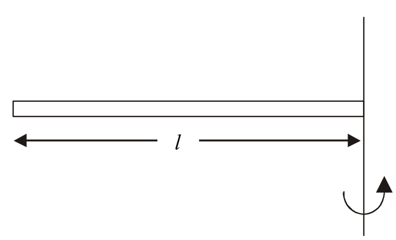

Fig. 8.9 **MOI of a rod**

Parallel axis theorem If MOI about an axis passing through COM of a body is known, the MOI of the body about an axis parallel to the axis passing through COM and at a distance x from it as illustrated in fig. 8.9 is

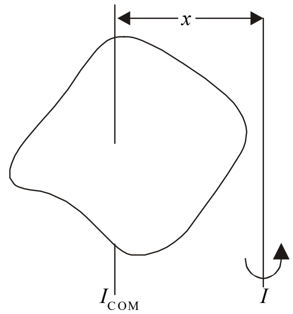

Fig. 8.9 **Parellel axis theorem illustration**

$I = I_{COM} + Mx^2$ where I_{COM} is the MOI about an axis passing through their COM.

Perpendicular Axis Theorem It can be applied only to plane lamina bodies. If x- and y- axes chosen in the plane of the body and z-axis be perpendicular to this plane, three being mutually perpendicular, then

$I_z = I_x + I_y$ where I_x and I_y are MOI about x-axis and y-axis respectively.

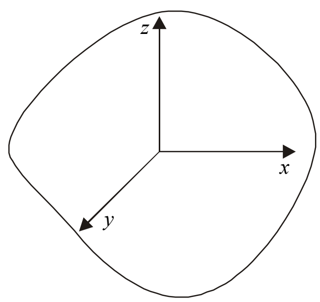

Fig. 8.10 **Perpendicular axis theorem illustration**

Angular velocity (instantaneous) $\omega = \dfrac{d\theta}{dt}$

Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

Linear velocity $\upsilon = r\omega$; tangential acceleration $a_t = r\alpha$

$$\omega = \omega_0 + \alpha t \quad ; \theta = \omega_0 t + \frac{1}{2}\alpha t^2 \;;\; \omega^2 = \omega_0^2 + 2\alpha\theta$$

Torque (τ) $\vec{\tau} = \vec{r} \times \vec{F} = I\alpha$

$|\vec{\tau}| = \Sigma$ Force $\times$ perpendicular from the axis of rotation.

$\vec{\tau} = \dfrac{dL}{dt}$ where L is angular momentum.

Note: Torque is moment of a force about a point. Though dimensions of torque are same as that of energy but it is not energy. Its unit is $N\text{–}m$. Dimensional formula is $[ML^2\ T^{-2}]$.

If line of action of a force passes through its COM then such a force will not form torque.

Angular Momentum is moment of momentum (linear) about a point, i.e., $\vec{L} = \vec{r} \times \vec{p}$

$\vec{L} = I\omega$; $|\vec{L}| = \Sigma p \times$ (perpendicular distance from axis of rotation).

Note: If external torque is zero then angular momentum is conserved.

Dimensional formula of $L = [ML^2\ T^{-1}]$. Its unit is kg m^2 s^{-1} and is same as that of Planck's constant h.

Angular impulse $J = \int_{t_1}^{t_2} \tau . dt = \Delta L = L_2 - L_1$

Rotational kinetic energy $= \dfrac{1}{2} I\omega^2$.

Note if a body only rotates about a fixed axis then it posseses only rotational KE. If, however, a body rolls then it possesses both rotational KE and linear KE, i.e. **Total KE**

$=$ **Rotational KE** $+$ **Linear KE** $= \dfrac{1}{2} I\omega^2 + \dfrac{1}{2} m\upsilon^2$

Work done $W = \int \vec{\tau} . d\vec{\theta}$; Rotational Power $P_{rot} = \vec{\tau} . \vec{\omega}$.

Acceleration of a body rolling down an incline plane: In Fig. 8.11

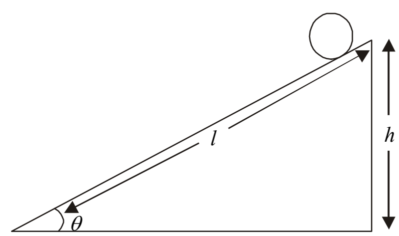

Fig. 8.11 **Acceleration of body rolling down an incline**

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{MR^2}} = \frac{g\sin\theta}{1 + \dfrac{k^2}{R^2}}$$

Velocity on reaching ground $\upsilon = a . t = \sqrt{\dfrac{2gh}{1 + \dfrac{k^2}{R^2}}}$

Time taken to reach the ground $t = \sqrt{\dfrac{2l\left(1 + \dfrac{k^2}{R^2}\right)}{g\sin\theta}}$

For a system to be in rotational equilibrium $\Sigma\vec{\tau} = 0$

For a system to be in linear equilibrium $\Sigma F = 0$

For total equilibrium (Rotational + Linear)

$\Sigma\tau = 0, \Sigma F = 0.$

Combined motion (Rotation + translation)

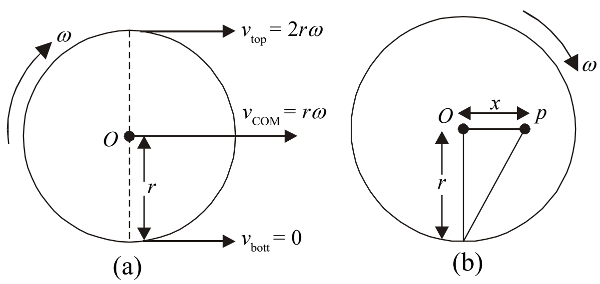

Fig. 8.12 **Velocities of different points of a wheel**

$$\vec{a}_{COM} = \frac{F_{ext}}{M} \text{ and } \alpha = \frac{\tau_{COM}^{ext}}{I_{COM}}$$

These equations together with initial conditions completely define the motion. τ_{COM}^{ext} is external torque about COM.

$$\upsilon_{COM} = r\omega$$

$$\upsilon_{bott} = 0 \; ; \; \upsilon_{top} = 2\,\upsilon_{COM} = 2r\omega \text{ [See Fig. 8.12 (a)]}$$

In Fig. 8.12 (b) in pure rolling $\upsilon_p = \omega\sqrt{r^2 + x^2}$.

Pure rolling $\upsilon_{COM} = r\omega$, the wheel completes 1 rotation and covers a distance = $2\pi r$.

Rolling with forward slipping If the wheel moves a distance > $2\pi r$ in one complete rotation then $\upsilon_{COM} > r\omega$ and motion is termed as rolling with forward slipping.

Rolling with backward slipping If the wheel moves a distance < $2\pi r$ in one complete rotation then $\upsilon_{COM} < r\omega$ and motion is known as rolling with backward slipping.

Angular momentum of a body in combined rotation and translation:

$L = L_{COM} + M\left(\vec{r}_0 \times \vec{\upsilon}_0\right)$. $M\left(\vec{r}_0 \times \vec{\upsilon}_0\right)$ is assumed to be the angular momentum as if mass is concentrated at COM and translating with υ_0. In an accelerating wheel force of friction acts in the direction of motion. So that frictional torque acts in a direction to oppose the accelerating torque.

If the wheel is rolling with forward slipping then force of friction acts in a direction opposite to the motion of the wheel until pure rolling begins.

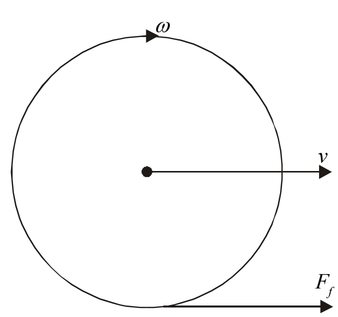

Fig. 8.13 **Friction in accelerating wheel**

Table 1. Equivalence between rotational and translation motion

Linear motion	Rotational motion
Displacement x	Angular displacement θ
Linear velocity $\upsilon = \frac{dx}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$
Acceleration $a = \frac{d\upsilon}{dt}$	Angular acceleration $\alpha = \frac{d\omega}{dt}$
mass m	MOI I
Linear momentum $p = m\upsilon$	Angular momentum $L = I\,\omega$
Force $F = ma$	Torque $\tau = I\,\alpha$
Impulse $I = \int F dt = \Delta p$	Rotational impulse $J = \int \tau dt$
Work $W = \int \vec{F}.d\vec{x}$	Work $W = \int \vec{\tau}.d\vec{\theta} = \Delta L$
KE = $\frac{1}{2} m\upsilon^2$	Rotational KE = $\frac{1}{2} I\,\omega^2$
Power $P = \vec{F} \cdot \vec{\upsilon}$	Rotational Power = $\vec{\tau} \cdot \vec{\omega}$

Three dimensional rotation is understood from gyrostat.

A spinning top shows

(i) spinning (ii) precession

(iii) nutation or wobbling. Hipparchus in 135 BC found that due to precession of earth ($T_{Precession}$ = 27,725 yrs) a change in the direction of the line of equinoxes occurs and phenomenon is called precession of equinoxes.

SHORT CUTS AND POINTS TO NOTE

1. MOI of a ring about a diameter = $\frac{MR^2}{2}$ (about XY).

   MOI of a ring about the tangent TT' parallel to diameter XY is $\frac{MR^2}{2} + MR^2$

   $= \frac{3}{2} MR^2$ (See Fig. 8.14).

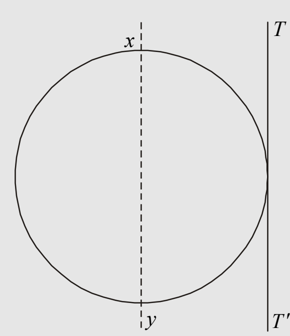

Fig. 8.14

2. MOI of the disc about one of the diameters = $\frac{MR^2}{4}$.
MOI of the disc about the tangent parallel to one of diameters = $\frac{5}{4}MR^2$

3. MOI of a ring about a tangent perpendicular to the plane of the ring = $2MR^2$ (See fig. 8.15).

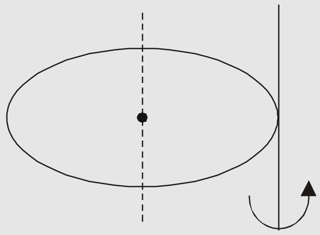

Fig. 8.15

MOI of a disc about a tangent perpendicular to the plane of the disc = $MR^2 + \frac{MR^2}{2} = \frac{3}{2}MR^2$

4. MOI is a tensor. Its value may vary with the direction. However, they are added like scalars.

5. MOI of hollow bodies is higher than MOI of solid bodies.

6. Acceleration of bodies rolling down an inclined plane is $a = \dfrac{g\sin\theta}{1+\dfrac{I}{MR^2}} = \dfrac{g\sin\theta}{1+\dfrac{k^2}{R^2}}$.

7. If bodies of equal radius roll down an inclined plane then a sphere will reach first (take minimum time) and ring will reach the last (take maximum time. Note, acceleration $\propto \frac{1}{I}$. Hence, more the MOI lesser will be the acceleration or higher is the time to roll down.
The minimum value of coefficient of friction required to roll down on incline plane is

$$\mu = \frac{\dfrac{I}{MR^2}\tan\theta}{1+\dfrac{I}{MR^2}}$$

8. While deciding about which axis MOI is maximum, consider $\sum m_i r_i^2$. The axis about which $\sum m_i r_i^2$ is large will have longer values of r for equal mass or heavier mass located farther.

9. Rotational KE = $\frac{1}{2}I\omega^2$ and total KE when a body is rolling is $KE_{Tot} = \frac{1}{2}I\omega^2 + \frac{1}{2}m\upsilon^2$.

10. x-component of the torque is

$$\hat{i}\,(F_z y - F_y z) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & y & z \\ 0 & F_y & F_z \end{vmatrix}.$$

Similarly, other components can be written y component is $-\hat{j}\,(F_z x - F_x z)$ and z component is $\hat{k}\,(F_y x - F_x y)$.

11. If mass of the pulley is m_p, thread is massless and pulley is smooth then

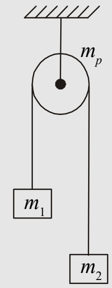

Fig. 8.16

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2 + \dfrac{m_p}{2}} = \frac{(m_2 - m_1)g}{\left(m_1 + m_2 + \dfrac{I}{R^2}\right)} \text{ where}$$

I is MOI of the pulley and R its radius.

12. The system is in rotational equilibrium if $\sum \vec{\tau} = 0$ and it is also in linear equilibrium if $\sum F = 0$.

13. Apply conservation of angular momentum if $\vec{\tau}_{ext} = 0$.

14. A body rolling with forward slipping has friction in a direction opposite to its motion untill pure rolling begins. υ_{COM} decreases and ω increases.

15. A body rolling with backward slipping has friction in the direction of motion so as to increase υ_{COM} until $\upsilon_{COM} = r\omega$. That is υ_{COM} increases and ω decreases.

16. When a body is accelerating with angular acceleration α, i.e., torque is applied then friction acts in the direction of motion as shown in fig. 8.17 so that frictional torque opposes the accelerating torque.

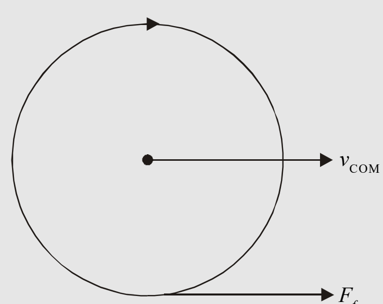

Fig. 8.17

17. In pure rolling $\upsilon_{COM} = r\omega$. Note, if a body is to roll down an incline plane then the incline must be rough. On smooth incline rolling cannot occur.

18. Though torque has dimensions of energy. It is not energy. Unit is N–m.

19. Velocity of precession $\omega_P = \frac{\tau_P}{L} = \frac{dL}{dt\,L}$

$\left\{\because \tau = \frac{dL}{dt}\right\}$ when ω_P increases body is about to fall.

20. Angular speed ω of a gyrostatic pendulum and time period $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$, and, $\omega = \sqrt{\frac{g}{l\cos\theta}}$.

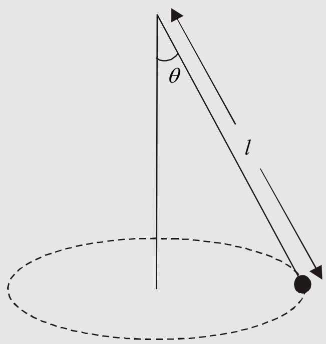

Fig. 8.20

21. If two rotating bodies having MOI I_1 and I_2 moving with speeds ω_1 and ω_2 join, then common angular velocity $\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$, loss in KE $= \frac{I_1 I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$.

CAUTION

1. Adding MOI like vectors or taking its components while finding MOI of a composite body.

⇒ MOI is a tensor. It is added like scalar.

2. Considering acceleration of a body rolling down an incline as $g \sin\theta$.

⇒ Note $a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$

3. Assuming in rotational motion $v = u + a\,t$,

$s = u\,t + \frac{1}{2}\,a\,t^2$ etc can be applied.

⇒ Apply $\omega = \omega_0 + \alpha\,t$, $\theta = \omega_0\,t + \frac{1}{2}\,\alpha\,t^2$,

$\omega^2 = \omega_0^2 + 2\,\alpha\,\theta$

if there is pure rotation. If there is rolling apply both. For linear motion of COM apply $V_{COM} = u_{COM} + a_{COM}\,t$ and so on. During rolling i.e. combined motion both $\omega = \omega_0 + \alpha\,t$ etc. and $V_{COM} = u_{COM} + a_{COM}\,t$ are applied. To combine

$a = r\alpha$ or $\upsilon = r\omega$ when pure rolling begins.

4. Considering frictional force stops rotation/rolling.

⇒ Rotational motion is stopped by frictional torque is not always true. If the body is rolling with forward slipping, friction acts in a direction to increase the rotational velocity or angular velocity.

Consider the case of a bicycle. The back wheel is paddled. The front wheel moves due to friction.

5. Considering solid bodies rotate more.

⇒ Hollow bodies have large MOI. They rotate more. This is the reason that all wheels are either made hollow or mass is concentrated at the rim.

