9 Gravitation

BRIEF REVIEW

Newton's Law of Gravitation Newton in 1665 formulated $F \propto m_1 m_2$

$$F \propto \frac{1}{r^2} \text{ or } F = \frac{Gm_1m_2}{r^2}$$

Where $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} and is called universal gravitational constant. The value of G was first measured by Cavendish in 1736. The value of G measured for small distances ($r < 200$ m) is less by about 1% and perhaps gives an indication of a fifth natural force. Note, gravitational field is independent of the nature of medium between the masses.

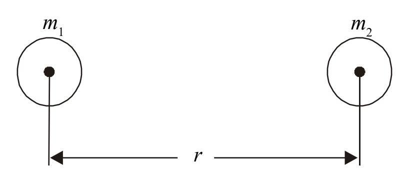


Fig. 9.1 Gravitational force between two masses

Gravitational Field Intensity Gravitational force per unit mass is called gravitational field intensity. Gravitational field intensity of earth is 'g'.

$$E_g = \frac{F}{m} = \frac{GM}{r^2} \text{ and } g = \frac{GM_e}{R_e^2}$$

Gravitational field intensity due to a ring at any point on the axial line as illustrated in Fig. 9.2 is $E_g = \frac{GMx}{(x^2+R^2)^{3/2}}$ E_g is maximum if $x = R/\sqrt{2}$.

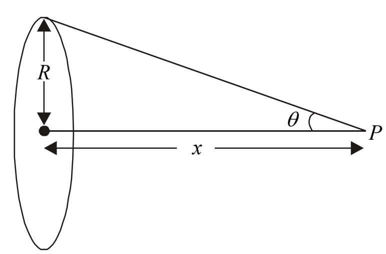


Fig. 9.2 Gravitational field intensity at a point on the axial line of ring.

Gravitational field due to a disc at any point on the axial line

$$E_g = \frac{2GM}{R^2}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right] = \frac{2GM}{R^2}[1-\cos\theta] \text{ in terms of}$$

angle θ.

Gravitational field intensity due to a shell

$$E_{g\text{ inside}} = 0, E_{g\text{ surface}} = \frac{GM}{R^2}$$

$$E_{g\text{ out}} = \frac{GM}{x^2} \quad x > R$$

See Fig. 9.3.

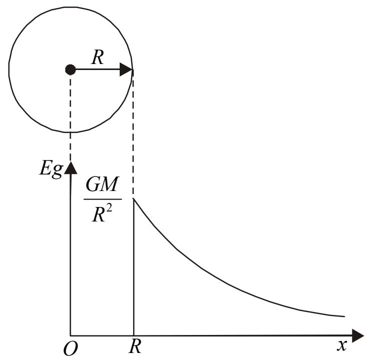


Fig. 9.3 Gravitertional field intensity due to a shell

Gravitational field intensity due to a solid sphere

$$E_{g\text{ inside}} = \frac{GMx}{R^3} \quad x < R$$

$$E_{g\text{ surface}} = \frac{GM}{R^2} \quad x = R$$

$$E_{g\text{ outside}} = \frac{GM}{R^2} \quad x > R$$

See Fig. 9.4.

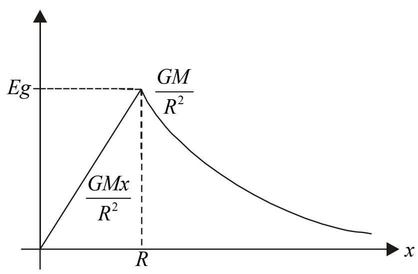


Fig. 9.4 Gravitational field intensity due to a solid sphere

Gravitational Potential (V_g) The amount of work done to bring a unit mass from infinity to that point under the influence of gravitational field of given mass M without changing the velocity $V_g = \dfrac{-GM}{r} = \int_{\infty}^{r} E_g \, .dx$

Gravitational potential due to a ring at any point on the axial line

$$V_g = \frac{-GM}{\sqrt{x^2 + R^2}}$$

Gravitational potential due to a shell

$$V_{\text{inside}} = V_{\text{surface}} = \frac{-GM}{R} \quad x \le R$$

$$V_{\text{outside}} = \frac{-GM}{x} \quad x > R$$

See Fig. 9.5.

