

# 3

# Motion in One and Two Dimensions

## BRIEF REVIEW

**Motion** A body is said to be in motion if it keeps changing its position with respect to its surroundings with the passage of time otherwise it is said to be at rest.

**Frame of Reference** A set of coordinates  $x, y, z$  and  $t$  is said to be a frame of reference. It is of two types, **inertial** and **noninertial**. **Inertial** frame of reference is one which is either fixed or moves with a uniform velocity in the same straight line. **Non-inertial** or **accelerated** frame of reference moves with an acceleration ' $a$ '. Newton's laws are valid only in inertial frame. Pseudo or inertial vectors are to be applied to make the frame of reference inertial from non-inertial so that Newton's laws may be applied.

**One-Dimensional Motion** If the particle changes its position only in one of the  $x, y$ , or  $z$  directions with respect to time, then the motion is said to be one-dimensional. Since the particle moves along a straight line, the motion may also be termed as linear or rectilinear.

**Speed** The time rate of change of distance is called speed, that is,  $v = \frac{dx}{dt}$  unit  $ms^{-1}$ .

**Velocity** The time rate of change of displacement is called velocity, that is,  $\vec{v} = \frac{d\vec{x}}{dt}$ . Unit  $ms^{-1}$ ,  $cm s^{-1}$  and  $ft s^{-1}$  in SI, CGS and FPS system, respectively,  $v = LT^{-1}$ .

**Displacement** The shortest distance between initial and final position of the particle is called displacement.

**Acceleration** The time rate of change of velocity is

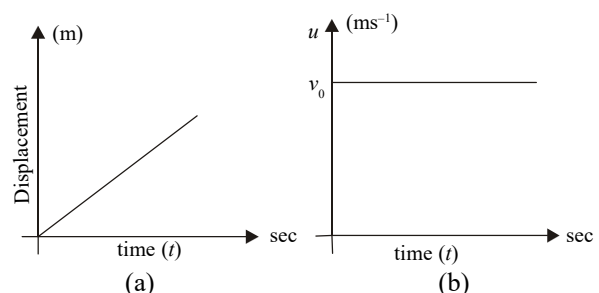
called acceleration,  $\vec{a} = \frac{d\vec{v}}{dt}$  units is  $ms^{-2}$ ,  $cm s^{-2}$  and  $ft s^{-2}$  in SI, CGS and FPS system, respectively,  $a = LT^{-2}$ . Speed, velocity or acceleration may be of four types. We define here velocity and others can be anticipated in similar terms.

(a) **Instantaneous Velocity** The velocity, at a particular instant of time is called instantaneous velocity, for example, velocity, at 4.82 s may be

$$\text{expressed as } \vec{v} = \left. \frac{d\vec{x}}{dt} \right|_{t=4.82s}$$

(b) **Uniform Velocity** If  $\frac{dx}{dt} = \text{constant}$  throughout

the motion and direction of motion does not vary throughout then such a velocity is called uniform velocity. Fig. 3.1 (a) shows displacement time graph and Fig. 3.1 (b) shows velocity time graph for a uniform velocity.

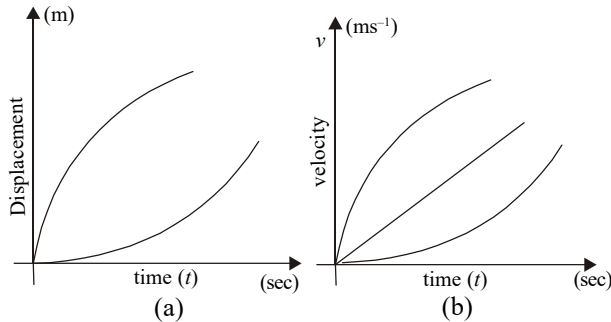


**Fig. 3.1** Uniform velocity

(c) **Variable Velocity** If  $\frac{dx}{dt}$  is not constant but

varies at different intervals of time or  $\frac{dx}{dt}$  is

constant but direction varies or both vary, then such a velocity is said to be variable velocity. Fig. 3.2 (a) illustrates  $x-t$  graph for a body moving with variable velocity and Fig. 3.2 (b) shows velocity  $V$  vs time variation for a body moving with variable velocity.



**Fig. 3.2** Variable velocity

(d) **Average Velocity** It is that uniform velocity with which if the body would have moved it would have covered the same displacement as it does otherwise by moving with variable velocity. Thus

$$v_{av} = \frac{\text{total displacement covered}}{\text{total time taken}}.$$

#### Average Velocity in Different Cases

(i) **Particles covering different displacement in different times:** Assume a particle covers  $s_1$  displacement in  $t_1$  and  $s_2$  in time  $t_2$  and so on then average velocity is

$$v_{av} = \frac{s_1 + s_2 + s_3 + \dots}{t_1 + t_2 + t_3 + \dots} = \frac{s_1 + s_2 + s_3 + \dots}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \dots}$$

Special case if  $s_1 = s_2 = s$ .

$$v_{av} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1 v_2}{v_1 + v_2} \text{ (harmonic mean)}$$

(ii) **Bodies moving with different velocity in different intervals of time.** A body moves with velocity  $v_1$  in time  $t_1$ ,  $v_2$  in time  $t_2$  and so on then  $v_{av}$  is given by

$$v_{av} = \frac{v_1 t_1 + v_2 t_2 + \dots}{t_1 + t_2 + \dots}$$

Special case if  $t_1 = t_2 = t_3 = \dots = t_n = t$

$$\text{Then } v_{av} = \frac{v_1 + v_2 + \dots + v_n}{n} \text{ (Arithmetic mean)}$$

#### Equations of Motion

$$(a) v = u + at \quad (b) s = ut + \frac{1}{2} at^2$$

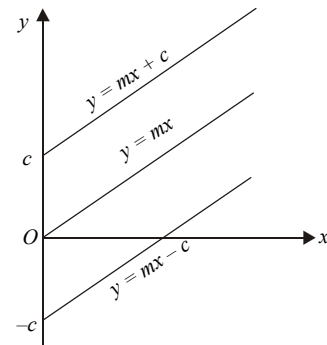
$$(c) v^2 - u^2 = 2as \quad (d) s_{nth} = u + \frac{s}{2} (2n - 1)$$

**The conditions under which these equations can be applied**

1. Motion should be 1-dimensional.
2. Acceleration should be uniform.
3. Frame of reference should be inertial.

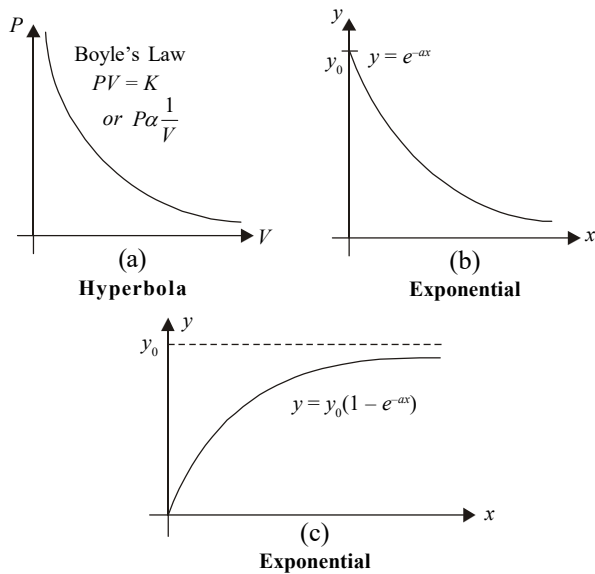
**While drawing graphs compare your equation with the following and then draw (matching the equation) graphs**

1.  $y = mx + c$  straight line with positive intercept on  $y$ -axis.  
 $y = mx$  straight line passing through origin.  
 $y = mx - c$  straight line with negative intercept (on  $-y$  axis).



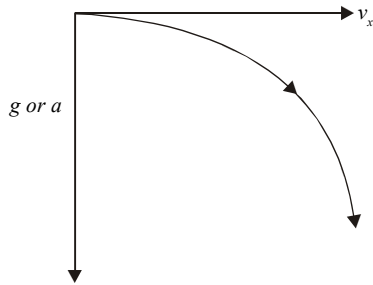
**Fig. 3.3** Straight lines

2.  $x^2 + y^2 = a^2$  circle with centre at origin.  
 $(x - h)^2 + (y - k)^2 = r^2$  a circle with centre at  $(h, k)$ .
3.  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  represents ellipse.
4.  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  shows hyperbola.
5.  $y = \frac{1}{x}$  or  $xy = k$  represents a rectangular hyperbola.  
 See Fig. 3.4 (a)
6.  $y = y_0 e^{-ax}$  and  $y = y_0 (1 - e^{-ax})$  represents exponential.  
 See Fig. 3.4 (b) and (c)

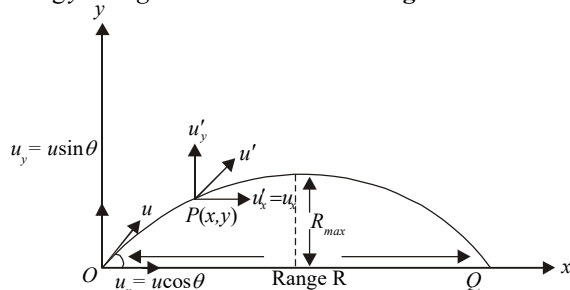
**Fig. 3.4**

While dealing with two-dimensional motion convert the problem into two one-dimensional motions. Separate  $v_x$  and  $v_y$  similarly  $a_x$  and  $a_y$ . Treat the motion in x- and y-directions.

**Projectile** A freely falling body having constant horizontal velocity may be termed as a projectile. In general, in one direction the motion be accelerated and in another direction the motion is uniform, then such a motion is called projectile motion. Fig. 3.5 shows acceleration in y-direction and uniform velocity in x-direction. Such bodies follow parabolic path.

**Fig. 3.5** Projectile motion

**Oblique Projectile Motion** Assume a projectile is fixed at an angle  $\theta$  with horizontal, with a velocity  $u$  from point  $O$  as shown in Fig. 3.6. Resolve velocity along x and y-axis. Along y-axis  $g$  acts then **maximum height attained**.

**Fig. 3.6** Projectile motion

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$$h_{\max} = \frac{u^2 \sin^2 \theta}{2g}.$$

**Time of Flight**  $T = \frac{2u \sin \theta}{g}.$

**Horizontal Range**  $R = \frac{u^2 \sin 2\theta}{g}$ . Note that the range will be same if projected at complement angles, i.e.  $\theta$  and  $(90 - \theta)$  with same velocity.

**Maximum Range**  $R_{\max} = \frac{u^2}{g}$  when  $\theta = 45^\circ$

**Trajectory**  $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$

or  $y = x \tan \theta \left[ 1 - \frac{x}{R} \right]$  Path is parabolic.

Instantaneous velocity =  $|v|$

$$= \sqrt{u_x^2 + v_y^2} = \sqrt{u_x^2 + (u_y - gt)^2}$$

$$= \sqrt{u^2 + g^2 t^2 - 2ugt \sin \theta}$$

$$\tan \beta = \frac{u_y - gt}{u_x}.$$

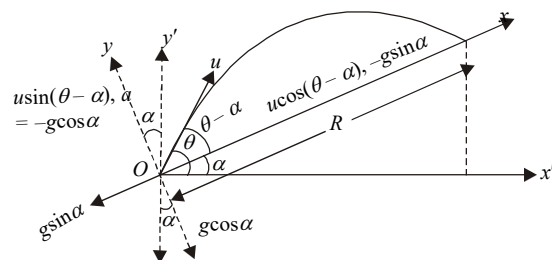
#### Range and time of flight along an inclined plane

Consider an inclined plane of inclination  $\alpha$ . Let a projectile be fixed at an angle  $\theta$  with the horizontal or at an angle  $(\theta - \alpha)$  with respect to incline plane as shown in Fig. 3.7.

The time of flight  $T' = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha}$

**Range**  $R' = \frac{2u^2 \sin(\theta - \alpha) \cos \theta}{g \cos^2 \alpha}$

$$R = \frac{u^2}{g \cos^2 \alpha} [\sin(2\theta - \alpha) - \sin \alpha]$$

**Fig. 3.7** Projectile motion along an incline

Range  $R'$  along the inclined is maximum if  $2\theta - \alpha = \pi/2$

or  $\theta - \alpha = \pi/2 - \theta$ . That is,  $R'$  is maximum when the direction of projection bisects the angle that the inclined plane makes

with  $Oy'$  and  $R'_{\max} = \frac{u^2}{g \cos^2 \alpha} \cdot [1 - \sin \alpha]$

**Note:** In projectile motion along the plane acceleration acts along  $x$  and  $y$  axis both.

### SHORT CUTS AND POINTS TO NOTE

1. Slope of  $x - t$  graph is velocity, slope of  $v - t$  graph is acceleration.

2. Average velocity  $v_{av} = \frac{\text{total displacement}}{\text{total time taken}}$

$$= \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{x_1 + x_2 + \dots}{t_1 + t_2}.$$

$$= \frac{x_1 + x_2 + \dots}{\frac{x_1}{v_1} + \frac{x_2}{v_2} + \dots}$$

If a body covers equal displacement with different

$$\text{velocities } \frac{1}{v_{av}} = \frac{1}{n} \left[ \frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_n} \right].$$

If a body moves half distance with  $v_1$  and other half

$$\text{with } v_2 \text{ then } v_{av} = \frac{2v_1 v_2}{v_1 + v_2}.$$

If a body moves with different velocities in equal intervals of time then

$$v_{av} = \frac{v_1 + v_2 + \dots + v_n}{n} \text{ (arithmetic mean)}$$

3. Area under  $v - t$  graph is displacement, area under  $a - t$  graph is velocity.
4. When a body leaves a moving body it acquires its velocity but not acceleration.

5. Instantaneous velocity  $v(t_1) = \left. \frac{dx}{dt} \right|_{t=t_1}$ .

6. Apply  $v = u + at$ ,  $s = ut + \frac{1}{2} at^2$ ,

$$v^2 - u^2 = 2as, s_{nth} = u + \frac{a}{2} (2n - 1) \text{ when}$$

- (i) motion is one dimensional or made so if two or three dimensional [by resolving].
- (ii) acceleration is uniform.
- (iii) frame of reference is inertial.

7. If acceleration is variable then start with  $\frac{dv}{dt} = f(t)$

$$\text{and } v = \int f(t) dt$$

8. If acceleration is variable and function of displacement or velocity. For example

$$a = -kv^2$$

$$\text{then } \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = -kv^2$$

$$\text{or } \int \frac{dv}{v} = \int -k dx.$$

9. Note carefully the graphs for  $v = u + at$  as shown in Fig. 3.8 (a) and (b).

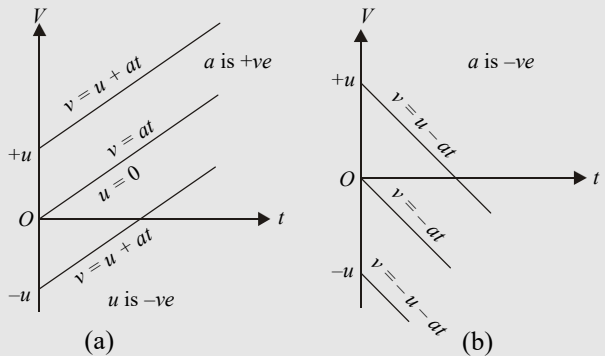


Fig. 3.8 Graphs for  $v = u + at$

10. Note the graph for  $s = at + \frac{1}{2} at^2$  carefully as shown in Fig. 3.9 (a), (b) and (c).

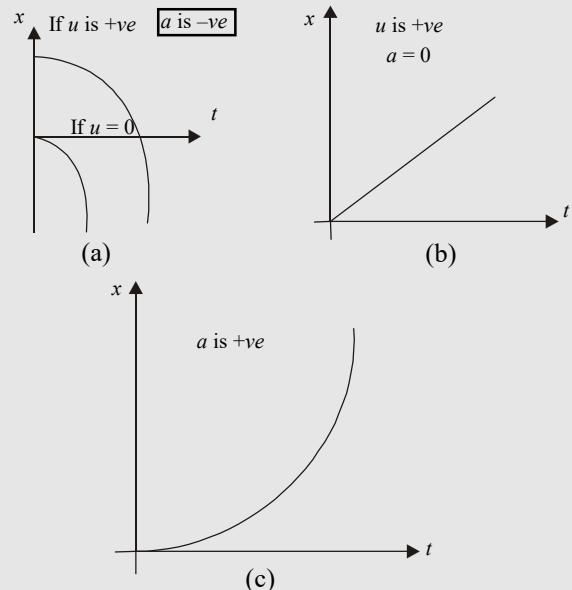


Fig. 3.9 Graphical representation of  $s = ut + \frac{1}{2} at^2$

11. If a particle starts from rest with an acceleration  $\alpha$ , after acquiring a maximum velocity the particle decelerates with  $\beta$  and finally comes to rest in time  $t$ , then

$$v_{\max} = \frac{\alpha\beta t}{\alpha + \beta} \text{ and distance covered } s = \frac{\alpha\beta t^2}{2(\alpha + \beta)}.$$

12. Time cannot be negative in physics.  
 13. If in a projectile motion, direct formulae is/are inapplicable, convert the problem into two one dimensional motions.

14. Average acceleration  $v_{av} = \frac{v_f - v_i}{t}$   

$$= \frac{(v_{fx}\hat{i} + v_{fy}\hat{j}) - (v_{ix}\hat{i} + v_{iy}\hat{j})}{t}$$

and direction  $\beta = \tan^{-1} \left( \frac{v_{fx} - v_{ix}}{v_{fy} - v_{iy}} \right).$

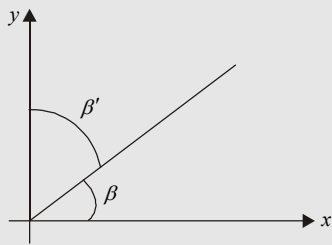
15. As far as possible apply vector laws to solve two dimensional problems if physical quantities involved are vectors.  
 16. Problems on relative velocity can even be solved using vector laws. Use  $v_{AB} = v_A - v_B$

or  $v_{AB} = (v_{Ax} - v_{Bx})\hat{i} + (v_{Ay} - v_{By})\hat{j}$ ;  $\tan \beta = \frac{v_{Ay} - v_{By}}{v_{Ax} - v_{Bx}}$

with respect to  $x$  direction

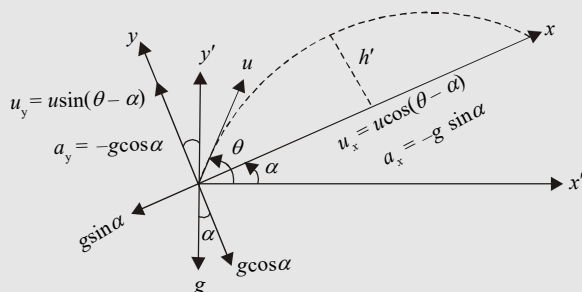
$$|v_{AB}| = \sqrt{(v_{Ax} - v_{Bx})^2 + (v_{Ay} - v_{By})^2};$$

$\tan \beta' = \frac{v_{Ax} - v_{Bx}}{v_{Ay} - v_{By}}$  with respect to  $y$ -direction.



**Fig. 3.10** Finding direction in two dimension motion

17. Whenever solving problems for inclined plane, consider axis along the plane as  $x$ -axis and perpendicular to it as  $y$ -axis. See Fig. 3.11.



**Fig. 3.11** Projectile motion along incline  
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**Note:**  $\left. \begin{aligned} u_x &= u \cos(\theta - \alpha) \\ a_x &= -g \sin \alpha \end{aligned} \right\} \text{ along the plane.}$

$\left. \begin{aligned} u_y &= u \sin(\theta - \alpha) \\ a_y &= -g \cos \alpha \end{aligned} \right\} \text{ perpendicular to the plane}$

i.e. use accelerated motion along both  $x$  and  $y$  axis.

$$\text{Time of flight} = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha} = \frac{2|u_y|}{|a_y|}$$

**Note**  $T = \frac{2|u_y|}{|a_y|}$  is true everywhere.

$$h' = \frac{u^2 \sin^2(\theta - \alpha)}{2g \cos \alpha} = \frac{u_y^2}{2|a_y|} \text{ is also true in all cases.}$$

18. To find radius of curvature of a projectile at any point  $R = \frac{v^2}{a_r}.$

The velocity  $v$  and radial or normal acceleration at that point is used in the above relation.

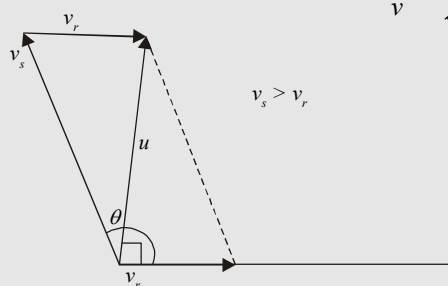
If  $v$  and  $a_r$  cannot be determined then use

$$R = \frac{\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2}}{\frac{d^2y}{dx^2}}$$

19. Equation of trajectory means the relation between  $x$  and  $y$ . Try to establish relation between  $x$  and  $y$  by eliminating  $t$ .  
 20. To cross the river along the shortest path the swimmer shall strike at an obtuse angle to the flow of river so that resultant velocity  $v$  is along the normal as illustrated in Fig. 3.12. provided  $v_{\text{swimmer}} > v_{\text{river}}$

From triangle law  $v = \sqrt{v_s^2 - v_r^2}$  where  $v_s$  = velocity of swimmer and  $v_r$  = velocity of river.

If the width of the river is  $l$  then  $t = \frac{l}{v} = \frac{R}{\sqrt{v_s^2 - v_r^2}}.$



**Fig. 3.12**

21. To cross the river in the shortest time (when  $v_{\text{swimmer}} > v_{\text{river}}$ ). Then the swimmer shall strike at right angle

to the flow of the river and  $t_{\min} = \frac{l}{v_{\text{swimmer}}}$ .

22. A particle is projected from the top of an incline as shown in Fig. 3.13  $a_x = g \sin \alpha$  and  $a_y = -g \cos \alpha$ .

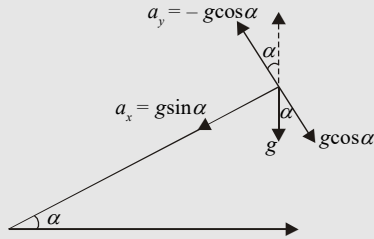


Fig. 3.13

23. Net acceleration in circular motion (Fig. 3.14)

$a_{\text{net}} = \sqrt{a_t^2 + a_r^2}$  where  $a_t$  is tangential acceleration and  $a_r$  is radial acceleration.

$$\tan \beta = \frac{a_r}{a_t}$$

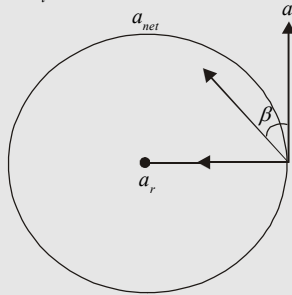


Fig. 3.14

24. If resultant of two motions is to be determined use

$$\begin{aligned} \vec{v}_R &= \vec{v}_1 + \vec{v}_2 = (v_{1x}\hat{i} + v_{1y}\hat{j}) + (v_{2x}\hat{i} + v_{2y}\hat{j}) \\ &= (v_{1x} + v_{2x})\hat{i} + (v_{1y} + v_{2y})\hat{j}. \end{aligned}$$

$$|\vec{v}_R| = \sqrt{(v_{1x} + v_{2x})^2 + (v_{1y} + v_{2y})^2} \text{ and}$$

$$\beta = \tan^{-1} \left( \frac{v_{1y} + v_{2y}}{v_{1x} + v_{2x}} \right) \text{ with respect to } x\text{-axis.}$$

25. If a particle is projected from the top of a tower or from a height  $h$  then consider the point of projection as origin. So that displacement is  $-h$ . Using  $-h =$

$$u \sin \theta t - \frac{gt^2}{2}. \text{ Find } t \text{ and range } = x = (u \cos \theta) t.$$

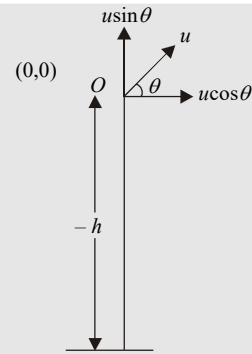


Fig. 3.15

26. If a person can throw a ball to maximum height  $h$  (vertically up) then the maximum horizontal distance up to which he can throw the ball is  $2h$ .
27. Once a particle is thrown in a gravitational field, it will return only after time of flight.
28. If a particle is thrown up, it will have same speed at the same height during ascent and during descent.
29. If  $v_{\text{swimmer}} < v_{\text{river}}$  then one has to reach the directly opposite point on crossing the river. The drifted part on foot or by other means the minimise drift or minimum total time as per given problem. To minimize put first derivative zero.
30. If the frame of reference is noninertial, make it inertial by applying pseudo vectors before applying Newton's laws or equation of motion.
31. Projectile attains maximum range when  $\theta$  (angle of projection) is  $45^\circ$ , on the same level. If projected from a height and the projectile reaches ground then

angle is less than  $45^\circ$  and is determined using  $\frac{dx}{d\theta} = 0$ , where

$$x = u_x (\text{time of flight})$$

$$\text{or } x = u_x (\text{time spent in gravitational field}).$$

32. Range will be same if a body is projected at  $\theta$  or  $(90 - \theta)$  (i.e. complimentary angle) with same velocity.

### CAUTION

- In uniform motion  $\vec{v} = \vec{v}_{av}$   
 $\Rightarrow$  Converse is, however, not true. That is if  $\vec{v}_{av} = \vec{v}$ , motion may or may not be uniform.
- Applying  $v = u + at$ ;  $s = ut + \frac{1}{2} at^2$  etc. even when acceleration is not uniform.  
 $\Rightarrow$  When acceleration is not uniform and motion is not circular/rotational, use  $\frac{dv}{dt} = a$  or  $v \frac{dv}{dx} = a$   
 or  $\frac{dx}{dt} = v$ .

3. Applying  $v = u + at$  etc. without modification when frame of reference is non-inertial.
- ⇒ For example, if the lift is moving up with an acceleration  $a$  then the effective acceleration for a body falling from the ceiling is  $(g + a)$ , i.e., apply vector algebra or relative acceleration.

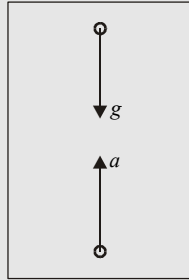


Fig. 3.16

4. Not differentiating between average and instantaneous velocities.
- ⇒ If a particle travels according to the equation  $x = t^2 + 2t + 5$  where  $x$  is in metres and  $t$  in seconds.
- Then  $v = \frac{dx}{dt}$  is instantaneous velocity. While  $v_{av} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$  where  $t_2$  and  $t_1$  are final and initial times.
5. Applying direct equations of projectile when starting or terminating points are not the same vertical height or vertical displacement between initial and terminating point is non-zero.
- ⇒ Apply one-dimensional motion approach—one along  $x$ - and the other along  $y$ -axis with time of flight as combining factor.

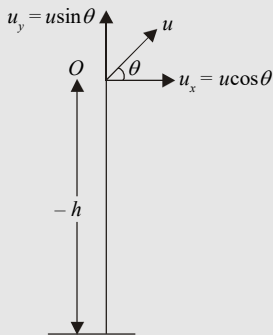


Fig. 3.17

6. Sticking to the origin at ground.
- ⇒ When the particle starts from a height  $h$ , consider it as origin so that the vertical displacement when it reaches the ground is  $h$ , i.e., use

$$-h = u \sin \theta t - \frac{gt^2}{2}$$

7. Not remembering common trigonometric formulae.

⇒ Remember trigonometric relations like

$$\sin 2\theta = 2 \sin \theta \cos \theta,$$

$$\sin (180 - \theta) = \sin \theta, \sin (A + B) + \sin (A - B) = 2 \sin A \cos B \text{ and } \sin A \sin B = \cos (A - B) - \cos (A + B)$$

$$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)].$$

8. Considering vertical distance given in problems in projectile motion as  $h_{\max}$ .

⇒ It is not necessary that vertical distance given be  $h_{\max}$ . If it is  $h_{\max}$ , then velocity at this point is only horizontal velocity, i.e., vertical component of velocity is zero at the highest point. Otherwise use

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

or  $y = x \tan \theta \left( 1 - \frac{x}{R} \right).$

Even in some cases these equations are not suitable, or make the problem lengthy then use

$$y = u_y t - \frac{1}{2} g t^2 \quad \text{and } x = u_x t$$

9. Considering  $x = u_x t$  along an inclined plane.

⇒ Along an inclined plane  $a_x$  is also present. Find out

$$a_x \text{ and then apply } x = u_x t + \frac{1}{2} a_x t^2.$$

10. Considering if the projectile strikes a wall or an obstacle its time of flight will change.

⇒ Time of flight remains fixed unless it is trapped somewhere.