6. Considering that a sphere can roll on a smooth inclined plane.

⇒ Minimum amount of coefficient of friction required is $\frac{2}{7}\tan\theta$, θ being angle of inclination.

7. Considering $\upsilon = r\omega$ in all cases of rolling.

⇒ $\upsilon = r\omega$ if there is pure rolling, i.e., rolling without slipping. In case of rolling with forward slipping $\upsilon > r\omega$ and in case of rolling with backward slipping $\upsilon < r\omega$.

8. When pulley is smooth but has mass, string is massless, considering mass of the pulley is redundant.

⇒ acceleration of blocks [see Fig. 8.18]

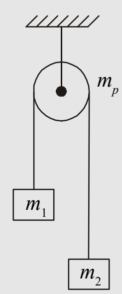

Fig. 8.18

$a = \frac{(m_2 - m_1)g}{\left(m_2 + m_1 + \frac{m_p}{2}\right)}$ where m_p is mass of the pulley. If MOI of the pulley is given then $a = \frac{(m_2 - m_1)g}{\left(m_2 + m_1 + \frac{I}{r^2}\right)}$

where I is MOI of the pulley.

9. Assuming perpendicular axis theorem (to find MOI) can be applied to any body.

⇒ Perpendicular axis theorem can be applied only to plane lamina. Parallel axis theorem is valid for all types of bodies.

10. Confused, what to do in rotational collision?

⇒ Conserve angular momentum.

SOLVED PROBLEMS

1. The moment of inertia of a uniform semicircular disc of mass M and radius R about a line perpendicular to the plane of disc and passing through the centre is

(a) $\dfrac{MR^2}{4}$ (b) $\dfrac{2}{5}MR^2$

(c) MR^2 (d) $\dfrac{MR^2}{2}$

[AIEEE 2005]

Solution (d) $2I = 2M\left(\dfrac{R^2}{2}\right)$ or $I = \dfrac{MR^2}{2}$.

2. A T shaped object with dimensions shown in fig, is lying on a smooth floor. A force F is applied at point P parallel to AB such that the object has only translational motion without rotation. Find location of P from C

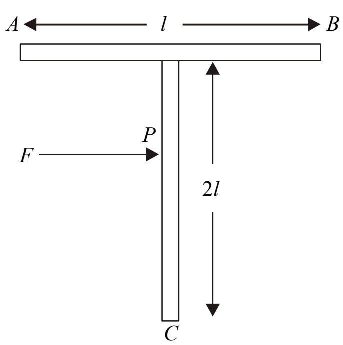

Fig. 8.19

(a) $\dfrac{2}{3}l$ (b) $\dfrac{3}{2}l$

(c) $\dfrac{4}{3}l$ (d) l

[AIEEE 2005]

Solution (c) P should be COM. Take C as origin.

$$x = \frac{2m(l) + m(2l)}{2m + m} = \frac{4l}{3}$$

3. A circular disc of radius $R/3$ is cut from a circular disc of radius R and mass 9 M as shown. Then MOI of the remaining disc about O perpendicular to the disc is

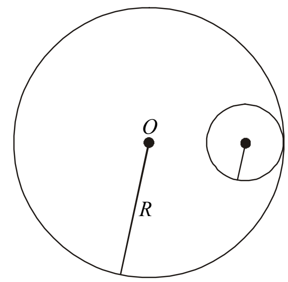

Fig. 8.20

. (a) $4MR^2$ (b) $9MR^2$

(c) $\dfrac{37}{9}MR^2$ (d) $\dfrac{40}{9}MR^2$

[IIT Screening 2005]

Solution (a) $I = \dfrac{9MR^2}{2} - \left[\dfrac{M}{2}\left(\dfrac{R}{3}\right)^2 + M\left(\dfrac{2R}{3}\right)^2\right]$

$= 4MR^2$

mass of hole made $= M = \dfrac{9M}{\pi R^2}\left[\pi\left(\dfrac{R}{3}\right)^2\right]$

4. A sphere is rolling on a frictionless surface as shown in Fig 8.21 with a translational velocity υ ms^{-1}. If it is to climb the inclined surface then υ should be

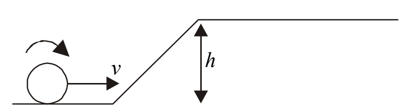

Fig. 8.21

(a) $\geq \sqrt{\dfrac{10}{7}gh}$ (b) $\geq \sqrt{2gh}$

(c) $2gh$ (d) $\dfrac{10}{7}gh$

[AIEEE 2005]

Solution (a) $\dfrac{1}{2}I\omega^2 + \dfrac{1}{2}m\upsilon^2 \geq mgh$

or $\dfrac{1}{2}\left(\dfrac{2}{5}mr^2\right)\omega^2 + \dfrac{1}{2}m\upsilon^2 \geq mgh$

or $\quad \upsilon \geq \sqrt{\frac{10}{7} gh}$

5. A horizontal platform is rotating with uniform angular velocity around the vertical axis passing through its centre. At some instant a viscous fluid of mass m is dropped at the centre and is allowed to spread out and finally fall. The angular velocity during this period

(a) decreases continuously

(b) decreases initially and increases again

(c) remains unaltered

(d) increases continuously

[AIIMS 2005]

Solution (b) Using conservation of angular momentum.

6. A ladder is leaned against a smooth wall and allowed to slip on a frictionless floor. Which figure represents trace of its COM?

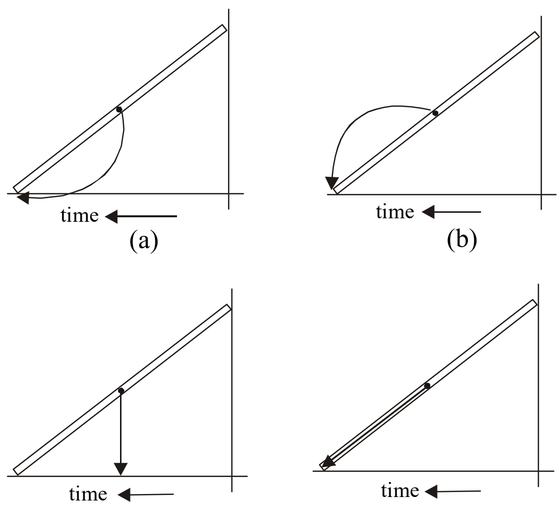

Fig. 8.22

[AIIMS 2005]

Solution (a)

7. The angular momentum of a rotating body changes from A_0 to $4A_0$ in 4 second. The Torque acting on the body is

(a) $\frac{3}{4} A_0$ (b) $4A_0$

(c) $3A_0$ (d) $\frac{3}{2} A_0$

[BHU PMT 2005]

Solution (a) $\tau = \frac{dL}{dt} = \frac{4A_0 - A_0}{4} = \frac{3A_0}{4}$

8. A wooden log of mass M and length L is hinged by a frictionless nail at O. A bullet of mass m strikes with velocity υ and sticks to it, find the angular velocity of the system immediately after collision.

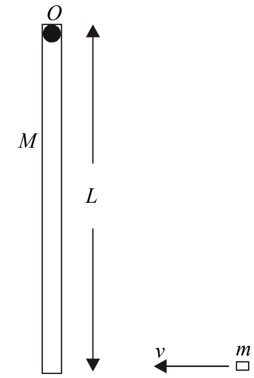

Fig. 8.23

[IIT Mains 2005]

Solution $(m\upsilon) L = \left(\frac{ML^2}{3} + mL^2\right) \omega$

or $\quad \omega = \frac{3m\upsilon}{(M+3m)L}$.

9. A cylinder of mass m and radius R rolls down an incline plane of inclination θ. Find the linear acceleration of the axis of the cylinder.

[IIT Mains 2005]

Solution $a = \frac{g\sin\theta}{1+\frac{I}{MR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3} g\sin\theta$

10. Two identical ladders, each of mass M and length l are resting on a rough horizontal surface as shown in Fig 8.24. A block of mass m hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B.

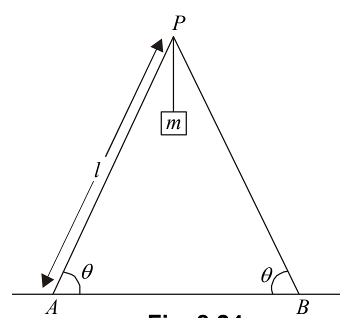

Fig. 8.24

[IIT Mains 2005]

Solution For equilibrium

(i) $\Sigma F_x = 0 \therefore F_{f_1} = -F_{f_2}$

or $\quad N_1 = N_2$

(ii) $\sum F_y = 0 \quad 2N = 2\,Mg + mg$

or $N = Mg + \dfrac{mg}{2}$

(iii) $\sum \tau = 0$ about P for either ladder

$$+\,Mg\,\frac{l}{2}\cos\theta - N\,l\cos\theta + F_f\,l\sin\theta = 0$$

or $$F_f = \frac{(M+m)g}{2}\cot\theta$$

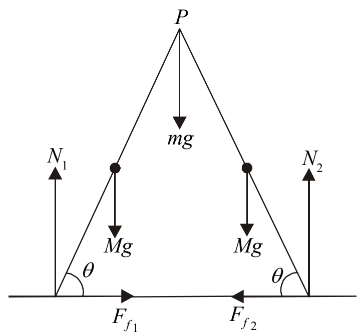

Fig. 8.25

11. An electric motor exerts a constant torque 10 *N–m* on a grind stone mounted on a shaft. MOI of the grind stone about the shaft is 2 kg m^2. If the system starts from rest find the work done in 8s.

(a) 1600 J (b) 1200 J

(c) 800 J (d) 600 J

Solution (a) $\alpha = \dfrac{\tau}{I} = \dfrac{10}{2} = 5$ rad/s^2.

$\omega = \omega_0 + \alpha t = 5\,(8) = 40$ rad s^{-1}

$W = \Delta KE = \dfrac{1}{2}I\omega^2 - 0 = \dfrac{1}{2}\times 2\times(40)^2 = 1600$ J

12. The power output of an automobile engine is advertised to be 200 hp at 600 rpm. Find the corresponding torque

(a) 137 Nm (b) 237 Nm

(c) 337 Nm (d) 287 Nm

Solution (b) $\tau = \dfrac{P}{\omega} = \dfrac{200\times 746}{600\times\dfrac{2\pi}{60}} = 237$ *N–m*.

13. A cable is wrapped several times around a uniform solid cylinder that can rotate about its axis. The cylinder has diameter 12 cm and mass 50 kg. The cable is pulled with a force 9 N. Assuming cable unwinds without stretching or slipping, find its acceleration

(a) 0.3 ms^{-2} (b) 0.32 ms^{-2}

(c) 0.36 ms^{-2} (d) 0.4 ms^{-2}

Solution (c) $\tau = I\,\alpha = F\,R$ or $\alpha = \dfrac{F\,R}{I}$

$= \dfrac{2\times 9}{50\times(.06)} = 6$ rad s^{-2}

$a_t = R\,\alpha = 0.06\times 6 = 0.36$ ms^{-2}

14. A turbine fan in a jet engine has MOI 2.5 kg m^2 about its axis of rotation. Its angular velocity is 40 t^2. Find the net torque at any instant.

(a) 100 t (b) 100 t^2

(c) 200 t (d) 200 t^2

Solution (c) $\tau = \dfrac{dL}{dt} = \dfrac{d}{dt}(I\,\omega) = \dfrac{d}{dt}(2.5\times 40\,t^2)$

$= 200\,t$

15. A fly wheel of mass 2 kg and radius 20 cm has an angular speed 50 rad s^{-1} when a clutch plate of mass 4 kg, radius having an angular speed 200 rad s^{-1} is combined with it. Find the common speed of rotation.

(a) 125 rad s^{-1} (b) 150 rad s^{-1}

(c) 175 rad s^{-1} (d) 100 rad s^{-1}

Solution (d) $\omega = \dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

$= \dfrac{2(.2)^2(50) + 4(.1)^2(200)}{2(.2)^2 + 4(.1)^2}$

$= \dfrac{12}{0.12} = 100$ rad s^{-1}

16. A bicycle wheel has mass of the rim 1 kg and 50 spokes each of mass 5 g. If radius of the wheel is 40 cm then find the MOI of the wheel.

(a) 0.160 kg m^2 (b) 0.174 kg m^2

(c) 0.18 kg m^2 (d) 0.196 kg m^2

Solution (b) $I = M\,R^2 + 50$ m $\dfrac{l^2}{3}$

$= 1\,(0.4)^2 + \dfrac{50\left(5\times 10^{-3}\right)}{3}\left(\dfrac{0.4}{3}\right)^2$

$= 0.16\left[1 + \dfrac{0.25}{3}\right] = 0.174$ Kgm^2

17. A 2 kg rock has velocity 12 ms^{-1} when at point P as shown in Fig 8.26. Find the angular momentum about point D.

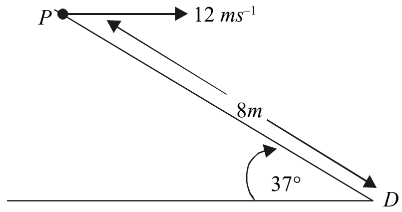

Fig. 8.26

(a) 115.2 kg m^2 s^{-1} (b) 125.2 kg m^2 s^{-1}

(c) 135 kg m^2 s^{-1} (d) none

Solution (a) $L = m\upsilon \times$ perpendicular distance

$$= 2 \times 12 \times 8 \times \frac{3}{5} = 115.2 \text{ kg m}^2 \text{ s}^{-1}$$

18. A beam of length l lies on the $+x$ axis with its left end on the origin. A cable pulls the beam in y-direction with a force $F = F_0\left(1 - \frac{x}{l}\right)$. If the axis is fixed at $x = 0$ then find the torque.

Fig. 8.27

(a) $-F_0 l$ (b) $\frac{F_0 l}{2}$

(c) $-\frac{F_0 l}{2}$ (d) none of these

Solution (b) Torque $d\tau = \int_0^l F\,x = \frac{F_0 l}{2}$

19. A solid cylinder of mass M and radius $2R$ is connected to a string through a frictionless yoke and axle. The string runs over a disk shaped pulley of mass M and radius R. The mass M is attached to the other end of the string. The cylinder rolls without slipping on the table top. Find the acceleration of the block after the system is released from rest.

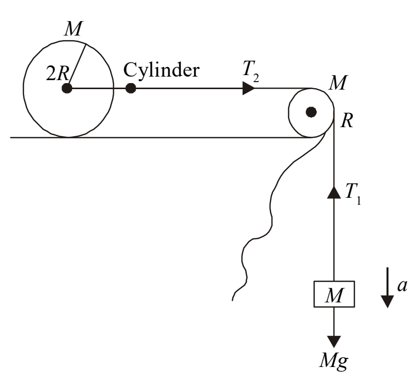

Fig. 8.28

(a) $\frac{2g}{5}$ (b) $\frac{g}{2}$

(c) $\frac{g}{3}$ (d) $\frac{g}{2}$

Solution (c) $2r\,\alpha_1 = r\,\alpha_2$ or $\alpha_2 = 2\,\alpha_1$

$$(T_2 - T_1)R = \left(\frac{MR^2}{2}\right)(2\alpha) \text{ or } T_2 - T_1 = Ma \quad ...(1)$$

$$Mg - T_1 = Ma \quad ....(2)$$

$$T_2(2R) = \frac{M(2R)^2}{2}(\alpha) \text{ or } T_2 = Ma \quad ....(3)$$

Adding (1), (2) and (3) $a = \frac{g}{3}$

20. A uniform disc of radius a has a hole of radius b at a distance c from the centre as shown. If the disc is free to rotate about a rod passing through the hole b, then find the MOI about the axis of rotation.