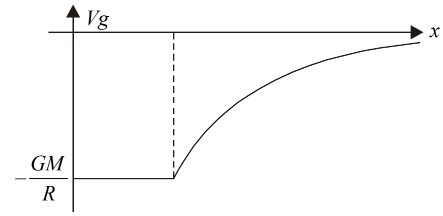


Fig. 9.5 Gravitational potential due to a shell

Gravitational potential due to a solid sphere

$$V_{\text{inside}} = \frac{-GM}{2R^3}\left[3R^2 - x^2\right] \quad x < R$$

$$V_{\text{surface}} = \frac{-GM}{R} \quad x = R \qquad \text{See Fig. 9.6}$$

$$V_{\text{outside}} = \frac{-GM}{x} \quad x > R$$

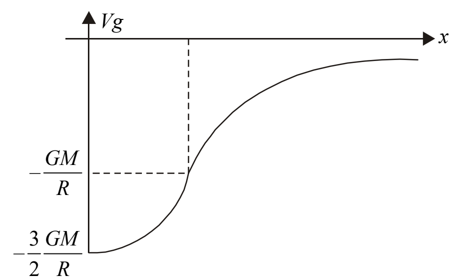


Fig. 9.6 Gravitational potential due to a solid sphere

Gravitational Potential Energy It is the amount of work done to bring a mass m from infinity to that point under the influence of gravitational field of a given mass M without changing the velocity $u_g = \dfrac{-GMm}{r}$. Note, that $u_g = mV_g$.

Work done $W = \Delta u_g$

Variation of 'g' due to height $g' = g\left(1 - \dfrac{2h}{R}\right)$ if $h << R$

$$g' = \frac{g}{\left(1 + h/R\right)^2} \quad \text{if } h \text{ is comparable to } R$$

Variation of 'g' due to depth $g' = g\left(1 - \dfrac{x}{R}\right)$ where x is depth

$g' = 0$ if $x = R$ i.e. at the centre of the earth.

Variation of 'g' with rotation of the earth lattitude

$g' = g\left(1 - \dfrac{R\omega^2}{g}\cos^2\lambda\right)$, i.e., g is maximum at the poles (where $\lambda = 90°$) and minimum at the equator (where $\lambda = 0$).

Orbital velocity $\upsilon_o = \sqrt{\dfrac{GM}{r}}$ Orbital velocity υ_o is the velocity with which a planet or a satellite moves in its orbit of radius r.

Escape velocity Escape velocity is the minimum velocity given to a body so that it escapes (from the surface of the earth/planet) from its gravitational field. $\upsilon_e = \sqrt{\dfrac{2GM}{r}}$.

Note $\upsilon_e = \sqrt{2}\ \upsilon_o$.

$$\text{Time period } T = \frac{2\pi r}{\upsilon_o} \text{ or } T^2 = \frac{4\pi^2 r^3}{GM}$$

$$KE = \frac{1}{2} m\upsilon_o^2 = \frac{GMm}{2r} \; ; PE = \frac{-GMm}{r}$$

$$\text{Total energy or Binding energy} = KE + PE = \frac{-GMm}{2r}.$$

Kepler's Laws

First law The planets revolve around the sun in the elliptical orbits with sun at one of the focus as illustrated in fig. 9.7 (a).

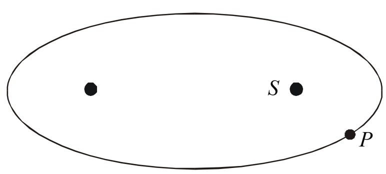


Fig. 9.7 (a) **Kepler's 1st law illustration**

Second law A line from the sun to the planet sweeps equal area in equal intervals of time as shown in fig. 9.7 (b). This law is based on conservation of angular momentum.

From Kepler's 2nd law one can easily derive

$$\frac{\upsilon_1}{\upsilon_2} = \frac{r_2}{r_1} = \frac{\upsilon_{\text{Perihelion}}}{\upsilon_{\text{aphelion}}} = \frac{r_{\text{aphelion}}}{r_{\text{Perihelion}}} \qquad \text{Fig. 9.7 (b)}$$

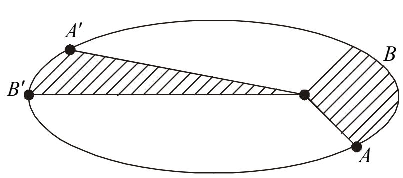
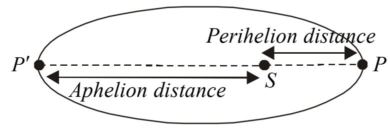


Fig. 9.7 (b) **Kepler's 2nd law illustration**

i.e., when the planet is closer to the sun it moves faster.