### SOLVED PROBLEMS

1. A ball is thrown up with a certain velocity so that it reaches a height  $h$ . Find the ratio of the times in which it is at  $\frac{h}{3}$ .

(a)  $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

(b)  $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(c)  $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

(d)  $\frac{1}{3}$

**Solution** (b)  $u^2 = 2gh$ ;  $\frac{h}{3} = \sqrt{2gh} t - \frac{1}{2} g t^2$  or  $g t^2 - 2$

$$\sqrt{2gh}t + \frac{2h}{3} = 0.$$

$$t = \frac{2\sqrt{2gh} \pm \sqrt{8gh - (8gh)/3}}{2g} \quad \text{or}$$

$$\begin{aligned} \frac{t_1}{t_2} &= \frac{2\sqrt{2gh} - 2\sqrt{2gh/3}(\sqrt{3}-1)}{2\sqrt{2gh} + 2\sqrt{2gh/3}(\sqrt{3}-1)} \\ &= \frac{\sqrt{3} - (\sqrt{3}-1)}{\sqrt{3} + \sqrt{3}-1} \\ &= \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} \end{aligned}$$

2. The displacement of a particle varies with time as  $x = a e^{-\alpha t} + b e^{\beta t}$  where  $a, \alpha, b, \beta$  are positive constants. The velocity of the particle
- will be independent of  $\alpha$  and  $\beta$
  - drop to zero when  $\alpha = \beta$
  - go on decreasing with time
  - go on increasing with time.

**Solution** (d)  $\frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$  as  $t$  increases  $\frac{-a\alpha}{e^{\alpha t}}$  decreases and  $b\beta e^{\beta t}$  increases.

3. Convert given  $v-x$  shown in Fig 3.18 to  $a-x$  graph.

(IIT Screening 2005)

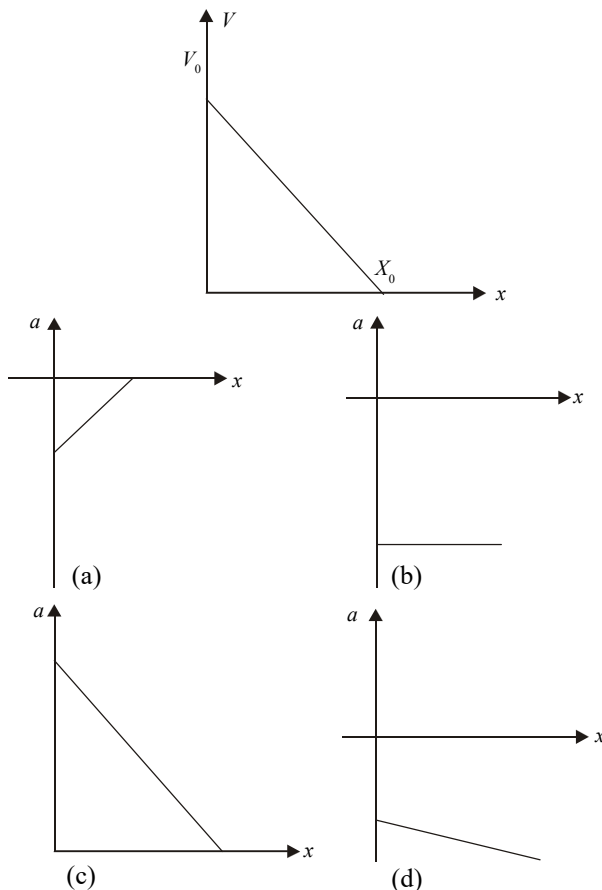


Fig. 3.18

**Solution** (a) equation of given curve is  $v = \left(1 - \frac{x}{x_0}\right) v_0$

$$a = \frac{dv}{dt} = -\frac{v_0}{x_0} \frac{dx}{dt} = \frac{-v_0^2}{x_0} \left(1 - \frac{x}{x_0}\right).$$

4. The relation between time  $t$  and distance  $x$  is  $t = ax^2 + bx$  where  $a$  and  $b$  are constant. The acceleration is
- $-2a bv^2$
  - $2 bv^3$
  - $-2 av^3$
  - $2 av^2$

(AIEEE 2005)

**Solution** (c)  $t = ax^2 + bx$  or  $\frac{dt}{dx} = 2ax + b$

$$\text{or } v = \frac{dx}{dt} = \frac{1}{2ax + b}.$$

$$\frac{dv}{dt} = \frac{-2a}{(2ax + b)^2} \frac{dx}{dt} = \frac{-2a}{(2ax + b)^3} = -2a v^3.$$

5. A car starting from rest accelerates at the rate  $f$  through a distance  $s$ , then continues at constant speed for time  $t$  and then decelerates at rate  $\frac{f}{2}$  to come to rest. If the total distance covered is 15 s, then

$$\begin{aligned} \text{(a) } s &= \frac{ft^2}{72} & \text{(b) } s &= \frac{ft^2}{4} \\ \text{(c) } s &= \frac{ft^2}{6} & \text{(d) } s &= \frac{ft^2}{2} \end{aligned}$$

(AIEEE 2005)

**Solution** (a)  $s = v_0 t_1$  and  $v_0 2t_1 = 2s$

Distance moved with uniform speed  $(15 - 3)s = 12s$

$$v_0 = \sqrt{2sf}; \quad 12s = v_0 t$$

$$12s = t \sqrt{2sf} \quad \text{or} \quad s = \frac{ft^2}{72}$$

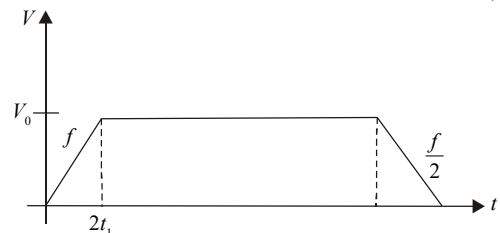


Fig. 3.19

6. A projectile can have the same range  $R$  for two angles of projection. If  $t_1$  and  $t_2$  are the times of flights in the



two cases, then product of the time of flights is proportional to

- (a)  $R^2$  (b)  $\frac{1}{R^2}$   
(c)  $\frac{1}{R}$  (d)  $R$

[AIEEE 2005]

**Solution** (d)  $t_1 = \frac{2u \sin \theta}{g}$ ,  $t_2 = \frac{2u \cos \theta}{g}$  and

$$t_1 t_2 = \frac{2u^2 \sin 2\theta}{g^2} = \frac{2R}{g}.$$

7. A particle is moving eastwards with a velocity  $5 \text{ ms}^{-1}$ . In 10 s, the velocity changes to  $5 \text{ ms}^{-1}$  northwards. The average acceleration in this time is

- (a)  $\frac{1}{\sqrt{2}} \text{ ms}^{-2} \text{ NE}$  (b)  $\frac{1}{2} \text{ ms}^{-2} \text{ N}$   
(c) zero (d)  $\frac{1}{\sqrt{2}} \text{ ms}^{-2} \text{ NW}$

[AIEEE 2005]

**Solution** (d)  $a_{av} = \frac{v_f - v_i}{t} = \frac{5\hat{i} - 5\hat{i}}{10}$

$$= a = \frac{1}{\sqrt{2}} \text{ ms}^{-2} \text{ NW}.$$

8. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at  $2 \text{ ms}^{-2}$ . He reaches the ground with a speed  $3 \text{ ms}^{-1}$ . At what height did he bail out?

- (a) 91 m (b) 182 m  
(c) 293 m (d) 111 m

**Solution** (c)  $v^2 = 2gh = 2 \times 10 \times 50$

$$= 50 + \left[ \frac{3^2 - 2 \times 10 \times 50}{-2(2)} \right] = 293 \text{ m}.$$

9. In Fig. 3.20 the position time graph of a particle of mass  $0.1 \text{ kg}$  is shown. Find the impulse at  $t = 2 \text{ sec}$ .

- (a)  $0.2 \text{ kg ms}^{-1}$  (b)  $-0.2 \text{ kg ms}^{-1}$   
(c)  $0.1 \text{ kg ms}^{-1}$  (d)  $-0.4 \text{ kg ms}^{-1}$

**Solution** (a)  $dp = F \cdot dt = m(v_f - 0) = 0.1(2) = 0.2 \text{ kg ms}^{-1}$

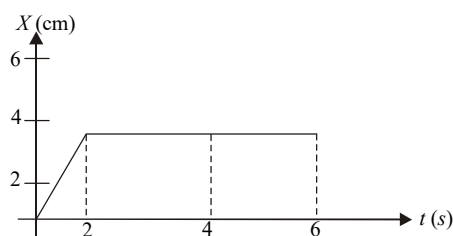


Fig. 3.20

10. When a ball is thrown up vertically with a velocity  $v_o$  it reaches a height  $h$ . If one wishes to triple the maximum height then the ball be thrown with a velocity

- (a)  $\sqrt{3} v_o$  (b)  $3 v_o$   
(c)  $9 v_o$  (d)  $\frac{3}{2} v_o$

[AIIMS 2005]

**Solution** (a)  $v^2 = 2gh$  or  $v = \sqrt{2gh}$ , i.e.,  $\frac{v_1}{v_2} = \sqrt{\frac{h_1}{h_2}}$

$$\therefore v_2 = \sqrt{3} v_o.$$

11. A car starts from rest, moves with an acceleration  $a$  and then decelerates at  $b$  for sometime to come to rest. If the total time taken is  $t$ , the maximum velocity is

- (a)  $\frac{abt}{a+b}$  (b)  $\frac{a^2 t}{a+b}$   
(c)  $\frac{at}{a+b}$  (d)  $\frac{b^2 t}{a+b}$

[BHU 2005]

**Solution** (a)  $v = 0 + at_1$ ;  $0 = at_1 - b(t - t_1)$

or  $t_1 = \frac{bt}{a+b} \therefore v_{\max} = \frac{abt}{a+b}.$

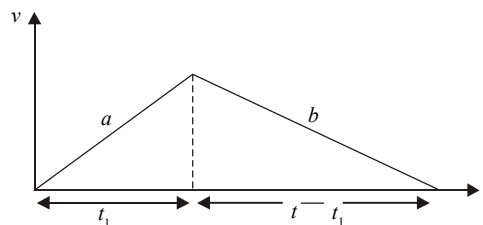


Fig. 3.21

12. From the top of a tower, two stones whose masses are in the ratio  $1 : 2$  are thrown, one straight up with an initial speed  $u$  and the second straight down with same speed  $u$ . Neglecting air resistance,

- (a) the heavier stone hits the ground with a higher speed.  
(b) the lighter stone hits the ground with a higher speed.

- (c) both the stones will have same speed when they hit the ground.  
 (d) the speed cannot be determined with the given data.

**Solution** (c)

13. Two runners start simultaneously from the same point on a circular 200 m track in the same direction. Their speeds are  $6.2 \text{ ms}^{-1}$  and  $5.5 \text{ ms}^{-1}$ . How far from the starting point the faster will overcome the slower?

- (a) 150 m away from the starting point  
 (b) 170 m away from the starting point  
 (c) 120 m away from the starting point  
 (d) none

**Solution** (b)  $200 = (6.2 - 5.5)t$  or  $t = 285.714 \text{ s}$

$$s = (6.2 \times 285.714) = 1770 \text{ m (faster)}, 1770 - 8 \times 200 = 170$$

Thus 170 m away from the starting point along the track in the direction of run.

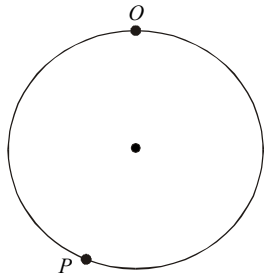


Fig. 3.22

14. A particle moves according to the equation  $x = 2t^2 - 5t + 6$ . Find (i) average velocity in the first 3 sec and (ii) velocity at  $t = 3 \text{ s}$ .

- (a)  $1 \text{ ms}^{-1}$ ,  $7 \text{ ms}^{-1}$  (b)  $4 \text{ ms}^{-1}$ ,  $3 \text{ ms}^{-1}$   
 (c)  $2 \text{ ms}^{-1}$ ,  $5 \text{ ms}^{-1}$  (d)  $3 \text{ ms}^{-1}$ ,  $7 \text{ ms}^{-1}$

**Solution** (a)  $x(3) = 2(3)^2 - 5(3) + 6 = 9$ ,  $x(0) = 6$

$$v_{av} = \frac{x(3) - x(0)}{3 - 0} = \frac{9 - 6}{3} = 1 \text{ ms}^{-1}$$

$$\left. \frac{dx}{dt} \right|_{t=3} = 4t - 5 = 4(3) - 5 = 7 \text{ ms}^{-1}$$

15. Plot acceleration time graph of the figure shown

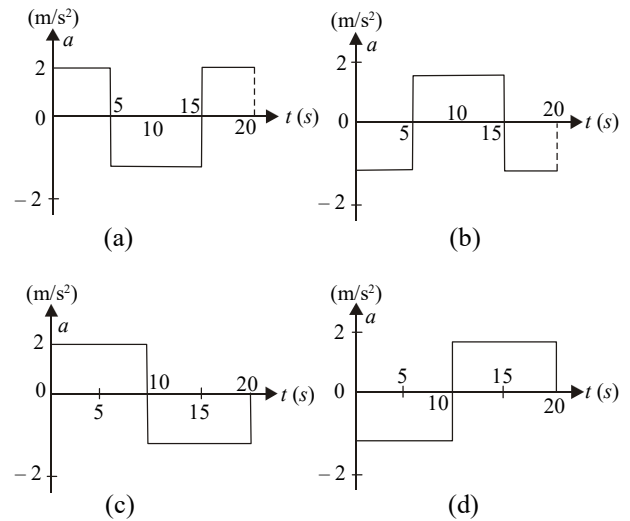
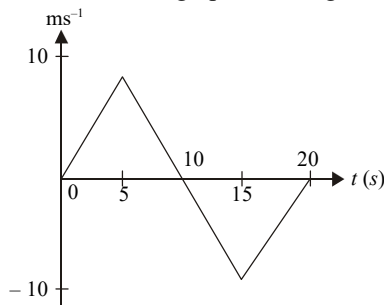


Fig. 3.23

**Solution** (a)

16. A girl after being angry throws her engagement ring from the top of a building 12 m high towards her boy friend with an initial horizontal speed of  $5 \text{ ms}^{-1}$ , speed with which the ring touches the ground is

- (a)  $5 \text{ ms}^{-1}$  (b)  $14.3 \text{ ms}^{-1}$   
 (c)  $1.5 \text{ ms}^{-1}$  (d)  $16.2 \text{ ms}^{-1}$

**Solution** (d)  $v_y^2 = 2ay = 2 \times 10 \times 12$   
 $v = \sqrt{25 + 240} = 16.2 \text{ ms}^{-1}$

17. The driver of a train A running at  $25 \text{ ms}^{-1}$  sights a train B on the same track with  $15 \text{ ms}^{-1}$ . The driver of train A applies brakes to produce a deceleration of  $1.0 \text{ ms}^{-2}$ . If the trains are 200 m apart, will the trains collide?

- (a) yes (b) no  
 (c) collision just avoided (d) none of these

**Solution** (c)  $v^2 - u^2 = 2as$  or  $s = \frac{25^2 - 15^2}{2 \times 1} = 200 \text{ m}$ .

18. Two cars A and B are 5 m long each. Car A is at any instant just behind B. A and B are moving at 54 km/h and 36 km/h. Find the road distance covered by the car A to overtake B.

- (a) 35 m (b) 30 m  
 (c) 32.5 m (d) 27.5 m

**Solution** (a)  $v_{AB} = 15 - 10 = 5 \text{ ms}^{-1}$

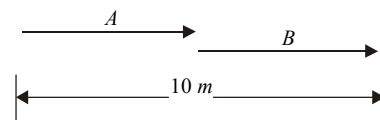


Fig. 3.24

$$x_{AB} = 10 \text{ m}; t = \frac{x_{AB}}{v_{AB}} = 2 \text{ s}$$

Road distance covered =  $v_A t$  + length of car  $A$   
 $= 15 \times 2 + 5 = 35$  m.

19. A flowerpot falls off a window sill and falls past the window below. It takes 0.5 s to pass through a 2.0 m high window. Find how high is the window sill from the top of the window?

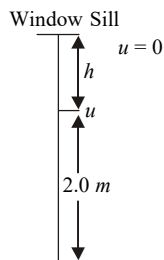


Fig. 3.25

- (a) 10 cm (b) 7.5 cm  
 (c) 12.5 cm (d) 15 cm

**Solution** (c)  $h = ut + \frac{1}{2} at^2$

or  $2.0 = u(.5) + 5 \left( \frac{1}{4} \right)$

or  $u = 1.5 \text{ ms}^{-1}$ .

Using  $v^2 - u^2 = 2gh$

$$h = \frac{1.5^2}{2 \times 10} = \frac{2.25}{20} = 0.125 \text{ m} = 12.5 \text{ cm}.$$

20. A particle moves according to the law  $a = -ky$ . Find the velocity as a function of distance  $y$ ,  $v_0$  is initial velocity.

- (a)  $v^2 = v_0^2 - ky^2$  (b)  $v^2 = v_0^2 - 2ky$   
 (c)  $v^2 = v_0^2 - 2ky^2$  (d) none

**Solution** (a)  $a = \frac{dv}{dt} = \frac{dv}{dy} \cdot \frac{dy}{dt}$

or  $\int_{v_0}^v v dv = \int_0^y -ky dy$  or  $v_0^2 - v^2 = ky^2$ .

21. A particle moves according to the equation  $\frac{dv}{dt} = \alpha - \beta v$  where  $\alpha$  and  $\beta$  are constants. Find velocity as a function of time. Assume body starts from rest.

- (a)  $v = \frac{\beta}{\alpha} (1 - e^{-\beta t})$  (b)  $v = \frac{\alpha}{\beta} (1 - e^{-\beta t})$   
 (c)  $v = \frac{\beta}{\alpha} e^{-\beta t}$  (d)  $v = \frac{\alpha}{\beta} e^{-\beta t}$

**Solution** (b)  $\int_0^v \frac{-\beta dv}{\alpha - \beta v} = -\beta \int_0^t dt$

or

$$\log_e \frac{(\alpha - \beta v)}{\alpha} = -\beta t \text{ or } v = \frac{\alpha}{\beta} (1 - e^{-\beta t})$$

22. A boat moves relative to water with a velocity  $v$  and river is flowing with  $2v$ . At what angle the boat shall move with the stream to have minimum drift?

- (a)  $30^\circ$  (b)  $60^\circ$   
 (c)  $90^\circ$  (d)  $120^\circ$

**Solution** (d) Let boat move at angle  $\theta$  to the normal as

shown in Fig. 3.26 then time to cross the river =  $\frac{l}{v \cos \theta}$ .

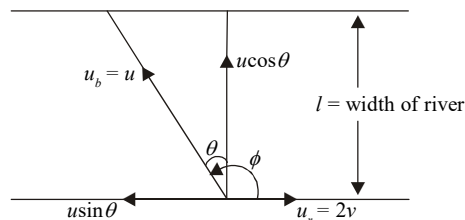


Fig. 3.26

drift  $x = (2v - v \sin \theta) \frac{l}{v \cos \theta}$  for  $x$  to be minimum.

$$\frac{dx}{d\theta} = 0 = l (2 \sec \theta \tan \theta - \sec^2 \theta) \text{ or } \sin \theta = \frac{1}{2}$$

or  $\theta = 30^\circ$  and  $\phi = 90 + 30 = 120^\circ$ .

23. A car starts moving rectilinearly from rest with  $5 \text{ ms}^{-2}$  for sometime, then uniformly and finally decelerates at  $5 \text{ ms}^{-2}$  and comes to a stop. The total time of motion equals 25 s. How long does the car move uniformly? Given  $V_{av} = 72 \text{ km/h}$  during motion.

- (a) 5 s (b) 10 s  
 (c) 15 s (d) 20 s

**Solution** (c) Total distance covered = area under  $v - t$  graph. From Fig

$$20 \times 25 = 5 t_1^2 + (25 - 2t_1) 5t_1$$

or  $5t_1^2 - 125t_1 + 500 = 0$

or  $(t_1 - 5)(t_1 - 20) = 0$

$\Rightarrow t_1 = 5 \text{ s}$  discard  $t_1 = 20 \text{ s}$ .

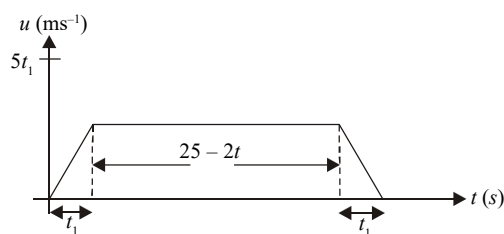


Fig. 3.27

24. A ship moves along the equator to the east with a speed 30 km/h. Southeastern wind blows  $60^\circ$  to the east with  $15 \text{ kmh}^{-1}$ . Find the wind velocity relative to the ship.

- (a)  $39.7 \text{ kmh}^{-1}, \tan^{-1} \frac{1}{5} \text{ N of W}$   
 (b)  $23.7 \text{ kmh}^{-1}, \tan^{-1} \frac{1}{3} \text{ N of W}$   
 (c)  $37.5 \text{ kmh}^{-1}, \tan^{-1} \frac{1}{5} \text{ N of E}$   
 (d) none

**Solution**

$$(a) v_{ws} = v_w - v_s$$

$$= (15 \cos 60^\circ \hat{i} + 15 \sin 60^\circ \hat{j}) - 30 \hat{i}$$

$$|v| = \sqrt{(39.5)^2 + (7.5)^2} = 39.7 \text{ kmh}^{-1}$$

$$\tan \beta = \frac{7.5}{37.5} = \frac{1}{5}$$

$$\beta = \tan^{-1} \frac{1}{5} \text{ North of West.}$$

25. A ball is thrown up with a velocity  $v_o$  and it returns to the spot of throw. Plot  $v-t$  and  $v-x$  graphs.

**Solution**

$$v = u + at$$

and  $a = -g$  is a straight line [Fig 3.]

$v^2 - u^2 = 2ax$  is parabolic [Fig 3.]

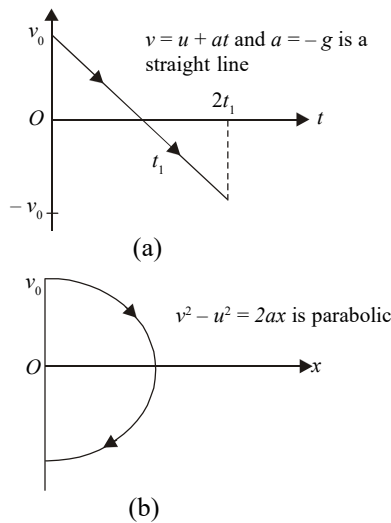


Fig. 3.28

26. From a tap 10 m high drops fall at regular intervals. When the first drop reaches the ground, the 5th drop is about to leave the tap. Find the separation between 2nd and 3rd drops.

- (a)  $\frac{35}{8} \text{ m}$  (b)  $\frac{31}{8} \text{ m}$   
 (c)  $\frac{27}{8} \text{ m}$  (d) none of these

**Solution**

$$\frac{1}{2} g t^2 = 10 \text{ or } t = \sqrt{2} \text{ s}$$

$$\text{time interval } \Delta t = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}} \text{ s.}$$

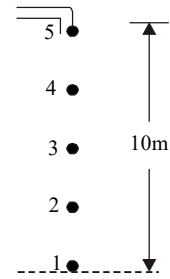


Fig. 3.29

$$x_2 - x_3 = \frac{1}{2} g \left[ \left( \frac{3}{2\sqrt{2}} \right)^2 - \left( \frac{2}{2\sqrt{2}} \right)^2 \right]$$

$$= 5 \left[ \frac{9}{8} - \frac{1}{2} \right] = \frac{25}{8} \text{ m}$$

27. When a ball is  $h$  metre high from a point O, its velocity is  $v_o$ . When it is  $h$  m below O, its velocity is  $2v$ . Find the maximum height from O it will acquire

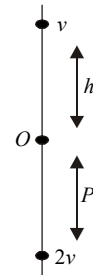


Fig. 3.30

- (a)  $\frac{2h}{3}$  (b)  $\frac{5h}{3}$   
 (c)  $\frac{3h}{2}$  (d)  $2h$

**Solution**

$$(b) (2v)^2 - v^2 = 2g(2h)$$

$$\text{or } \frac{v^2}{2g} = \frac{2}{3} h;$$

$$h_{\max} = h + \frac{2h}{3} = \frac{5h}{3}$$

28. The first stage of the rocket launches a satellite to a height of 50 km and velocity attained is  $6000 \text{ kmh}^{-1}$  at which point its fuel exhausted. How high the rocket will reach?

- (a) 138.9 km (b) 188.9 km  
 (c) 88.9 km (d) 168.9 km

**Solution**

$$(b) h = \frac{v^2}{2g} + 50 \text{ km}$$

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$$= \frac{(5000/3)^2}{20 \times 1000} + 50 = 188.9 \text{ km.}$$

29. A particle moves according to the equation  $t = \sqrt{x} + 3$ , when the particle comes to rest for the first time

- (a) 3 s (b) 2.5 s  
(c) 3.5 s (d) none of these

**Solution** (a)  $x = (t-3)^2$

$$v = \frac{dx}{dt} = -2(t-3) = 0 \text{ or } t = 3 \text{ s.}$$

30. A particle of mass  $m$  is projected with a velocity  $6\hat{i} + 8\hat{j}$ . Find the change in momentum when it just touches ground.

- (a) 0 (b) 12 m  
(c) 16 m (d) 20 m

**Solution** (c)  $\Delta p = m(v_f - v_i) = m[(6\hat{i} - 8\hat{j}) - (6\hat{i} + 8\hat{j})] = -16m\hat{j}$   
 $|\Delta p| = 16m$

31. A particle is projected with a velocity  $6\hat{i} + 8\hat{j}$ , 3 m away from a vertical wall. After striking the wall it lands at ..... away from the wall.

- (a) 3 m (b) 3.3 m  
(c) 5.5 m (d) 6.6 m

**Solution** (d)  $T = \frac{2u_y}{a_y} = \frac{2 \times 8}{10} = 1.6 \text{ s}$

$$t = \frac{3}{6} = 0.5 \text{ s.}$$

$$x = u_x(T-t) = 6(1.6 - 0.5) = 6.6 \text{ m}$$

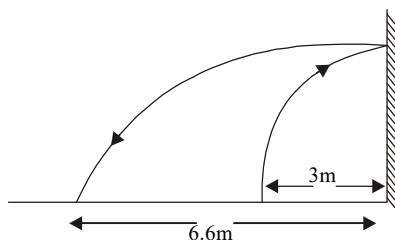


Fig. 3.31

32. The radius vector of a point  $A$  relative to the origin varies as  $r = at\hat{i} + bt^2\hat{j}$  where  $a$  and  $b$  are positive constants. Find the equation of trajectory.

- (a)  $y = \frac{b}{a^2} x^2$  (b)  $y^2 = \frac{b}{a^2} x$

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- (c)  $y = \frac{a^2}{b} x^2$  (d) none of these

**Solution** (a)  $x = at, y = bt^2$  or  $y = b \left(\frac{x}{a}\right)^2$

33. A particle moves in the  $xy$  plane as  $v = a\hat{i} + bx\hat{j}$  where  $\hat{i}$  and  $\hat{j}$  are the unit vectors along  $x$  and  $y$  axis. The particle starts from origin at  $t = 0$ . Find the radius of curvature of the particle as a function of  $x$ .