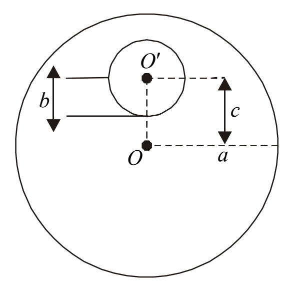

Fig. 8.29

(a) $\frac{M}{2}\left(a^2 + b^2 + \frac{2c^2a^2}{a^2 - b^2}\right)$

(b) $M\left(a^2 + b^2 + \frac{c^2a^2}{a^2 - b^2}\right)$

(c) $\frac{M}{2}\left(a^2 + b^2 + \frac{c^2a^2}{a^2 - b^2}\right)$ (d) none

Solution (a) Let ρ be the mass per unit area. Then MOI of the disc about O' $I = \pi\rho a^2\left(\frac{a^2}{2}\right) + \pi\rho a^2(c^2) - \pi b^2\rho\left(\frac{b^2}{2}\right)$

$$= \frac{\pi\rho a^2}{2}[a^2 + 2c^2] - \frac{\pi\rho b^4}{2}$$

$$= \frac{\pi\rho}{2}[a^2(a^2 + 2c^2) - b^4)$$

and $$\rho = \frac{M}{\pi(a^2 - b^2)}$$

$$= \frac{M}{2}\left[\frac{a^4 - b^4}{a^2 - b^2} + \frac{2a^2c^2}{a^2 - b^2}\right]$$

$$= \frac{M}{2}\left[a^2b^2 + \frac{2c^2a^2}{a^2 - b^2}\right]$$

21. Find the MOI of a uniform square plate of mass m and edge a about one of its diagonals.

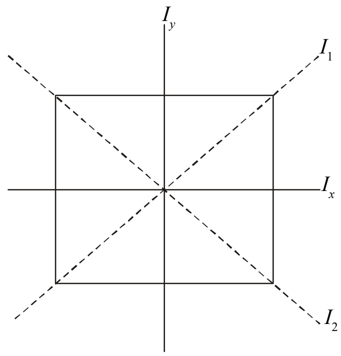

Fig. 8.30

(a) $\frac{Ma^2}{6}$ (b) $\frac{Ma^2}{3}$

(c) $\frac{Ma^2}{9}$ (d) $\frac{Ma^2}{12}$

Solution (d) $I_z = I_x + I_y = I_1 + I_2 = 2\,I_x = 2\,I_1$

or $I_1 = I_x = \frac{I_z}{2}$ $\quad I_z = \frac{M}{12}\,(a^2 + a^2) = \frac{Ma^2}{6}$

$\therefore I_1 = \frac{Ma^2}{12}$

22. Two spheres each of mass M and radius R are in contact as shown. Find the MOI if they are rotated about the common tangent.

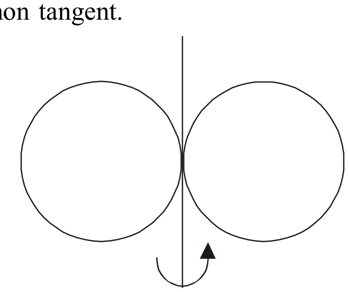

Fig. 8.31

(a) $\frac{7}{5}MR^2$ (b) $\frac{2}{3}\,MR^2$

(c) $\frac{4}{5}\,MR^2$ (d) $\frac{4}{3}\,MR^2$

Solution (b) $I = \left(\frac{2}{5}MR^2 + MR^2\right) \times 2 = \frac{14}{5}\,MR^2.$

23. A boy of mass M stands on a platform of radius R capable to rotate freely about its axis. The moment of inertia of the platform is I. The system is at rest. The friend of the boy throws a ball of mass m with a velocity υ horizontally. The boy on the platform catches it. Find the angular velocity of the system in the process.

(a) $\frac{m\upsilon R}{(M+m)R^2}$ (b) $\frac{m\upsilon R}{I + MR^2}$

(c) $\frac{m\upsilon R}{I + mR^2}$ (d) $\frac{m\upsilon R}{I + (M+m)R^2}$

Solution (d) $m\upsilon R = \left[I + (M+m)R^2\right]\,\omega$

or $$\omega = \frac{m\upsilon R}{I + (M+m)R^2}$$

24. A ball of steel rolls down an incline of inclination θ. Find the ratio of rotational KE to linear KE.

(a) $\frac{2}{7}$ (b) $\frac{2}{3}$

(c) $\frac{2}{5}$ (d) $\frac{5}{7}$

Solution (c) $\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}m\upsilon^2} = \frac{2}{5}\,\frac{MR^2\omega^2}{M\upsilon^2} = \frac{2}{5}$

25. The pulley shown in fig has MOI 0.5 kg m^2 and radius 10 cm. Assuming no friction any where, find the acceleration of 4 kg block

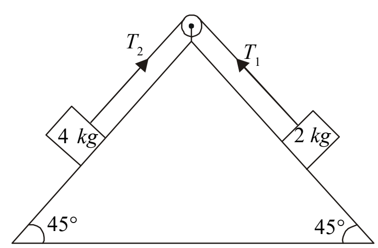

Fig. 8.32

(a) 1.11 ms^{-2} (b) 0.75 ms^{-2}

(c) 0.5 ms^{-2} (d) 0.25 ms^{-2}

Solution (d) $4\,g\cos 45 - T_1 = 4\,a$ ...(1)

$(T_1 - T_2)\,R = I\,\alpha$

or $T_1 - T_2 = \frac{I}{R^2}\,a$ ...(2)

$T_2 - 2\,g\cos 45 = 2\,a$ ... (3) Adding (1), (2) and (3)

$2\,g\cos 45 = 6a + \dfrac{I}{R^2}\; a$ or $a = \dfrac{\sqrt{2}\times 10}{6+50} = 0.25\ \text{ms}^{-2}$

26. A spherical shell of radius R is rolling down an incline of inclination θ without slipping. Find minimum value of coefficient of friction.

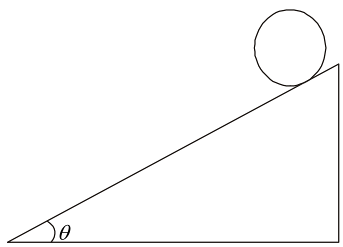

Fig. 8.33

(a) $\dfrac{2}{7}\tan\theta$ (b) $\dfrac{2}{5}\tan\theta$

(c) $\dfrac{2}{3}\tan\theta$ (d) none

Solution (b) $F_f\, r = I\,\alpha$

or $\mu mg\cos\theta = \dfrac{2}{3}\, Ma$

$\mu mg\cos\theta = \dfrac{2}{3}\, M\, \dfrac{g\sin\theta}{1+\dfrac{2}{3}} \;=\; \mu = \dfrac{2}{5}\tan\theta$

Shortcut $\mu = \dfrac{\dfrac{I}{MR^2}\tan\theta}{1+\dfrac{I}{MR^2}} = \dfrac{2}{5}\tan\theta$

TYPICAL PROBLEMS

27. A ball of radius r lies at the bottom of a vertical ring of radius R, find the minimum velocity to be given so that the ball completes the loop rolling without slipping.

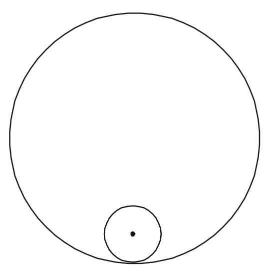

Fig. 8.34

(a) $\sqrt{5g(R-r)}$ (b) $\sqrt{\dfrac{27}{10}g(R-r)}$

(c) $\sqrt{\dfrac{27}{7}g(R-r)}$ (d) $\sqrt{\dfrac{27}{5}g(R-r)}$

Solution (c) $\dfrac{1}{2}\; m\upsilon_{\text{bott}}^2 + \dfrac{1}{2}\; I\omega_{\text{bott}}^2 = m\,g\,2\,(R-r) + \dfrac{1}{2}$

$m\upsilon_{\text{top}}^2 + \dfrac{1}{2}\; I\omega_{\text{top}}^2$

$m\upsilon_{\text{bott}}^2 + \dfrac{2}{5}\; m\upsilon_{\text{bott}}^2 = 4\,mg\,(R-r) + m\upsilon_{\text{top}}^2 + \dfrac{2}{5}\; m\upsilon_{\text{top}}^2$

$\dfrac{7}{5}\; m\upsilon_{\text{bott}}^2 = 4\,mg\,(R-r) + \dfrac{7}{5}\; m\upsilon_{\text{top}}^2 = 4\,mg\,(R-r) + \dfrac{7}{5}$

$m\,[g\,(R-r)]$

or $\upsilon_{\text{bott}} = \sqrt{\dfrac{27}{7}g(R-r)}$

28. The pulley shown in Fig 8.35 has radius 20 cm and MOI 0.2 kg m^2. Spring used has force constant 50 N m^{-1}. The system is released from rest. Find the velocity of 1kg block when it has descended 10 cm.

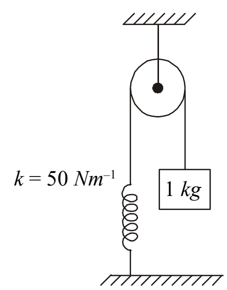

Fig. 8.35

(a) $1/2\ \text{ms}^{-1}$ (b) $1/\sqrt{2}\ \text{ms}^{-1}$

(c) $1/\sqrt{3}\ \text{ms}^{-1}$ (d) none

Solution (a) $mgx = \dfrac{1}{2}\, kx^2 + \dfrac{1}{2}\, I\omega^2 + \dfrac{1}{2}\, m\upsilon^2$

$1\,(10)\,(.1) = \dfrac{1}{2}\left[50(.1)^2 + (.2)\left(\dfrac{\upsilon}{.2}\right)^2 + 1\left(\upsilon^2\right)\right]$

$\Rightarrow 2 = 0.5 + 6\,\upsilon^2$

or $\upsilon = 1/2\ \text{ms}^{-1}$

29. A solid sphere rolling on a rough horizontal surface with a linear speed υ collides elastically with a fixed, smooth vertical wall. Find the speed of the sphere after it has began pure rolling in the backward direction.

(a) $\frac{2}{7}\,\upsilon$ (b) $\frac{3}{7}\,\upsilon$

(c) $\frac{4}{7}\,\upsilon$ (d) none

Solution (b) $F_f r = I\,\alpha = \frac{2}{5}\,M r^2\,\alpha$ or

$$\alpha = \frac{5F_f}{2mr} \text{ (just before collision)}$$

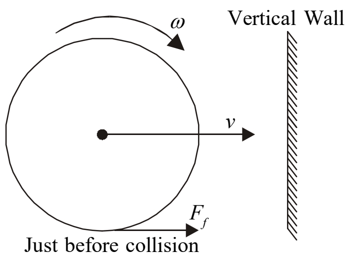

Fig. 8.36 (a)

after collision

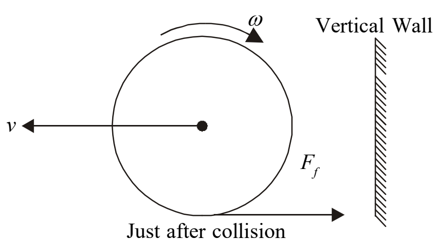

Fig. 8.36 (b)

$$\upsilon_f = \upsilon - \frac{F_f}{m}\,t \quad \ldots(1)$$

$$\omega_f = -\omega + \alpha\,t = -\omega + \frac{5F_f}{2mr}\,t.$$

or $$r\omega_f = -r\omega + \frac{5}{2}\,\frac{F_f}{m}\,t$$

or $$\upsilon_f = -\upsilon + \frac{5}{2}\,\frac{F_f}{m}\,t \quad \ldots(2)$$

From Eq. (1) and (2) $\upsilon_f = -\upsilon + \frac{5}{2}\,(\upsilon - \upsilon_f)$ or $\upsilon_f = \frac{3}{7}\,\upsilon$

30. A thin spherical shell lying on a rough horizontal surface is hit by a cue in such a way that line of action passes through the centre of the shell. As a result shell starts moving with a linear speed υ without any initial angular velocity. Find the linear velocity to the shell when it starts pure rolling.

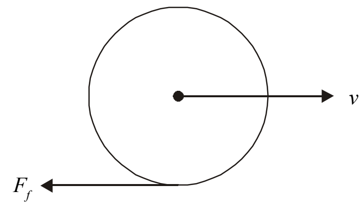

Fig. 8.37

(a) $\frac{3}{5}\,\upsilon$ (b) $\frac{2}{5}\,\upsilon$

(c) $\frac{4}{5}\,\upsilon$ (d) none

Solution (a) $\upsilon_f = \upsilon - \frac{F_f}{m}\,t \quad \ldots(1) \qquad F_f r = I\,\alpha$

or $$F_f r = \frac{2}{3}\,m r^2\,\alpha$$

or $$\alpha = \frac{3}{2}\,\frac{F_f}{mr} \quad \omega = 0 + \alpha\,t$$

or $$\omega = \frac{3}{2}\,\frac{F_f}{mr}\,t$$

or $$r\omega = \upsilon_f = \frac{3F_f}{2m}\,t \ \ldots(2)$$

From Eq (1) and (2) $\upsilon_f = \frac{3}{5}\,\upsilon.$

31. A uniform rod pivoted at upper end is released when it is making an angle of 60°. Find the radial force acting on a particle of mass dm at its tip when it makes an angle of 37° with the vertical.

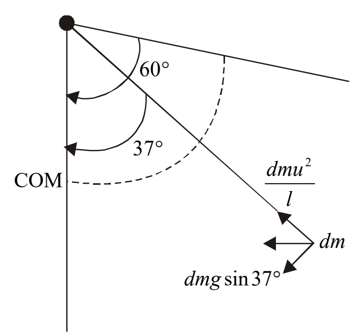

Fig. 8.38

(a) 0.6 dm g (b) 0.8 dm g

(c) 0.9 dm g (d) none

Solution (c) $\frac{1}{2}\,I\,\omega^2 = mg\,\frac{l}{2}\,(\cos 37 - \cos 60)$

or $\dfrac{Ml^2}{3}\,\omega^2 = mg\,l\,(.8-.5)$

$\dfrac{m\upsilon^2}{l} = 0.9\,mg$ or $\dfrac{\upsilon^2}{l} = 0.9$ g

$F_{\text{rad}} = dm\,\dfrac{\upsilon^2}{l} = 0.9\,(dm)$ g

32. A uniform rod of length L rests against a smooth roller as shown in Fig 8.39 (a). Find the friction coefficient between the ground and the lower end if the minimum angle that the rod makes with the horizontal is θ.

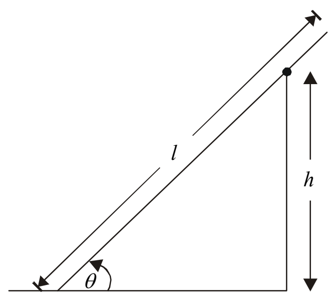

Fig. 8.39 (a)

(a) $\mu = \dfrac{l\cos\theta\sin^2\theta}{2h - l\cos^2\theta\sin\theta}$ (b) $\mu = \dfrac{l\sin\theta\cos^2\theta}{2h - l\cos\theta\sin^2\theta}$

(c) $\mu = \dfrac{l\sin\theta\cos\theta}{2h - l\cos\theta\sin\theta}$ (d) none

Solution (a) Balance horizontal forces $\mu N_2 = N_1 \sin\theta$

Balance vertical forces $N_1 \cos\theta + N_2 = mg$

or $N_1\left(\cos\theta + \dfrac{\sin\theta}{\mu}\right) = mg$

or $N_1 = \dfrac{\mu mg}{\mu\cos\theta + \sin\theta} = mg$

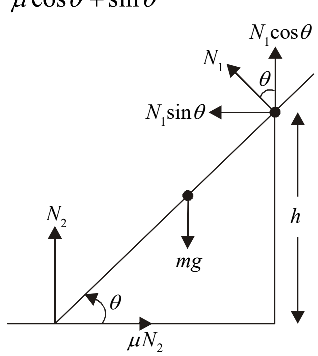

Fig. 8.39 (b)

Apply $\sum\tau = 0$ about point A.