Closest distance of a planet from the sun is called **perihelion distance**. **Farthest** distance of the planet from the sun is called **aphelion distance**.



Closest distance of a planet from the sun is called perihelion distance. Farthest distance of the planet from the sun is called Aphelion distance.

Fig. 9.7 (c) **Perihelion and aphelion distance illustration**

Third law The square of the time period of a planet is proportional to the cube of the semimajor axis, i.e., $T^2 \propto r^3$.

If e is the eccentricity of an elliptical orbit then

r is semi-major axis.

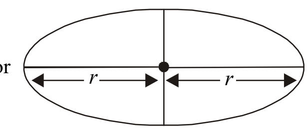


Fig. 9.7 (d) **Kepter's 3rd law illustration**

$\dfrac{r_{\text{aphelion}}}{r_{\text{Perihelion}}} = \dfrac{1+e}{1-e}$. $r_{\text{aphelion}} + r_{\text{Perihelion}} = 2r$ r being semimajor axis.

Schwarzschild radius $R_S = \dfrac{2GM}{c^2}$ where c is speed of light with radius R_S.

Event horizon The surface of the sphere with radius R_s surrounding a blackhole is called **event horizon.** Since, light cannot escape from with in this sphere, we can not see events occuring inside.

Weightlessness in a satellite $\dfrac{GMm}{r^2} - N = \left(\dfrac{GM}{r^2}\right) m$

or $N = 0$ where N is normal contact force exerted by the surface. That is in a satellite surface does not exert any force on the body. Hence, apparent weight of the body is zero.

SHORT CUTS AND POINTS TO NOTE

1. Gravitational force is only attractive force and $F = \dfrac{Gm_1m_2}{r^2}$. The force is conservative. If $r \le 10^{-8}$ m then intermolecular forces dominate. Gravitational force is the weakest of the known natural forces.

   This force does not depend on the nature of medium present in between the two masses.

2. Gravitational field intensity $E_g = \dfrac{GM}{r^2}$ is force per unit mass. Gravitation field intensity of the earth = $g = \dfrac{GM}{R^2}$.

3. Aryabhatt in 5th century AD first described that the earth revolves around the sun and the moon revolves around the earth.

4. The moon takes 27.3 days to complete one revolution around the earth. The mean radius of the orbit is 3.85×10^5 km.

5. The value of G was measured by Cavendish for the first time. The value of G is about 1 % less when distance < 200 m, indicating the possibility of a 5th natural force.

6. Gravitational field intensity due to a ring along the axial line is $E_g = \dfrac{GMx}{\left(R^2 + x^2\right)^{3/2}}$. E_g will be maximum if $x = \dfrac{R}{\sqrt{2}}$. $E_{centre} = 0$.

7. Gravitational field intensity due to a disc

$$E = \frac{2GM}{R^2}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

$$= \frac{2GM}{R^2}[1 - \cos\theta].$$

$$E_{centre} = \frac{2GM}{R^2}.$$ See Fig. 9.8.

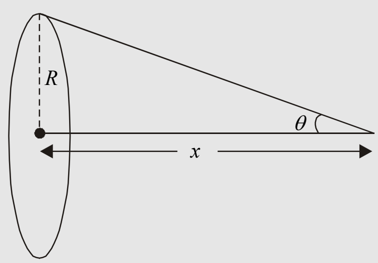


Fig. 9.8

8. Gravitational field intensity due to a shell

$E_{g,\ inside} = 0 \quad x < R$

$$E_{g,\ surface} = \frac{GM}{R^2} \quad x = R$$

$$E_{g,\ outside} = \frac{GM}{x^2} \quad x > R$$

as illustrated in Fig. 9.9

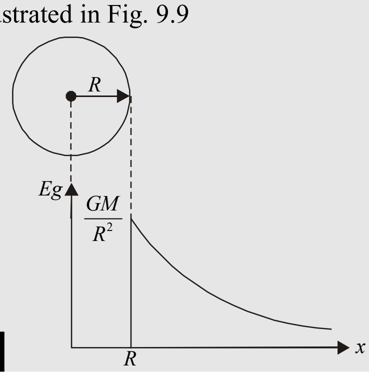


Fig. 9.9

9. Gravitational field intensity due to a solid sphere

$$E_{g,\ inside} = \frac{GMx}{R^3} \quad x < R$$

$$E_{g,\ surface} = \frac{GM}{R^2} \text{ for } x = R;$$

$$E_{g,\ outside} = \frac{GM}{x^2} \quad x > R.$$ See Fig. 9.10.