- (a)  $\frac{a^2 + b^2 x^2}{ba}$  (b)  $\frac{a}{b} \left[ 1 + \left( \frac{bx}{a} \right)^2 \right]^{\frac{3}{2}}$

- (c)  $\frac{b}{a} \left[ 1 + \left( \frac{ax}{b} \right)^2 \right]^{\frac{3}{2}}$  (d) none of these

**Solution** (b)  $\frac{dv}{dt} = a$  or  $x = at$

$$\frac{dy}{dt} = bat \text{ or } y = \frac{bat^2}{2} \text{ or } y = \frac{bx^2}{2a}$$

$$\frac{dy}{dx} = \frac{b}{a} x \text{ and } \frac{d^2y}{dx^2} = \frac{b}{a}$$

$$R = \frac{\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[ 1 + \left( \frac{b}{a} x \right)^2 \right]^{\frac{3}{2}}}{\frac{b}{a}}$$

$$= \frac{a}{b} \left[ 1 + \left( \frac{b}{a} x \right)^2 \right]^{\frac{3}{2}}$$

34. A man riding on a flat car moving with  $10 \text{ ms}^{-1}$ . He attempts to throw a ball through a stationary hoop 5 m above his hand such that the ball moves horizontally through the hoop. He throws the ball with  $12 \text{ ms}^{-1}$  with respect to himself. Find the horizontal distance from where he throws the ball.

- (a) 15 m (b) 14.2 m  
(c) 16.7 m (d) 18.2 m

**Solution** (c)  $h_{\max} = 5 = \frac{u_y^2}{2g} \therefore u_y = 10$

$$u_x = \sqrt{12^2 - 10^2} = \sqrt{44}.$$

$$v_x = 10 + \sqrt{44} ; \frac{T}{2} = \frac{u_y}{g} = 1 \text{ s} ;$$

$$x = v_x \cdot \frac{T}{2} = 10 + \sqrt{44} .$$

$$= 16.7 \text{ m}$$

35. A body standing on a long railroad car throws a ball straight upwards, the car is moving on the horizontal road with an acceleration  $1 \text{ ms}^{-2}$ . The vertical velocity given is  $9.8 \text{ ms}^{-1}$ . How far behind the boy the ball will fall on the railroad car?

- (a) 1 m (b)  $\frac{3}{2}$  m  
(c)  $\frac{7}{4}$  m (d) 2 m

**Solution** (d)  $T = \frac{2u_y}{g} = 2 \times \frac{9.8}{9.8} = 2 \text{ s} ;$

$$x = \frac{1}{2} a_x t^2 = \frac{1}{2} (1) (2)^2 = 2 \text{ m}.$$

36. Find the average velocity of a projectile between the instant it crosses one third the maximum height. It is projected with  $u$  making an angle  $\theta$  with the vertical.

- (a)  $u \cos \theta$  (b)  $u$   
(c)  $u \sin \theta$  (d)  $u \tan \theta$

**Solution** (c) Note carefully the vertical velocities at the same height are in opposite directions and therefore their average sum = 0. It is horizontal velocity which is uniform and hence  $v_{av} = u \sin \theta (= u_x)$ .

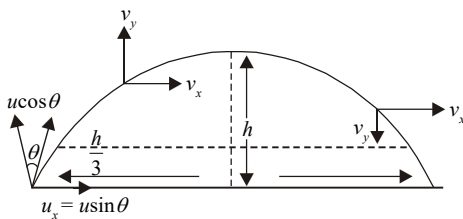


Fig. 3.32

37. A person is standing on a truck moving with  $14.7 \text{ ms}^{-1}$  on a horizontal road. He throws a ball so that it returns to him when the truck has moved 58.8 m. Find the speed of the ball and angle of projection as seen by a man standing on the road.

- (a)  $22.5 \text{ ms}^{-1}$ ,  $53^\circ$  (b)  $24.5 \text{ ms}^{-1}$ ,  $53^\circ$   
(c)  $19.6 \text{ ms}^{-1}$ , vertical (d) none of these

**Solution** (b)

$$T = \frac{58.8}{14.7} = 4 \text{ s}$$

$$T = \frac{2u_y}{g} = 4 \therefore u_y = 19.6 \text{ ms}^{-1}$$

$$v = \sqrt{14.7^2 + 19.6^2}$$

$$= 24.5 \text{ ms}^{-1}$$

$$\tan \beta = \frac{v_y}{v_x} = \frac{19.6}{14.7} = \frac{4}{3} \text{ or } \beta = 53^\circ \text{ wrt horizontal.}$$

38. Six persons are situated at the corners of a hexagon of side  $l$ . They move at a constant speed  $v$ . Each person maintains a direction towards the person at the next corner. When will the persons meet?

- (a)  $\frac{l}{v}$  (b)  $\frac{2l}{3v}$   
(c)  $\frac{3l}{2v}$  (d)  $\frac{2l}{v}$

**Solution** (d)  $t = \frac{l}{v_{AB}} = \frac{l}{v_A - v_B \text{ in the direction of A}}$

$$= \frac{l}{v - v \cos 60} = \frac{2l}{v} .$$

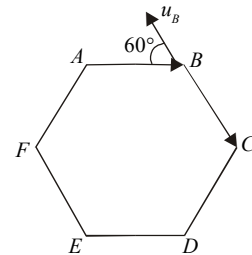


Fig. 3.33

39. The compass needle of the airplane shows it is heading due North and speedometer indicates a velocity  $240 \text{ km h}^{-1}$ . Wind is blowing  $100 \text{ km h}^{-1}$  due east. Find the velocity of airplane with respect to earth.

- (a)  $260 \text{ ms}^{-1}$ ,  $23^\circ$  E of N (b)  $260 \text{ ms}^{-1}$ ,  $23^\circ$  W of N  
(c)  $260 \text{ ms}^{-1}$ ,  $32^\circ$  E of N (d) none

**Solution** (a)  $v_{AE} = 100 \hat{i} + 240 \hat{j}$

$$v_{AE} = \sqrt{(240)^2 + 100^2} = 260 \text{ ms}^{-1} ;$$

$$\phi = \tan^{-1} \left( \frac{100}{240} \right) = 23^\circ \text{ E of N.}$$

40. In an exhibition, you win a prize if you toss a coin into a small dish placed. The dish is on a sheep 2.1 m away at a height  $h$  from the hand. The coin is tossed into the dish if its velocity is  $6.4 \text{ ms}^{-1}$  at an angle of  $60^\circ$ . Find  $h$ .

- (a) 1.2 m (b) 1.35 m  
(c) 1.5 m (d) 1.65 m

**Solution** (c)  $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$

$$= 2.1 \tan 60 - \frac{9.8(2.1)^2}{2 \times 6.4^2 \times \left(\frac{1}{2}\right)^2}$$

$$= 2.1 \sqrt{3} - \frac{4.9 \times 4.4}{10.24} = 1.5 \text{ m}$$

## TYPICAL PROBLEMS

41. A projectile is launched from a height  $h$  making an angle  $\theta$  with the horizontal with speed  $v_o$ . Find the horizontal distance covered by it before striking the ground.

**Solution**  $-h = v_o \sin \theta t - \frac{1}{2} g t^2$

or  $g t^2 - 2 v_o \sin \theta t - 2h = 0$ .

$$t = \frac{2v_o \sin \theta + \sqrt{4v_o^2 \sin^2 \theta + 8gh}}{2g}$$

$$x = \frac{v_o \cos \theta}{2} \left[ v_o \sin \theta + \sqrt{v_o^2 \sin^2 \theta + 2gh} \right]$$

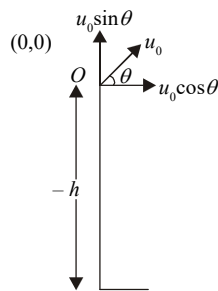


Fig. 3.34

42. A baseball is projected with a velocity  $v$  making an angle  $\theta$  with the incline of inclination  $\alpha$  as shown in Fig 3.35 (a). Find the condition that the ball hits the incline at right angle.

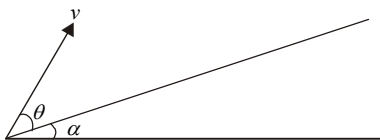


Fig. 3.35 (a)

- (a)  $\cot \theta = \tan \alpha$  (b)  $\sin \theta = \cos \alpha$   
(c)  $\tan \theta = \sin \alpha$  (d)  $\cot \theta = \cos \alpha$

**Solution** (a)  $T = \frac{2u_y}{|a_y|} = \frac{2v \sin \theta}{g \cos \alpha}$ . It will hit vertically the

incline if  $v_x = 0$ .

$$0 = v \cos \theta T - g \sin \alpha T^2$$

or  $v \cos \theta \left( \frac{2v \sin \theta}{g \cos \alpha} \right) - \frac{g \sin \alpha}{2} \left( \frac{2v \sin \theta}{g \cos \alpha} \right)^2 = 0$

$$\frac{2v^2 \sin \theta}{g \cos \alpha} [\cos \theta \cos \alpha - \sin \alpha \sin \theta] = 0$$

or  $\cot \theta = \tan \alpha$

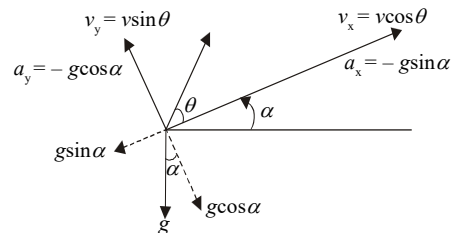


Fig. 3.35 (b)

43. An elevator is moving with  $2.5 \text{ ms}^{-1}$ . A bolt in the elevator ceiling 3 m above the elevator falls. How long does it take for the bolt to fall on the floor of elevator?
- (a) 0.731 s (b) 0.762 s  
(c) 0.782 s (d) 8.31 s

**Solution** (c)  $\frac{1}{2} g t^2 = 3$   $u_{\text{initial}} = v_{\text{rel}} = 0$  because bolt will also get a velocity  $2.5 \text{ ms}^{-1}$ .

$$t = \sqrt{6} = 0.782 \text{ s}$$

44. A point moves on the  $xy$  plane according to the law  $x = a \sin \omega t$  and  $y = a (1 - \cos \omega t)$  where  $a$  and  $\omega$  are positive constants. Find the distance covered in time  $t_o$ .

(a)  $a \omega t_o$  (b)  $\sqrt{2a^2 + 2a^2 \cos \omega t_o}$

(c)  $2a \frac{\sin \omega t_o}{2}$  (d)  $2a \frac{\cos \omega t_o}{2}$

**Solution**  $v_x = a \omega \cos \omega t$  and  
 $v_y = a \omega \sin \omega t$

$$\text{or } v = a\omega \cos \omega t \hat{i} + a\omega \sin \omega t \hat{j}$$

$$\text{or } |v| = a\omega$$

$$s = |v| t_o = a\omega t_o$$

45. A particle moves with a deceleration  $\propto \sqrt{v}$ . Initial velocity is  $v_o$ . Find the time after which it will stop.

$$(a) \frac{\sqrt{v_o}}{k}$$

$$(b) \frac{\sqrt{v_o}}{2k}$$

$$(c) \frac{2\sqrt{v_o}}{k}$$

(d) none of these

**Solution** (c)  $\frac{dv}{dt} = -k\sqrt{v}$

$$\text{or } \int_{v_o}^0 \frac{dv}{\sqrt{v}} = \int_0^t -k dt \quad \text{or } t = \frac{2\sqrt{v_o}}{k}$$

46. A particle moves according to the equation  $v = a\sqrt{x}$ . Find the average velocity in the first  $s$  metres of the path.

$$(a) \frac{\sqrt{s}}{a}$$

$$(b) \frac{\sqrt{s}}{2a}$$

$$(c) \frac{2a}{\sqrt{s}}$$

$$(d) \frac{2\sqrt{s}}{a}$$

**Solution** (d)  $\frac{dx}{dt} = a\sqrt{x}$

$$\text{or } \int_0^s \frac{dx}{\sqrt{x}} = \int_0^t a dt \quad \text{or } t = \frac{2\sqrt{s}}{a}$$

$$v_{av} = \frac{s}{t} = \frac{2\sqrt{s}}{a}$$

47. Particle  $A$  moves uniformly with velocity  $v$  so that vector  $v$  is continually aimed at point  $B$  which moves rectilinearly with a velocity  $u < v$ . At  $t = 0$ ,  $v$  and  $u$  are perpendicular. Find the time when they converge. Assume  $A$  and  $B$  are separated by  $l$  at  $t = 0$ .

**Solution**  $A$  approaches  $B$  with a velocity  $= v - u \cos \alpha$ .

$$\frac{dx}{dt} = v - u \cos \alpha$$

$$\int_0^l dx = \int_0^t (v - u \cos \alpha) dt$$

$$\text{or } \frac{l - vt}{u} = \int -\cos \alpha dt$$

$$ut = \int v \cos \alpha dt \quad \text{or } ut = \frac{-v(l - vt)}{u}$$

$$\text{or } (v^2 - u^2) = lv$$

$$\text{or } t = \frac{lv}{v^2 - u^2}$$

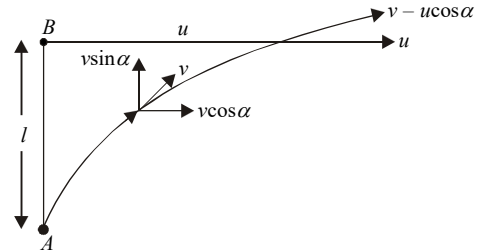


Fig. 3.36

48. From point  $A$  located on a highway, one has to get by a car as soon as possible to point  $B$  located in the field at a distance  $l$  from point  $D$ . If the car moves  $n$  times slower in the field, at what distance  $x$  from  $D$  one must turn off the highway.

**Solution** Let  $v$  be the velocity in the field and  $nv$  in the velocity on the highway.

$$\text{Then } t_1 = \frac{AD - x}{nv} \quad \text{and } t_2 = \frac{\sqrt{l^2 + x^2}}{v}$$

$$\text{For } t \text{ to be minimum } \frac{d}{dx} (t_1 + t_2) = 0$$

$$\frac{d}{dx} \left[ \frac{1}{v} \left\{ \left( \frac{AD - x}{n} \right) - \sqrt{l^2 + x^2} \right\} \right]$$

$$= \frac{1}{n} - \frac{x}{\sqrt{l^2 + x^2}} = 0$$

$$\text{or } l^2 + x^2 = n^2 x^2$$

$$\text{or } x = \frac{l}{\sqrt{n^2 - 1}}$$

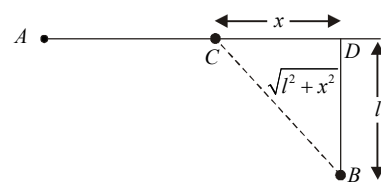


Fig. 3.37

49. A swimmer wishes to cross a 1 km wide river flowing at 5 km h<sup>-1</sup>. His speed in still waters is 3 km h<sup>-1</sup>. He has to reach directly opposite in minimum possible time. If he does not reach directly opposite by swimming, he has to walk that distance at 5 km h<sup>-1</sup>. Find the time taken.

**Solution** time  $t_1$  to cross river  $= \frac{1}{3 \cos \theta}$



$$x = v_x \cdot t_1$$

$$x = (5 - 3 \sin \theta) \times \frac{1}{3 \cos \theta}$$

$$\text{time } t_2 \text{ to reach } P \text{ by walking} = \frac{x}{5} = \frac{5 - 3 \sin \theta}{15 \cos \theta}.$$

$$\text{For time to be minimum } \frac{d(t_1 + t_2)}{d\theta} = 0$$

$$= \frac{d}{d\theta} \left( \frac{1}{3 \cos \theta} + \frac{5 - 3 \sin \theta}{15 \cos \theta} \right)$$

$$\text{or } \frac{2}{3} \sec \theta \tan \theta - \frac{1}{5} \sec^2 \theta = 0$$

$$\text{or } \sin \theta = \frac{3}{5}$$

$$t = \frac{1}{3 \cos \theta} + \frac{5 - 3 \sin \theta}{15 \cos \theta}$$

$$= \frac{1}{3 \left( \frac{4}{5} \right)} + \frac{5 - 3 \times \frac{3}{5}}{15 \times \frac{4}{5}} = \frac{5}{12} + \frac{16}{12 \times 5}$$

$$= \frac{41}{60} = 0.66 \text{ h}$$

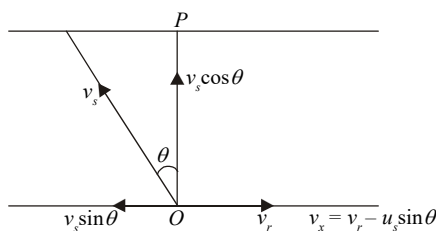


Fig. 3.38

50. A cannon and a target are 5.1 km apart located on the same level. Find the time to hit the target. Speed of cannon is  $240 \text{ ms}^{-1}$ . Assume no air drag.

- (a) 24.6 s, 42.6 s                      (b) 24.69 s, 48 s  
(c) 23 s, 46.1 s                      (d) 21 s, 41 s.

**Solution** (a)  $5.1 = \frac{u^2 \sin 2\theta}{g}$

$$\sin 2\theta = \frac{5100 \times 9.8}{240 \times 240}$$

$$= \frac{170}{192} = 0.88$$

$$2\theta = 61^\circ 42' \text{ or } \theta = 30^\circ 51'$$

$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 240 \times \frac{1}{2}}{9.8}$$

$$= 24.69 \text{ or } T = \frac{240 \times \sqrt{\frac{3}{2}}}{9.8} = 42.6 \text{ s.}$$

51. A particle moves according to the equation  $y = ax - bx^2$ . Only gravitation field is present. Find the initial velocity.

**Solution**  $a_x = 0$ ;  $a_y = -g$ ;  $y = ax - bx^2$  differentiating

$$\frac{dy}{dt} = a \frac{dx}{dt} - 2bx \frac{dx}{dt}; \left. \frac{dy}{dt} \right|_{x=0} = a \frac{dx}{dt}$$

differentiating again.

$$\frac{d^2y}{dt^2} = \frac{ad^2x}{dt^2} - 2bx \frac{d^2x}{dt^2} - 2b \left( \frac{dx}{dt} \right)^2$$

$$\therefore \frac{d^2x}{dt^2} = 0 \text{ and } \frac{d^2y}{dt^2} = -g = -2b \left( \frac{dx}{dt} \right)^2$$

$$\therefore \frac{dx}{dt} = \sqrt{\frac{g}{2b}} \cdot \left. \frac{dy}{dt} \right|_{x=0} = a \sqrt{\frac{g}{2b}};$$

$$v_{\text{initial}} = \sqrt{v_x^2 + v_y^2} = \sqrt{\frac{g}{2b}} (1+a)^2.$$

52. A particle moves along the trajectory of parabola  $y = ax^2$ . Assuming speed to be constant, find the radius of curvature.

- (a)  $\frac{1}{2a}$                                       (b)  $\frac{1}{a}$   
(c)  $\frac{1}{2ax}$                                       (d)  $\frac{2}{a}$

**Solution** (a)  $y = ax^2$ ;  $\frac{dy}{dt} = 2ax \frac{dx}{dt}$  or

$$\frac{d^2y}{dt^2} = 2a \left( \frac{dx}{dt} \right)^2 + 2ax \frac{d^2x}{dt^2}.$$

As the speed is constant only acceleration present is

$$a_r = \frac{v^2}{r}; \left. \frac{d^2y}{dt^2} \right|_{x=0} = 2a \left( \frac{dx}{dt} \right)^2.$$

$$2av^2 = \frac{v^2}{R} \text{ or } R = \frac{1}{2a}.$$

53. A boy is on the shore of a river. He is in the line of 1 m long boat and is 5.5 m away from the centre. He wishes to throw a stone into the boat. He can throw with a velocity  $10 \text{ ms}^{-1}$ . Find the minimum and maximum angles for a successful shot.

**Solution**  $5 = \frac{u^2 \sin 2\theta}{g}$  or  $2\theta = 30^\circ$ ,  $\theta = 15^\circ$  or  $75^\circ$

$6 = \frac{u^2 \sin 2\theta}{g}$  or  $2\theta = 37^\circ$ ,  $\theta = 18.5^\circ$  or  $71.5^\circ$ .

Allowed range  $15^\circ$  to  $18.5^\circ$  and  $71.5^\circ$  to  $75^\circ$ , Disallowed range  $18.5^\circ$  to  $71.5^\circ$

54. Car A is moving due east with speed  $v_1$ . At a certain time it is  $l_1$  away from the crossing. Car B at that instant is  $l_2$  away from the crossing and moving due north with a speed  $v_2$ . Find the shortest distance between them.

**Solution** Let after time  $t$  they are closest then

$$x^2 = (l_1 - v_1 t)^2 + (l_2 - v_2 t)^2$$

for  $x$  to be minimum  $\frac{dx}{dt} = 0$

or  $2(l_1 - v_1 t)(-v_1) + 2(l_2 - v_2 t)(-v_2) = 0$

or  $t = \frac{l_1 v_1 + l_2 v_2}{v_1^2 + v_2^2}$

and  $x = \sqrt{\left[l_1 - v_1 \left(\frac{l_1 v_1 + l_2 v_2}{v_1^2 + v_2^2}\right)\right]^2 + \left[l_2 - v_2 \left(\frac{l_1 v_1 + l_2 v_2}{v_1^2 + v_2^2}\right)\right]^2}$

$$= \frac{\sqrt{(l_1 v_2 - l_2 v_1 v_2)^2 + (l_2 v_1^2 - l_1 v_1 v_2)^2}}{(v_1^2 + v_2^2)}.$$

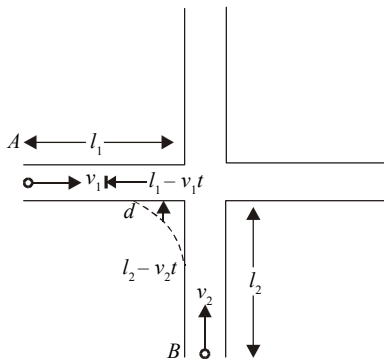


Fig. 3.39

55. Pick the correct statements.
- (A) Average speed of a particle can never be less than the magnitude of average velocity.
- (B) We may come across  $\left|\frac{d\vec{v}}{dt}\right| \neq 0$  but  $\frac{d}{dt} |\vec{v}| = 0$ .
- (C) The average velocity of a particle is zero in a time interval but instantaneous velocity is never zero in that interval.

- (a) only A and B are correct  
 (b) only B and C are correct  
 (c) only A and C are correct  
 (d) A, B and C are correct

**Solution** (d)

56. The velocity of a particle is zero at  $t = 0$  then which of the statement is incorrect?

- (a) The acceleration at  $t = 0$  must be zero  
 (b)  $\left.\frac{dv}{dt}\right|_{t=0}$  may be zero  
 (c) If  $a = 0$  in the interval  $0 - 10$  s the speed is also zero in the same interval.  
 (d) If the speed is zero in the interval  $0 - 10$  s then  $a = 0$ , in  $0 - 10$  s.

**Solution** (a)

57. An object may have

- (A) varying speed without having varying velocity  
 (B) varying velocity without having varying speed.  
 (C) nonzero acceleration without having varying velocity  
 (D) nonzero acceleration without having varying speed  
 (a) A and B are correct  
 (b) B and C are correct  
 (c) B and D are correct  
 (d) none of these

**Solution** (c)

58. A particle moves according to the equation  $x = a \sin \omega t$  and  $y = a(1 - \cos \omega t)$ . The path of the particle is

- (a) circle (b) parabola  
 (c) hyperbola (d) cycloid  
 (e) ellipse

**Solution** (d)

59. A girl throws a water filled balloon on the eve of Holi with a velocity  $12 \text{ ms}^{-1}$  at an angle of  $60^\circ$  with the horizontal. It hits a car at the same height (of throw) approaching her at  $15 \text{ ms}^{-1}$ . Find the distance of the car from her.

- (a) 13 m (b) 31 m  
 (c) 44 m (d) 18 m

**Solution** (c)  $T = \frac{2u \sin \theta}{g} = \frac{2 \times 12 \times \sqrt{3}}{10} = 2.07 \text{ s};$

$$R = \frac{u \sin 2\theta}{g} = \frac{12 \times 12 \times \sqrt{3}}{10 \times 2} = 13 \text{ m}$$

Distance of car from her =  $13 + 15(2.07) = 44 \text{ m}$

60. A motorcyclist does a daredevil stunt. He jumps of a 40 m wide river. The take ramp is inclined at  $53^\circ$  as shown in Fig. 3.40. Motorcycle is 2m long. Find the minimum velocity for a successful attempt.

**Solution**  $-15 = u \sin 53^\circ t - \frac{1}{2} g t^2$

and  $u \cos 53^\circ t = 42$  or  $u = \frac{42}{0.6t} = \frac{70}{t}$

$5t^2 - 15t - 56 = 0$  or  $t^2 - 3t - 11.2 = 0$

$t = \frac{3 + \sqrt{9 + 44.8}}{2} = \frac{10.33}{2} = 5.16 \text{ s}$

$u = \frac{70}{5.16} = 13.8 \text{ ms}^{-1}$ .

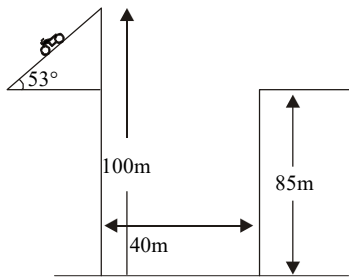


Fig. 3.40

61. A particle starts from origin at  $t = 0$  and moves in the  $x$ - $y$  plane with a constant acceleration  $\alpha$  in the  $y$ -direction. The equation of motion is  $y = kx^2$ . Its velocity component along  $x$ -direction is

(a) variable (b)  $\sqrt{\frac{2\alpha}{k}}$

(c)  $\frac{\alpha}{2k}$  (d)  $\sqrt{\frac{\alpha}{2k}}$

**Solution** (d)  $y = kx^2$  or  $\frac{dy}{dt} = 2kx \frac{dx}{dt}$

and  $\frac{d^2y}{dt^2} = 2k \left( \frac{dx}{dt} \right)^2 + 2kx \frac{d^2x}{dt^2}$ .

$\therefore \frac{d^2x}{dt^2} = 0 \therefore \frac{dx}{dt} = \sqrt{\frac{\alpha}{2k}}$

62. A particle moves with an initial velocity  $v_o$  and retardation  $kv$ , where  $v$  is the velocity at any time  $t$ .

(a) The particle will cover a total distance  $\frac{v_o}{k}$

(b) The particle comes to rest at  $t = \frac{1}{k}$

- (c) Particle continues to move for a long time

(d) at time  $\frac{1}{\alpha}$ ,  $v = \frac{v_o}{2}$

**Solution** (c)  $\frac{v dv}{dx} = -kv$  or  $\int_{v_o}^0 dv = -k \int_0^{x_p} dx$

or  $x_o = \frac{v_o}{k}$ .

$\frac{dv}{dt} = -kv$  or  $\int_{v_o}^v \frac{dv}{v} = - \int_0^t k dt$

or  $v = v_o e^{-kt} \therefore v \rightarrow 0$  when  $t \rightarrow \infty$ .

63. A particle moves along  $x$ -axis. It is at rest, At  $t = 0$ ,  $x = 0$  and comes to rest at  $t = 1$  and  $x = 1$ . If  $\alpha$  denotes instantaneous acceleration, then

- (a)  $\alpha$  cannot remain positive in the interval  $0 \leq t \leq 1$

- (b)  $|\alpha| \leq 2$  at any point in the path

- (c)  $|\alpha|$  must be  $\geq 4$  at some point in the path

- (d)  $\alpha$  must change sign during motion but no other assertion can be made

**Solution** (a), (c)  $\frac{1}{2} \alpha t^2 = 1$  taking  $t \leq \frac{1}{2\sqrt{2}}$ ;  $d \geq 4$ .

64. Take  $z$  = axis as vertical and  $x$ - $y$  plane as horizontal. A particle 'A' is projected with velocity  $4\sqrt{2} \text{ ms}^{-1}$  making an angle  $45^\circ$  to the horizontal. Particle B is projected at  $5 \text{ ms}^{-1}$  at an angle  $\theta = \tan^{-1} \left( \frac{1}{3} \right)$  to  $y$ -axis in  $y$ - $z$  plane then velocity of B wrt A.