$N_1 \sin\theta\, h + N_1 \cos\theta\, h\cot\theta = mg\,\dfrac{l}{2}\cos\theta$

$h\,N_1\left[\sin^2\theta + \cos^2\theta\right] = \dfrac{mgl\cos\theta\sin\theta}{2}$

$\dfrac{h\mu mg}{\mu\cos\theta + \sin\theta} = \dfrac{mgl}{2}\cos\theta\sin\theta$

$2\,\mu h = \mu l\cos^2\theta\sin\theta + l\cos\theta\sin^2\theta$

$\mu(2h - l\cos^2\theta\sin\theta) = l\cos\theta\sin^2\theta$

or $\mu = \dfrac{l\cos\theta\sin^2\theta}{2h - l\cos^2\theta\sin\theta}$

33. When a force 6 N is exerted at 30° to a wrench at a distance of 8 cm from a nut as shown in Fig 8.40, it is just able to loosen it. What force F is required to loosen the nut if applied 16 cm away to the wrench and normal to the wrench.

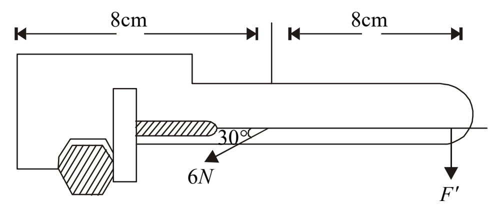

Fig. 8.40

(a) 3 N (b) $\sqrt{3}$ N

(c) 1.5 (d) none

Solution (c) $8 \times 6 \sin 30 = F \times 16$ or $F = 1.5$ N

34. Particles of mass 1 g, 2 g, 3 g, ..., 100 g are kept at the marks 1 cm, 2 cm, 3 cm, ... 100 cm respectively on a metre scale. Find the MOI of the system of particles about a perpendicular bisector of the metre scale.

Solution 0 g, 100 g; 1 g, 99 g ; 2 g, 98 g ; 3 g, 97 g ; .... are equally spaced from the axis of rotation.

$\therefore\ I = (100)\,[1^2 + 2^2 + ... + 50^2]$

$= 0.1\,[1^2 + 2^2 + ... + 50^2] \times 10^{-4}$ kg m^2

$= 0.1 \times \dfrac{50\times51\times101}{6} \times 10^{-4} = 0.43$ kg m^2

$\left\{\text{use} \sum n^2 = \dfrac{n(n+1)(2n+1)}{6}\right\}$.

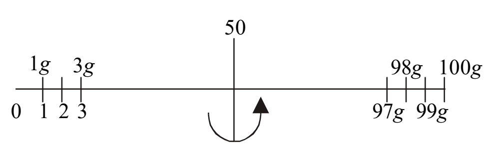

Fig. 8.41

35. A spring wrapped on a wheel of MOI 0.2 kg m^2 and radius 10 cm over a light pulley to support a block of mass 2 kg as shown in fig. Find the acceleration of the block.

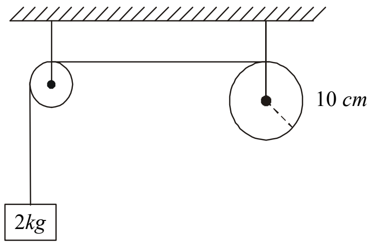

Fig. 8.42

(a) 0.89 ms^{-2} (b) 1.12 ms^{-2}

(c) 0.69 ms^{-2} (d) none

Solution (a) $2a = 2g - T$ ... (1)

$T \cdot r = I\alpha$

or $T = \dfrac{I}{r^2}a = \dfrac{0.2a}{(0.1)^2}$ ... (2)

From equations (1) and (2)

$$a = \frac{2g}{(2+20)} = 0.89 \text{ ms}^{-2}$$

36. A paper roll of M kg and radius R rests against the wall and is held in place with brackets as shown. Assume no friction. MOI of the paper and the rod is I about the axis. The other end of the bracket is connected to the wall by a smooth hinge making an angle θ as shown. Neglect weight of the bracket. The kinetic friction coefficient is μ_k. A Force F is applied on the paper to unroll it. Find the angular acceleration

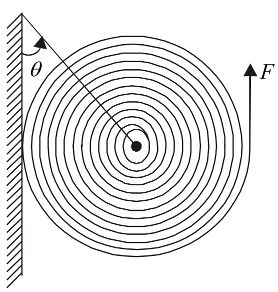

Fig. 8.43

Solution T and $M\,g$ pass through COM and hence do not contribute to torque.

$F \cdot R + \mu N R = I\,\alpha$

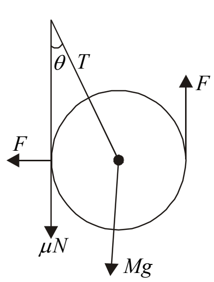

Fig. 8.44

$$(F + \mu Ma) = \frac{I}{R^2}\,a \qquad \text{(using } a = r\alpha\text{)}$$

$$a = \frac{F}{\dfrac{I}{R^2} - \mu M}$$

37. The door of an almirah is 6 ft high, 1.5 ft wide and weights 8 kg. The door is supported by two hinges situated at a distance of 1 ft from the ends. Assuming forces exerted on the hinges are equal, the magnitude of the force is

(a) 15 N (b) 10 N

(c) 28 N (d) 43 N

Solution (d) $\tan\theta = \dfrac{2}{3/4} = \dfrac{8}{3}$ or $\cos\theta = \dfrac{3}{\sqrt{73}}$

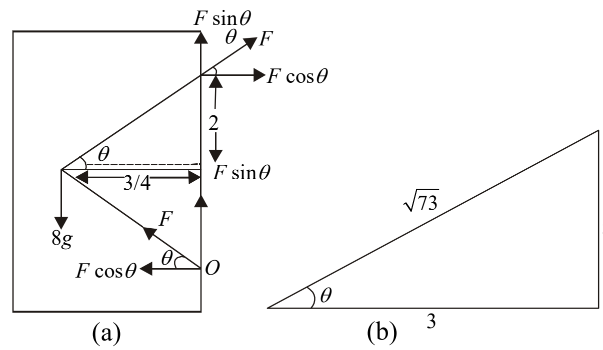

Fig. 8.45

Taking torque about O

$$8g\left(\frac{3}{4}\right) = 4F\cos\theta$$

$$6g = 4F\left(\frac{3}{\sqrt{73}}\right) \text{ or } F = 5\sqrt{73} = 43\,N$$

38. A uniform rod of length $6a$ and mass 8 m lies on a smooth horizontal table. Two point masses m and 2 m moving in the same horizontal plane with speeds 2 v and v strike the rod as shown in the Fig 8.46. Find the velocity of centre of mass and angular velocity about COM. Also find KE just after collision.

Solution Conserve momentum as external force is zero.

$-2mv + m\,2v + 0 = (2m + m + 8m) \times v'$

$v' = 0$ that is, velocity of COM is zero

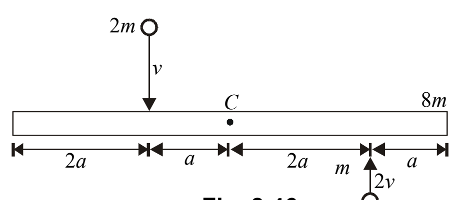

Fig. 8.46

$\tau_{ext} = 0 \therefore$ angular momentum is conserved

$$2mva + m(2v)(2a) = \left[2ma^2 + m(2a)^2 + \frac{8m(6a)^2}{12}\right]\omega$$

or $\quad \omega = \dfrac{v}{5a}$.

$$\text{KE after collision} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(30ma^2\right)\left(\frac{v}{5a}\right)^2$$

$$= \frac{3}{5}mv^2.$$

39. A uniform ball of radius r rolls without slipping down from the top of a sphere of radius R. The angular velocity of the ball when it breakes from the sphere is ... . Assume initial velocity negligible.

Solution $\quad \dfrac{mv^2}{(R+r)} = mg\cos\theta;\ mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$

$$mg(R+r)(1-\cos\theta) = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

$$\frac{10}{7}mg\left(1-\cos\theta\right) = mg\cos\theta$$

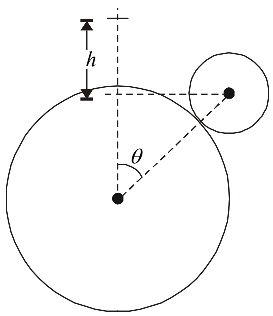

Fig. 8.47

$$mv^2 = \frac{10}{7}mg\left(R+r\right)\left(1-\cos\theta\right) \quad \frac{10}{7} = \frac{17}{7}\cos\theta$$

or $\quad \cos\theta = \dfrac{10}{17}$

$$v = \sqrt{g(R+r)\cos\theta} = \sqrt{\frac{10}{17}g(R+r)}$$

and $\quad \omega = \dfrac{v}{r} = \sqrt{\dfrac{10g\left(R+r\right)}{17r^2}}$

40. A spool with a thread wound on it, of mass m, rests on a rough horizontal surface. Its MOI about its own axis is $\gamma m R^2$ where R is outer radius of spool and γ is a constant. Radius of the wound thread is equal to r. The spool is pulled without sliding by the thread with a constant force F directed at an angle α. Find acceleration component along x axis.

Solution $\quad F\cos\alpha - F_f = m\,a_x \quad \text{... (1)}$

$$F_f R - Fr = I\,\alpha = \gamma m R^2\,\alpha$$

or $\quad F_f - F\dfrac{r}{R} = \gamma m\,a_x \quad \text{... (2)}$

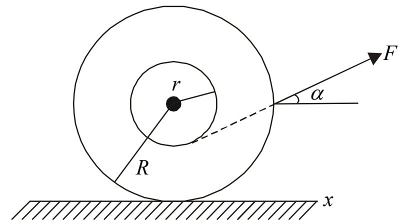

Fig. 8.48

Adding (1) and (2), we get $a_x = \dfrac{F\left(\cos\alpha - \dfrac{r}{R}\right)}{(1+\gamma)m}$

(**Note** for acceleration to be produced $\cos\alpha > r/R$)

41. A uniform rod of mass m and length l is hinged at its upper end. It is released from a horizontal position. When it becomes vertical, what force does it exert on the hinge?

(a) $\dfrac{3}{2}\,mg$ $\qquad$ (b) $2\,mg$

(c) $\dfrac{5}{2}\,mg$ $\qquad$ (d) mg

Solution $\quad N - mg = \dfrac{m\upsilon^2}{r} \quad \text{...(1)}$

$$N = mg + \frac{m\left(\frac{l}{2}\omega\right)^2}{l/2} = mg + \frac{3}{2}mg = \frac{5}{2}mg$$

$$\frac{1}{2}I\omega^2 = mg\,\frac{l}{2}$$

$$\frac{1}{2}\frac{ml^2}{3}\omega^2 = mg\,\frac{l}{2}$$

or $\quad \dfrac{m\upsilon^2}{l} = 3\,mg \quad \text{... (2)}$

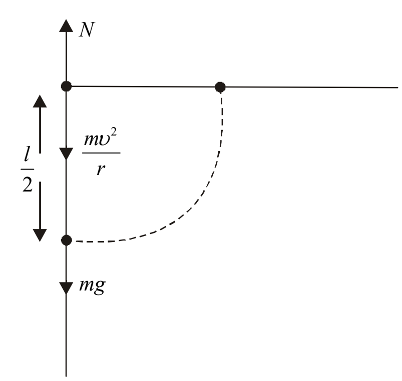

Fig. 8.49

42. Two uniform solid spheres of unequal masses and unequal radii released from rest from the same height on a rough inclined plane. The spheres roll without slipping then

(a) heavier sphere reaches the bottom first.

(b) bigger sphere reaches the bottom first.

(c) both reach the bottom with same velocity.

(d) the two spheres reach the bottom together.

Solution (c), (d) $= a = \dfrac{g\sin\theta}{1+\dfrac{I}{Mr^2}}$

$$= \frac{5}{7}\, g\sin\theta \text{ and } \upsilon^2 = 2a\, l$$

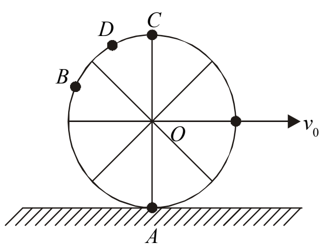

Fig. 8.50

43. Consider a bicycle wheel rolling on a smooth horizontal surface with linear speed υ_0. Then

(a) speed of A is zero.

(b) speed of B, C and D are equal and equal to υ_0.

(c) speed of B > speed of O.

(d) speed of $C = 2\ \upsilon_0$.

Solution (a), (c) and (d)

44. A sphere is rolled on a rough horizontal surface. It gradually slows down and stops. The force of friction tends to

(a) increase linear velocity.

(b) decrease linear velocity.

(c) increase angular speed.

(d) decrease angular speed.

Solution (b) and (c) See Fig 8.51

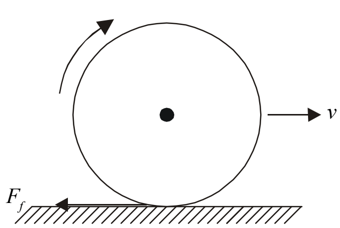

Fig. 8.51

F_f (Force of friction) acts in opposite direction to linear motion. It forms the torque in clockwise direction which aids rotation or angular velocity increases.

45. A bicycle is being peddelled then force of friction acts

(a) on front wheel in forward direction.

(b) on back wheel in backward direction.

(c) on front wheel in backward direction.

(d) on back wheel in forward direction.

Solution (c) and (d) See Fig.

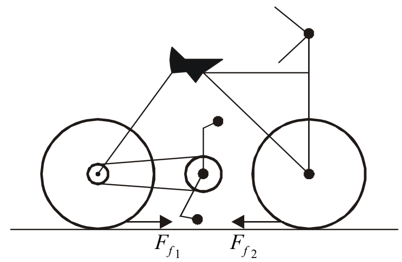

Fig. 8.52

F_{f1} on backward wheel tends to counter rotational torque responsible for forward motion. F_{f2} on forward wheel helps in providing rotational motion.

46. (a) Fig 8.53 (a) shows rolling with forward slipping

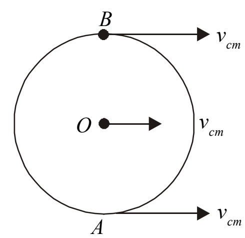

Fig. 8.53 (a)

(b) Fig 8.53 (b) shows pure rotation about O

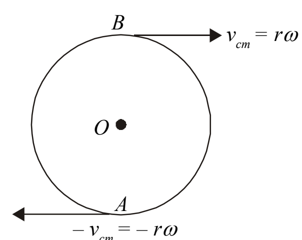

Fig. 8.53 (b)

(c) Fig 8.53 (c) shows pure rolling

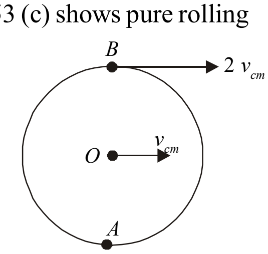

Fig. 8.53 (c)

(d) Fig 8.53 (c) shows rolling with backward slipping

Solution (b) and (c). Fig. 1 represents pure translation.

PASSAGE 1

Read the following passage and answer the questions given at the end.

A student holds a rim-loaded bicycle wheel, rotating at a relatively high angular speed ω with its shaft horizontal as shown in Fig 8.54 (a). His physics teacher now asks him to turn the shaft rapidly (in time Δt) so that the shaft points at a small angle $\Delta\theta$ above the horizontal as shown in Fig 8.54 (b). He also asks the student to keep the shaft in a vertical plane all the times.

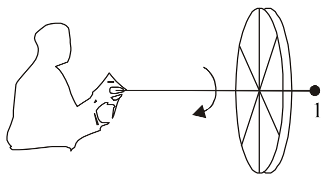

Fig. 8.54 (a)

While following the instructions student finds the wheel swerves around to his right, rather violently when he turns the shaft of the wheel upwards. He almost fails to keep the shaft in a vertical plane.

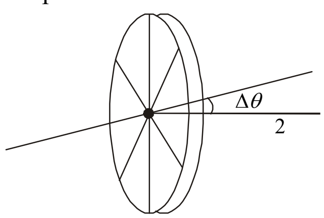

Fig. 8.54 (b)

1. When the student holds the shaft horizontal which torque he has to exert?
   (a) Torque to balance the turning effect of force of gravity
   (b) Torque to balance the motional torque
   (c) Torque to balance revolutionary torque
   (d) none

Solution He has to exert a torque to balance the turning effect of force of gravity on the centre of mass directed towards horizontal axis and emerges perpendicular outwards. This torque is to be applied whether or not the wheel is rotating.