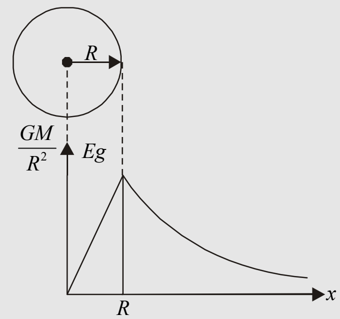


Fig. 9.10

10. Gravitational Potential $V_g = \dfrac{-GM}{r}$ due to a point mass at any point P distance r from the point mass. Negative sign shows force is attractive.

11. Gravitational Potential due to a ring at any point P on the axial line $V_g = \dfrac{-GM}{\sqrt{R^2 + x^2}}$.

12. Gravitational Potential due to a shell

$$V_{inside} = V_{surface} = \frac{-GM}{R}.$$

$$V_{outside} = \frac{-GM}{x} \quad x > R$$ See Fig. 9.11.

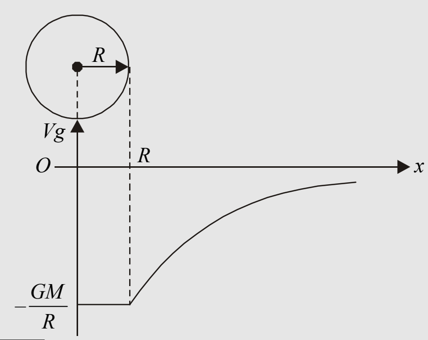


Fig. 9.11

13. Gravitational Potential due to a solid sphere

$$V_{inside} = \frac{-GM}{2R^3}\left[3R^2 - x^2\right] \quad x < R$$

$V_{surface} = \dfrac{-GM}{R}$ and $V_{out} = \dfrac{-GM}{x}$ $x > R$ See Fig. 9.12.

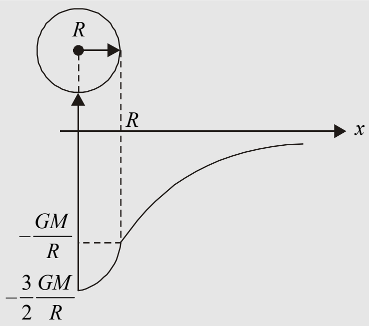


Fig. 9.12

14. Gravitational potential energy $U_g = \frac{-GMm}{r}$ $= m\,V_g$. Negative sign indicates force is attractive. Work done to raise a body of mass m to a height $= nR$ where R is radius of the earth is $\left(W = \frac{mgR}{1+\frac{1}{n}}\right)$

n could be an integer or fraction.

15. W = Δ *PE* because gravitational force is conservative, $F = \frac{-dU}{dr}$. At equilibrium $\frac{dU}{dr} = 0$.

16. Gravitational field intensity $g = \frac{GM}{R^2}$ (due to the earth) is valid upto 10 km above the surface of the earth. With height or depth g decreases.

17. Variation of g with height $g' = g\left(1 - \frac{2h}{R}\right)$ if $h < \frac{R}{10}$.

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} \text{ if } h > \frac{R}{10}.$$

Note, g, never becomes zero with height. Therefore in space we come across the term microgravity and not weightlessness. ($g \to 0$ only if $h \to \infty$).

18. Variation of g with lattitude

$g' = g\left[1 - \frac{R\omega^2\cos^2\lambda}{g}\right]$. At poles $\lambda = 90°$, $g' = g$ and is maximum.

At equator $g' = g\left(1 - \frac{R\omega^2}{g}\right)$ $\because$ $\lambda = 0$. g is minimum at equator. If earth rotates (spins) at a rate 17 times the present value the weight of a body at equator will become zero.

19. Variation of g with depth $g' = g\left(1 - \frac{x}{R}\right)$. If $x = R$, i.e., at the centre of the earth $g' = 0$. The body will becomes weightless at the centre of the earth.