- (a) has its initial magnitude  $5 \text{ ms}^{-1}$

- (b) magnitude will change with time

- (c) lies in the  $xy$  plane

- (d) will initially make an angle  $\left( \theta + \frac{\pi}{2} \right)$  with  $+x$  axis.

**Solution** (b), (c), (d)

65. Two particles moving initially in the same direction undergo a one dimensional, elastic collision. Their relative velocities before and after collision are  $\vec{v}_1$  and  $\vec{v}_2$ . Then

(a)  $|\vec{v}_1| = |\vec{v}_2|$

(b)  $\vec{v}_1 = -\vec{v}_2$  only if the two are of equal mass

(c)  $\vec{v}_1 \cdot \vec{v}_2 = |\vec{v}_1|^2$

(d)  $|\vec{v}_2 \cdot \vec{v}_1| = |\vec{v}_2|^2$

**Solution** (a), (c), (d) In head on collision

$$e = 1 = - \frac{\text{relative velocity after collision}}{\text{relative velocity before collision}}$$

66. Pick up the correct statements.

- (a) average speed  $\geq$  average velocity.  
 (b) It is possible to have  $\left| \frac{d\vec{v}}{dt} \right| \neq 0, \frac{d|v|}{dt} = 0$   
 (c) Even though  $v_{av} = 0$  in a given interval while instantaneous velocity is not zero at any point in the interval.  
 (d) If  $v_{av} = 0$  the acceleration must be present.

**Solution** (a), (b), (c)

67. A particle moves along  $x$ -axis according to the equation

$$x = u(t-2) + a(t-2)^2$$

- (a) The initial velocity of the particle is  $u$   
 (b) The acceleration of the particle is  $a$   
 (c) The acceleration of the particle is  $2a$   
 (d) at  $t = 2$  s, particle is at origin.

**Solution** (c), (d)  $\frac{dx}{dt} = u + 2a(t-2)$  and  $\frac{d^2x}{dt^2} = 2a$ .

### PASSAGE 1

Read the passage and answer the questions given at the end.

A projectile is any body that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance. A batted baseball, a thrown football, a package dropped from an airplane, and a bullet shot from a rifle are all projectiles. The path followed by a projectile is called its trajectory.

Represent the projectile as a single particle with an acceleration (due to gravity) that is constant in both magnitude and direction. Neglect the effects of air resistance and the curvature of the Earth and its rotation. Like all models, this one has limitations. Curvature of the earth has to be considered in the flight of long range missiles and air resistance is of crucial importance to a sky diver.

1. If air resistance is considered, then the maximum height achieved by the projectile  
 (a) decreases (b) increases  
 (c) remains unchanged  
 (d) very difficult to answer as no data provided

**Solution** (a)

2. Air resistance is proportional to

- (a)  $v$  (b)  $v^2$   
 (c)  $v^{-2}$  (d)  $v^3$

**Solution** (b) To a good approximation air resistance  $\propto v^2$ .

3. Comparing with no air-resistance curve, for the motion of a baseball with effect of air resistance, the correct curve will be

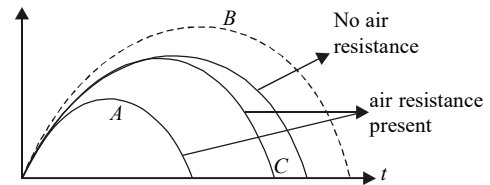


Fig. 3.41

- (a) A (b) B  
 (c) C (d) none

**Solution** (a)

4. A gun is fired horizontally on the bull's eye at a height  $h$   
 (a) The bullet hits the bull's eye  
 (b) The bullet moves left or right of the Bull's eye due to jerk experienced on firing  
 (c) The bullet misses the bull's eye and hits upward  
 (d) The bullet misses the target and hits downwards

**Solution** (d) due to gravity it follows projectile path (parabolic) and moves downward.

### PASSAGE 2

Read the passage and answer the questions at the end.

Journey in a train is adventurous particularly when you have a seat. The girl sitting near the window ate a banana and dropped the peel from the window. Her copassenger looking through the window found that it dropped vertically down and touched the ground in 0.2 s. After some time she requested her sister sitting on the upper berth to drop a chocolate bar. The sister dropped the bar, but it fell in front of the girl instead of reaching her hand. She was angry but the co-passenger calmed her by saying that she dropped exactly in line of your hand but as the train is accelerating it did not reach you and fell in front of you.

1. Is the co-passenger's explanation to the girl correct?

**Solution** No, the train is actually retarding. When the girl on the upper berth released the chocolate train was faster and the chocolate acquired the same horizontal velocity but the train retarded and became slow. Therefore, the girl sitting on the lower berth (due to motion of train) covered lesser distance and the chocolate covered longer distance and fell in front of her hands.

2. An observer standing outside the train finds the banana peel moving

- (a) vertically down (b) in parabolic path  
(c) horizontally (d) cycloid

**Solution** (b)

3. If the train would have moved with uniform velocity the chocolate will fall

- (a) behind her hands (b) towards left  
(c) towards right (d) in her hands

**Solution** (d)

4. If a projectile has velocity > escape velocity which trajectory it will follow

- (a) elliptic (b) hyperbolic  
(c) vertical straight (d) parabolic

**Solution** (b)

5. Two particles are thrown with  $8 \text{ ms}^{-1}$  as shown in Fig. 3.42 one horizontally from a height of 40 m and the other from a height of 50 m in making an angle  $60^\circ$  with the vertical. They strike in mid air. Find the coordinates of strike point and distance between the buildings.

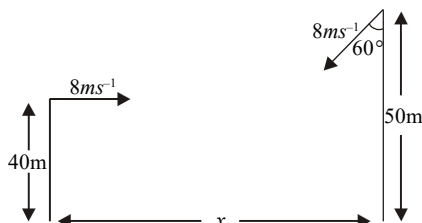


Fig. 3.42

**Solution**  $\frac{1}{2}gt^2 = y$ ;  $y + 10 = 8 \cos 60^\circ t + \frac{1}{2}gt^2$

$$y_1 = \frac{10}{2} \times (2.5)^2 = 31.25 \text{ m};$$

$$y_2 = y_1 + 10 = 41.25 \text{ m}$$

$$x_1 = 8(2.5) = 20 \text{ m};$$

$$x_2 = 8 \sin 60^\circ \times 2.5 = 17.32 \text{ m}$$

$$x = 20 + 17.32 = 37.32 \text{ m}$$

### PASSAGE 3

Read the following passage and answer the questions given at the end.

We interpret average and instantaneous velocity in terms of the slope of a graph of position versus time. In the same way, we can get additional insight into the concepts of average and instantaneous acceleration from a graph with instantaneous velocity  $v$  on the vertical axis and time  $t$  on the horizontal axis; that is, a  $v_x - t$  graph as shown in Fig. 3.44. The points on the graph labelled  $p_1$  and  $p_2$  correspond to

points  $P_1$  and  $P_2$  as shown in Fig. 1. The average acceleration  $a_{av-x} = \Delta v_x / \Delta t$  during this interval is the slope of the line  $p_1 p_2$ . As point  $p_2$  as shown in Fig 2 approaches point  $P_1$ , point  $P_2$  in the  $v_x - t$  graph of as shown in Fig 2. approaches point  $p_1$  and the slope of the line  $p_1 p_2$  approaches the slope of the line tangent to the curve at point  $p_1$ . Thus, on a graph of velocity as a function of time, the instantaneous acceleration at any point is equal to the slope of the tangent to the curve at that point. As shown in Fig 3.44 the instantaneous acceleration varies with time.

Note that by itself, the algebraic sign of the acceleration doesn't tell you whether a body is speeding up or slowing down. You must compare the signs of the velocity and the acceleration. When  $v_x$  and  $a_x$  have the same sign, the body is speeding up. If both are positive, the body is moving in the positive direction with increasing speed. If both are negative, the body is moving in the negative direction with a velocity that is becoming more and more negative, and again the speed is increasing. When  $v_x$  and  $a_x$  have opposite signs, the body is slowing down. If  $v_x$  is positive and  $a_x$  is negative, the body is moving in the positive direction with decreasing speed; if  $v_x$  is negative and  $a_x$  is positive, the body is moving in the negative direction with a velocity that is becoming less negative, and again the body is slowing down. As Fig 3.45 illustrates all these possibilities.

A Grand Prix car at two points on the straightway.

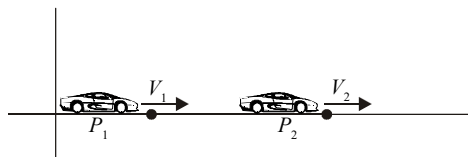


Fig. 3.43

A  $v_x - t$  graph of the motion in Fig. 1. The average acceleration between  $t_1$  and  $t_2$  equals the slope of the line  $p_1 p_2$ . The instantaneous acceleration at  $P_1$  equals the slope of the tangent at  $P_1$ .

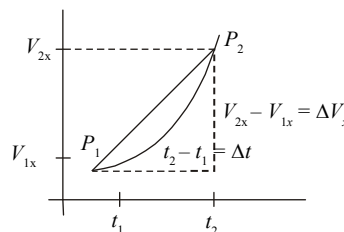


Fig. 3.44

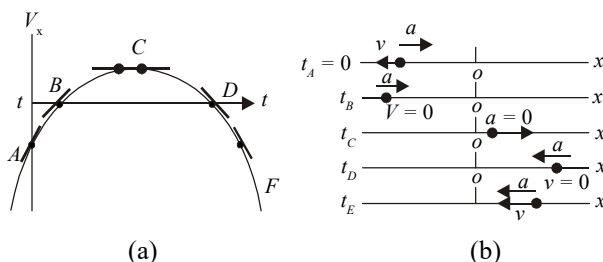


Fig. 3.45

- Does the slope of  $v-t$  indicate accelerated motion?
  - Accute angle indicates accelerated motion and obtuse angle retarded motion
  - Accute angle represents retarded motion and obtuse angle represents accelerated motion.
  - Accelerated motion always gives curved path.
  - If acceleration is uniform path will be linear.
- Motion along  $AB$  is

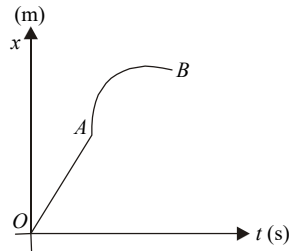


Fig. 3.46

- accelerated
  - retarded
  - uniform velocity
  - circular
- Plot  $v-t$  graph for given  $a-t$  graph.

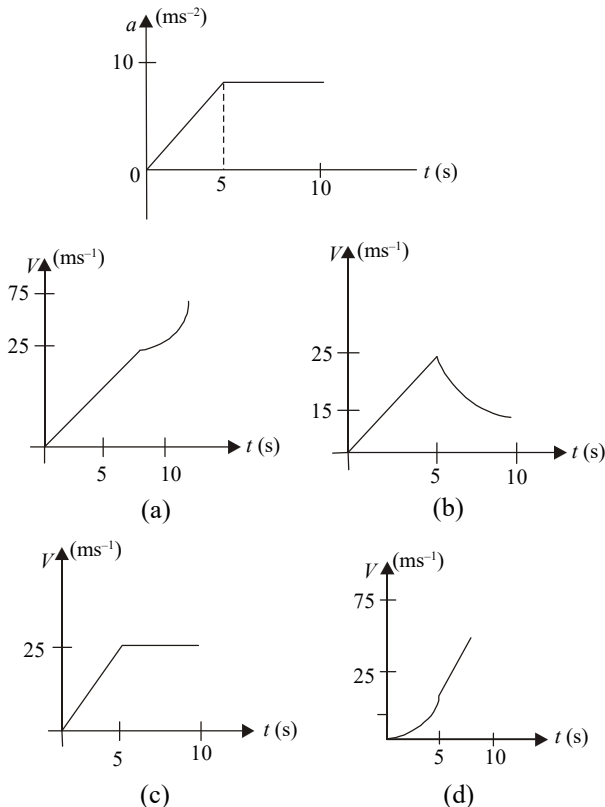


Fig. 3.47

### Solution

- (a)  $a = \tan \theta$  if  $\theta$  is accute  $\tan \theta$  is +ve; if  $\theta$  is obtuse  $\tan \theta$  is -ve.
- (b)  $\therefore$  slope  $\tan \theta$  is negative.
- (d)

### PASSAGE 4

Read the following passage and answer the questions given at the end.

Average velocity of a particle during an interval is defined as

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\text{displacement}}{\text{time elapsed}}$$

Suppose a particle is moving in such a way that its average velocity measured for a number of different intervals, does not turn out to be constant. This particle is said to move with variable velocity. Therefore, we define instantaneous velocity.

Velocity can vary by a change in magnitude, by a change in direction or path.

The limiting value of  $\lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t}$  is called instantaneous

velocity, i.e.,  $\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t}$

$$= \frac{d\vec{r}}{dt}$$

The magnitude  $v$  of the instantaneous velocity is called speed and is the absolute value of  $\vec{v}$ . That

is,  $v = |\vec{v}| = \left| \frac{d\vec{r}}{dt} \right|$ . Note speed shall remain positive.

- A particle moves half the distance with velocity  $u$ . Rest of the half distance is covered with velocities  $v_1$  and  $v_2$  in equal intervals of time. Find the average velocity.

$$(a) v_{av} = \frac{2u(v_1 + v_2)}{2u + v_1 + v_2} \quad (b) \frac{2u(v_1 + v_2)}{u + v_1 + v_2}$$

$$(c) \frac{2u(v_1 + v_2)}{u + 2(v_1 + v_2)} \quad (d) \frac{u + \frac{v_1 + v_2}{2}}{2}$$

(e) none

- The  $OA$  and  $AB$  part of the graph correspond to

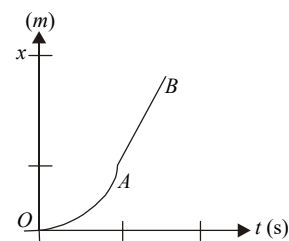


Fig. 3.48

- (a) uniform retardation, variable acceleration
- (b) uniform acceleration, uniform velocity
- (c) constant velocity, uniform acceleration
- (d) uniform acceleration, varying velocity.

3. Find the average speed between 0 – 10 s.

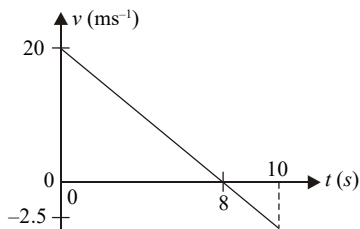


Fig. 3.49

- (a) 8.25 ms<sup>-1</sup>
- (b) 7.75 ms<sup>-1</sup>
- (c) 8.75 ms<sup>-1</sup>
- (d) 7.92 ms<sup>-1</sup>

### Solution

$$1. \quad (a) \quad v_{av, net} = \frac{2uv'_{av}}{u + v'_{av}} = \frac{2u \left( \frac{v_1 + v_2}{2} \right)}{u + \left( \frac{v_1 + v_2}{2} \right)}$$

2. (b)

$$3. \quad (a) \quad \text{Avg. speed} = \frac{\text{distance covered}}{\text{time taken}}$$

$$= \frac{80 + 2.5}{10} = 8.25 \text{ ms}^{-1}.$$

### PASSAGE 5

Read the following passage and answer the questions given at the end.

If a particle is moving in such a way that its average acceleration turns out to be different for a number of different time intervals, the particle is said to have variable acceleration. The acceleration can vary in magnitude, or in direction or both. In such cases we find acceleration at any instant, called the instantaneous acceleration. It is defined as

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

That is the acceleration of a particle at time  $t$  is the

limiting value of  $\frac{\Delta v}{\Delta t}$  at time  $t$  as  $\Delta t$  approaches zero. The

direction of the instantaneous acceleration  $\vec{a}$  is the limiting direction of the vector change in velocity  $\Delta v$ .

1. The limiting direction of vector change implies

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- (a) the direction of the tangent at the limiting point.
- (b) the direction of normal at the limiting point.
- (c) direction of final velocity.
- (d) direction of initial velocity.

2. A particle is travelling due north with 5 ms<sup>-1</sup>. It turns east in 5 sec. and continues to move with 5 ms<sup>-1</sup>. Find the average acceleration during turning.

- (a) 0
- (b)  $\sqrt{2} \text{ ms}^{-2} \text{ NE}$
- (c)  $\sqrt{2} \text{ ms}^{-2} \text{ SE}$
- (d)  $2 \text{ ms}^{-2} \text{ SE}$

3. A particle is moving along a straight line with 10 ms<sup>-1</sup>. It takes a U-turn in 5 s and continues to move along with same velocity 10 ms<sup>-1</sup>. Find the average acceleration during turning.

- (a) 0
- (b)  $-2 \text{ ms}^{-2}$
- (c)  $-4 \text{ ms}^{-2}$
- (d) none of these

4. A particle travels according to the equation such that its acceleration  $a \propto -v^2$ . Find the distance covered when its velocity falls from  $v_0$  to  $v$ .

$$(a) \quad x = \frac{-1}{k} \left[ \frac{1}{v_0} - \frac{1}{v} \right] \quad (b) \quad x = \frac{v_0}{k}$$

$$(c) \quad x = k \log_e \frac{v_0}{v} \quad (d) \quad x = \frac{1}{k} \log_e \frac{v_0}{v}$$

### Solution

1. (a)

$$2. \quad (c) \quad a_{av} = \frac{5i - 5j}{5} = \sqrt{2} \text{ ms}^{-2} \text{ SE.}$$

$$3. \quad (c) \quad a_{av} = \frac{v_f - v_i}{t} = \frac{-10\hat{i} - (10\hat{i})}{5} = -4\hat{i} \text{ ms}^{-2}.$$

$$4. \quad (d) \quad \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = -kv^2 \text{ or } \int_{v_0}^v \frac{dv}{v^2} = -\int_0^x kx$$

$$\text{or } x = \frac{1}{k} \log \frac{v_0}{v}.$$

### PASSAGE 6

Read the following passage and answer the questions given at the end.

$$\vec{a} = \frac{d\vec{v}}{dt} = \hat{i} \frac{dv_x}{dt} + \hat{j} \frac{dv_y}{dt}.$$

$$\text{or } \vec{a} = a_x \hat{i} + a_y \hat{j}.$$

$a_x$  and  $a_y$  are called the scalar components of the acceleration  $\vec{a}$ . In one-dimensional motion one of the



acceleration  $a_x$  or  $a_y$  will be zero. If motion is along vertical direction then  $a_y = g$  and we term it as motion under gravity. The sparkling example of two-dimensional motion under gravity is projectile motion. A projectile has uniform horizontal velocity and falls freely under gravity. Therefore, the path appears to be parabolic. Circular motion is another example of two-dimensional motion. In uniform circular motion, particle has time period fixed. If  $T$  is the time period then angular velocity  $\omega = \frac{2\pi}{T}$ . If the angular velocity varies with time the particle is said to possess nonuniform angular velocity.

- A particle moves according to the equation  $x = a \sin \omega t$  and  $y = a \cos \omega t$ . Then its resultant motion is
  - uniform circular motion and  $\omega$  is clockwise.
  - uniform circular motion and  $\omega$  is anticlockwise.
  - nonuniform circular motion with  $\omega$  clockwise.
  - nonuniform circular motion with  $\omega$  anticlockwise.
- In uniform circular motion
  - acceleration is variable.
  - acceleration is uniform.
  - the direction and magnitude of acceleration both vary.
  - if force applied is doubled in circular motion, then angular velocity becomes double.
- Which of the following is correct?
  - Radius of curvature of projectile motion is uniform throughout.
  - Radius of curvature of projectile motion varies at each point.
  - Radius of curvature is maximum at the time of projection.
  - Radius of curvature is maximum at the highest point.
- A particle moves under gravity. It implies
  - its horizontal velocity is zero.
  - its horizontal displacement is zero.
  - it has only vertical displacement.
  - it may have horizontal displacement but acceleration is along vertical direction.
- A particle moves according to the law  $x = at^2$  and  $y = bt^2$  then
  - its acceleration is under gravity
  - it move in a curved path in  $XY$  plane.
  - it moves in a straight line path in  $X-Y$  plane.
  - it describes ellipse.

- A particle moves from  $A$  to  $B$  diametrically opposite in a circle of radius 5 m with a velocity  $10 \text{ ms}^{-1}$ . Find the average acceleration.
  - zero
  - $\frac{40}{\pi} \text{ ms}^{-2}$
  - $\frac{20}{\pi} \text{ ms}^{-2}$
  - none

### Solution

- (a) at  $t = 0, x = 0, y = a$

$$\text{at } \omega t = \frac{\pi}{2} \quad x = a, y = 0$$

- (a)
- (d)
- (a)
- (b)

$$6. \quad (b) \quad a_{av} = \frac{\Delta v}{\Delta t} = \frac{2v \sin \theta/2}{\pi/2} = \frac{40}{\pi} \text{ ms}^{-2}.$$

### PASSAGE 7

Read the following passage and answer the questions given at the end.

The velocity of a particle is the rate at which its position varies with time. The position of a particle in a particular reference frame is given by a position vector drawn from the origin of that frame of reference. At  $t = t_1$  assume the particle is at  $A$  and its position in  $xy$  plane being described by position vector  $\vec{r}_1$ . At a later time  $t_2$  the particle reaches at point  $B$  as shown in Fig 3.50 and its position vector is  $\vec{r}_2$ .

The displacement vector  $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$  describes the change in position of the particle from  $A$  to  $B$  in time  $t_2 - t_1$ . Average velocity of the particle during this interval is

$$\vec{v}_{av} = \frac{\text{displacement (a vector)}}{\text{time elapsed (a scalar)}}$$

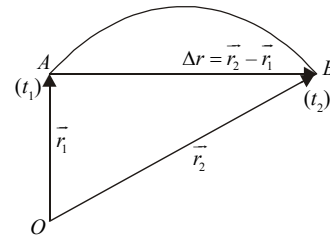


Fig. 3.50

- The average velocity defined above is called average the average acceleration.



- (a) the motion between  $A$  and  $B$  is known.  
 (b) the motion between  $A$  and  $B$  is erratic.  
 (c) the motion between  $A$  and  $B$  may have been steady or erratic.  
 (d) the motion between  $A$  and  $B$  is steady.
2. A particle moves from  $A$  to  $B$  such that  $x = t_2 + t - 3$ . Its average velocity from  $t = 2$  s to  $t = 5$  s is  
 (a)  $6 \text{ ms}^{-1}$  (b)  $8 \text{ ms}^{-1}$   
 (c)  $8.5 \text{ ms}^{-1}$  (d)  $7 \text{ ms}^{-1}$
3. A particle moves according to the law  $t = ax^2 + bx + c$   
 (a) The average velocity is same between  $x = 3$  m to  $x = 9$  m.  
 (b) The average velocity cannot be defined between  $x = 3$  m to  $x = 9$  m.  
 (c) The average acceleration is uniform throughout.  
 (d) The acceleration depends on cube of the velocity.
4. A body throws a ball to his friend 20 m away. The ball reaches to the friend in 4 s. The friend then throws the ball back to boy and it reaches the boy in 5 s.  
 (a) The average velocity is  $40/9 \text{ ms}^{-1}$ .  
 (b) The average acceleration is zero.  
 (c) The average velocity is zero but average acceleration is nonzero.  
 (d) Average acceleration of the motion cannot be defined.

**Solution**

1. (c)
2. (b)  $v_{av} = \frac{x(5) - x(2)}{5 - 2} = \frac{(25 + 5 - 3) - (4 + 2 - 3)}{3}$
3. (d)  $t = ax^2 + bx + c$

$$\frac{dt}{dx} = 2ax + b$$

$$\text{or } v = \frac{dx}{dt} = \frac{1}{2ax + b}$$

$$\text{acceleration} = \frac{dv}{dt} = \frac{-2a}{(2ax + b)^2} \cdot \frac{dx}{dt} = -2a v^3$$

4. (c) as  $g$  acts.

**PASSAGE 8**

Read the following passage and answer the questions given at the end.

P. Kirkpatrick in February 1944 *American Journal of Physics* published a paper titled 'Bad Physics in Athletic Measurements'. He suggested in this paper how the choice of Olympic venue affects the records of athletes. In 1936 Jesse Owens (United States) established a world record in running long jump of 8.09 m at the Olympic games of Berlin ( $g = 9.8128 \text{ ms}^{-2}$ ). If the Olympics was held at Melbourne ( $g = 9.7999 \text{ ms}^{-2}$ ) his record would have been 8.1065 m. Similarly, he quoted many other examples.

1. Is the claim of Mr. Kirkpatrick right?  
 (a) Yes  
 (b) No  
 (c) cannot say  
 (d) may be correct or may be not
2. Find the change in range of a projectile if acceleration due to gravity changes by  $dg$ .  
 (a)  $dR = R \frac{dg}{g}$  (b)  $dR = -\frac{Rdg}{g}$   
 (c)  $dR = \frac{Rdg}{2g}$  (d)  $dR = \frac{-Rdg}{2g}$
3. Which of the sport is most affected by variation of ' $g$ '?  
 (a) swimming (b) jump  
 (c) horse riding (d) shooting  
 (e) all are equally affected.

**Solution**

1. (a)  $R' = \frac{R}{g'} = \frac{8.09 \times 9.8128}{9.7999} = 8.1065 \text{ m}$
2. (b)  $R = \frac{u^2 \sin 2\theta}{g}$  ... (1)  
 differentiating (1)  
 $\frac{dR}{dg} = -\frac{u^2 \sin 2\theta}{g^2}$   
 or  $\frac{dR}{R} = -\frac{dg}{g}$
3. (b)

## QUESTIONS FOR PRACTICE

- A car moves in a semicircular track of radius 700 m. If it starts from one end of the track and stops at the other end, the displacement of car is  
 (a) 2200 m (b) 700 m  
 (c) 1400 m (d) none of these
- Displacement-time graph of a body is shown below. Velocity-time graph of the motion of the body is

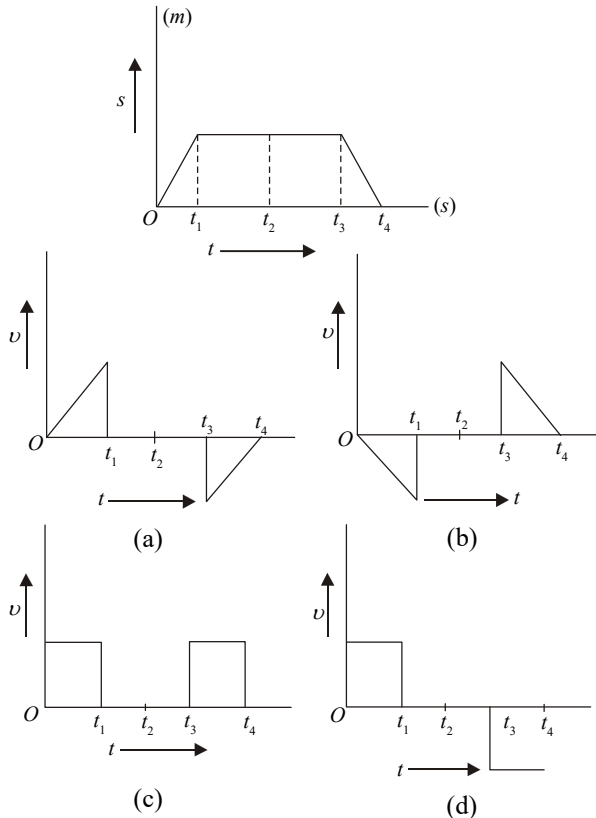


Fig. 3.51

- For Figure of Q.3, the acceleration-time graph of the motion of the body is

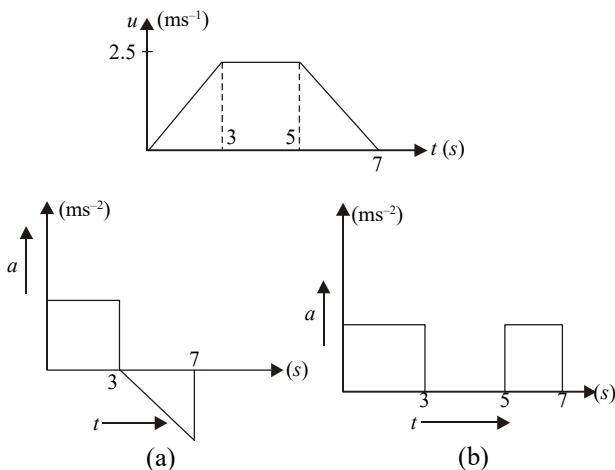


Fig. 3.52

- A boy can throw a stone to maximum height of 50 m. To what maximum range can he throw this stone and to what height so that the maximum range is maintained?  
 (a) 100 m, 100 m (b) 100 m, 25 m  
 (c) 200 m, 50 m (d) 100 m, 50 m
- A player throws a ball upwards with an initial speed of  $29.4 \text{ ms}^{-1}$ . The height to which the ball rises and the time taken to reach the player's hands are  
 (a) 22.05 m, 38 s (b) 44.1 m, 6 s  
 (c) 29.4 m, 6 s (d) 54.5 m, 9 s
- It was known that a shell fired with a given velocity and at an angle of projection  $\frac{5\pi}{36}$  radian can strike a target but a hill was found to obstruct its path. The angle of projection to hit the target should be  
 (a) Data is insufficient (b)  $\frac{13\pi}{16}$  radian  
 (c)  $\frac{10\pi}{36}$  radian (d)  $\frac{23\pi}{26}$  radian
- For the velocity-time curve shown below, the distance covered by a body from 5th to 7th second of its motion is ----- fraction of the total distance covered by it.