2. Which torque he has to apply while tilting the shaft upwards?

Solution The student has to apply an average torque τ_{av} $= \dfrac{\Delta L}{\Delta t} = \dfrac{L\sin\theta}{\Delta t}$. It has the direction of ΔL i.e. nearly vertical as illustrated in Fig 8.55. If he fails to do so shaft will not remain in the vertical plane.

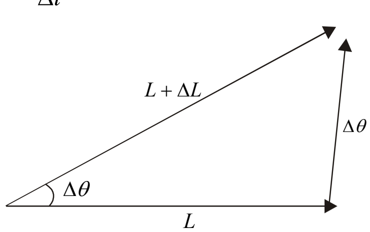

Fig. 8.55

PASSAGE 2

Read the following passage and answer the questions given at the end.

Consider a rapidly spinning top with its axis slightly inclined to the vertical. The weight acts vertical at the centre of mass and the reaction at the point of contact constitutes a torque $\vec{C}$. This torque acts perpendicular to the momentum vector $\vec{L}$ and hence, produce a precession of the spin axis about the vertical axis. The precession acts in a direction of the torque $\vec{C}$.

The precession velocity $\Omega = \dfrac{C}{L}$.

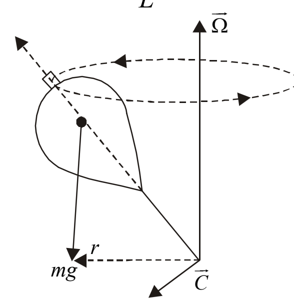

Fig. 8.56

In actual practice there is wobbling about the mean precession. The wobbling is not visible when the spinning is fast. The slower the spinning speed, the more obvious the rotation is. The time period of precession of earth is 27,725 years.

1. Why does the spinning top inclined to the vertical not fall due to its weight?

Solution If the top were not spinning the torque due to gravitational force will tend to lower down the centre of mass and hence, the top would have fallen. However, in case of spinning top, the torque being engaged in producing precession is unable to topple down the top.

2. What is nutation or wobbling?

Solution The top along with spinning and precession shows wobbling i.e., during precession it lowers down and rises. If the spinning speed is large wobbling will soon be damped. However, if the spinning speed is large, it is largely visible.

PASSAGE 3

Read the following passage and answer the questions given at the end.

The Crab Nebula is a cloud of glowing gas about 10 light years across, located about 6500 light years from the earth. It is the remnant of a star that underwent a super nova explosion seen on earth in 1054 AD. Energy is released by the Crab Nebula at a rate of about 5×10^{31} W, about 10^5 times the rate at which the sun radiates energy. The Crab Nebula obtains its energy from the rotational kinetic energy of a rapidly spining neutron star at its center. This object rotates once every 0.0331 s and its period is increasing by 4.22×10^{-13} s for each second of time that elapses. Theories of supernova predict that the neutron star in the Crab Nebula has a mass about 1.4 times that of the sun. Assume neutron star as a uniform solid sphere

1. Assuming energy lost by neutron star is equal to the rate at which energy is released by Nebula. Find the moment of inertia.

(a) 8.5×10^{25} kgm^2 (b) 2×10^{31} kgm^2
(c) 2×10^{36} kgm^2 (d) 2×10^{38} kgm^2

2. Find the radius of neutron star.

(a) 18.3 km (b) 13.6 km
(c) 3×10^3 km (d) 13.6×10^4 km

3. What is the linear speed at the equator of a neutron star?

(a) 2.6×10^6 ms^{-1} (b) 2.6×10^7 ms^{-1}
(c) 2.6×10^8 ms^{-1} (d) none of these

4. What is the density of neutron star?

(a) 2.4×10^6 kgm^{-3} (b) 2.4×10^{12} kg m^{-3}
(c) 2.4×10^{17} kgm^{-3} (d) none

Solution **1.** (d) $\frac{1}{2} I(\omega_1^2 - \omega_2^2) = 5 \times 10^{31}$

$$\frac{1}{2} I \times 4\pi^2 \left(\frac{1}{T_1^2} - \frac{1}{T_2^2}\right) = 5 \times 10^{31}$$

$$\Rightarrow \quad \frac{1}{2} I(2\pi^2)\left[\frac{(T_2 - T_1)(T_2 + T_1)}{T_1^2 . T_2^2}\right] = 5 \times 10^{31}$$

or $\quad I(20) = \dfrac{5\times10^{31}\times(.0331)^3}{4.22\times10^{-13}}$

or $\quad I = \dfrac{34.3\times5\times10^{38}}{20\times4.22} = 2 \times 10^{38}$

Solution **2.** (b) $\frac{2}{5}$ $(1.4 \times 2 \times 10^{30})\, R^2$

$= 1.8 \times 10^8$

or $\quad R = 1.36 \times 10^4$ or 13.6 km

Solution **3.** (a) $v = R\omega$

$= 1.36 \times 10^4 \times \dfrac{2\pi}{.0331}$

$= 2.6 \times 10^6$ ms^{-1}

Solution **4.** (c) $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$

$= \dfrac{3\times1.4\times2\times10^{30}}{4\times3.14\times(1.36\times10^4)^3}$

$= \dfrac{6\times10^7}{(1.36)^3}$

$= 2.4 \times 10^{17}$ kgm^{-3}

PASSAGE 4

Read the following passage and answer the questions given at the end.

Kitchen food processor has become a necessity now-a-days in every kitchen. An old man has a food processor which works on a dc motor. At $t = 0$ the direction of current in the motor of the processor is reversed. It results into an angular displacement.

$$\theta(t) = 250\,t - 20\,t^2 - 150\,t^3$$

1. When does the velocity become zero?

(a) 0.7 s (b) 0.6 s
(c) 0.5 s (d) 0.3 s

2. Find the angular accelaration when the angular velocity is zero?

(a) –630 rad s^{-2} (b) –40 rad s^{-2}
(c) –670 rad s^{-2} (d) none

3. Find average velocity in the time interval 0 to the time calculated in question 55 ?

(a) 113.75 rad s^{-1} (b) 143.25 rad s^{-1}
(c) 162.5 rad s^{-1} (d) none

4. How fast was the motor shaft at $t = 0$?

(a) 125 rad s^{-1} (b) 250 rad s^{-1}
(c) 375 rad s^{-1} (d) 0

Solution **1.** (a) $\dfrac{d\theta}{dt} = 250 - 40t - 450t^2 = 0$

or $\quad 45t^2 + 4t - 25 = 0$

$$t = \frac{-4 \pm \sqrt{16+4500}}{90} = \frac{63}{90} = 0.7\, s$$

Solution **2.** (c) $\dfrac{d^2\theta}{dt^2}$

$= -40 - 900t = -40 - 900(.7) = -670$ rad/s

Solution **3.** (c) $\theta(0) = 0$, $\theta(.7)$

$= 250\,(.7) - 20\,(.7)^2 - 150\,(.7)^3$

$= 175 - 9.8 - 51.45 = 113.75$

$$\omega_{av} = \frac{(113.75 - 0)}{.7} = 162.5 \text{ rad } s^{-1}$$

Solution **4.** (b) $\left.\dfrac{d\theta}{dt}\right|_{t=0}$

$= 250$ rad s^{-1}

QUESTIONS FOR PRACTICE

1. A square plate lies in the xy plane with its centre at the origin and its edges parallel to the x and y axes. Its moments of inertia about the x, y and z axes are I_x, I_y, and I_z respectively, and about a diagonal it is I_D.

(a) $I_x = I_y = \frac{1}{2} I_z$ (b) $I_x = I_y = 2I_z$

(c) $I_D = I_x$ (d) $I_D = I_z$

2. Four identical rods, each of mass m and length l, are joined to form a rigid square frame. The frame lies in the xy plane, with its centre at the origin and the sides parallel to the x and y axes. Its moment of inertia about

(a) the x-axis is $\frac{2}{3}ml^2$

(b) the z-axis is $\frac{4}{3}ml^2$

(c) an axis parallel to the z-axis and passing through a corner is $\frac{10}{3}ml^2$

(d) one side is $\frac{5}{2}ml^2$

3. P is the centre of mass of four point masses A, B, C and D, which are coplanar but not collinear.

(a) P may or may not coincide with one of the point masses.

(b) P must lie within the quadrilateral $ABCD$.

(c) P must lie within or on the edge of at least one of the triangles formed by taking A, B, C and D three at a time.

(d) P must lie on a line joining two of the points A, B, C, D.

4. When slightly different weights are placed on the two pans of a beam balance, the beam comes to rest at an angle with the horizontal. The beam is supported at a single point P by a pivot.

(a) The net torque about P due to the two weights is nonzero at the equilibrium position.

(b) The whole system does not continue to rotate about P because it has a large moment of inertia.

(c) The centre of mass of the system lies below P.

(d) The centre of mass of the system lies above P.

5. A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is

(a) 2, if their lines of action pass through the centre of mass of the body.

(b) 3, if their lines of action are not parallel.

(c) 3, if their lines of action are parallel.

(d) 4, if their lines of action are parallel and all the forces have the same magnitude.

6. A block with a square base measuring $a \times a$, and height h, is placed on an inclined plane. The coefficient of friction is μ. The angle of inclination (θ) of the plane is gradually increased. The block will

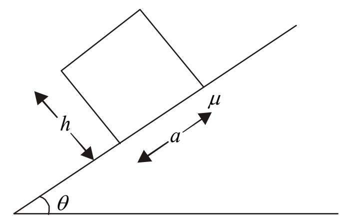

Fig. 8.57

(a) topple before sliding if $\mu > \frac{a}{h}$.

(b) topple before sliding if $\mu < \frac{a}{h}$.

(c) slide before toppling if $\mu > \frac{a}{h}$.

(d) slide before toppling if $\mu < \frac{a}{h}$.

7. Two men support a uniform horizontal beam at its two ends. If one of them suddenly lets go, the force exerted by the beam on the other man will

(a) remain unaffected.

(b) increase.

(c) decrease.

(d) become unequal to the force exerted by him on the beam.

8. A uniform rod kept vertically on the ground falls from rest. Its foot does not slip on the ground.

(a) No part of the rod can have acceleration greater than g in any position.

(b) At any one position of the rod, different points on it have different accelerations.

(c) Any one particular point on the rod has different accelerations at different positions of the rod.

(d) The maximum acceleration of any point on the rod, at any position, is 1.5 g.

9. A man spinning in free space changes the shape of his body, e.g., by spreading his arms or curling up. By doing this, he can change his

(a) moment of inertia.

(b) angular momentum.

(c) angular velocity.

(d) rotational kinetic energy.

10. A man standing on a platform holds weights in his outstretched arms. The system rotates freely about a central vertical axis. If he now draws the weights inwards close to his body,

(a) the angular velocity of the system will increase.

(b) the angular momentum of the system will decrease.

(c) the kinetic energy of the system will increase.

(d) he will have to expend some energy to draw the weights in.

11. A horizontal disc rotates freely about a vertical axis through its centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc. After some time, the two rotate with a common angular velocity.

(a) Some friction exists between the disc and the ring.

(b) The angular momentum of the 'disc plus ring' is conserved.

(c) The final common angular velocity is $\frac{2}{3}$rd of the initial angular velocity of the disc.

(d) $\frac{2}{3}$rd of the initial kinetic energy changes to heat.

12. Two horizontal discs of different radii are free to rotate about their central vertical axes. One is given some angular velocity, the other is stationary. Their rims are now brought in contact. There is friction between the rims.

(a) The force of friction between the rims will disappear when the discs rotate with equal angular speeds.

(b) The force of friction between the rims will disappear when they have equal linear velocities.

(c) The angular momentum of the system will be conserved.

(d) The rotational kinetic energy of the system will not be conserved.

13. A constant external torque τ acts for a very brief period Δt on a rotating system having moment of inertia I.

(a) The angular momentum of the system will change by $\tau\,\Delta t$.

(b) The angular velocity of the system will change by $\frac{\tau\Delta t}{I}$.

(c) If the system was initially at rest, it will acquire rotational kinetic energy $\frac{(\tau\Delta t)^2}{2I}$.

(d) The kinetic energy of the system will change by $\frac{(\tau\Delta t)^2}{2I}$.

14. Two identical spheres A and B are free to move and to rotate about their centres. They are given the same impulse J. The lines of action of the impulses pass through the centre of A, and away from the centre of B.

(a) A and B will have the same speed.

(b) B will have greater kinetic energy than A.

(c) They will have the same kinetic energy, but the linear kinetic energy of B will be less than that of A.

(d) The kinetic energy of B will depend on the point of impact of the impulse on B.

15. The motion of a sphere moving on a rough horizontal surface changes from pure sliding (without rolling) to pure rolling (without slipping). In this process, the force of friction

(a) initially acts opposite to the direction of motion and later in the direction of motion.

(b) causes linear retardation.

(c) causes angular acceleration.

(d) stops acting when pure rolling begins.

16. A disc of circumference s is at rest at a point A on a horizontal surface when a constant horizontal force begins to act on its centre. Between A and B there is sufficient friction to prevent slipping, and the surface is smooth to the right of B. $AB = s$. The disc moves from A to B in time T. To the right of B,

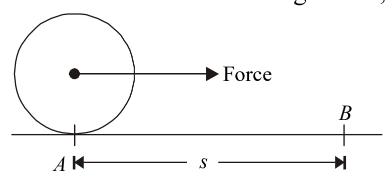

Fig. 8.58

(a) the angular acceleration of the disc will disappear, linear acceleration will remain unchanged.

(b) linear acceleration of the disc will increase.

(c) the disc will make one rotation in time $T/2$.

(d) the disc will cover a distance greater than s in a further time T.

17. A solid sphere starts from rest at the top of an incline of height h and length l, and moves down. The force of friction between the sphere and the incline is F. This is insufficient to prevent slipping. The kinetic energy of the sphere at the bottom of the incline is W.

(a) The work done against the force of friction is $F\,l$.

(b) The heat produced is $F\,l$.

(c) $W = mgh - F\,l$.

(d) $W > (mgh - F\,l)$.

18. A ring (R), a disc (D), a solid sphere (S) and a hollow sphere with thin walls (H), all having the same mass but different radii, start together from rest at the top of an inclined plane and roll down without slipping.

(a) All of them will reach the bottom of the incline together.

(b) The body with the maximum radius will reach the bottom first.

(c) They will reach the bottom in the order S, D, H, R.

(d) All of them will have the same kinetic energy at the bottom of the incline.

19. A solid sphere rolls without slipping on a rough horizontal floor, moving with a speed v. It makes an elastic collision with a smooth vertical wall. After impact,

(a) it will move with a speed v initially.

(b) its motion will be rolling without slipping.

(c) its motion will be rolling with slipping initially and its rotational motion will stop momentarily at some instant.

(d) its motion will be rolling without slipping only after some time.

20. A sphere S rolls without slipping, moving with a constant speed on a plank P. The friction between the upper surface of P and the sphere is sufficient to prevent slipping, while the lower surface of P is smooth and rests on the ground. Initially, P is fixed to the ground by a pin N. If N is suddenly removed,

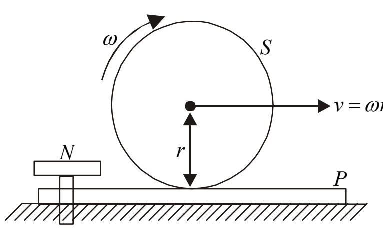

Fig. 8.59

(a) S will begin to slip on P.

(b) P will begin to move backwards.

(c) the speed of S will decrease and its angular velocity will increase.

(d) there will be no change in the motion of S and P will still be at rest.

21. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is v.

(a) $0 \le v \le 2\,u$.

(b) $v = u$, if CP is horizontal.

(c) $v = u$, if CP makes an angle of 60° with the horizontal and P is below the horizontal level of C.

(d) $v = \sqrt{2u}$, if CP is horizontal.