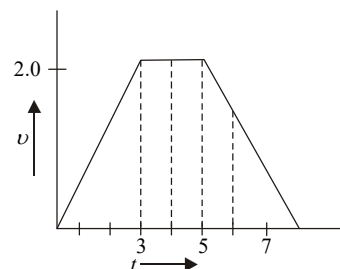


Fig. 3.53

- 2/9
- 9/2
- 1/2
- 2/3

8. A vehicle moves west with a speed of  $50 \text{ ms}^{-1}$  and then towards north with a speed of  $50 \text{ ms}^{-1}$  only. Total time taken by the body is 5s. What is the average acceleration of the body?

(a) 0 (b)  $50 \text{ ms}^{-2}$   
(c)  $14 \text{ ms}^{-2}$  (d)  $20.4 \text{ ms}^{-2}$

9. A body is projected at an angle  $\theta$  with the vertical with kinetic energy KE. What is the kinetic energy of the particle at the highest point?

(a)  $\text{KE} \cos^2 \theta$  (b)  $\text{KE} \sin^2 \theta$

(c)  $\frac{\text{KE}}{2}$  (d)  $\text{KE} \tan^2 \theta$

10. A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is

(a)  $\tan^{-1} \frac{3}{2}$  (b)  $\tan^{-1} \frac{2}{3}$

(c)  $\tan^{-1} \frac{1}{2}$  (d)  $\tan^{-1} \frac{3}{4}$

11. If the velocity of the motorcycle  $v$  is constant, then determine the velocity of the mass as a function of  $x$ . Given that ends  $P$  and  $R$  are coincident on  $Q$  when  $x = 0$ .

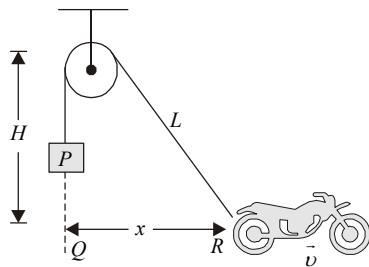


Fig. 3.54

(a)  $\frac{xv}{\sqrt{H^2 + x^2}}$  (b)  $\frac{H^2 + x^2}{xv}$

(c)  $\sqrt{\frac{H^2 + x^2}{vx}}$  (d)  $\frac{H^2 + x^2}{(vx)^2}$

12. Three points are located at the vertices of an equilateral triangle having each side as  $\alpha$ . All the points move simultaneously with speed  $u$  such that first point continually heads for second, the second for the third and the third for the first. Time taken by the points to meet at the centre is

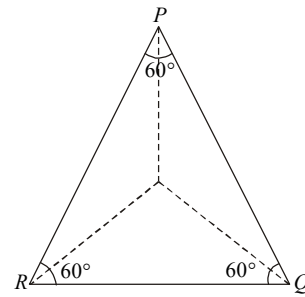


Fig. 3.55

(a)  $\frac{\alpha}{3u}$  (b)  $\frac{2\alpha}{3u}$

(c)  $\frac{\alpha^2}{u^3}$  (d)  $\frac{3\alpha}{2u}$

13. A wall clock has a 5 cm long minute hand. The average velocity of the tip of the hand reaching 0600 hrs. to 1830 hrs. is

(a)  $2.2 \times 10^{-14} \text{ cms}^{-1}$  (b)  $1.2 \times 10^{-4} \text{ cms}^{-1}$

(c)  $5.6 \times 10^{-3} \text{ cms}^{-1}$  (d)  $3.2 \times 10^{-3} \text{ cms}^{-1}$

14. A particle leaves the origin at  $t = 0$  and moves in the positive  $x$ -axis direction. Velocity of the particle

at any instant is given by  $v = u \left( 1 - \frac{t}{t'} \right)$ . If  $u = 10$

$\text{ms}^{-1}$  and  $t' = 5 \text{ s}$  find the  $x$  coordinate of the particle at an instant of 10 s.

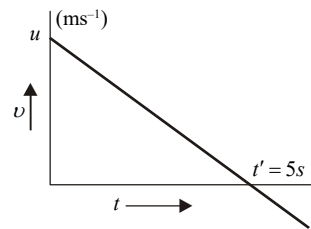


Fig. 3.56

(a) 0 (b) 10 m  
(c) 20 m (d) -10 m

15. The acceleration of a particle is increasing linearly with time  $t$  as  $\beta t$ . If the particle starts from origin with initial velocity  $u$ , the distance travelled by it in  $t$  second is

(a)  $ut + \frac{1}{2} \beta t^3$  (b)  $ut + \frac{1}{2} \beta t^3$

(c)  $ut + \frac{1}{3} \beta t^3$  (d)  $ut + \frac{\beta t^3}{6}$

16. A drunkard takes a step of 1 m in 1 s. He takes 5 steps forward and 3 steps backward and so on. The time taken by him to fall in a pit 13 m away from the start is

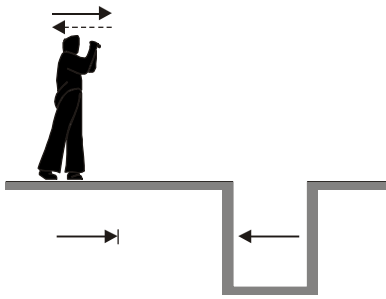


Fig. 3.57

- (a) 26 s (b) 31 s  
(c) 37 s (d) 41 s

17. A jeep moves at uniform speed of  $60 \text{ kmh}^{-1}$  on a straight road blocked by a wall. The jeep has to take a sharp perpendicular turn along the wall. A rocket flying at uniform speed of  $100 \text{ kmh}^{-1}$  starts from the wall towards the jeep when the jeep is 30 km away.

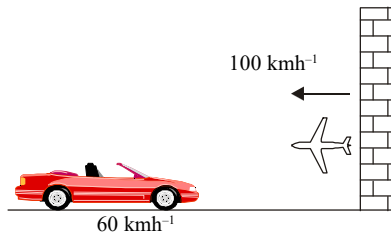


Fig. 3.58

The rocket reaches the windscreen and returns to wall. Total distance covered by the rocket is

- (a) 100 km (b) 50 km  
(c) 25 km (d) 75 km

18. A marble rolls down from top of a staircase with constant horizontal velocity  $u$ .

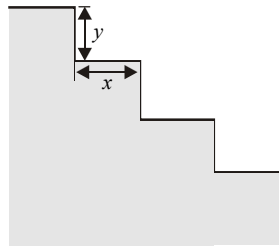


Fig. 3.59

If each step is  $y$  metre high and  $x$  metre wide, the marble just hits the edge of the  $n$ th step when  $n =$

- (a)  $\frac{xu^2}{gy^2}$  (b)  $\frac{u^2}{gy}$   
(c)  $\frac{2yu^2}{gx^2}$  (d)  $\frac{4yu^2}{gx^2}$

19. A particle experiences a fixed acceleration for 6s after starting from rest. It covers a distance of  $s_1$  in first two seconds,  $s_2$  in the next 2 seconds and  $s_3$  in the last 2 seconds then  $s_3 : s_2 : s_1$  is

- (a) 1 : 3 : 5 (b) 5 : 3 : 1  
(c) 1 : 2 : 3 (d) 3 : 2 : 1

20. A football dropped from a height onto an elastic net, stretched horizontally much above the ground rebounds. The graph for the motion is

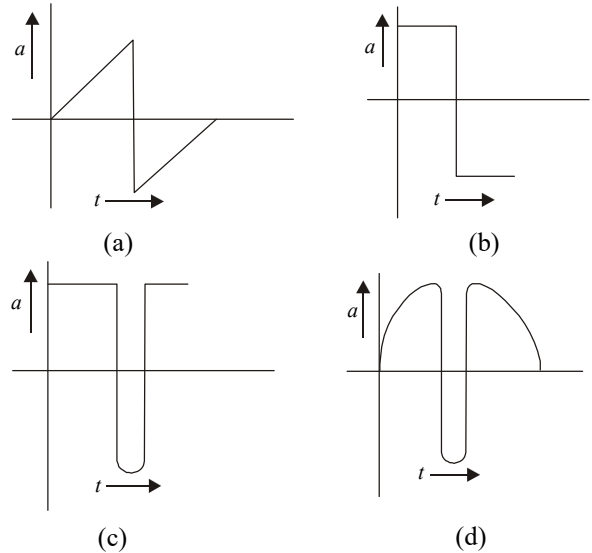


Fig. 3.60

21. A projectile is required to hit a target whose coordinates relative to horizontal and vertical axes through the point of projection are  $(\alpha, \beta)$ . If the gun velocity is  $\sqrt{2g\alpha}$ , it is impossible to hit the target if

- (a)  $\beta > \frac{3}{4} \alpha$  (b)  $\beta \geq \frac{1}{4} \alpha$   
(c)  $\beta \leq \frac{3}{4} \alpha$  (d)  $\beta \geq \frac{3}{4} \alpha$

22. A stone is allowed to fall from the top of a tower and cover half the height of the tower in the last second of its journey. The time taken by the stone to reach the foot of the tower is

- (a)  $(2 - \sqrt{2}) s$  (b)  $4 s$   
(c)  $(2 + \sqrt{2}) s$  (d)  $(2 \pm \sqrt{2}) s$

23. A balloonist is ascending at a velocity of  $12 \text{ ms}^{-1}$  and acceleration  $2 \text{ ms}^{-2}$ . A packet is dropped from it when it is at a height of 65 m from the ground, it drops a packet. Time taken by the packet to reach the ground is

- (a) 5 s (b) -5 s  
(c) 7 s (d)  $\frac{13}{5}$  s

24. A particle is projected up an inclined plane of inclination  $\alpha$  to the horizontal. If the particle strikes the plane horizontally then  $\tan \alpha =$

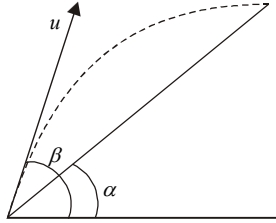


Fig. 3.61

- (a)  $\tan \beta$  (b)  $2 \tan \beta$   
(c)  $3 \tan \beta$  (d)  $\frac{1}{2} \tan \beta$

25. Three balls roll down three different frictionless paths as shown below. If the masses of the balls are 1 kg, 2 kg and 3 kg then their respective velocities  $u_1$ ,  $u_2$  and  $u_3$  on reaching the ground are such that

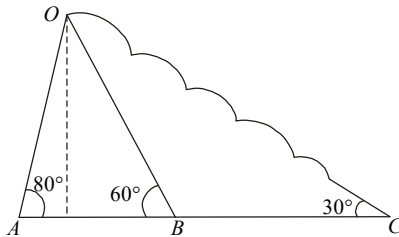


Fig. 3.62

- (a)  $u_1 > u_2 > u_3$  (b)  $u_1 < u_2 < u_3$   
(c)  $u_1 = 2u_2 = 3u_3$  (d)  $u_1 = u_2 = u_3$

26. A girl standing on a pedstal at rest throws a ball upwards with maximum possible speed of  $50 \text{ ms}^{-1}$ . If the platform starts moving up at  $5 \text{ ms}^{-1}$  and the girl again throws the similar ball in similar way. The time taken in the previous case and the time taken in the second case to return to her hands are

- (a) 10 s, 5 s (b) 5 s, 10 s  
(c) 10 s, 10 s (d) 5 s, 5 s

27. An aeroplane drops a parachutist. After covering a distance of 40 m, he opens the parachute and retards at  $2 \text{ ms}^{-2}$ . If he reaches the ground with a speed of  $2 \text{ ms}^{-1}$ , he remains in the air for about

- (a) 16 s (b) 3 s  
(c) 13 s (d) 10 s

28. A huge rectangular compartment falls vertically down with acceleration  $a$ . A gun fixed at corner  $P$  requires to hit  $Q$ . The bullet fired by it will hit  $Q$  if



Fig. 3.63

- (a)  $a = g$  (b)  $a > g$   
(c)  $a < g$  (d) any of (a), (b) or (c)

29. A helicopter is flying at 3564 m above ground. If an angle of  $30^\circ$  is subtended at a ground point by the helicopter position 100 s apart, what is the speed of the helicopter?

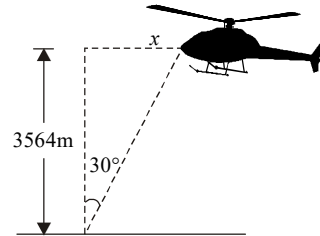


Fig. 3.64

- (a)  $100 \text{ ms}^{-1}$  (b)  $150 \text{ ms}^{-1}$   
(c)  $20 \text{ ms}^{-1}$  (d)  $25 \text{ ms}^{-1}$

30. A particle is thrown vertically upwards from ground. It takes time  $t_1$  to reach a height  $h$ . It continues to move and takes time  $t_2$  to reach the ground. Its maximum height is

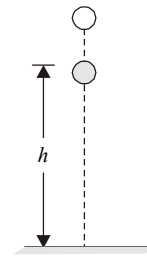


Fig. 3.65

- (a)  $\frac{g}{2} \frac{t_1 + t_2}{2}$  (b)  $\frac{g}{2} \sqrt{t_1^2 + t_2^2}$

- (c)  $\frac{g}{8} (t_1 + t_2)^2$  (d)  $g (t_1^2 + t_2^2)$

31. A tram moves with a velocity  $u$ . A particle moving horizontally with a speed  $v$  enters through corner  $B$  perpendicular to  $u$ . This particle strikes the diagonally opposite corner  $A$ . If the dimensions of tram are  $16 \times 2.4 \times 3.2 \text{ m}^3$ , the value of  $u$  is

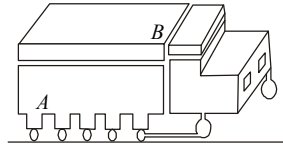


Fig. 3.66

- (a)  $3 \text{ ms}^{-1}$  (b)  $20 \text{ ms}^{-1}$   
(c)  $15 \text{ ms}^{-1}$  (d)  $30 \text{ ms}^{-1}$

32. A driver driving a truck at a constant speed of  $20 \text{ ms}^{-1}$  suddenly saw a parked car ahead of him by  $95 \text{ m}$ . He could apply the brake after some time to produce retardation of  $2.5 \text{ ms}^{-2}$ . An accident was just avoided, his reaction time is



Fig. 3.67

- (a)  $0.5 \text{ s}$  (b)  $0.75 \text{ s}$   
(c)  $0.8 \text{ s}$  (d)  $1 \text{ s}$

33. A water tap leaks such that water drops fall at regular intervals. Tap is fixed  $5 \text{ m}$  above the ground. First drop reaches the ground and at that very instant third drop leaves the tap. At this instant the second drop is at a height of

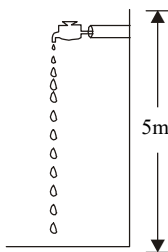


Fig. 3.68

- (a)  $3 \text{ m}$  (b)  $4.5 \text{ m}$   
(c)  $3.75 \text{ m}$  (d)  $2.5 \text{ m}$

34. A particle moves along a straight line as per equation  $x^2 = \alpha t^2 + 2\beta t + \gamma$ , where  $x$  is the distance travelled and  $\alpha, \beta, \gamma$  are constants. Its acceleration varies as
- (a)  $x^{-3}$  (b)  $x^{3/2}$   
(c)  $x^{-2/3}$  (d)  $x^2$

35. A tank moves uniformly along  $x$ -axis. It fires a shot from origin at an angle of  $30^\circ$  with horizontal while moving along positive  $x$ -axis and the second shot is also fired similarly except that the tank moves along negative  $x$ -axis. If the respective range of the shot are  $250 \text{ m}$  and  $210 \text{ m}$  along  $x$ -axis, then

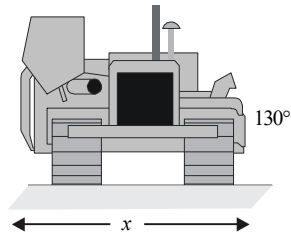


Fig. 3.69

the initial velocity values are

- (a)  $12 \text{ m/s}$  and  $21 \text{ m/s}$  (b)  $53 \text{ m/s}$  and  $49 \text{ m/s}$   
(c)  $95 \text{ m/s}$  and  $58 \text{ m/s}$  (d)  $79 \text{ m/s}$  and  $19 \text{ m/s}$

36. Engine of a vehicle can give it an acceleration of  $1 \text{ ms}^{-2}$  and its brakes can retard it at  $3 \text{ ms}^{-2}$ . The minimum time in which the vehicle can make a journey between stations  $A$  and  $B$  having a distance of  $1200 \text{ m}$  is

- (a)  $55.6 \text{ s}$  (b)  $65.6 \text{ s}$   
(c)  $50.6 \text{ s}$  (d)  $56.5 \text{ s}$

37. On a rainy day, a raindrop falls from very high clouds and faces retardation due to air. This retardation is directly proportional to the instantaneous speed of the drop. An expression for the distance travelled by the drop in time  $t$  is

- (a)  $s = \frac{g}{\alpha^2} (e^{-\alpha t} - 1) + \frac{g}{\alpha} t$   
(b)  $s = \frac{gt}{\alpha}$   
(c)  $s = \frac{g}{\alpha^2} (e^{-\alpha t} - 1)$   
(d)  $s = \frac{gt}{\alpha^2} + \frac{g}{\alpha} (e^{-\alpha t} - 1)$

38. A marble starts falling from rest on a smooth inclined plane of inclination  $\alpha$ . After covering distance  $h$  the ball rebounds off the plane. The distance from the impact point where the ball rebounds for the second time is

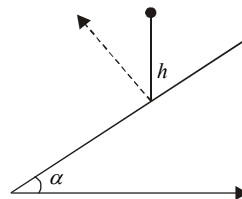


Fig. 3.70

- (a)  $8h \cos \alpha$  (b)  $8h \sin \alpha$   
(c)  $2h \tan \alpha$  (d)  $4h \sin \alpha$

39. What is the maximum range that a ball thrown with a speed of  $40 \text{ ms}^{-1}$  can cover without hitting the  $25 \text{ m}$  high ceiling of a long hall?

- (a) 150.5 m (b) 100.25 m  
(c) 110.3 m (d) 200.5 m

40. Two canons installed at the top of a cliff 10 m high fire a shot each with speed  $5\sqrt{3} \text{ ms}^{-1}$  at some interval. One canon fires at  $60^\circ$  with horizontal whereas the second fires horizontally. The coordinates of point of collision of shots are

- (a)  $3\sqrt{5} \text{ m}, 3 \text{ m}$  (b)  $\frac{1}{5\sqrt{3}} \text{ m}, \frac{1}{5} \text{ m}$   
(c)  $\frac{1}{3} \text{ m}, \frac{1}{3\sqrt{5}} \text{ m}$  (d)  $5\sqrt{3} \text{ m}, 5 \text{ m}$

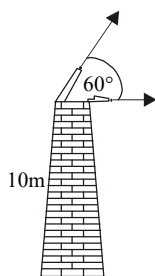


Fig. 3.71

41. A lift moves upward with an acceleration of  $1.2 \text{ ms}^{-2}$ . A nail falls from the ceiling of the lift 3 m above the floor of the lift. Distance of its fall with reference to the shaft of the lift is  
(a) 0.75 m (b) 0.5 m  
(c) 1 m (d) 1.5 m
42. A body travels 200 cm in the first two seconds and 220 cm in the next four seconds. What will be the velocity at the end of 7th second from the start?  
(a)  $10 \text{ cms}^{-1}$  (b)  $20 \text{ cms}^{-1}$   
(c)  $15 \text{ cms}^{-1}$  (d)  $5 \text{ cms}^{-1}$
43. A car accelerates from rest at a constant rate  $\alpha$  for some time after which it decelerates at a constant rate  $\beta$  to come to rest. If the total time elapsed is  $t$ , the maximum velocity acquired by the car is given by

- (a)  $\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)t$  (b)  $\left(\frac{\alpha^2 - \beta^2}{\alpha\beta}\right)t$   
(c)  $\left(\frac{\alpha + \beta}{\alpha\beta}\right)t$  (d)  $\left(\frac{\alpha\beta}{\alpha + \beta}\right)t$

44. A particle is moving in a plane with velocity given by  $\vec{u} = u_0 \hat{i} + (a\omega \cos \omega t) \hat{j}$ , where  $\hat{i}$  and  $\hat{j}$  are unit vectors along  $x$  and  $y$  axis, respectively. If the particle is at origin at  $t = 0$ , the distance from origin at time  $3\pi/2\omega$  is

- (a)  $a^2 + \omega^2$  (b)  $\left[\left(\frac{3\pi u_0}{2\omega}\right)^2 + a^2\right]^{1/2}$   
(c)  $\sqrt{a^2 + \left(\frac{2}{3}\pi u_0\right)^2}$  (d)  $\sqrt{a^2 + \left(\frac{\pi u_0}{\omega}\right)^2}$

[Based on Roorkee 1985]

45. A train moves from one station to another in 2 hrs time, its speed during the motion is shown in graph below.

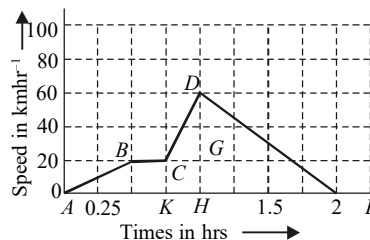


Fig. 3.72

The maximum acceleration during the journey and distance covered during the time interval from 0.75 hour to 1.00 hour are

- (a)  $60 \text{ km hr}^{-2}$ , 100 km (b)  $160 \text{ km hr}^{-2}$ , 10 km  
(c)  $10 \text{ km hr}^{-2}$ , 10 km (d)  $260 \text{ km hr}^{-2}$ , 1 km
46. The displacement  $x$  of a particle moving in one dimension is related to time by the equation  $t = (\sqrt{x} + 3)$ , where  $x$  is in  $m$  and  $t$  in  $s$ . The displacement, when velocity is zero, is  
(a) 1 m (b) 2 m  
(c) 4 m (d) zero
47. A soldier jumps out from an aeroplane with a parachute. After dropping through a distance of 19.6 m, he opens the parachute and decelerates at the rate of  $1 \text{ ms}^{-2}$ . If he reaches the ground with a speed of  $4.6 \text{ ms}^{-1}$ , how long was he in air?  
(a) 10 s (b) 12 s  
(c) 15 s (d) 17 s
48. A boat having a speed of 5 km/hr in still waters, crosses a river of width 1 km along the shortest possible path in 15 minutes. The speed of the river water in km/h is  
(a) 1 (b) 3  
(c) 4 (d)  $\sqrt{41}$

[I.I.T. 1988]

49. A particle moving in a straight line covers half the distance with speed  $3 \text{ ms}^{-1}$ . The other half is covered in two equal time intervals with speeds  $4.5 \text{ ms}^{-1}$  and  $7.5 \text{ ms}^{-1}$ , respectively. The average speed of the particle during motion is

- (a)  $4 \text{ ms}^{-1}$  (b)  $5 \text{ ms}^{-1}$   
 (c)  $5.5 \text{ ms}^{-1}$  (d)  $4.8 \text{ ms}^{-1}$

[I.I.T. 1992]

50. A particle in uniformly accelerated motion travels  $a$ ,  $b$  and  $c$  distances in  $x$ th,  $y$ th, and  $z$ th second of its motion, respectively. Then  $a(y-z) + b(z-x) + c(x-y) =$

- (a) 1 (b) 0  
 (c) 2 (d) 3

51. The driver of a train moving with a speed  $v_1$  sights another train at a distance  $s$ , ahead of him moving in the same direction with a slower speed  $v_2$ . He applies the brakes and gives a constant deceleration  $a$  to his train. For no collision,  $s$  is

- (a)  $\frac{(v_2 - v_1)^2}{2a}$  (b)  $> \left( \frac{v_1 - v_2}{2a} \right)^2$   
 (c)  $< \frac{(v_1 - v_2)^2}{2a}$  (d)  $< \frac{v_1 - v_2}{2a}$

52. The muzzle velocity of a certain rifle is  $330 \text{ ms}^{-1}$ . At the end of one second, a bullet fired straight up into the air will travel a distance of

- (a)  $(330 - 4.9) \text{ m}$  (b)  $330 \text{ m}$   
 (c)  $(330 + 4.9) \text{ m}$  (d)  $(330 - 9.8) \text{ m}$

53. Four persons  $K$ ,  $L$ ,  $M$  and  $N$  are initially at rest at four corners of a square of side  $d$ . Each person now moves with a uniform speed  $v$  in such a way that  $K$  always moves directly towards  $L$ ,  $L$  directly towards  $M$ ,  $M$  directly towards  $N$  and  $N$  directly towards  $K$ . Show that the four persons will meet at time  $t =$

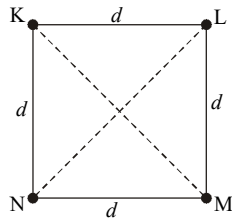


Fig. 3.73

- (a)  $\frac{4d}{v}$  (b)  $\frac{3d}{v}$   
 (c)  $\frac{2d}{v}$  (d)  $\frac{d}{v}$

[Based on I.I.T. 1984]

54.  $A$ ,  $B$ ,  $C$  and  $D$  are points in a vertical line such that  $AB = BC = CD$ . If a body falls from rest at  $A$ , prove that the time of descent through  $AB$ ,  $BC$  and  $CD$  are in the ratio of  $1 : \sqrt{3} : 2$ .

- (a)  $(\sqrt{2}-1) : (\sqrt{3}-\sqrt{2})$   
 (b)  $(\sqrt{3}-1) : (\sqrt{3}-\sqrt{2})$   
 (c)  $(\sqrt{3}-\sqrt{2}) : (\sqrt{2}-1)$   
 (d)  $(1-\sqrt{2}) : (\sqrt{2}-\sqrt{3})$

[Roorkee 1987]

55. A car is travelling at a velocity of  $10 \text{ kmh}^{-1}$  on a straight road. The driver of car throws a parcel with a velocity of  $10\sqrt{2} \text{ kmh}^{-1}$  when car is passing by a man standing on the side of a road. If parcel just reaches the man, the direction of throw makes following angle with the direction of car

- (a)  $135^\circ$  (b)  $45^\circ$   
 (c)  $\tan^{-1}(\sqrt{2})$  (d)  $\tan^{-1}(1/\sqrt{2})$

[Roorkee 1992]

56. A ball is dropped vertically from a height  $d$  above the ground. It hits the ground and bounces up vertically to a height  $d/2$ . Neglecting subsequent motion and air resistance, its velocity  $v$  varies with height  $h$  above the ground as

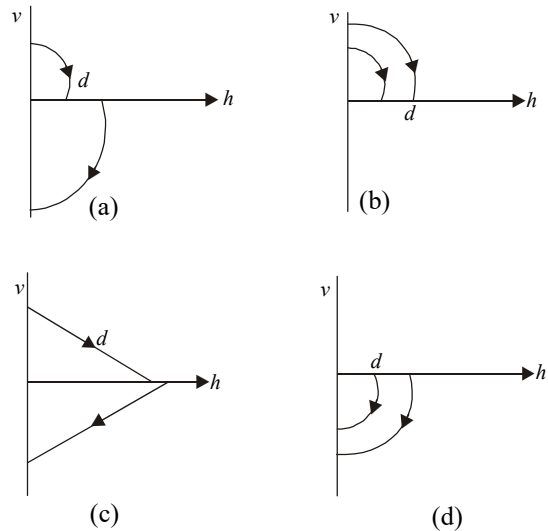


Fig. 3.74

[I.I.T. Screening 2000]

57. An observer moves with a constant speed along the line joining two stationary objects. He will observe that the two objects

- (a) have the same speed  
 (b) have the same velocity



- (c) move in the same direction
- (d) move in opposite directions

58. Which of the following statements are true for a moving body?

- (a) If its speed changes, its velocity must change and it must have some acceleration.
- (b) If its velocity changes, its speed must change and it must have some acceleration.
- (c) If its velocity changes, its speed may or may not change, and it must have some acceleration.
- (d) If its speed changes but direction of motion does not change, its velocity may remain constant.