22. A ring rolls without slipping on a horizontal surface. At any instant, its position is as shown in the figure.

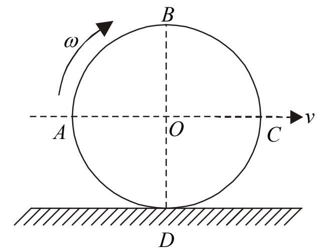

Fig. 8.60

(a) Section ABC has greater kinetic energy than section ADC.

(b) Section BC has greater kinetic energy than section CD.

(c) Section BC has the same kinetic energy as section DA.

(d) The sections AB, BC, CD and DA have the same kinetic energy.

23. A wheel of radius r rolls without slipping with a speed v on a horizontal road. When it is at a point A on the road, a small blob of mud separates from the wheel at its highest point and lands at point B on the road.

(a) $AB = v\sqrt{r/g}$ (b) $AB = 2v\sqrt{r/g}$

(c) $AB = 4v\sqrt{r/g}$

(d) If $v > \sqrt{4rg}$, the blob of mud will land on the wheel and not on the road.

24. In the figure, the disc D does not slip on the surface S. The pulley P has mass, and the string does not slip on it. The string is wound around the disc.

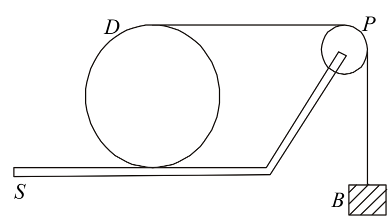

Fig. 8.61

(a) The acceleration of the block B is double the acceleration of the centre of D.

(b) The force of friction exerted by D on S acts to the left.

(c) The horizontal and the vertical sections of the string have the same tension.

(d) The sum of the kinetic energies of D and B is less than the loss in the potential energy of B as it moves down.

25. In the figure, the blocks have unequal masses m_1 and m_2 ($m_1 > m_2$). m_1 has a downward acceleration a. The pulley P has a radius r, and some mass. The string does not slip on the pulley.

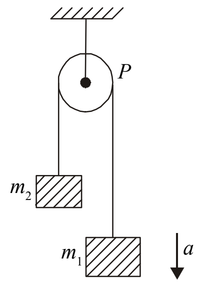

Fig. 8.62

(a) The two sections of the string have unequal tensions.

(b) The two blocks have accelerations of equal magnitude.

(c) The angular acceleration of P is a/r.

(d) $a < \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right) g.$

26. Two particles A and B, of mass m each, are joined by a rigid massless rod of length l. A particle P of mass m, moving with a speed u normal to AB, strikes A and sticks to it. The centre of mass of '$A + B + P$' system is C.

(a) The velocity of C before impact is $u/3$.

(b) The velocity of C after impact is $u/3$.

(c) The velocity of '$A + P$' immediately after impact is $u/2$.

(d) The velocity of B immediately after impact is zero.

27. In the previous question, immediately after the impact,

(a) $AC = l/3$.

(b) the angular momentum of the '$A + B + P$' system about C is $\dfrac{1}{3}\ mul.$

(c) the moment of inertia of the '$A + B + P$' system about C is $\dfrac{2}{3}\ ml^2$.

(d) the angular velocity of the '$A + B + P$' system is $u/2l.$

28. In Q. No. 26, immediately after impact,

(a) the velocity of '$A + P$' with respect to C is $u/6$, to the right.

(b) the angular velocity of '$A + P$' with respect to C is $u/2l$, clockwise.

(c) the velocity of B with respect to C is $u/3$, to the left.

(d) the angular velocity of B with respect to C is $u/2l$, clockwise.

29. A thin uniform rod of mass m and length l is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Immediately after impact,

(a) the angular momentum of the rod is Jl.

(b) the angular velocity of the rod is $3J/ml$.

(c) the kinetic energy of the rod is $3J^2/2m$.

(d) the linear velocity of the midpoint of the rod is $3J/2m$.

30. The density of a rod gradually decreases from one end to the other. It is pivoted at an end so that it can move about a vertical axis through the pivot. A horizontal force F is applied on the free end in a direction perpendicular to the rod. The quantities, that do not depend on which end of the rod is pivoted, are

(a) angular acceleration.

(b) angular velocity when the rod completes one rotation.

(c) angular momentum when the rod completes one rotation.

(d) torque of the applied force.

31. Consider a wheel of a bicycle rolling on a level road at a linear speed v_0 (Figure 8.63).

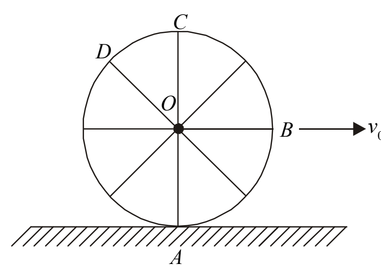

Fig. 8.63

(a) the speed of the particle A is zero.

(b) the speed of B, C and D are all equal to v_0.

(c) the speed of C is $2\ v_0$.

(d) the speed of B is greater than the speed of O.

32. Two uniform solid spheres having unequal masses and unequal radii are released from rest from the same height on a rough incline. If the spheres roll without slipping,

(a) the heavier sphere reaches the bottom first.
(b) the bigger sphere reaches the bottom first.
(c) the two spheres reach the bottom together.
(d) the information given is not sufficient to tell which sphere will reach the bottom first.

33. A hollow sphere and a solid sphere having same mass and same radii are rolled down a rough inclined plane.
(a) The hollow sphere reaches the bottom first.
(b) The solid sphere reaches the bottom with greater speed.
(c) The solid sphere reaches the bottom with greater kinetic energy.
(d) The two spheres will reach the bottom with same linear momentum.

34. A sphere cannot roll on
(a) a smooth horizontal surface.
(b) a smooth inclined surface.
(c) a rough horizontal surface.
(d) a rough inclined surface.

35. In rear-wheel drive cars, the engine rotates the rear wheels and the front wheels rotate only because the car moves. If such a car accelerates on a horizontal road, the friction
(a) on the rear wheels is in the forward direction.
(b) on the front wheels is in the backward direction.
(c) on the rear wheel has larger magnitude than the friction on the front wheels.
(d) on the car is in the backward direction.

36. A sphere can roll on a surface inclined at an angle θ if the friction coefficient is more than $\frac{2}{7} g \sin\theta$. Suppose the friction coefficient is $\frac{1}{7} g \sin\theta$. If a sphere is released from rest on the inclined,
(a) it will stay at rest.
(b) it will make pure translational motion.
(c) it will translate and rotate about the centre.
(d) the angular momentum of the sphere about its centre will remain constant.

37. A sphere is rolled on a rough horizontal surface. It gradually slows down and stops. The force of friction tries to
(a) decrease the linear velocity.
(b) increase the angular velocity.
(c) increase the linear momentum.
(d) decrease the angular velocity.

38. Figure (8.64) shows a smooth inclined plane fixed in a car accelerating on a horizontal road. The angle of incline θ is related to the acceleration a of the car as $a = g \tan\theta$. If the sphere is set in pure rolling on the incline

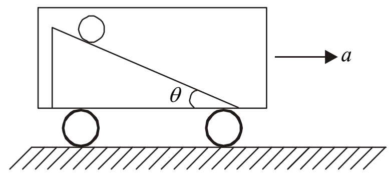

Fig. 8.64

(a) it will continue pure rolling.
(b) it will slip down the plane.
(c) its linear velocity will increase.
(d) its linear velocity will decrease.

39. Torque per unit moment of inertia is equivalent to
(a) angular velocity. (b) angular acceleration.
(c) radius of gyration. (d) inertia.

40. A circular disc starts slipping without rolling down an inclined plane then its velocity will be
(a) gh (b) $2gh$
(c) $\sqrt{gh}$ (d) $\sqrt{2gh}$

41. A spherical shell first rolls and then slips down an inclined plane. The ratio of its acceleration in two cases will be
(a) 5/3 (b) 3/5
(c) 15/13 (d) 13/15

42. The moment of inertia of a ring about its geometrical axis is I, then its moment of inertia about its diameter will be
(a) $2I$ (b) $I/2$
(c) I (d) $I/4$

43. A car is moving with a speed of 72 kmh^{-1}. The radius of its wheel is 50 cm. If its wheels come to rest after 20 rotations as a result of application of brakes, then the angular retardation produced in the car will be
(a) 23.5 $rads^{-2}$ (b) 0.25 $rads^{-2}$
(c) 6.35 $rads^{-2}$ (d) zero

44. The unit of moment of inertia is
(a) Joule/sec
(b) Joule/second/radian
(c) Joule/second^2/radian^2
(d) Joule/radian

45. A solid cylinder of mass 0.1 kg and radius 0.025 metre is rolling on a horizontal smooth table with uniform velocity of 0.1 ms^{-1}. Its total energy will be
(a) 7.5×10^{-2} Joule (b) 7.5×10^{-3} Joule
(c) 7.5×10^{-4} Joule (d) 0.07×10^{-4} Joule

46. A ring is rolling on an inclined plane. The ratio of the linear and rotational kinetic energies will be

(a) 2 : 1 (b) 1 : 2
(c) 1 : 1 (d) 4 : 1

47. The angular momentum and the moment of the inertia are respectively

(a) vector and tensor quantities.
(b) scalar and vector quantities.
(c) scalar and scalar quantities.
(d) vector and vector quantities.

48. In an arrangement four particles, each of mass 2 gm are situated at the coordinates point (3, 2, 0), (1, –1, 0), (0, 0, 0) and (–1, 1, 0). The moment of inertia of this arrangement about the Z-axis will be

(a) 8 units (b) 19 units
(c) 43 units (d) 34 units

49. The kinetic energy of rotation of a particle is 18 Joule. If the angular momentum vector coincides with the axis of rotation and the moment of inertia of the particle about this axis is 0.01 Kgm^2, then its angular momentum will be

(a) 0.06 J-sec (b) 0.6 J-sec
(c) 0.006 J-sec (d) zero

50. The relation between the linear velocity and angular velocity is

(a) $\vec{\omega} = \vec{r}\times\vec{v}$ (b) $\vec{v} = \vec{r}\times\vec{\omega}$
(c) $\vec{v} = \vec{\omega}\times\vec{r}$ (d) $\vec{\omega} = \vec{v}\times\vec{r}$

51. The moment of inertia of a body about a given axis of rotation depends upon.

(a) the distribution of mass.
(b) distance of the body from the axis of rotation.
(c) shape of the body.
(d) all of the above.

52. A rigid body is rotating about a vertical axis at n rotations per minute. If the axis slowly becomes horizontal in t seconds and the body keeps on rotating at n rotations per minute then the torque acting on the body will be, if the moment of inertia of the body about axis of rotation is I.

(a) Zero (b) $\dfrac{2\tau nl}{60t}$
(c) $\dfrac{2\sqrt{2}\tau nl}{60t}$ (d) $\dfrac{4\tau nl}{60t}$

53. A particle is revolving in a circle of radius r. Its displacement after completing half the revolution will be

(a) πr (b) $2r$
(c) $2\pi r$ (d) $\dfrac{r}{2}$

54. The relation between angular momentum an angular velocity is

(a) $\vec{J} = \vec{r}\times\vec{\omega}$ (b) $\vec{J} = \vec{\omega}\times\vec{r}$
(c) $\vec{J} = \dfrac{1}{\vec{\omega}}$ (d) $\vec{J} = I\vec{\omega}$

55. Two metallic discs have same mass and same thickness but different densities. The moment of inertia about the geometrical axis will be more of the disc

(a) with lower density.
(b) with higher density.
(c) M.I. of both the discs will be same.
(d) nothing can be said.

56. Minimum time period in a compound pendulum is obtained when

(a) $1 = \pm\dfrac{K}{2}$ (c) $1 = \pm K$
(b) $1 = \pm\dfrac{K}{\sqrt{2}}$ (d) $1 = 0$

57. The moment of inertia of a diatomic molecule about anaxis passing through its center of mass and perpendicular to the line joining the two atoms will be (μ = Reduced mass of the system)

(a) μr^2 (b) $\dfrac{\mu r^2}{2}$
(c) 0 (d) $\dfrac{3}{4}\mu r^2$

58. Which of the following quantities is zero about the center of mass of a body

(a) mass. (b) moment of mass.
(c) acceleration. (d) angular acceleration.

59. A ring of mass 10 Kg and diameter 0.4 meter is rotating about its geometrical axis at 1200 rotations per minute. Its moment of inertia and angular momentum will be respectively

(a) 0.4 Kg/m^2 and 50.28 Joule/sec
(b) 50.28 Kg/m^2 and 0.4 Joule/sec
(c) 0.4 Joule/sec and 50.28 Kg/m^2
(d) 0.4 Kg/m^2 and zero

60. Two rotating bodies have same angular momentum but their moments of inertia are I_1 and I_2 respectively ($I_1 > I_2$). Which body will have higher kinetic energy of rotation.

(a) first.
(b) second.

(c) both will have same kinetic energy.

(d) not possible to predict.

61. A chain couples and rotates two wheels in a bicycle. The radii of bigger and smaller wheels are 0.5m and 0.1 respectively. The bigger wheel rotates at the rate of 200 rotations per minute, then the rate of rotation of smaller wheel will be—

(a) 1000 *r*pm (b) $\frac{50}{3}$

(c) 200 *r*pm (d) 40 *r*pm

62. The moment of inertia of a fly-wheel is 4 Kg/m^2. A torque of 10 Newton-meter is applied on it. The angular acceleration produced will be—

(a) 25 radians/sec^2 (b) 0.25 radians/sec^2

(c) 2.5 radian/sec^2 (d) zero

63. The value of angular momentum of the earth rotating about its own axis is—

(a) 7×10^{33} Kg/m^2/sec. (b) 7×10^{33} Kg /m^2/sec.

(c) 0.7×10^{33} Kg/m^2/sec. (d) zero

64. The work done in rotating a body from angle θ_1 to angle

θ_2 will be—

(a) $\frac{\tau}{(\theta_1 - \theta_2)}$ (b) $\tau(\theta_2 - \theta_1)$

(c) Zero (d) $\frac{(\theta_1 - \theta_2)}{\tau}$

65. A girl sits near the edge of a rotating circular platform. If the girl moves from circumference towards the center of the platform then the angular velocity of the platform will

(a) decrease. (b) increase.

(c) remain same. (d) becomes zero.

66. The moment of inertia of a hollow sphere of mass 1Kg and inner and outer diameters 0.2 and 0.4 meter respectively about its diametric axis will be

(a) zero (b) 0.177 Kg/m^2

(c) 0.0177 Kg/m^2 (d) 177 Kg/m^2

67. A gramophone disc is rotating at 78 rotations per minute. Due to power cut, it comes to rest after 30 second. The angular retardation of the disc will be

(a) 0.27 radians/sec^2. (b) 0.127 radians/sec^2.

(c) 12.7 radians/sec^2. (d) zero.

68. A long thread is wrapped round a reel. If one end of thread is held in hand and the reel is allowed to fall under gravity, them the acceleration of the reel will be

(a) g (b) $\frac{3}{2}g$

(c) $\frac{3}{2}g$ (d) zero

69. The moment of inertia of a circular disc of mass 200 gm and radius 5cm about a tangential axis normal to the plane of the disc will be

(a) 750 g/cm^2 (b) 7500 g/cm^2

(c) 75 g/cm^2 (d) zero

70. A rod with rectangular cross-section oscillates about a horizontal axis passing through one of its ends and it behaves like a seconds pendulum. Its length will be

(a) 1.5 *m* (b) 1 *m*

(c) 3 *m* (d) 2 *m*

71. The ratio of kinetic energies of two spheres rolling with equal center of mass velocities is 2 : 1. If their radii are in the ratio 2 : 1, then the ratio of their masses will be

(a) 2 : 1 (b) 1 : 8

(c) 1 : 7 (d) $2\sqrt{2}$: 1

72. Out of the following bodies of same mass, which one will have maximum moment of inertia about an axis passing through its center of gravity and perpendicular to its place?

(a) ring of radius *r*.

(b) disc of radius *r*.

(c) square frame of sides 2*r*.

(d) square lamina of side 2*r*.