59. Let  $v$  and  $a$  denote the velocity and acceleration respectively of a body

- (a)  $a$  can be nonzero when  $v = 0$ .
- (b)  $a$  must be zero when  $v = 0$ .
- (c)  $a$  may be zero when  $v \neq 0$ .
- (d) The direction of  $a$  must have some correlation with the direction of  $v$ .

60. Let  $\vec{v}$  and  $\vec{a}$  denote the velocity and acceleration, respectively, of a body in one-dimensional motion.

- (a)  $|\vec{v}|$  must decrease when  $\vec{a} < 0$ .
- (b) Speed must increase when  $\vec{a} > 0$ .
- (c) Speed will increase when both  $\vec{v}$  and  $\vec{a} < 0$ .
- (d) Speed will decrease when  $\vec{v} < 0$  and  $\vec{a} > 0$ .

61. The figure shows the velocity ( $v$ ) of a particle plotted against time ( $t$ ).

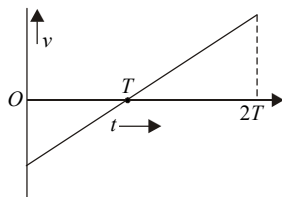


Fig. 3.75

- (a) The particle changes its direction of motion at some point.
- (b) The acceleration of the particle remains constant.
- (c) The displacement of the particle is zero.
- (d) The initial and final speeds of the particle are the same.

62. A particle moves along the  $x$ -axis as follows: it starts from rest at  $t = 0$  from a point  $x = 0$  and comes to rest at  $t = 1$  at a point  $x = 1$ . No other information is available about its motion for the intermediate time

( $0 < t < 1$ ). If  $\alpha$  denotes the instantaneous acceleration of the particle, then

- (a)  $\alpha$  cannot remain positive for all  $t$  in the interval  $0 \leq t \leq 1$
- (b)  $|\alpha|$  cannot exceed 2 at any point in its path
- (c)  $|\alpha|$  must be  $\geq 4$  at some point or points in its path
- (d)  $\alpha$  must change sign during the motion, but no other assertion can be made with the information given

63. The displacement ( $x$ ) of a particle depends on time

( $t$ ) as  $x = \alpha t^2 - \beta t^3$ .

- (a) The particle will return to its starting point after time  $\alpha/\beta$ .
- (b) The particle will come to rest after time  $2\alpha/3\beta$ .
- (c) The initial velocity of the particle was zero but its initial acceleration was not zero.
- (d) No net force will act on the particle at  $t = \alpha/3\beta$ .

64. A particle moves with an initial velocity  $v_0$  and retardation  $\alpha v$ , where  $v$  is its velocity at any time  $t$ .

- (a) The particle will cover a total distance  $v_0/\alpha$ .
- (b) The particle will come to rest after a time  $1/\alpha$ .
- (c) The particle will continue to move for a very long time.
- (d) The velocity of the particle will become  $v_0/2$  after a time  $1/\alpha$ .

65. A particle starts from the origin of coordinates at time  $t = 0$  and moves in the  $xy$  plane with a constant acceleration  $\alpha$  in the  $y$ -direction. Its equation of motion is  $y = \beta x^2$ . Its velocity component in the  $x$ -direction is

- (a) variable
- (b)  $\sqrt{\frac{2\alpha}{\beta}}$
- (c)  $\frac{\alpha}{2\beta}$
- (d)  $\sqrt{\frac{\alpha}{2\beta}}$

66. In Fig. 3.76, pulley  $P$  moves to the right with a constant speed  $u$ . The downward speed of  $A$  is  $v_A$ , and the speed of  $B$  to the right is  $v_B$ .

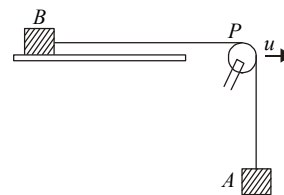


Fig. 3.76

- (a)  $v_B = v_A$
- (b)  $v_B = u + v_A$

- (c)  $v_B + u = v_A$   
 (d) The two blocks have accelerations of the same magnitude.

67. In Fig. 3.77, the blocks are of equal mass. The pulley is fixed. In the position shown,  $A$  moves down with a speed  $u$ , and  $v_B$  = the speed of  $B$ .

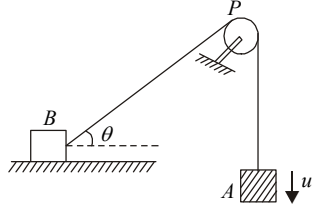


Fig. 3.77

- (a)  $B$  will never lose contact with the ground.  
 (b) The downward acceleration of  $A$  is equal in magnitude to the horizontal acceleration of  $B$ .  
 (c)  $v_B = u \cos \theta$   
 (d)  $v_B = u / \cos \theta$
68. Two particles  $A$  and  $B$  start simultaneously from the same point and move in a horizontal plane.  $A$  has an initial velocity  $u_1$  due east and acceleration  $a_1$  due north.  $B$  has an initial velocity  $u_2$  due north and acceleration  $a_2$  due east.
- (a) Their paths must intersect at some point.  
 (b) They must collide at some point.  
 (c) They will collide only if  $a_1 u_1 = a_2 u_2$ .  
 (d) If  $u_1 > u_2$  and  $a_1 < a_2$ , the particles will have the same speed at some point of time.
69. Two particles are projected from the same point with the same speed, at different angles  $\theta_1$  and  $\theta_2$  to the horizontal. They have the same horizontal range. Their times of flights are  $t_1$  and  $t_2$ , respectively.

- (a)  $\theta_1 + \theta_2 = 90^\circ$       (b)  $\frac{t_1}{t_2} = \tan \theta_1$   
 (c)  $\frac{t_1}{t_2} = \tan \theta_2$       (d)  $\frac{t_1}{\sin \theta_1} = \frac{t_2}{\sin \theta_2}$

70. A cart moves with a constant speed along a horizontal circular path. From the cart, a particle is thrown up vertically with respect to the cart.
- (a) The particle will land somewhere on the circular path.  
 (b) The particle will land outside the circular path.  
 (c) The particle will follow an elliptical path.  
 (d) The particle will follow a parabolic path.

71. A man on a moving cart, facing the direction of motion, throws a ball straight up with respect to himself.
- (a) The ball will always return to him.  
 (b) The ball will never return to him.  
 (c) The ball will return to him if the cart moves with a constant velocity.  
 (d) The ball will fall behind him if the cart moves with some acceleration.

72. A small ball is connected to a block by a light string of length  $l$ . Both are initially on the ground. There is sufficient friction on the ground to prevent the block from slipping. The ball is projected vertically up with a velocity  $u$ , where  $2gl < u^2 < 3gl$ . The centre of mass of the 'block + ball' system is  $C$ .



Fig. 3.78

- (a)  $C$  will move along a circle.  
 (b)  $C$  will move along a parabola.  
 (c)  $C$  will move along a straight line.  
 (d) The horizontal component of the velocity of the ball will first increase and then decrease.
73. A large rectangular box  $ABCD$  falls vertically with an acceleration  $a$ . A toy gun fixed at  $A$  and aimed towards  $C$  fires a particle  $P$ .

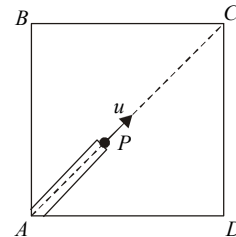


Fig. 3.79

- (a)  $P$  will hit  $C$  if  $a = g$ .  
 (b)  $P$  will hit the roof  $BC$  if  $a > g$ .  
 (c)  $P$  will hit the wall  $CD$  or the floor  $AD$  if  $a < g$ .  
 (d) May be either (a), (b) or (c), depending on the speed of projection of  $P$ .
74. A railway compartment is 16 m long, 2.4 m wide and 3.2 m high. It is moving with a velocity  $v$ . A particle moving horizontally with a speed  $u$ , perpendicular to the direction of  $v$ , enters through a hole at an upper corner  $A$  and strikes the diagonally opposite corner  $B$ . Assume  $g = 10 \text{ m/s}^2$ .

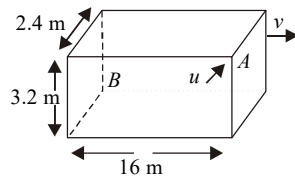


Fig. 3.80

- (a)  $v = 20 \text{ m/s}$   
 (b)  $u = \text{m/s}$   
 (c) To an observer inside the compartment path is parabolic.  
 (d) To a stationary observer outside the compartment path is parabolic.
75. Two shells are fired from a cannon with a speed  $u$  each, at angles of  $\alpha$  and  $\beta$ , respectively, to the horizontal. The time interval between the shots is  $T$ . They collide in mid-air after time  $t$  from the first shot. Which of the following conditions must be satisfied?
- (a)  $\alpha > \beta$   
 (b)  $t \cos \alpha = (t - T) \cos \beta$   
 (c)  $(t - T) \cos \alpha = t \cos \beta$   
 (d)  $(u \sin \alpha) t - \frac{1}{2} g t^2 = (u \sin \beta) (t - T) - \frac{1}{2} g (t - T)^2$
76. A man who can swim at a speed  $v$  relative to the water wants to cross a river of width  $d$ , flowing with a speed  $u$ . The point opposite him across the river is  $P$ .
- (a) The minimum time in which he can cross the river is  $\frac{d}{v}$ .  
 (b) He can reach the point  $P$  in time  $\frac{d}{v}$ .  
 (c) He can reach the point  $P$  in time  $\frac{d}{\sqrt{v^2 - u^2}}$   
 (d) He cannot reach  $P$  if  $u > v$ .
77. A river is flowing from west to east at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still waters, wants to swim across the river in the shortest time. He should swim in a direction
- (a) due north (b)  $30^\circ$  east of north  
 (c)  $30^\circ$  north of west (d)  $60^\circ$  east of north
78. In the arrangement shown in figure, the ends  $P$  and  $Q$  of an inextensible string move downwards with uniform speed  $u$ . Pulleys  $A$  and  $B$  are fixed. The mass  $M$  moves upwards with a speed.

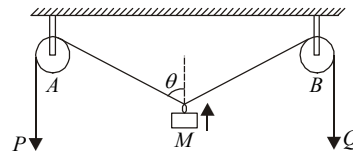


Fig. 3.81

- (a)  $2u \cos \theta$  (b)  $u/\cos \theta$   
 (c)  $2u/\cos \theta$  (d)  $u \cos \theta$
79. Consider the motion of the tip of the minute hand of a clock. In one hour
- (a) the displacement is zero  
 (b) the distance covered is zero  
 (c) the average speed is zero  
 (d) the average velocity is zero
80. A particle moves along the  $X$ -axis as
- $$x = u(t - 2 \text{ s}) + a(t - 2 \text{ s})^2.$$
- (a) the initial velocity of the particle is  $u$ .  
 (b) the acceleration of the particle is  $a$ .  
 (c) the acceleration of the particle is  $2a$ .  
 (d) at  $t = 2 \text{ s}$  particle is at the origin.
81. Pick the correct statements:
- (a) Average speed of a particle in a given time is never less than the magnitude of the average velocity.  
 (b) It is possible to have a situation in which  $\left| \frac{d\vec{v}}{dt} \right| \neq 0$  but  $\frac{d}{dt} |\vec{v}| = 0$ .  
 (c) The average velocity of a particle is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval.  
 (d) The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval. (Infinite accelerations are not allowed).
82. An object may have
- (a) varying speed without having varying velocity  
 (b) varying velocity without having varying speed  
 (c) nonzero acceleration without having varying velocity  
 (d) nonzero acceleration without having varying speed
83. Mark the correct statements for a particle going on a straight line.

- (a) If the velocity and acceleration have opposite sign, the object is slowing down.
- (b) If the position and velocity have opposite sign, the particle is moving towards the origin.
- (c) If the velocity is zero at an instant, the acceleration should also be zero at that instant.
- (d) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.

84. The velocity of a particle is zero at  $t = 0$ .

- (a) The acceleration at  $t = 0$  must be zero.
- (b) The acceleration at  $t = 0$  may be zero.
- (c) If the acceleration is zero from  $t = 0$  to  $t = 10$  s, the speed is also zero in this interval.
- (d) If the speed is zero from  $t = 0$  to  $t = 10$  s the acceleration is also zero in this interval.

85. Mark the correct statements:

- (a) The magnitude of the velocity of a particle is equal to its speed.
- (b) The magnitude of average velocity in an interval is equal to its average speed in that interval.
- (c) It is possible to have a situation in which the speed of a particle is always zero but the average speed is not zero.
- (d) It is possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero.

86. The velocity-time plot for a particle moving on a straight line is shown in the figure.

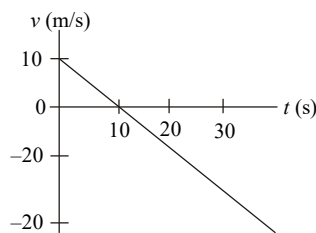


Fig. 3.82

- (a) The particle has a constant acceleration.
- (b) The particle has never turned around.
- (c) The particle has zero displacement.
- (d) The average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s.

87. Fig. 3.75 shows the position of a particle moving on the  $X$ -axis as a function of time.

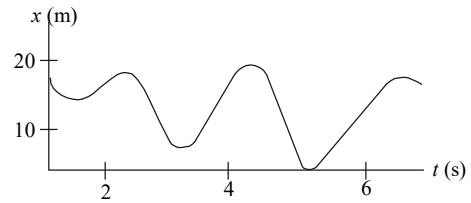


Fig. 3.83

- (a) The particle has come to rest 6 times.
- (b) The maximum speed is at  $t = 6$  s.
- (c) The velocity remains positive for  $t = 0$  to  $t = 6$  s.
- (d) The average velocity for the total period shown is negative.

88. Two extremely small blocks are lying on a smooth uniform rod of mass  $M$  and length  $L$ . Initially the blocks are lying at the centre. The whole system is rotating with an angular velocity  $\omega_0$  about an axis passing through the centre and perpendicular to the rod. When the blocks reach the ends of the rod, then the angular velocity of the rod will be—

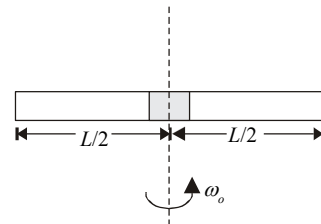


Fig. 3.84

- (a)  $\frac{M\omega_0}{M+2m}$
- (b)  $\frac{M\omega_0}{M+4m}$
- (c)  $\frac{M\omega_0}{M+6m}$
- (d)  $\frac{M\omega_0}{M+8m}$

89. A rocket is projected vertically upwards, whose time velocity graph is shown in. The maximum height reached by the rocket is -

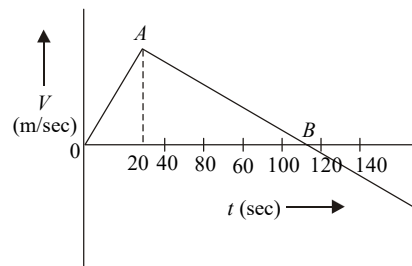


Fig. 3.85

- (a) 1 km
- (b) 10 km
- (c) 20 km
- (d) 60 km

90. Three blocks of mass  $m_1$ ,  $m_2$  and  $m_3$  are lying in contact with each other on a horizontal frictionless plane as shown in the figure. If a horizontal force  $F$  is applied on  $m_1$  then the force at the contact plane of  $m_1$  and  $m_2$  will be

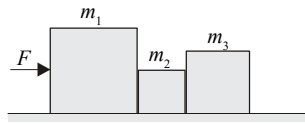


Fig. 3.86

- (a)  $\frac{F(m_2 + m_3)}{(m_1 + m_2 + m_3)}$  (b)  $\frac{m_1 + F}{(m_1 + m_2 + m_3)}$   
 (c)  $m_1 F$  (d)  $\frac{F(m_1 + m_2)}{(m_1 + m_2 + m_3)}$

91. A bullet is fired from a rifle. If the rifle recoils freely then the kinetic energy of the rifle will be  
 (a) equal to that of the bullet.  
 (b) less than that of the bullet.  
 (c) more than that of the bullet  
 (d) zero

92. A small disc is lying on the top of a hemispherical bowl of radius  $R$ . The minimum speed to be imparted to the disc so that it may leave the bowl without slipping is

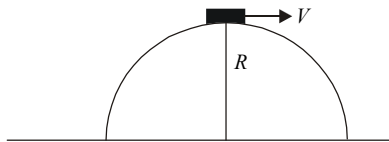


Fig. 3.87

- (a)  $v = \sqrt{g} \frac{R}{2}$  (b)  $v = 2 \sqrt{g} R$   
 (c)  $v = \sqrt{g} R$  (d)  $v = \sqrt{2g} R$

93. The quantity which remains constant for a body moving in a horizontal circle, is  
 (a) kinetic energy (b) acceleration  
 (c) force (d) velocity.

94. Force  $F$  varies with time in accordance with the following figure. The mean force will be

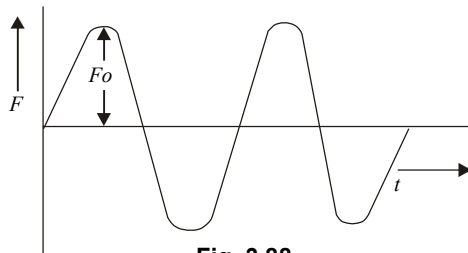


Fig. 3.88

- (a)  $F_0$  (b)  $\frac{F_0}{2}$   
 (c)  $2 F_0$  (d) Zero

95. An object of mass 150 kg. is to be lowered with the help of a string whose breaking strength is 100 Kg/ wt. What should be the minimum acceleration of the body so that the string may not break ?

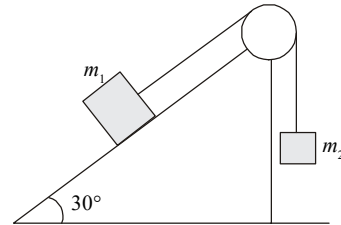


Fig. 3.89

- (a)  $2 \text{ m/s}^2$  (b)  $4 \text{ m/s}^2$   
 (c)  $3.33 \text{ m/s}^2$  (d)  $4.5 \text{ m/s}^2$

96. Two blocks of mass 8 kg. and 5 kg are connected by a heavy rope of mass 3 kg. An upward force of 180 N is applied as shown in the figure. The tension in the string at point  $P$  will be

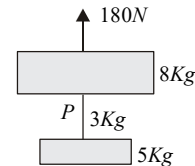


Fig. 3.90

- (a) 60 N (b) 90 N  
 (c) 120 N (d) 150 N

97. A body is released from the top of a tower. The body covers a distance of 24.5 m in the last second of its motion. The height of tower is

- (a) 59.8 m (b) 44.1 m  
 (c) 39.2 m (d) 49 m

98. A meter scale is suspended freely from one of its ends. Its other end is given a horizontal velocity  $v$  such that it completes one revolution in the vertical circle. The value of  $v$  is

- (a)  $\pi\sqrt{3} \text{ m/s.}$  (b)  $\pi\sqrt{6} \text{ m/s.}$   
 (c)  $\pi\sqrt{2} \text{ m/s.}$  (d)  $\pi \text{ m/s.}$

99. A block slips with constant velocity on a plane inclined at an angle  $\theta$ . The same block is pushed up the plane with an initial velocity  $v_0$ . The distance covered by the block before coming to rest is—

- (a)  $\frac{v_0^2}{2g \sin \theta}$  (b)  $\frac{v_0^2}{4g \sin \theta}$   
 (c)  $\frac{v_0^2 \sin^2 \theta}{2g}$  (d)  $\frac{v_0^2 \sin^2 \theta}{4g}$

100. A ball is dropped from a height of 19.6 m. The distance covered by it in the last second is

- (a) 19.6 m (b) 14.7 m  
 (c) 4.8 m (d) 9.8 m

101. A particle is projected upwards. The times corresponding to height  $h$  while ascending and while descending are  $t_1$  and  $t_2$  respectively. The velocity of projection will be

- (a)  $gt_1$  (b)  $gt_2$   
 (c)  $gt(t_1 + t_2)$  (d)  $\frac{g(t_1 + t_2)}{2}$

102. A frictionless wire is fixed between  $A$  and  $B$  inside a sphere of radius  $R$ . A small ball slips along the wire. The time taken by the ball to slip from  $A$  to  $B$  will be—

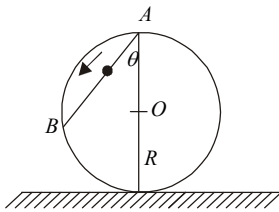


Fig. 3.91

- (a)  $\frac{2\sqrt{gr}}{g \cos \theta}$  (b)  $\frac{2\sqrt{gR \cos \theta}}{g}$   
 (c)  $2\sqrt{R/g}$  (d)  $\frac{gR}{\sqrt{g \cos \theta}}$

103. Starting from rest, a body takes 4 seconds in slipping from top to bottom of an inclined plane. The time taken by the same body in covering one quarter distance on the same plane from rest will be

- (a) 1 s (b) 2 s  
 (c) 4 s (d) 1.6 s

104. A 150 meter long train is moving towards north with a velocity of 10 m/s. A parrot is flying in the south direction parallel to tram at 5 m/s. The time taken by the parrot in crossing the train is

- (a) 12 s (b) 8 s  
 (c) 15 s (d) 10 s

105. A uniform stationary sphere starts rolling down from the upper end of the surface as shown in the figure,

and it reaches the lower right end  $H = 27$  m and  $h = 20$  m. The sphere will fall on the ground level at the following distance from  $A$

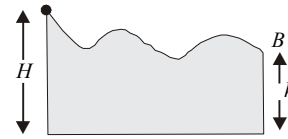


Fig. 3.92

- (a) 10 m (b) 20 m  
 (c) 30 m (d) 40 m

106. A body of mass  $m$  starts moving with velocity  $V_0$  at point  $A$  on a frictionless path as shown in the figure. The speed of the body at point  $B$  will be

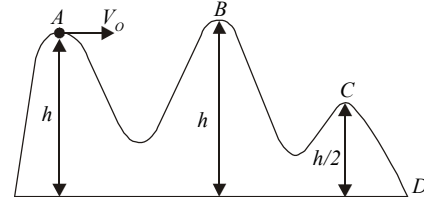


Fig. 3.93

- (a) Zero (b)  $V_0$   
 (c)  $\frac{V_0}{2}$  (d)  $2V_0$

107. In the above problem the speed at point  $C$  will be

- (a)  $2V_0$  (b)  $V_0$   
 (c)  $\frac{V_0}{2}$  (d)  $\sqrt{V_0^2 + gh}$

108. A passenger train is moving with speed  $V_1$ , on rails. The driver of this train observes another goods train moving in the same direction with speed  $v_2$  ( $v_1 > v_2$ ). If on applying brakes, the retardation produced is  $a$ , then the minimum distance covered by the passenger train so that it may not collide with the goods train will be

- (a)  $\frac{(v_1^2 - v_2^2)}{2a}$  (b)  $\frac{(v_1 + v_2)}{2a}$   
 (c)  $\frac{(v_1 - v_2)}{2a}$

(d) information is incomplete

109. Figure 3.86 represents a painter in a swing by the side of a building. When the painter pulls the string then the force applied on the surface is 450 N, whereas the weight of painter is 1000 N. If the weight of the swing is 250 N then the acceleration produced in the swing will be

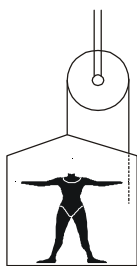


Fig. 3.94

- (a)  $4 \text{ m/s}^2$  (b)  $2 \text{ m/s}^2$   
(c)  $5 \text{ m/s}^2$  (d)  $6 \text{ m/s}^2$
110. The length of the arm of a nut-cracker is 15 cm. A force of 22.5 kg. weight is required to cut a nut without cracker. Where should the nut be placed on the cracker in order to cut it by a force of 2.25 kg/wt
- (a) 1 cm from fulcrum  
(b) 1.5 cm from fulcrum  
(c) 0.5 cm from fulcrum  
(d) 2.0 cm from fulcrum
111. Two balls  $A$  and  $B$  are simultaneously thrown.  $A$  is thrown from the ground level with a velocity of  $20 \text{ ms}^{-1}$  in the upward direction and  $B$  is thrown from a height of 40 m in the downward direction with the same velocity. Where will the two balls meet?
- (a) 15 m (b) 25 m  
(c) 35 m (d) 45 m
112. A body falls freely from the top of a tower. It covers 36% of the total height in the last second before striking the ground level. The height of the tower is
- (a) 50 m (b) 75 m  
(c) 100 m (d) 125 m
113. Two blocks are attached to the two ends of a string passing over a smooth pulley as shown in the figure. The acceleration of the block will be (in  $\text{m/s}^2$ )

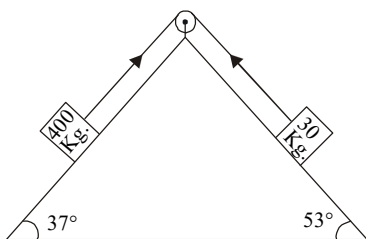


Fig. 3.95

- (a) 0.33 (b) 1.32  
(c) 1 (d) 0.66
114. In the above problem the tension in the string will be

- (a) 456 N (b) 850 N  
(c) 1000 N (d) 2000 N

115. A ball is thrown from a height of 12.5 m from the ground level in the horizontal direction. It falls at a horizontal distance of 200 m. The initial velocity of the ball is
- (a) 40 m/s (b) 80 m/s  
(c) 120 m/s (d) 20 m/s
116. The distance traveled by a body in fourth second is twice the distance traveled in seconds. If the acceleration of the body is  $3 \text{ m/s}^2$ , then its initial velocity is
- (a)  $\frac{3}{2} \text{ m/s}$  (b)  $\frac{5}{2} \text{ m/s}$   
(c)  $\frac{7}{2} \text{ m/s}$  (d)  $\frac{9}{2} \text{ m/s}$
117. The diameter of the tap of a fire brigade pump is 5 cm. Water is thrown by this pump at a horizontal speed of 18 m/s on a wall. If water rebounds back from the wall, then the force exerted by water on wall will be
- (a)  $2.35 \times 10^5 \text{ dyne}$  (b)  $5.76 \times 10^6 \text{ dyne}$   
(c)  $6.36 \times 10^7 \text{ dyne}$  (d)  $10^7 \text{ dyne}$
118. A 30 kg sphere, moving with a velocity 48 m/s, splits into two parts after explosion. The masses of these fragments are 18 kg and 12 kg. If the heavier fragment comes to rest after explosion, the velocity of second fragment will be
- (a) 80 m/s (b) 100 m/s  
(c) 110 m/s (d) 120 m/s
119. A bullet of mass 20 gm and moving with a velocity of 200 m/s strikes a sound and comes to rest after penetrating 3 cm inside it. The force exerted by the sand on the bullet will be
- (a)  $11.2 \times 10^8 \text{ dyne}$  (b)  $15.7 \times 10^8 \text{ dyne}$   
(c)  $13.3 \times 10^8 \text{ dyne}$  (d)  $18.6 \times 10^8 \text{ dyne}$
120. A bullet, moving with a velocity of 200 cm/s penetrates a wooden block and comes to rest after traversing 4 cm inside it. What velocity is needed for traversing a distance of 6 cm in the same block
- (a) 104.3 cm/s (b) 136.2 cm/s  
(c) 244.9 cm/s (d) 272.7 cm/s
121. The diameter of a solid disc is 0.5 m and its mass is 16 kg. What torque is required to increase its angular velocity from zero to 120 rotations/minute in 8 seconds?



- (a)  $\frac{\pi}{4}$  N/m (b)  $\frac{\pi}{2}$  N/m  
(c)  $\pi$  N/m (d)  $\pi$  N/m

122. In the above problem, at what rate is the work done by the torque at the end of eighth second?

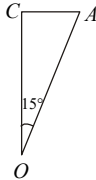
- (a)  $\pi$  W (b)  $\pi^2$  W  
(c)  $\pi^3$  W (d)  $\pi^4$  W

123. Two projectiles each of mass  $m$  are projected with same velocity  $v$  making an angle  $\alpha$  and  $\beta$  from the same point in opposite directions. Find the change in their momentum at any instant.

- (a)  $2mv \sin(\alpha + \beta)$  (b)  $2mv \sin \frac{\alpha + \beta}{2}$   
(c)  $2mv \cos(\alpha + \beta)$  (d)  $2mv \cos \frac{(\alpha + \beta)}{2}$

124. An aircraft is flying at a height of 2800 m above the ground. The angle subtended by it in 10 s is  $30^\circ$ . Find the speed of the aircraft

- (a)  $150 \text{ ms}^{-1}$   
(b)  $100 \text{ ms}^{-1}$   
(c)  $140 \text{ ms}^{-1}$   
(d)  $125 \text{ ms}^{-1}$



125. A rifle with a muzzle velocity  $1500 \text{ ft s}^{-1}$  shoots a bullet at a small target 150 ft away. How high above the target must the gun be aimed so that the bullet hits the target?

- (a) 2.02 inch (b) 1.72 inch  
(c) 1.82 inch (d) 1.92 inch

### PASSAGE 1

Read the following passage and answer the questions given at the end.

Electrons, nuclei, atoms and molecules like all forms of matter, will fall under the influence of gravity. Consider separately the beam of electrons, of nuclei, of atoms and of molecules travelling a horizontal distance of 1 m. Let the average speed of electrons be  $3 \times 10^7 \text{ ms}^{-1}$ , for a thermal neutron  $2.2 \times 10^5 \text{ ms}^{-1}$ , for a neon atom  $5.8 \times 10^2 \text{ ms}^{-1}$  and for an oxygen molecule  $4.6 \times 10^2 \text{ ms}^{-1}$ . The beams move through vacuum horizontally with initial velocities mentioned above. A golf ball is also projected horizontally with  $20 \text{ ms}^{-1}$  in vacuum.

- Out of the given beams which deviates maximum in travelling 2 m?  
(a) electron beam (b) neutron beam  
(c) neon atom (d) oxygen atom
- Find the deviation of golf ball in travelling through 2 m.  
(a) 2 cm (b) 5 cm  
(c) 8 cm (d) 3.6 cm

- Is there any effect of electron-electron repulsion?  
(a) Yes (b) No  
(c) insufficient data to reply (d) none

### Solution

- (d) Deviation  $y = \frac{1}{2} g t^2$  and  $t = \frac{x}{v}$  or  $y = \frac{1}{2} g \left( \frac{x}{v} \right)^2$ .
- (b)  $y = \frac{1}{2} g \left( \frac{2}{20} \right)^2 = 5 \text{ cm}$
- (b) Since the net velocity has already taken into account the repulsion, no effect of repulsion is to further added.

### PASSAGE 2

Read the following passage and answer the questions given at the end.

The instantaneous acceleration of a body, that is, its acceleration at one instant of time or at one point of its path, is defined in the same way as instantaneous velocity. Let the second point  $Q$  in Fig. 3.96 be taken closer and closer to the first point  $P$ , and let the average acceleration be computed over shorter and shorter intervals of time. The instantaneous acceleration at the first point is defined as the limiting value of the average acceleration when the second point is taken closer and closer to the first.

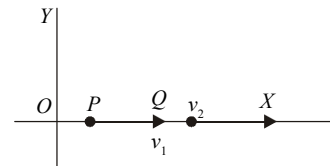


Fig. 3.96

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- A particle travels according to the equation  $t = \alpha x^2 + \beta x + \gamma$ . The acceleration at any instant is  
(a)  $2\alpha x^3$  (b)  $-2\alpha x^3$   
(c)  $-2\alpha$  (d)  $-2\alpha x^2$
- A particle moves according to the equation  $x = \alpha t^3 + \beta t^2$ . Find instantaneous acceleration at any time  $t_1$   
(a)  $6\alpha + 2\beta$  (b)  $3\alpha t + 2\beta$   
(c)  $3\alpha t + \beta$  (d)  $6\alpha t + 2\beta$

### PASSAGE 3

Read the following passage and answer the questions given at the end.

We consider a particle moving along the  $x$ -axis as in Fig. 3.97. Its distance from the origin  $O$  is described by the coordinate  $x$ , which varies with time. At a time  $t$ , the particle is at point  $P$ , where its coordinate is  $x_1$ , and at time  $t_2$  it is at

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point  $Q$ , where its coordinate is  $x_2$ . The displacement during the time interval from  $t_1$  to  $t_2$  is the vector from  $P$  to  $Q$ ; the  $x$ -component of this vector is  $(x_2 - x_1)$  and all other components are zero.

It is convenient to represent the quantity  $x_2 - x_1$ , the change in  $x$ , by means of a notation using the Greek letter  $\Delta$  (capital delta) to designate a change in any quantity. Thus we write

$$\Delta x = x_2 - x_1,$$

in which  $\Delta x$  is not a product but is to be interpreted as a single symbol representing the change in the quantity  $x$ . Similarly, we denote the time interval from  $t_1$  to  $t_2$  as

$$\Delta t = t_2 - t_1$$

The average velocity of the particle is defined as the ratio of the displacement  $\Delta x$  to the time interval  $\Delta t$ . We represent average velocity by the letter  $v$  with a bar ( $\bar{v}$ ) to signify average value. Thus

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

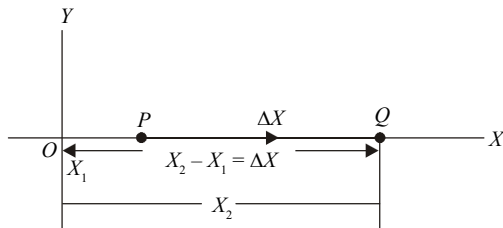


Fig. 3.97

1. A particle moves half the time of its journey with  $u$ . The rest of the half time it moves with two velocities  $V_1$  and  $V_2$  such that half the distance it covers with  $V_1$  and the other half with  $V_2$ . Find the net average velocity. Assume straight line motion.

$$(a) \frac{u(V_1 + V_2) + 2V_1V_2}{2(V_1 + V_2)} \quad (b) \frac{2u(V_1 + V_2)}{2u + V_1 + V_2}$$

$$(c) \frac{u(V_1 + V_2)}{2V_1} \quad (d) \frac{2V_1V_2}{u + V_1 + V_2}$$

2. A particle moves according to the equation  $x = t^2 + 3t + 4$ , the average velocity in the first 5 s is

$$(a) 8 \text{ ms}^{-1} \quad (b) 7.6 \text{ ms}^{-1} \\ (c) 6.4 \text{ ms}^{-1} \quad (d) 5.8 \text{ ms}^{-1}$$

3. A particle moves according to the law shown in Fig. 3.90. Find the average velocity in 6 seconds.

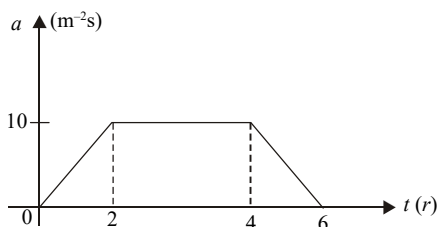


Fig. 3.98

$$(a) 20 \text{ ms}^{-1} \quad (b) 12.8 \text{ ms}^{-1} \\ (c) 16.3 \text{ ms}^{-1} \quad (d) 18.9 \text{ ms}^{-1}$$

4. The resistive force suffered by a motor boat is  $\propto V^2$  when the engine was shut down. When the velocity is  $V_0$  find the average velocity at any time  $t$ .

$$(a) V_{av} = \frac{V_0 + V}{2} \quad (b) \frac{VV_0}{2(V_0 + V)}$$

$$(c) \frac{VV_0 \log_e \frac{V_0}{V}}{(V_0 - V)} \quad (d) \frac{2VV_0 \log_e \frac{V_0}{V}}{(V_0 + V)}$$

5. A particle moves according to Fig. 3.91. Its KE vs time graph is

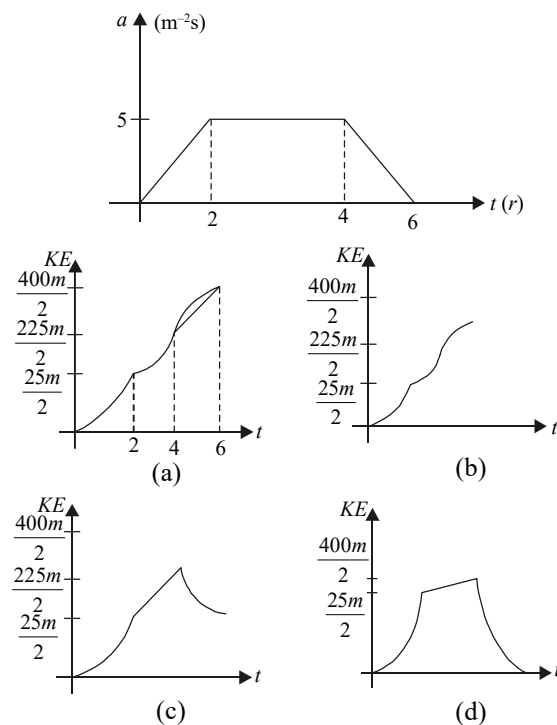


Fig. 3.99

### Solution

$$1(a) V_{av} = \frac{u + \frac{2V_1V_2}{V_1 + V_2}}{2} = \frac{u(V_1 + V_2) + 2V_1V_2}{2(V_1 + V_2)}$$

$$2(a) V_{av} = \frac{x(5) - x(0)}{5 - 0} = \frac{5^2 + 3(5) + 4 - 4}{5} = 8 \text{ ms}^{-1}$$

$$3(a) \frac{d^2x}{dt^2} = 5t, \quad \frac{dx}{dt} = \frac{5t^2}{2}$$

$$x = \int_0^2 \frac{5t^2}{2} dt = \frac{5t^3}{6} \Big|_0^2 = \frac{20}{3} \text{ m} \quad (0 - 25)$$

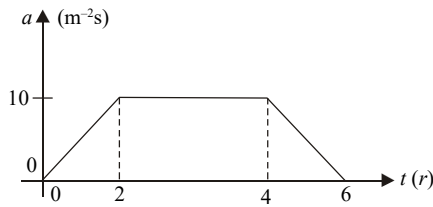


Fig. 3.100

$$x_2 = ut + \frac{1}{2} at^2 = 10(2) + 10(2)^2 = 60 \text{ m}$$

$$\frac{d^2x}{dt^2} = -5t, \quad \frac{dx}{dt} = \left[ 30 - \frac{5t^2}{2} \right]; \quad x_3 = \int_0^2 \left( 30 - \frac{5t^2}{2} \right) dt$$

$$x_3 = 30(2) - \frac{5t^3}{6} = 60 - \frac{20}{3} = \frac{160}{3}$$

$$v_{av} = \frac{x_1 + x_2 + x_3}{t} = \frac{20/3 + 60 + \frac{160}{3}}{6}$$

$$= \frac{360}{3 \times 6} = 20 \text{ ms}^{-1} \square 18.9 \text{ ms}^{-1}$$

$$4(c) \quad \frac{dV}{dt} = -kV^2, \quad - \int_{V_0}^V \frac{dV}{V^2} = \int_0^t k dt \quad \left[ \frac{1}{V} - \frac{1}{V_0} \right] = kt$$

$$\text{or} \quad t = \frac{1}{K} \left[ \frac{1}{V} - \frac{1}{V_0} \right]$$

$$\frac{dV}{dx} \cdot \frac{dx}{dt} = -kV^2 \quad \text{or} \quad \int_{V_0}^V \frac{dV}{V} = \int_0^x -k dx$$

$$\log_e V_0 - \log_e V = kx \quad \frac{\log_e V_0 / V}{k} = x$$

$$\text{or} \quad v_{av} = \frac{x}{t} = \frac{(\log_e V_0 / V) \times k}{k \left[ \frac{1}{V} - \frac{1}{V_0} \right]}$$

$$\text{or} \quad V_{av} = \frac{V V_0 \log_e \frac{V_0}{V}}{(V_0 - V)}$$

#### PASSAGE 4

Read the following passage and answer the questions given at the end.

Radar is used for ranging of the projectiles. A radar observer on the ground is watching an approaching projectile. At a certain instant, he gathers the following information. The projectile has reached maximum altitude and is moving

horizontally with a speed  $v$ , the straight line distance of the projectile is  $l$ . The line of sight to the projectile is an angle  $\theta$  above the horizontal.  $D$  is the distance between the observer and the point of impact of the projectile. Assume observer lies in the plane of the trajectory and the Earth is flat in that part.

1. Find  $D$  in terms of  $l$ ,  $v$  and  $\theta$ .

(a)  $\frac{gl^2}{v^2} \cot \theta$  (b)  $\frac{gl^2}{v^2} \tan \theta$

(c)  $\frac{gl^2}{2v^2} \tan \theta$  (d)  $\frac{gl^2}{2v^2} \cot \theta$

2. Does the projectile pass over his head before reaching him?

(a) Yes (b) No

(c) insufficient data to reply

**Solution** 1. (d)  $l = \frac{u^2 \sin \alpha \cos \alpha}{g} = \frac{v}{g} v_y$

$$v_y = \frac{gl}{v}$$

$$h = \frac{v_y^2}{2g} = \frac{g^2 l^2}{2v^2 g} = \frac{gl^2}{2v^2}$$

$$\frac{D}{h} = \cot \theta$$

$$\text{or} \quad D = h \cot \theta = \frac{gl^2}{2v^2} \cot \theta$$

2(c) If  $\theta < \alpha$ , the angle of projection of projectile, then the projectile will fall before reaching him.

#### PASSAGE 5

Read the following passage and answer the questions given at the end.

A determined student waited to test the law of gravity himself and jumps off the top of CN Tower in Toronto (553 m high) and falls freely. His initial velocity is zero. A rocketeer arrives at the scene 5.0 s later and dives off the top of the tower to save him. The rocketeer leaves the tower with an initial speed  $v_0$ . In order to catch the student and to prevent injury to him, the rocketeer should catch the student at a sufficiently great height above the ground so that rocketeer and student slow down and arrive at the ground with zero velocity. The upward acceleration that accomplishes this is provided by the rocketeer's jet pack, which he turns on when he catches the student, before that rocketeer is in free fall. To prevent discomfort to the student, the magnitude of acceleration should not exceed  $5g$ .

1. What is the minimum height above the ground at which the rocketeer should catch the student?

- (a) 92.1 m (b) 460.9 m  
 (c) 78.8 m (d) 82.3 m
2. What must be the rocketeer's minimum downward speed?
- (a) 90.2 ms<sup>-1</sup> (b) 75.4 ms<sup>-1</sup>  
 (c) 82.3 ms<sup>-1</sup> (d) 65.5 ms<sup>-1</sup>

**Solution** 1. (a) 2. (b)

**EXPLANATION**

$$h = \frac{1}{2}gt^2 = u(t-5) + \frac{1}{2}g(t-5)^2$$

$$\text{or } \frac{1}{2}gt^2 = u(t-5) + \frac{1}{2}gt^2 + \frac{25g}{2} - 5gt$$

$$\text{or } u(t-5) + \frac{25g}{2} - 5gt = 0$$

$$2 \times 5g(553 - h) = g^2t^2$$

$$\text{or } gt^2 = 10(553 - h)$$

$$\text{or } gt^2 = 10\left(553 - \frac{1}{2}gt^2\right)$$

$$6gt^2 = 5530$$

$$\text{or } t^2 = \frac{553}{6}$$

$$\text{or } t = 9.6$$

$$h = \frac{gt^2}{2} = \frac{5 \times 553}{6} \text{ or } 553 - h = 553 \left(1 - \frac{5}{6}\right) = 92.1 \text{ m}$$

$$u(t-5) = 5gt - \frac{25g}{2}$$

$$\text{or } u = \frac{50(9.6) - 125}{4.6} = 75.4 \text{ ms}^{-2}$$

**Answers to Questions for Practice**

|             |           |             |             |               |             |               |
|-------------|-----------|-------------|-------------|---------------|-------------|---------------|
| 1. (c)      | 2. (d)    | 3. (c)      | 4. (b)      | 5. (b)        | 6. (b)      | 7. (a)        |
| 8. (c)      | 9. (c)    | 10. (b)     | 11. (a)     | 12. (b)       | 13. (a)     | 14. (a)       |
| 15. (d)     | 16. (c)   | 17. (b)     | 18. (c)     | 19. (b)       | 20. (c)     | 21. (a)       |
| 22. (d)     | 23. (a)   | 24. (d)     | 25. (d)     | 26. (c)       | 27. (a)     | 28. (a)       |
| 29. (c)     | 30. (c)   | 31. (b)     | 32. (b)     | 33. (c)       | 34. (a)     | 35. (b)       |
| 36. (d)     | 37. (a)   | 38. (b)     | 39. (a)     | 40. (d)       | 41. (c)     | 42. (a)       |
| 43. (d)     | 44. (b)   | 45. (b)     | 46. (d)     | 47. (d)       | 48. (b)     | 49. (d)       |
| 50. (b)     | 51. (b)   | 52. (a)     | 53. (d)     | 54. (b)       | 55. (a)     | 56. (a)       |
| 57. (a,b,c) | 58. (a,c) | 59. (a,c)   | 60. (c,d)   | 61. (a,b,c,d) | 62. (a,c)   | 63. (a,b,c,d) |
| 64. (a,c)   | 65. (d)   | 66. (b,d)   | 67. (a,d)   | 68. (a,c,d)   | 69. (a,b,d) | 70. (b,d)     |
| 71. (c,d)   | 72. (a,d) | 73. (a,b,c) | 74. (a,b,d) | 75. (a,b,d)   | 76. (a,c,d) | 77. (a)       |
| 78. (b)     | 79. (a,d) | 80. (c,d)   | 81. (a,b)   | 82. (b,d)     | 83. (a,b,d) | 84. (b,c,d)   |
| 85. (a)     | 86. (a,d) | 87. (a)     | 88. (c)     | 89. (d)       | 90. (a)     | 91. (b)       |
| 92. (c)     | 93. (a)   | 94. (d)     | 95. (c)     | 96. (b)       | 97. (b)     | 98. (b)       |
| 99. (b)     | 100. (b)  | 101. (d)    | 102. (c)    | 103. (b)      | 104. (d)    | 105. (b)      |
| 106. (b)    | 107. (d)  | 108. (a)    | 109. (b)    | 110. (b)      | 111. (a)    | 112. (d)      |
| 113. (b)    | 114. (a)  | 115. (a)    | 116. (d)    | 117. (c)      | 118. (a)    | 119. (c)      |
| 120. (c)    | 121. (c)  | 122. (b)    | 123. (d)    | 124. (a)      | 125. (d)    |               |

**EXPLANATION**

- 4.(b) For maximum height, the boy has to throw the stone at  $\theta = 90^\circ$ , then

$$H = \frac{u^2}{2g} \text{ or } u^2 = 2gH$$

$$\text{or } u^2 = 2 \times 9.8 \times 50 = 980 \text{ (ms}^{-1}\text{)}^2$$

$$\text{Maximum range } R = \frac{u^2}{g} = \frac{980}{9.8} = 100 \text{ m}$$

Greatest height for maximum range is given by

$$H = \frac{u^2}{2g} \sin^2 45 = \frac{u^2}{2g} \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{u^2}{4g}$$

$$= \frac{980}{4 \times 9.8} = 25 \text{ m}$$

$$5.(b) \text{ From } u = \sqrt{2gh} \quad h = \frac{u^2}{2g} = 44.1 \text{ m}$$

$$h' = ut + \frac{1}{2}gt^2 = 29.4t - \frac{1}{2}(9.8)t^2 = 0$$

$$\therefore t = 6 \text{ s}$$

Using  $g = \frac{u}{t}$ , where  $t$  is time taken to reach the highest point, we get

$$t = \frac{u}{g} = \frac{29.4}{9.8} = 3 \text{ s}$$

By symmetry, the time taken by the ball to reach from the highest point to the hands of the player is 3 s.

$$\therefore \text{total time} = 3 + 3 = 6 \text{ s.}$$

6.(b) The other angle of projection for the same range is  $90^\circ - \theta$ .

$$\text{i.e. } \frac{\pi}{2} - \frac{5\pi}{36} = \frac{13\pi}{36} \text{ radian}$$

8.(c) Change in velocity

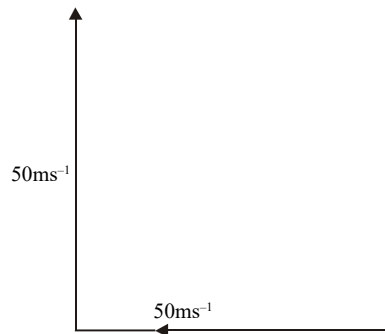


Fig. 3.101

$$= \sqrt{50^2 + 50^2}$$

$$= \sqrt{5000} = 70.7 \text{ ms}^{-1}$$

$$\therefore \text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$= \frac{70.7}{5} = 14.14 \text{ ms}^{-2}.$$

9. (c) Let  $v$  be the velocity of projection then  $KE = \frac{1}{2}mv^2$ , but velocity of body at highest point is  $v_x = v \sin \theta$ .  
 $\therefore$  kinetic energy of the body at the highest point is equal to

$$\frac{1}{2}m(v \sin \theta)^2 = \frac{1}{2}mv^2 \sin^2 \theta = KE \sin^2 \theta$$

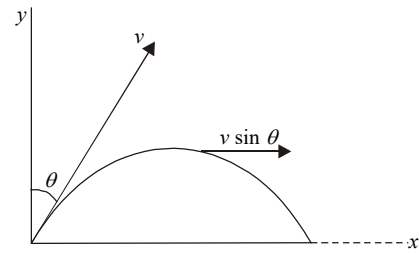


Fig. 3.102

$$\text{for } \theta = 45^\circ,$$

$$K.E._{\text{top}} = KE \sin^2 45^\circ = \frac{KE}{2}.$$

10. (b) Using  $R = \frac{u^2 \sin 2\theta}{g}$  we get

$$\frac{g}{u^2} = \frac{\sin 2\theta}{R}$$

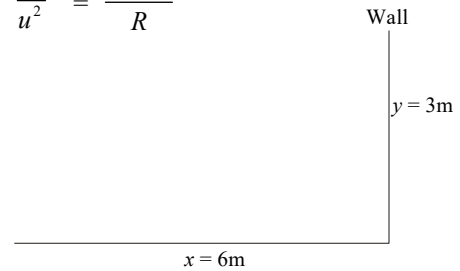


Fig. 3.103

$$\text{Range, } R = 6 + 18 = 24 \text{ m}$$

$$\therefore \frac{g}{u^2} = \frac{\sin 2\theta}{24}$$

$$\text{As } y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$\text{for } x = 6 \text{ m, } y = 3 \text{ m}$$

$$\text{and } \frac{g}{u^2} = \frac{\sin 2\theta}{24} \text{ we get}$$

$$3 = 6 \tan \theta - \frac{\sin 2\theta}{2 \times 24} \frac{6^2}{\cos^2 \theta}$$

Using  $\sin 2\theta = 2 \sin \theta \cos \theta$ , we get

$$\tan \theta = \frac{2}{3} \text{ or } \theta = \tan^{-1}(2/3)$$

11. (a) Here  $L = \sqrt{H^2 + x^2}$  or  $L^2 = H^2 + x^2$

Differentiating,

$$2L \left( \frac{dL}{dt} \right) = 0 + 2x \frac{dx}{dt}$$

But  $\frac{dL}{dt}$  is velocity of the mass and  $\frac{dx}{dt}$  is velocity of motorcycle.

$$\therefore L v_m = x v$$

Or 
$$v_m = \frac{xv}{L} = \frac{xv}{\sqrt{H^2 + x^2}}$$

12. (b) Net velocity  $= u - (-u \cos 60^\circ) = \frac{3}{2} u$

$$\text{Integrating } \alpha = \int \frac{3}{2} u dt \quad \text{or} \quad \alpha = \frac{3u}{2} t$$

or 
$$t = \frac{2\alpha}{3u}$$

13. (a) Displacement of minutes hand = 10 cm

Time difference between 0600 hrs to 1830 hrs is 12 hr and 30 minutes i.e. 45000 s

$$\therefore \text{Average velocity} = \frac{10}{45000} \\ = 2.2 \times 10^{-4} \text{ cms}^{-1}$$

14. (a)  $v = u \left(1 - \frac{t}{t'}\right)$  or  $\frac{dx}{dt} = u \left(1 - \frac{t}{t'}\right)$

$$\text{Integrating, } x = u \left(t - \frac{t^2}{2t'}\right) + C$$

$$\text{at } t = 0, x = 0 \text{ and } C = 0$$

$$\therefore x = u \left(t - \frac{t^2}{2t'}\right) = 10 t \left(1 - \frac{t}{10}\right)$$

$$\text{Putting } t = 10$$

$$x = 10 \times 10 \left(1 - \frac{10}{10}\right) = 0$$

15. (d) Using the relation,

$$S = ut + \frac{k}{(n+1)(n+2)} t^{n+2}$$

for non-uniform accelerated motion, we get

$$S = ut + \frac{\beta}{(1+1)(1+2)} t^{(1+2)}$$

(There  $a = \beta t$  so comparing it with  $a = kt^n$  we get  $n = 1$ )

$$\therefore S = ut + \frac{\beta}{6} t^3.$$

16. (c) When he takes 8 steps, the displacement is  $(5 - 3) = 2\text{m}$ .

Time taken for 8 steps = 8 s

$$\therefore \text{Average velocity} = \frac{2}{8} = \frac{1}{4} \text{ ms}^{-1}$$

In the last 5 steps the drunkard will not be able to come backward because he would fall in the pit.

$$\therefore \text{Total displacement required} \\ = 13 - 5 = 8 \text{ m}$$

$$\text{Time required} = \frac{8}{1/4} = 32 \text{ s}$$

17. (b) The time taken by jeep to cover a distance of 30 km

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{30}{60} = \frac{1}{2} \text{ hr.}$$

Total distance covered by rocket in this duration = speed  $\times$  time.

$$= 100 \times \frac{1}{2} = 50 \text{ km}$$

18. (c) Total horizontal distance to be covered

$$= x \times n = nx$$

Total vertical distance to be covered

$$= y + n = ny$$

$$\text{but } nx = ut \dots (i)$$

and 
$$ny = \frac{1}{2} gt^2 \dots (ii)$$

Substituting the value of  $t$  from (i) in (ii),

$$ny = \frac{1}{2} g \frac{n^2 x^2}{u^2}$$

Or 
$$n = \frac{2yu^2}{gx^2}$$

19. (b)  $S_1 = \frac{1}{2} a t^2 = \frac{1}{2} a \times 2^2 = 2a \dots (i)$

$$S_1 + S_2 = \frac{1}{2} a \times (2+2)^2 = 8a \dots (ii)$$

$$S_1 + S_2 + S_3 = \frac{1}{2} a \times (2+2+2)^2 = 18a \dots (iii)$$

Total time = 32 + time required to cover last 5 steps = 32 + 5 = 37 s.

20. (c) The football falls on the net with constant acceleration. Firstly, the net makes the acceleration drop but due to elasticity it again accelerates the football which ultimately gains constant acceleration.