73. A particle is executing uniform circular motion with angular momentum *J*. If its kinetic energy is reduced to half and its angular frequency is doubled then its angular momentum becomes

(a) 2 *J* (b) 4 *J*

(c) $\frac{J}{2}$ (d) $\frac{J}{4}$

74. The angle covered by a body in n^{th} second is

(a) $\omega_0 + \frac{\alpha}{2}(2n-1)$ (b) $\omega_0 - \frac{\alpha}{2}(2n-1)$

(c) $\omega_0 + \frac{\alpha}{2}(n-1)$ (d) $\omega_0 - \frac{\alpha}{2}(n-1)$

75. A particle of mass m is tied to the end of a string passing through a hollow tube. The particle is revolved with angular velocity ω. The force required to be applied at the lower end of the string in order to maintain dynamic equilibrium will be

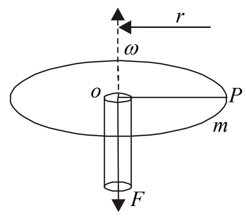

Fig. 8.65

(a) $m\omega^2 r$ (b) mg

(c) $\dfrac{m\omega^2 r}{2}$ (d) $m\omega^2 r + mg$

76. A Yo-Yo is a toy in the form of a disc with a concentric shaft. A string is wound on the shaft. If it is suspended from the free end, then the string unwinds and winds so that the Yo-Yo falls down and rises up again and again. The ratio of the tension in the string while descending and ascending is

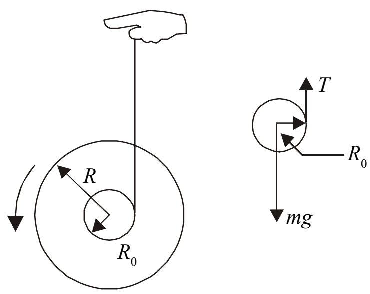

Fig. 8.66

(a) 1 : 2 (b) 1 : 1

(c) $R : R_0$ (d) $R_0 : R$

77. Two masses of 200 gm and 300 gm are attached to the 20 cm and 70 cm marks of a light meter scale respectively. The moment of inertia of this system about an axis passing through 50 cm mark will be

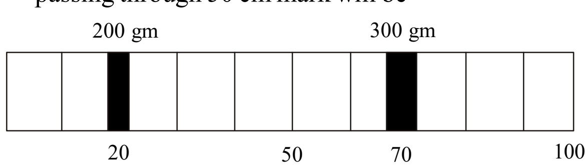

Fig. 8.67

(a) 0.3 Kg/m^2 (b) 0.03 Kg/m^2

(c) 0.15 Kg/m^2 (d) zero

78. The ratio of angular frequency and linear frequency is

(a) 2π (b) Π

(c) $\dfrac{1}{2\pi}$ (d) $\dfrac{\pi}{2}$

79. A mass M is moving with constant velocity parallel to x-axis. Its angular momentum about the origin

(a) is zero. (b) increases.

(c) decreases. (d) remains constant.

80. Which of the following bodies of same mass has maximum moment of inertia about its geometric axis?

(a) a bar pendulum. (b) a solid sphere.

(c) a circular ring. (d) a circular disc.

81. Which of the following relations is wrong ?

(a) $\vec{J} = \vec{r} \times \vec{P}$ (b) $\vec{a} = \vec{r} \times \vec{\alpha}$

(c) $\vec{v} = \vec{\omega} \times \vec{r}$ (d) $\tau = \dfrac{d\vec{J}}{dt}$

82. If the position vector of a particle is $\hat{r} = \left(3\hat{i} = 4\hat{j}\right)$ metre and its angular velocity is $\vec{\omega} = \left(\hat{j} = 2\hat{k}\right)$ rad/sec then its linear velocity is (in m/s)

(a) $-\left(8\vec{i} - 6\vec{j} + 3\vec{k}\right)$ (c) $\left(3\vec{i} - 6\vec{j} + 8\vec{k}\right)$

(b) $-\left(3\vec{i} - 6\vec{j} + 6\vec{k}\right)$ (d) $\left(6\vec{i} - 8\vec{j} + 3\vec{k}\right)$

83. Moon is revolving round the earth as well as it is rotating about its own axis. The ratio of its angular momenta in two cases will be—(orbital radius of moon $= 3.82 \times 10^8$ m and radius of moon $= 1.74 \times 10^6$ m)

(a) $1.22 \times 10^{5/4}$ (b) $1.22 \times 10^{5/3}$

(c) $1.22 \times 10^{5/2}$ (d) $1.22 \times 10^{5/1}$

84. The moment of inertia of a solid cylinder of mass M, length L and radius R about the diameter of one of its faces will be

(a) $M\left(\dfrac{L^2}{12} + \dfrac{R^2}{4}\right)$ (b) $M\left(\dfrac{L^2}{3} + \dfrac{R^2}{4}\right)$

(c) zero (d) $\dfrac{MR^2}{2}$

85. Equal torques are applied about a central axis on two rings of same mass and same thickness but made up of different materials. If ratio of their densities is 4 : 1 then the ratio of their angular acceleration will be :

(a) 16 : 1 (b) 1 : 16

(c) 8 : 1 (d) 1 : 12

86. A circular hoop of mass M and radius R is suspended from a nail in the wall. Its moment of inertia about an axis along the nail will be

(a) zero (b) MR^2

(c) $2MR^2$ (d) $\dfrac{MR^2}{2}$

87. The direction of $\vec{\tau}$ is

(a) parallel to the plane of $\vec{r}$ and $\vec{F}$.

(b) perpendicular to the plane of $\vec{r}$ and $\vec{F}$.

(c) parallel to the plane of $\vec{r}$ and $\vec{P}$.

(d) perpendicular to the plane of $\vec{r}$ and $\vec{P}$.

88. The moment of inertia of a ring of mass 2 Kg about a tangential axis lying in its own plane is 3Kg/m^2. The radius of the ring is

(a) 1 m (b) 3 cm

(c) 3 mm (d) 6 m

89. A fly-wheel of moment of inertia 0.4 Kg/m^2 and radius 0.2 m is free to rotate about a central axis. If a string is wrapped around it and it is pulled with a force of 10 Newton then its angular velocity after four seconds will be

(a) 5 radians/sec. (b) 20 radians/sec.

(c) 10 radians/sec. (d) 0.8 radians/sec.

90. The equation of motion of a compound pendulum is

(a) $\frac{d^2x}{dt^2}+\omega^2x=0$ (b) $\frac{d^2\theta}{dt^2}+\omega^2\theta=0$

(c) $F=-Kx$ (d) $\frac{d\theta}{dt^2}+\omega^2\theta=0$

91. A block of mass 12 Kg is attached to a string wrapped around a wheel of radius 10 cm. The acceleration of the block moving down an inclined plane is measured at 2 m/s^2 The tension in the string is

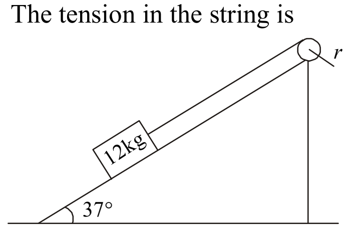

Fig. 8.68

(a) 24.5 Newton (b) 68.7 Newton

(c) 23.4 Newton (d) 46.8 Newton

92. In the above problem the moment of inertia of the wheel is

(a) 0.23 Kg/m/s^2 (b) 0.46 Kg/m/s^2

(c) 0.92 Kg/m/s^2 (d) 0.69 Kg/m/s^2

93. In Q. No. 41, the angular speed of the wheel after the 3 seconds after start will be (in radians/sec)

(a) 10 (b) 20

(c) 40 (d) 60

94. A uniform solid cylinder of mass M and radius R rotates about a frictionless horizontal axle. Two similar masses suspended with the help of two ropes wrapped around the cylinder. If the system is released from test then the tension in each rope will be

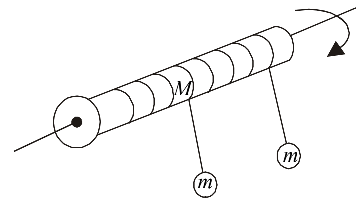

Fig. 8.68

(a) $\frac{Mmg}{(M+m)}$ (b) $\frac{Mmg}{(M+2m)}$

(c) $\frac{Mmg}{(M+3m)}$ (d) $\frac{Mmg}{(M+4m)}$

95. In the above problem the acceleration of each mass will be

(a) $\frac{4mg}{(M+2m)}$ (b) $\frac{4mg}{(M+4m)}$

(c) $\frac{2mg}{(M+m)}$ (d) $\frac{2mg}{(M+2m)}$

96. In the Q. No. 44, the angular velocity of the cylinder, after the masses fall down through distance h, will be

(a) $\frac{1}{R}\sqrt{8mgh/(M+4m)}$ (b) $\frac{1}{R}\sqrt{8mgh/(M+m)}$

(c) $\frac{1}{R}\sqrt{mgh/(M+m)}$ (d) $\frac{1}{R}\sqrt{8mgh/(M+2m)}$

97. A massless string is wrapped round a disc of mass M and radius R. Another end is tied to a mass m which is initially at height h from ground level as shown in the figure. If the mass is released then its velocity while touching the ground level will be

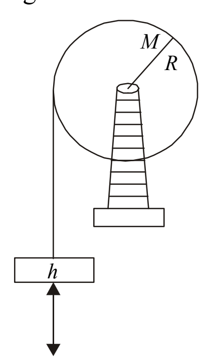

Fig. 8.69

(a) $\sqrt{2gh}$ (b) $\sqrt{2gh}\frac{M}{m}$

(c) $\sqrt{2gh\ \ m/M}$ (d) $\sqrt{4mgh/2m+M}$

98. The centres of four spheres each of mass m and diameter $2a$ are at the four corners of a square of side b. The moment of inertia of the system about one side of the square will be

(a) $\frac{2}{5}m\left[4a^2+5b^2\right]$ (b) $\frac{2}{5}m\left[5a^2+4b^2\right]$

(c) $\frac{2}{5}m\left[a^2+b^2\right]$ (d) $m\left[\frac{8}{5}a^2+b^2\right]$

99. In the above question the moment of inertia of the system about the diagonal of square will be

(a) $\frac{2}{5}m\left[4a^2+5b^2\right]$ (b) $\frac{2}{5}m\left[5a^2+4b^2\right]$

(c) $\frac{2}{5}m\left[a^2+b^2\right]$ (d) $m\left[\frac{8}{5}a^2+b^2\right]$

100. In Q. No. 98 the moment of inertia of the system about an axis passing through one corner of the square and perpendicular to its plane will be

(a) $\frac{4}{5}m\left[2a^2+5b^2\right]$ (b) $\frac{5}{4}m\left[a^2+2b^2\right]$

(c) $\frac{2}{5}m\left[3a^2+4b^2\right]$ (d) $\frac{3}{4}m\left[2a^2+4b^2\right]$

101. A solid cylinder of mass 2 Kg and radius 0.2 m is rotating about its own axis without friction with angular velocity 3 rad/s. A particle of mass 0.5 Kg and moving with a velocity of 5 m/s strikes the cylinder and sticks to it as shown in. The angular momentum of the cylinder before collision will be

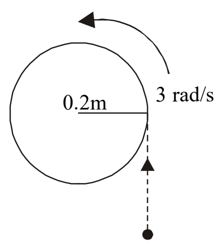

Fig. 8.70

(a) 0.12 Joule/s (b) 12 Joule/s

(c) 1.2 Joule/s (d) 1.12 Joule/s

102. In the above question the angular velocity of the system after the particle sticks to it will be

(a) 0.3 radians/sec. (b) 5.3 radians/sec

(c) 10.3 radians/sec (d) 8.3 radians/sec

103. In Q. No. 51, the energy of the system in the beginning is

(a) 1.43 J (b) 2.43 J

(c) 3.43 J (d) 8.3 J

104. The rotational kinetic energy of two bodies of moments of inertia 9 Kg/m^2 and 1 Kg/m^2 are same. The ratio of their angular momenta is

(a) 3 : 1 (b) 1 : 3

(c) 9 : 1 (d) 1 : 9

105. The moment of inertia of a circular disc about its own axis is 4 Kg/m^2. Its moment of inertia about the diameter will be

(a) 4 Kg/m^2 (b) 2 Kg/m^2

(c) zero (d) 8 Kg/m^2

106. A hollow cylinder is rolling on an inclined plane, inclined at an angle of 30° to the horizontal. It's speed after traveling a distance of 10 m will be

(a) 49 m/sec (b) 0.7 m/sec

(c) 7 m/sec (d) zero.

107. The moment of inetia of a spherical shell about a tangential axis is

(a) $\frac{2}{5}MR^2$ (b) $\frac{7}{5}MR^2$

(c) $\frac{2}{3}MR^2$ (d) $\frac{5}{3}MR^2$

108. A body with moment of inertia 3 Kg/m^2 is at rest. A torque of 6 Newton/metre applied on it rotates the body for 20 second. The angular displacement of the body is

(a) 800 radians (b) 600 radians

(c) 400 radians (d) 200 radians

109. The ratio of the angular velocities of the hour hand and minute hand of a watch is

(a) 1 : 1 (b) 1 : 12

(c) 43200 : 1 (d) 720 : 1

110. The second equation of motion in rotatory motion is

(a) $S = ut + \frac{at^2}{2}$ (b) $\theta = \omega_1 t + \frac{\alpha t^2}{2}$

(c) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ (d) $\omega_2 = \omega_1 + \alpha t$

111. Which of the following pairs do not match?

(a) Rotational power Joule.

(b) Torque/Newton meter.

(c) Angular displacement/Radian.

(d) Angular acceleration/Radian/sec.

112. A stone tied to one end of the string is revolved round a rod in such a way that the string winds over the rod and gets shortened. In this process which of the following quantities remains constant ?

(a) mass. (b) momentum.

(c) angular momentum. (d) kinetic energy.

113. A solid sphere of mass 0.1 Kg and radius 2 cm rolls down an inclined plane 1.4 m in lengdi (slope l in 10). Starting from rest its final velocity will be

(a) 1.4 m/sec (b) 0.14 m/sec

(c) 14 m/sec (d) 0.7 m/sec

114. Four particles each of mass m are lying symmetrically on the rim of a disc of mass M and radius R. The moment of inertia of this system about an axis passing through one of the particles and perpendicular to the plane of the disc is

(a) $16\,\mathrm{m}R^2$ (b) $(3\,\mathrm{M}+16\,\mathrm{m})\,\dfrac{R^2}{2}$

(c) $(3\,\mathrm{m}+16\,\mathrm{M})\,\dfrac{R^2}{2}$ (d) zero

115. A cockroach of mass m is moving on the rim of a disc with velocity V in the anticlockwise direction. The moment of inertia of the disc about its own axis is I and it is rotating in the clockwise direction with angular speed ω. If the cockroach stops moving then the angular speed of the disc will be

(a) $\dfrac{I\omega}{I+mR^2}$ (b) $\dfrac{I\omega+mVR}{1+mR^2}$

(c) $\dfrac{I\omega-mVR}{I+mR^2}$ (d) $\dfrac{I\omega-mVR}{I}$

116. If the force applied on a particle is zero then the quantities which are conserved are

(a) only momentum.

(b) only angular momentum.

(c) momentum and angular momentum.

(d) only potential energy.

117. A body starts rolling down an inclined plane of length L and height h. This body reaches the bottom of the plane in time t. The relation between L and t is

(a) $t \propto L$ (b) $\dfrac{t\infty 1}{L}$

(c) $t \propto L^2$ (d) $\dfrac{t\infty 1}{L^2}$

118. In hydrogen atom an electron revolves in a circular path. If the radius of its path is 0.53 A° and it makes 7×10^{15} revolutions per second, its angular momentum about proton is

(a) 11.2×10^{-35} Joule/sec (b) 11.2×10^{-34} Joule/sec

(c) 11.2×10^{-33} Joule/sec (d) 11.2×10^{-33} Joule/sec

119. The expressions for the tangential and centripetal accelerations are

(a) $r\alpha$ and $\omega^2 r$ (b) $\omega^2 r$ and $r\alpha$

(c) ωr and $r\alpha$ (4) $r\alpha$ and ωr

120. The unit of J/P is

(a) meter/sec (b) meter

(c) Joule (d) Joule/sec

121. If the tangential and centripetal accelerations are tangents and along the centre, respectively, then the resultant acceleration (a) will be

(a) $a = a_t + a_c$ (b) $a = \sqrt{a_t^2 + a_c^2}$

(c) $a = a_t - a_c$ (d) $a = a_c - a_t$

122. The dimensions of $\tau/\propto$ are

(a) ML^{-2} (b) ML^2

(c) M^2L^2 (d) $M^{-2}L^{-2}$

123. The block of mass M is initially moving to the right without friction with speed V_1. It passes over the cylinder to the dashed position, when it first makes contact with the cylinder, it slips on the cylinder but the friction is large enough so that slipping ceases before M losses contact with the cylinder. The final velocity of the V_2 of the block will be, if the radius of the cylinder is R and its moment of inertia is 1 and initially it is at rest.