22. (d) Using  $\frac{h}{2} = \frac{g}{2} (2n-1)$  and  $h = \frac{1}{2} g n^2$

(where  $n$  = total time and last moment of motion is  $n$ th second).

We get  $n^2 - 4n + 2 = 0$

Solving,  $n = (2 \pm \sqrt{2})$  s

23. (a) Using  $h = ut + \frac{1}{2} g t^2$  we get

$$-65 = 12t - \frac{1}{2} \times 10 \times t^2$$

Or  $5t^2 - 12t - 65 = 0$

Solving  $t = 5$  s or  $-13/5$  s

Time cannot be negative, thus  $t = 5$  s.

24. (d) As the particle strikes the plane horizontally, its velocity is parallel to horizontal axis so its vertical velocity is given by

$$0 = u \sin \beta - gt \quad (v = u + at)$$

or  $u \sin \beta = g \left[ \frac{2u \sin(\beta - \alpha)}{g \cos \alpha} \right]$

( $\because t$  is time of flight)

or  $\sin \beta \cos \alpha = 2 [\sin \beta \cos \alpha - \cos \beta \sin \alpha]$

or  $2 \cos \beta \sin \alpha = \sin \beta \cos \alpha$

or  $2 \frac{\sin \alpha}{\cos \alpha} = \frac{\sin \beta}{\cos \beta}$

or  $2 \tan \alpha = \tan \beta$  or  $\tan \alpha = \frac{1}{2} \tan \beta$

25. (d) Since the vertical and downward distances travelled by the balls are equal ( $h$ ) therefore their respective velocities are same i.e. each ball has velocity  $= \sqrt{2gh}$ .

26. (c) Time taken to reach the highest point,

$$t = \frac{u}{g} = \frac{50}{10} = 5 \text{ s}$$

Total time taken to reach back

$$= 5 + 5 = 10 \text{ s}$$

When platform starts moving with uniform speed, the acceleration is nil. Motion of the ball and the girl w.r.t. each other is unaffected so total time taken in this case also is 10 s.

27. (a) Using  $h = \frac{1}{2} g t^2$ , we get,  $t_1 = \sqrt{\frac{2h}{g}}$ .

Let  $t_1$  be the time taken from instants of jumping to the opening of parachute, then

$$t_1 = \sqrt{\frac{2 \times 40}{9.8}} = 2.86 \text{ s}$$

His velocity at this point is given by,

$$\begin{aligned} v_1^2 &= 2gh_1 = 2 \times 9.8 \times 40 \\ &= 784 \text{ or } v_1 = 28 \text{ ms}^{-1} \end{aligned}$$

For the remaining journey,

$$v = v_1 + at_2$$

or  $t_2 = \frac{v - v_1}{a} = \frac{2 - 28}{-2} = 13 \text{ s}$

$$\therefore \text{total time} = t_1 + t_2 = 2.86 + 13$$

$$= 15.86 \approx 16 \text{ s}$$

28. (a) The huge rectangular box is supposed to have an upward acceleration  $a$  so that reaction depends upon  $(g-a)$  i.e. no net acceleration if  $g=a$ . The compartment now behaves as if stationary and the bullet has no net deflecting acceleration so it will hit the aim.

29. (c) Here  $30^\circ = x/3564$

or  $x = 3564 \times \frac{1}{\sqrt{3}} = 2000 \text{ m}$

$$\text{Speed of the helicopter} = \frac{2000}{100} = 20 \text{ ms}^{-1}$$

30. (c) Let  $u$  be the velocity, then time taken by the particle to reach ground is,

$$t = t_1 + t_2 = 2 \times \frac{u}{g}$$

or  $u = \frac{g}{2} (t_1 + t_2)$

$$\text{Now, } H = \frac{u^2}{2g}$$

or  $H = g^2 \frac{(t_1 + t_2)^2}{4 \times 2g} = \frac{g}{4} (t_1 + t_2)^2$

31. (b) Considering the vertical motion,

$$3.2 = \frac{1}{2} \times 10 \times t^2 \text{ i.e., } t = 0.8 \text{ s}$$

The particle covers 2.4 m due to velocity component  $v$  and it covers 16 m due to velocity  $u$  in 0.8 s

$$\therefore u = \frac{16}{0.8} = 20 \text{ ms}^{-1}$$

32. (b) Let the driver apply brakes at Y, then

$$S_1 = 20 \times t \quad \dots\dots(i)$$

where  $t$  is the time taken by the driver to react to the situation

Using  $2aS = v^2 - u^2$  we get,

$$-2 \times 2.5 \times S_2 = 0 - 20^2$$

( $\because$  of retardation)

$$\text{or } S_2 = \frac{400}{5} = 80 \text{ m}$$

$$\text{But } 95 = S_1 + S_2$$

$$\text{or } S_1 = 95 - S_2 = 95 - 80 = 15 \text{ m}$$

$$\text{From (i) } t = \frac{S_1}{20} = \frac{15}{20} = 0.75 \text{ s}$$

33. (c) Distance covered by first drop is 5 m. If  $T$  is the time interval between drops then time taken by first drop to cover this distance is  $2T$ .

$$\therefore S = \frac{1}{2} g (2T)^2 = 2g T^2 \quad \dots \text{(i)}$$

Distance covered by second drop in time  $T$ ,

$$D = \frac{1}{2} g t^2 \quad \dots \text{(ii)}$$

From (i) and (ii)

$$D = \frac{1}{4} (2g T^2) = \frac{5}{4} = 1.25 \text{ m}$$

$$\therefore \text{Distance of second drop from ground} \\ = 5 - 1.25 = 3.75 \text{ m}$$

34. (a) Given  $x^2 = (\alpha t^2 + 2\beta t + \gamma)$

$$\therefore x = (\alpha t^2 + 2\beta t + \gamma)^{1/2}$$

$$v = \frac{dx}{dt} = \frac{1}{2} (\alpha t^2 + 2\beta t + \gamma)^{-1/2} (2\alpha t + 2\beta)$$

$$= (\alpha t^2 + 2\beta t + \gamma)^{-1/2} (\alpha t + \beta)$$

$$\text{and } a = \frac{dv}{dt} = -1/2 (\alpha t^2 + 2\beta t + \gamma)^{-3/2}$$

$$(2\alpha t + 2\beta) (\alpha t + \beta) + (\alpha t^2 + 2\beta t + \gamma)^{-1/2} (\alpha)$$

$$= -(\alpha t^2 + 2\beta t + \gamma)^{-3/2} (\alpha t + \beta)^2 + \alpha (\alpha t^2 + 2\beta t + \gamma)^{-1/2}$$

$$= (\alpha t^2 + 2\beta t + \gamma)^{-3/2} (-\alpha^2 t^2 - \alpha^2 - 2\alpha\beta t + \alpha^2 t^2 + 2\alpha\beta t + \alpha\gamma)$$

$$= \frac{\alpha\gamma - \beta^2}{(\alpha t^2 + 2\beta t + \gamma)^{3/2}} = \frac{\alpha\gamma - \beta^2}{x^3}$$

Thus  $a \propto x^{-3}$

35. (b) Let  $u$  be the velocity of projectile w.r.t. tank's velocity  $v$ , then

for  $x$  axis,

$$U_x = u \cos 30^\circ + v$$

$$U_y = u \sin 30^\circ$$

$$\text{and } T = \frac{2u \sin 30^\circ}{g}$$

$$\text{Range, } R_1 = U_x T$$

$$= \frac{2u \sin 30^\circ}{g} (u \cos 30^\circ + v)$$

for  $y$  axis,

$$U'_x = u \cos 30^\circ - v$$

$$\text{and } U'_y = u \sin 30^\circ$$

$$T = \frac{2u \sin 30^\circ}{g}$$

$$\text{Range, } R_2 = TU'_x$$

$$= \frac{2u \sin 30^\circ}{g} (u \cos 30^\circ - v)$$

$$\text{Then, } R_1 + R_2 = \frac{4u^2}{g} (\sin 30^\circ \cos 30^\circ)$$

$$R_1 - R_2 = \frac{4u}{g} v \sin 30^\circ$$

Eliminating  $u$ , we get

$$v^2 = \frac{g}{4 \tan 30^\circ} \frac{(R_1 - R_2)^2}{(R_1 + R_2)}$$

$$= \frac{10}{4 \tan 30^\circ} \frac{(250 - 200)^2}{(250 + 200)}$$

$$= 24 \text{ m}^2 \text{ s}^{-2}$$

$$\text{Or } v = 4.9 \text{ ms}^{-1}$$

36. (d) During acceleration,  $\frac{v}{t_1} = 1$   $t_1 = v$

$$\text{During retardation, } \frac{v}{t_2} = 3 \text{ or } t_2 = v/3$$

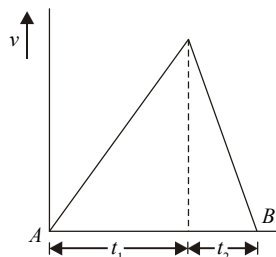


Fig. 3.104

Using  $s = \frac{1}{2} v(t_1 + t_2)$  [Displacement = area of  $\Delta$ ]

$$\text{or } 1200 = \frac{1}{2} v(v + v/3)$$

$$\text{or } 1200 = \frac{2v^2}{3}$$

$$\text{or } v^2 = \frac{3600}{2} = 1800$$

$$\text{or } v = 42.4$$

$$\therefore t_1 + t_2 = 42.4 + \frac{42.4}{3} = 56.5 \text{ s}$$

37. (a) Let the retardation produced by instantaneous opposition

$= \alpha v$  (where  $\alpha$  is a constant)

Net instantaneous acceleration  $= g - \alpha v$

$$\text{i.e. } \frac{dv}{dt} = (g - \alpha v)$$

$$\text{Integrating, } \int_0^v \frac{dv}{(g - \alpha v)} = \int_0^t dt$$

$$\text{In } \frac{g - \alpha v}{g} = -\alpha t \text{ i.e. } \frac{g - \alpha v}{g} = e^{-\alpha t}$$

$$\text{i.e. } v = \frac{g}{\alpha} (1 - e^{-\alpha t})$$

$$\text{i.e. } \frac{dS}{dt} = \frac{g}{\alpha} (1 - e^{-\alpha t})$$

$$\text{i.e. } dS = \frac{g}{\alpha} (1 - e^{-\alpha t}) dt$$

$$\text{Integrating, } \int_0^S dS = \frac{g}{\alpha} \int_0^t (1 - e^{-\alpha t}) dt$$

$$= \frac{g}{\alpha} \int_0^t dt - \frac{g}{\alpha} \int_0^t e^{-\alpha t} dt$$

$$\text{or } S = \frac{g}{\alpha} t + \frac{g}{\alpha^2} e^{-\alpha t} - \frac{g}{\alpha^2}$$

$$\text{or } S = \frac{g}{\alpha^2} (e^{-\alpha t} - 1) + \frac{g}{\alpha} t$$

38. (b) Velocity before strike  $u = \sqrt{2gh}$

Component of acceleration along the inclined plane  $= g \sin \alpha$  and the perpendicular component  $= g \cos \alpha$

$$\text{Using } S = ut + \frac{1}{2} at^2$$

for vertical direction we get,

$$0 = v \cos \alpha t - \frac{1}{2} g \cos \alpha t^2 \text{ and}$$

for horizontal direction,

$$x = u \sin \alpha t + \frac{1}{2} g \sin \alpha t^2$$

$$= u \sin \alpha \frac{2u}{g} + \frac{1}{2} g \sin \alpha \left( \frac{2u}{g} \right)^2 \left( Q t = \frac{2u}{g} \right)$$

$$= \frac{2u^2 \sin \alpha}{g} + \frac{2u^2 \sin \alpha}{g} = \frac{4u^2 \sin \alpha}{g}$$

$$= 4 \times \frac{2gh \times \sin \alpha}{g} = 8h \sin \alpha$$

39. (a) Using  $H = \frac{u^2 \sin^2 \alpha}{2g}$

$$\text{and } R = \frac{2u^2 \sin \alpha \cos \alpha}{g}$$

$$\text{we get } R^2 = \frac{4u^4}{g^2} \sin^2 \alpha \cos^2 \alpha$$

Eliminating  $\alpha$ , we get

$$R^2 = \frac{4u^2}{g} \frac{2gH}{u^2} \left( 1 - \frac{2gH}{u^2} \right)$$

$$= \frac{8H}{g} (u^2 - 2gH)$$

$$\text{or } R = \left[ \frac{8H}{g} (u^2 - 2gH) \right]^{1/2}$$

$$= \left[ \frac{8 \times 25}{9.8} (40^2 - 2 \times 9.8 \times 25) \right]^{1/2}$$

40. (d) Let  $t_1$  and  $t_2$  be the times of shot from cannon at  $60^\circ$  and the shot from horizontal cannon to reach the point of collision.



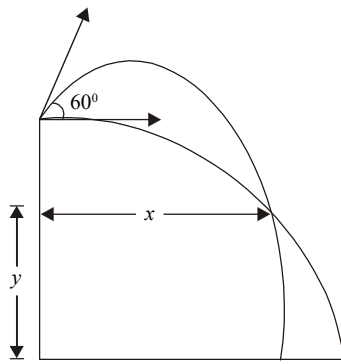


Fig. 3.105

$$\therefore x = (5\sqrt{3} \cos 60) t_1 = 5\sqrt{3} t_2$$

$$\begin{aligned} \text{and } y &= 10 + 5\sqrt{3} \sin 60 t_1 - \frac{1}{2} g t_1^2 \\ &= 10 - \frac{1}{2} g t_2^2 \end{aligned}$$

41. (c) Let  $y_1$  be the distance covered by the fan and  $y_2$  be the distance covered by lift just before the fan reaches the floor of the lift,

$$\therefore y_1 + y_2 = 3$$

$$\text{but } y_1 = -2.4 t + \frac{1}{2} 9.8 t^2$$

(initial velocity of fan is upward and acceleration is downward)

$$\text{and } y_2 = 2.4 t + \frac{1}{2} \times 1.2 t^2$$

$$= 2.4 t + 0.6 t^2$$

$$\therefore y_1 + y_2 = 4.9 t^2 + 0.6 t^2 = 5.5 t^2$$

$$\text{or } 3 = 5.5 t^2$$

$$\text{or } t = \sqrt{\frac{3}{5.5}} = 0.74 \text{ s}$$

$$\begin{aligned} \text{Now } y_1 &= -2.4 \times 0.74 + 4.9 \times 0.74^2 \\ &= -1.68 + 2.68 = 1 \text{ m} \end{aligned}$$

42. (a) Using  $s = ut + \frac{1}{2} at^2$  we get

$$200 = u(2) + \frac{1}{2} a(2)^2 \quad \dots\dots(i)$$

After 4 s i.e. for  $t = 6$  s, distance covered is  $220 + 200 = 420$  cm

$$\therefore 420 = u(6) + \frac{1}{2} a(6)^2 \quad \dots\dots(ii)$$

Solving (i) and (ii)

$$a = -15 \text{ cm s}^{-2}$$

$$\text{and } u = 115 \text{ cm s}^{-1}$$

$$\begin{aligned} \therefore \text{Velocity after } 7 \text{ s} &= u + at \\ &= 115 - 15 \times 7 = 10 \text{ cm s}^{-1} \end{aligned}$$

43. (d) Using,  $v = u + at_1$ , we get,

$$t_1 = \frac{v}{\alpha} \quad (\because u = 0)$$

For retarded motion,

$$0 = v - \beta/2 \quad \text{or } t_2 = v/\beta$$

$$\text{Total time } t = t_1 + t_2 = \frac{v}{\alpha} + \frac{v}{\beta}$$

$$= v \left( \frac{\alpha + \beta}{\alpha\beta} \right) \text{ or } u = \left( \frac{\alpha\beta}{\alpha + \beta} \right) t$$

44. (b) Given,  $u = u_0 \hat{i} + (a\omega \cos \alpha t) \hat{j}$

Thus velocity along y axis,  $U_y = a \cos \alpha t$  and velocity along x axis,  $u_x = u_0$ .

Displacement at time  $t$  in horizontal direction,

$$x = \int u_0 dt = u_0 t \quad \left( Q \ v = \frac{dx}{dt} \right)$$

$$\text{and } y = \int a \cos \alpha t dt = a \sin \alpha t$$

Eliminating  $t$ ,  $y = a \sin (\alpha x/u_0)$

At time  $3\pi/2\omega$ ,  $x = u_0 (3\pi/2\omega)$

$$\text{and } y = a \sin \frac{3\pi}{2} = -a$$

Thus distance of particle from origin

$$S = \sqrt{\left[ \frac{3\pi u_0}{2\omega} \right]^2 + a^2} \quad (Q \ R = \sqrt{x^2 + y^2})$$

45. (b) Maximum acceleration during journey = slope of  $CD$

$$= \frac{DG}{CG} = \frac{60 - 20}{1 - 0.75} = \frac{40}{0.25} = 160 \text{ km hr}^{-2}$$

Distance covered in the asked interval = Area of rectangle  $KCGH$  + Area of  $\Delta CDG$ .

$$= (0.25 \times 20) + \frac{1}{2} (40 \times 0.25)$$

$$= 5 + 5 = 10 \text{ km}$$

46. (d) Here  $t = \sqrt{x} + 3$  or  $\sqrt{x} = t - 3$

$$\text{or } x = (t - 3)^2 = t^2 - 6t + 9$$

$$\frac{dx}{dt} = v = 2t - 6 = 0$$

or  $t = 3 \text{ s}$

$$3 = \sqrt{x} + 3 \text{ or } x = 0$$

47. (d) Let  $t_1$  be the time before opening of parachute.

Using  $h = ut + \frac{1}{2}gt^2$ , we get

$$19.6 = 0 + \frac{1}{2} \times 9.8 \times t_1^2$$

or  $t_1^2 = \frac{19.6}{4.9} = 4$  or  $t_1 = 2 \text{ s}$

Taking  $v_1$  as velocity attained after falling through 19.6 m and using  $v^2 - u^2 = 2gh$ , we have

$$v_1^2 - 0 = 2 \times 9.8 \times 19.6$$

Again taking  $t_2$  as time taken after opening of parachute and using  $v = u + at$ , we get

$$4.6 = 19.6 - 1 \times t_2$$

or  $t_2 = 19.6 - 4.6 = 15 \text{ s}$

$$\therefore \text{total time} = t_1 + t_2 = 2 + 15 = 17 \text{ s.}$$

48. (b) Speed of boat  $v = 5 \text{ km/h}$

Speed of boat in flowing water  $v_b = \frac{1}{1/4} = 4 \text{ km/h}$

Speed of the river  $v_r = \sqrt{v^2 - v_b^2}$

$$= \sqrt{5^2 - 4^2} = 3 \text{ km h}^{-1}$$

49. (d) Here  $t_1 = \frac{x}{2 \times 3} = \frac{x}{6}$ ;  $x_1 = 4.5 t_2$

Also  $\frac{x}{2} = 7.5 t_2 = x_1 + x_2 = 12 t_2$

or  $t_2 = \frac{x}{24}$

$$t = \frac{x}{24} + \frac{x}{6} = \frac{5x}{24}$$

$$v = \frac{x}{t} = \frac{24}{5} = 4.8 \text{ ms}^{-1}$$

50. (b) Using  $S_n = u + \frac{a}{2} (2n - 1)$  we get

$$a = u + \frac{a'}{2} (2x - 1) \quad \dots\dots(i)$$

[ $d$  = uniform acceleration]

$$b = u + \frac{a'}{2} (2y - 1) \quad \dots\dots(ii)$$

$$c = u + \frac{a'}{2} (2z - 1) \quad \dots\dots(iii)$$

or  $a = dx + \left(u - \frac{a'}{2}\right) \quad \dots\dots(iv)$

From (iv)  $ay = dxy + \left(u - \frac{a'}{2}\right)y$

and  $az = dxy + \left(u - \frac{a'}{2}\right)z$

Subtracting,  $a(y - z)$

$$= d(xy - xz) + \left(u - \frac{a'}{2}\right)(y - z)$$

Similarly,  $b(z - x)$

$$= a'(yz - yx) + \left(u - \frac{a'}{2}\right)(z - x)$$

and  $c(x - y) = d(zx - yz) + \left(u - \frac{a'}{2}\right)(x - y)$

Adding above 3 equations

$$a(y - z) + b(z - x) + c(x - y) =$$

$$a'(xy - xz + yz - yx + xz - yz) + \left(u - \frac{a'}{2}\right)$$

$$(y - z + z - x + x - y) = 0$$

51. (b) To avoid collision, the faster train should come to rest after covering a distance  $d$ . Using  $v^2 - u^2 = 2aS$  we get

$$0 - V_n^2 = 2(-a)d \quad (\text{where } V_n = v_1 - v_2)$$

or  $d = \frac{V_n^2}{2a} = \frac{(v_1 - v_2)^2}{2a}$

Collision can be avoided if

$$d > \frac{(v_1 - v_2)^2}{2a}$$

52. (a) Using  $h = ut + at^2$ , we get

$$h = 330 \times 1 - \frac{1}{2} \times 9.8 \times 1$$

$$= (330 - 4.9) \text{ m.}$$

53. (d) Initial distance between any two persons =  $d$

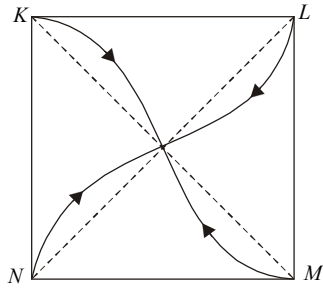


Fig. 3.106

Considering  $KL$ ,

velocity component of  $L$  along  $LK$ ,

$$v \cos 90^\circ = 0$$

Thus speed between  $K$  and  $L$  is  $v$

$$\therefore \text{time } t = \frac{d}{v}.$$

54. (a) Let  $t_1, t_2$  and  $t_3$  be the times to cover  $AB, BC$  and  $CD$

$$\text{For } AB, h = \frac{1}{2} g t_1^2$$

$$\text{For } AC, 2h = \frac{1}{2} g (t_1 + t_2)^2$$

$$\text{For } AD, 3h = \frac{1}{2} g (t_1 + t_2 + t_3)^2$$

$$\begin{aligned} \therefore t_1 : (t_1 + t_2) : (t_1 + t_2 + t_3) \\ = 1 : \sqrt{2} : \sqrt{3} \end{aligned}$$

$$\text{Thus } t_1 : t_2 : t_3 :: 1 : (\sqrt{2} - 1) : \sqrt{3} - \sqrt{2}.$$

55. (a) When the car is exactly opposite, the man at least distance, the bag will reach him when thrown with velocity  $v_b$ .

$$\sin \theta = \frac{v_b}{v_c} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\text{or } \theta = 45^\circ$$

$$\therefore \text{total angle between } v_c \text{ and } v_b \text{ is } 90 + 45 = 135^\circ$$

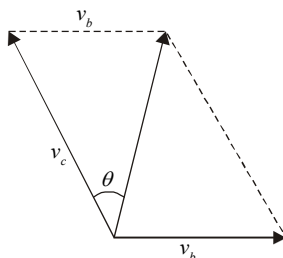


Fig. 3.107

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56. (a) Using  $v_2 - u_2 = 2gh$ , we find that when ball is dropped,  $v^2 = 2gh$  ( $u = 0$ )

The variation is **parabolic**.

At  $h = 0$ , velocity is maximum.

Now direction of velocity is reversed and goes on decreasing such that it becomes zero at  $d/2$ .

57. Superimpose on the observer and the two objects a velocity equal and opposite to the velocity of the observer. The system now reduces to what is seen by the observer.

$$64. (d) v \frac{dv}{dx} = -\alpha v$$

$$\text{or } \int_{v_0}^0 dv = -\alpha \int_0^{x_0} dx$$

$$\text{or } x_0 = v_0 / \alpha \cdot \frac{dv}{dt} = -\alpha v$$

$$\text{or } \int_{v_0}^v \frac{dv}{v} = -\alpha \int_0^t dt$$

$$\text{or } v = v_0 e^{-\alpha t} \quad \therefore v = 0 \text{ for } t \rightarrow \infty.$$

$$65. y = \beta x^2 \quad \text{or} \quad \dot{y} = 2\beta x \dot{x}$$

$$\text{or } \ddot{y} = 2\beta (\dot{x})^2 = \alpha$$

$$\text{or } \dot{x} = \sqrt{\alpha/2\beta}.$$

[ $\ddot{x} = 0$  as it has acceleration only in the  $y$ -direction.]

66. At any instant of time, let the length of the string  $BP = l_1$  and the length  $PA = l_2$ . In a further time  $t$ , let  $B$  move to the right by  $x$  and  $A$  move down by  $y$ , while  $P$  moves to the right by  $ut$ . As the length of the string must remain constant,

$$l_1 + l_2 = (l_1 - x + ut) + (l_2 + y)$$

$$\text{or } x = ut + y$$

$$\text{or } \dot{x} = u + \dot{y}.$$

$\dot{x}$  = speed of  $B$  to the right =  $v_B$ ,  $\dot{y}$  = downward speed of  $A = v_A$ .

$$\therefore v_B = u + v_A. \quad \text{Also, } \dot{v}_B = \dot{v}_A$$

$$\text{or } a_B = a_A.$$

67. Let the lengths of the sections of the string be  $BP = l_1$  and  $PA = l_2$  in the position shown. Let  $B$  move through a small horizontal distance  $x$  to  $B'$  and  $A$  move down through a distance  $y$  to  $A'$ . Length of the string

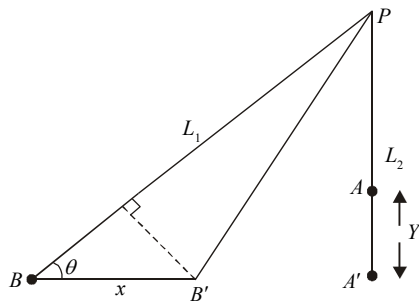


Fig. 3.108

$$= BP + PA = B'P + PA'.$$

$$l_1 + l_2 = (l_1 - x \cos \theta) + l_2 + y$$

$$\text{or } \dot{y} = \dot{x} \cos \theta$$

$$\text{or } v_A = v_B \cos \theta = u.$$

69. Projectiles with the same initial speed have the same horizontal range for complimentary angles of projection. Here,  $\theta_1 = 90 - \theta_2$ .

72.

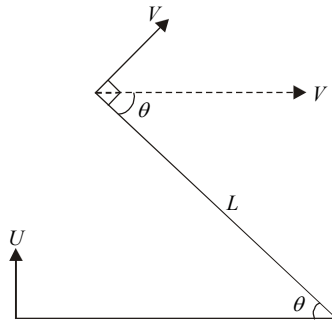


Fig. 3.109

As the block does not move, the ball moves along a circular path of radius  $l$ . The centre of mass of the system always lies somewhere on the string.

Let  $v$  = speed of ball when the string makes an angle  $\theta$  with the horizontal.

$$\frac{1}{2} mv^2 = \frac{1}{2} mu^2 - mgl \sin \theta$$

The horizontal component of  $v = V = v \sin \theta = \sin \theta \sqrt{u^2 - 2gl \sin \theta}$ .

For  $V$  to be maximum,  $\frac{dV}{d\theta} = 0$ , which gives  $\sin \theta$

$$= \frac{u^2}{3gl}.$$

73. Superimpose an upward acceleration  $a$  on the system. The box becomes stationary. The particle has an upward acceleration  $a$  and a downward acceleration  $g$ . If  $a = g$ , the particle has no acceleration and will hit  $C$ . If  $a > g$ , the particle has a net upward acceleration, and if  $a < g$ , the particle has a net downward acceleration.
74. Consider the vertical motion of the particle after entering the compartment. Let it reach the floor in time  $t$ .

$$3.2 = \frac{1}{2} (10)t^2 \quad \text{or} \quad t = 0.8 \text{ s}$$

Due to the velocity component  $u$ , which remains constant, it covers a distance of 2.4 m in 0.8 s.

$$\therefore u = \frac{2.4 \text{ m}}{0.8 \text{ s}} = 3 \text{ m/s}.$$

75. For two shells to collide in air, they must reach the same point in the same time. First shell is fired at  $\alpha$  angle and has a longer time of flight  $t$ . The second fired at  $\beta$  has less time of flight as  $\beta < \alpha$ .  $\therefore$  b and d are correct.