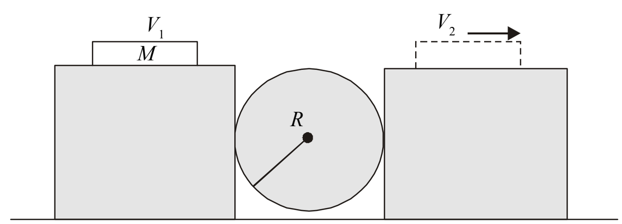

Fig. 8.71

(a) $V_2 = \dfrac{MV_1}{M+\dfrac{m}{2}}$ (b) $V_2 = \dfrac{mV_1}{M+\dfrac{M}{2}}$

(c) $V_2 = V_1$ (d) $V_2 = \dfrac{V_1 I}{MR^2}$

124. A solid sphere rests on a horizontal surface, A horizontal impulse is applied at height h from the centre (Fig 8.72).

The sphere starts rotating just after the application of impulse. The ratio ω_3 will be

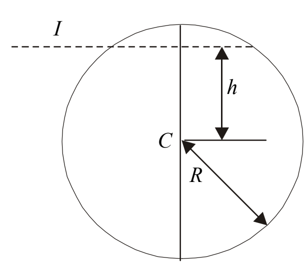

Fig. 8.72

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

125. A solid cube of side l is made to oscillate about a horizontal axis passing through one of its edges. Its time period will be

(a) $2\pi\sqrt{\frac{2\sqrt{2}}{3}\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{2}{3}\frac{l}{g}}$

(c) $2\pi\sqrt{\frac{\sqrt{3}}{2}\frac{l}{g}}$ (d) $2\pi\sqrt{\frac{2}{\sqrt{3}}\frac{l}{g}}$

126. The radius of gyration of a plane lamina of mass M, length L and breadth B about an axis passing through its center of gravity and perpendicular to its plane will be

(a) $\sqrt{(L^2+B^2)/12}$ (b) $\sqrt{(L^2+B^2)/8}$

(c) $\sqrt{(L^2+B^2)/2}$ (d) $\sqrt{(L^3+B^3)/12}$

127. Two spheres each of mass 1Kg are attached to the two ends of a rod of mass 1Kg and length 1m. The moment of inertia of the system about an axis passing through its centre of gravity and perpendicular to the rod will be

(a) 1 Kgm^2 (b) $\frac{1}{2}$ Kgm^2

(c) $\frac{7}{12}$ Kgm^2 (d) $\frac{12}{7}$ Kgm^2

128. The diameter of a solid disc is 0.5m and its mass is 16Kg. What torque will increase its angular velocity from zero to 120 rotations/minute in 8 seconds ?

(a) $\frac{\pi}{4}$ N/m (b) $\frac{\pi}{2}$ N/m

(c) $\frac{\pi}{3}$ N/m (d) π N/m

129. In the above problem at what rate is work done by the torque at the end of 8th second ?

(a) π Watt (b) π^2 Watt

(c) π^3 Watt (d) π^4 Watt

130. Two point masses are lying on a smooth uniform mass M and length L. Initially the masses are in the middle of the rod. The system is rotating about an axis passing through the center and perpendicular to the rod with angular velocities ω. No external force acts on the system. When the masses reach the ends of the rod, then the angular velocity of the system will be

(a) $\frac{M\omega_0}{M+2m}$ (b) $\frac{M\omega_0}{M+4m}$

(c) $\frac{M\omega_0}{M+6m}$ (d) $\frac{M\omega_0}{M+8m}$

Answers to Questions for Practice

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(b,c,d)	**6.**	(a,d)	**7.**	(c)
8.	(b,c,d)	**9.**	(a,c,d)	**10.**	(a,c,d)	**11.**	(a,b,d)	**12.**	(b,d)	**13.**	(a,b,c)	**14.**	(a,b,d)
15.	(b,c,d)	**16.**	(b,c,d)	**17.**	(a,d)	**18.**	(c,d)	**19.**	(c,d)	**20.**	(d)	**21.**	(a,c,d)
22.	(a,b)	**23.**	(c)	**24.**	(a,b,d)	**25.**	(a,b,c,d)	**26.**	(a,b,c,d)	**27.**	(a,b,c,d)	**28.**	(a,b,c,d)
29.	(a,b,c,d)	**30.**	(d)	**31.**	(a,c,d)	**32.**	(c)	**33.**	(b)	**34.**	(b)	**35.**	(a,b,c)
36.	(c)	**37.**	(a,b)	**38.**	(a)	**39.**	(b)	**40.**	(d)	**41.**	(c)	**42.**	(a)
43.	(a)	**44.**	(a)	**45.**	(b)	**46.**	(c)	**47.**	(c)	**48.**	(d)	**49.**	(c)
50.	(c)	**51.**	(d)	**52.**	(c)	**53.**	(b)	**54.**	(d)	**55.**	(a)	**56.**	(b)
57.	(a)	**58.**	(b)	**59.**	(a)	**60.**	(b)	**61.**	(a)	**62.**	(c)	**63.**	(a)
64.	(b)	**65.**	(b)	**66.**	(c)	**67.**	(a)	**68.**	(b)	**69.**	(b)	**70.**	(a)
71.	(a)	**72.**	(c)	**73.**	(d)	**74.**	(a)	**75.**	(a)	**76.**	(b)	**77.**	(b)
78.	(a)	**79.**	(d)	**80.**	(c)	**81.**	(b)	**82.**	(a)	**83.**	(d)	**84.**	(b)
85.	(a)	**86.**	(c)	**87.**	(b)	**88.**	(a)	**89.**	(b)	**90.**	(b)	**91.**	(d)
92.	(a)	**93.**	(d)	**94.**	(d)	**95.**	(a)	**96.**	(a)	**97.**	(a)	**98.**	(b)
99.	(a)	**100.**	(d)	**101.**	(a)	**102.**	(d)	**103.**	(a)	**104.**	(a)	**105.**	(c)
106.	(d)	**107.**	(a)	**108.**	(b)	**109.**	(c)	**110.**	(d)	**111.**	(c)	**112.**	(b)
113.	(b)	**114.**	(d)	**115.**	(c)	**116.**	(a)	**117.**	(b)	**118.**	(c)	**119.**	(c)
120.	(b)	**121.**	(b)	**122.**	(b)	**123.**	(a)	**124.**	(b)	**125.**	(a)	**126.**	(a)
127.	(c)	**128.**	(a)	**129.**	(b)	**130.**	(c)						

EXPLANATION

7(c) When the beam is supported at A and B, the force exerted by each man = $mg/2$.

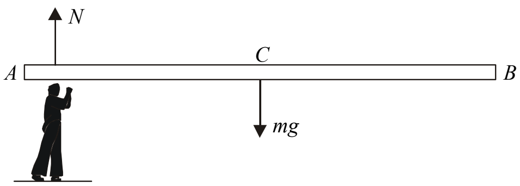

Fig. 8.73

When the support at B is withdrawn, taking torque about A,

$\tau = (mg)\, l/2 = I\alpha = (ml^2/3)\, \alpha$

or $\alpha = 3g/2l$.

The instantaneous linear acceleration of the centre of mass is

$a_{CM} = (\alpha)\,(AC) = (3g/2l)\; l/2 = 3g/4$.

Let N = force exerted on the beam at A.

$\therefore \quad mg - N = ma_{CM} = m(3g/4)$ or $N = \dfrac{1}{4}\, mg$.

8 (b, c, d) Taking torque about A, when the rod has fallen through an angle θ,

$$\tau = mg\,\frac{1}{2}\,\sin\theta = I\alpha = \left(\frac{1}{3}ml^2\right)\alpha$$

or
$$\alpha = \frac{3g}{2l}\,\sin\theta$$

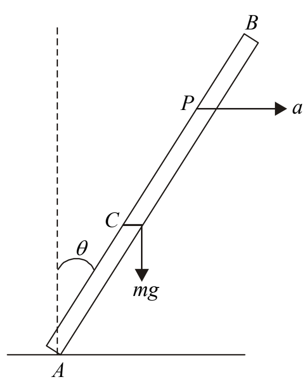

Fig. 8.74

For any point P on the rod, at a distance r from A, the linear acceleration is

$$a = r\alpha = \frac{3gr}{2l}\,\sin\theta.$$

11 (a, b, d) Let ω_1 = the initial angular velocity of the disc.

ω_2= the final common angular velocity of the disc and the ring. For the disc, $I_1 = \dfrac{1}{2}\, mr^2$.

For the ring, $I_2 = mr^2$.

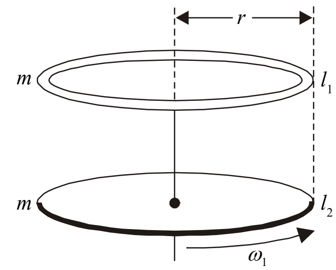

Fig. 8.75

By conservation of angular momentum,

$$L = I_1\omega_1 = (I_1 + I_2)\,\omega_2 \text{ or } \omega_2 = \frac{I_1\omega_1}{I_1 + I_2} = \omega_1/3.$$

Initial kinetic energy = $E_1 = \dfrac{1}{2}\, I_1\, \omega_1^2$.

Final kinetic energy = $E_2 = \dfrac{1}{2}\,(I_1 + I_2)\,\omega_2^2$.

Heat produced = loss in kinetic energy = $E_1 - E_2$.

Ratio of heat produced to initial kinetic energy

$$= \frac{E_1 - E_2}{E_1} = \frac{2}{3}.$$

12 (b, d) The force of friction between the two surfaces in contact disappears when there is no relative (linear) motion between them. Angular momentum will not be conserved as the discs will have final angular velocities in opposite directions.

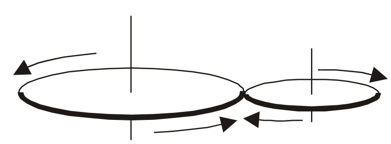

Fig. 8.76

13 (a, b, c) Let L = angular momentum.

$$\tau = \frac{dL}{dt} \text{ or } dL = \tau\, dt.$$

For constant torque,

$\Delta L = \tau\,\Delta t = I\,\Delta\omega$ or $I\omega$ if $\omega_i = 0$

$$\text{Rotational kinetic energy} = \frac{1}{2}\, I\omega^2 = \frac{(\Delta L)^2}{2I}.$$

14 (a, b, d) $J = mv$ for both. A has no angular motion. For B, angular momentum imparted by $J = L = Jh$.

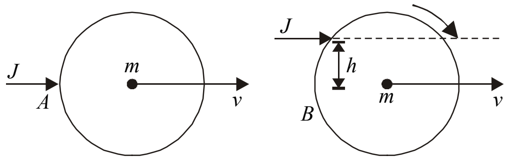

Fig. 8.77

16 (b, c, d) Let P = external force F = force of friction between A and B.

a_1 = acceleration between A and B, a_2 = acceleration beyond B.

$P - F = ma_1$ and $P = ma_2$. $\therefore a_2 > a_1$.

Let α = angular acceleration between A and B. For one rotation,

$$\theta = 2\pi = \frac{1}{2}\alpha T^2$$

or $T = (4\pi/\alpha)^{1/2}$ = time of travel from A to B.

Angular velocity at $B = \omega_B = \alpha T$.

For one rotation to the right of B,

$\theta = 2\pi = \omega_B t$ or

$$t = \frac{2\pi}{\alpha T} = \frac{\frac{1}{2}T^2}{T} = \frac{T}{2}.$$

17 (a, d) In rolling with slipping, the force of friction produces a torque which gives an angular acceleration to the body. Hence, part of the work done against friction is converted to rotational kinetic energy, which adds to the total kinetic energy. Only the remaining part of the work done against friction is converted to heat.

18 (c, d) In rolling without slipping, no work is done against friction. Hence, loss in gravitational potential energy of a body is equal to its total kinetic energy, i.e., linear plus rotational kinetic energies. Also, $v = \omega r$.

Total kinetic energy $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}[mv^2 + (mk^2)(v^2/r^2)]$

$= \frac{1}{2}mv^2(1 + k^2/r^2)$, where k = radius of gyration.

As all the bodies have the same final total kinetic energy, their final velocities will depend only on the ratio k/r. Bodies with smaller values of k/r will have greater v and hence reach the bottom earlier.

19 (a, c, d) After impact, the force of friction will act in a direction opposite to that of the motion. The body will have retained its initial angular motion (clockwise). The force of friction will cause linear retardation, reducing v. It will also cause an anticlockwise angular acceleration, which will reduce ω to zero and then introduce anticlockwise ω till rolling without slipping begins. F will then disappear.

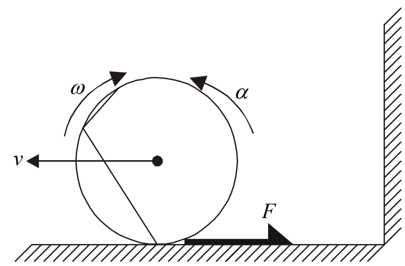

Fig. 8.78

20 (d) In rolling without slipping, at constant speed, there is no force of friction between the surfaces.

Therefore, removing the pin causes no change to the system.

21 (a, c, d) Every point on the ring has a horizontal velocity u due to its linear motion, and in addition a velocity u, tangential to the ring, due to its rotational motion. The resultant of these two is the velocity of the point with respect to the ground.

Hence, $v_A = 0$, $v_B = 2u$, $v_D = \sqrt{2}\,u$, $v_E = u$.

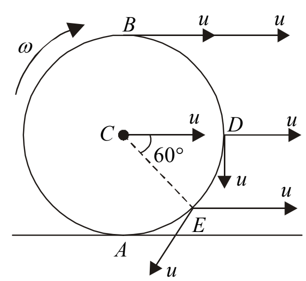

Fig. 8.79

23 (c) At the point of leaving the wheel, the blob of mud is at a height $2r$ above the road and has a horizontal velocity $2v$ use results of Q.21

Let t = time of travel from D to B. Then, $2r = \frac{1}{2}gt^2$

or $t = 2\sqrt{r/g}$ and $AB = (2v)\,t$.

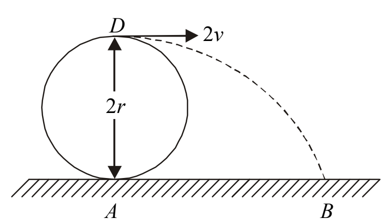

Fig. 8.80

24 (a, b, d) For a pulley with mass, the strings on its two sides do not have the same tension when in motion.

Also, the rotating pulley has some kinetic energy derived from the loss in potential energy of the block B as it moves down.

26. (a, b, c, d) As all three particles are part of the system, any impact between them produces no change in the velocity of their centre of mass. Also, '$A + B$' can exert a force on B only along the rod, i.e., normal to u.

29. (a, b, c, d) Angular momentum = linear momentum × perpendicular distance from the point of rotation

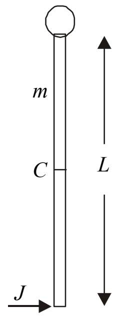

Fig. 8.81

or $L = Jl$

Also, $I = ml^2/3$.

$\therefore$ $\omega = L/I = 3J/ml$.

$$\text{Kinetic energy} = \frac{L^2}{2I}$$

$$= \frac{J^2t^2}{2(ml^2/3)}$$

$$= \frac{J^2t^2}{2(ml^2/3)}$$

$$v_c = \omega.$$

$$\frac{1}{2} = \frac{3J}{2m}.$$