20. Kepler's first law: The planets revolve around the sun in elliptical orbits with sun at one of the foci.

Second law The line joining the sun and the planet sweeps equal area in equal interval of time or the areal velocity is constant. The law is based on conservation of angular momentum and leads to

$\frac{\upsilon_1}{\upsilon_2} = \frac{r_2}{r_1}$ or $\frac{\upsilon_{\text{Perihelion}}}{\upsilon_{\text{aphelion}}} = \frac{r_{\text{aphelion}}}{r_{\text{Perihelion}}}$. See fig (9.13) if e is eccentricity of the orbit then

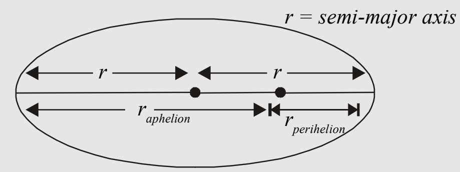


Fig. 9.13

$$\frac{r_{\text{aphelion}}}{r_{\text{Perihelion}}} = \frac{1+e}{1-e};\quad r_{\text{aphelion}} + r_{\text{Perihelion}} = 2r$$

- If $e < 1$ and $\upsilon > \upsilon_{\text{escape}}$ (or total energy $KE + PE > 0$). The path of the satellite is hyperbolic and it escapes.
- If $e < 1$, total energy is negative (< 0) or $\upsilon < \upsilon_{\text{escape}}$ the satellite moves in an elliptical path.
- If $e = 0$, total energy is negative, i.e. $\upsilon < \upsilon_{\text{escape}}$, the satellite moves in a circular path.
- If $e = 0$, total energy is zero or $\upsilon = \upsilon_e$, the satellite will acquire parabolic path.

Third law $T^2 \propto r^3$ where r is semimajor axis.

21. Orbital velocity $= \upsilon_o = \sqrt{\frac{GM}{r}}$ where r is radius of the orbit. $r = R_e + h$ for a satellite.

$\frac{\upsilon_{o1}}{\upsilon_{o2}} = \sqrt{\frac{R_e + h_2}{R_e + h_1}}$ if $\upsilon_o < \upsilon < \upsilon_e$ then the path is elliptical.

22. Escape velocity $\upsilon_e = \sqrt{\frac{2GM}{r}} = \sqrt{2}\,\upsilon_o$ i.e. if the velocity of a satellite revolving around earth (or that of a planet revolving around the sun) is increased by 41.4% then it will escape away. If $\upsilon > \upsilon_e$ satellite takes hyperbolic path and escapes from the gravitational field of the earth.

23. Time period of revolution $T^2 \propto r^3$.

$$T = \frac{2\pi r}{\upsilon} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

or $$T^2 = \frac{4\pi^2}{GM} r^3$$

24. Total energy or binding energy of a body revolving around the earth/planet or the sun is BE ($= E_{\text{Tot.}}$)

$$= KE + PE = \frac{GMm}{2r} - \frac{GMm}{r} = \frac{-GMm}{2r}$$

i.e., $KE + PE = -KE = \frac{PE}{2}$ or $PE = -2\,KE$.

25. The path of the projectiles thrown to lower height is parabolic and thrown to larger heights is elliptical.

26. Geostationary or communication satellites have circular orbit. They are situated at a height 36000 km above the surface of the earth (r = 42400 km from the centre of the earth). Minimum number of communication satellites to cover whole globe is 3 as one satellite covers nearly 41% area. Maximum number of communication satellites = 180 which can be operative at a time (at a slot of 2° each).

27. **Schwarzchild radius** It is the distance surrounding a blackhole where even the light cannot escape $R_S = \frac{2GM}{c^2}$. The surface of the sphere surrounding black hole upto a radius R_S is called event horizon. We cannot see events occuring in this region.

28. Coroilis force = 2 $m\upsilon\omega$. When a body of mass m move along a diameter with a velocity υ on a turn table rotating with angular speed ω the coroilis force is experienced.

CAUTION

1. Not remembering that gravitational field intensity depends upon shape, geometry and distance.

⇒ $E_g = 0$ at the centre of a ring $E_g = \frac{GMx}{(r^2 + x^2)^{3/2}}$ at any point on the axial line.

$E_g = 0$ inside the shell (only due to shell). The presence of other body will cause E_g.

If a part of the body is cut, E_g will vary not only due to the fact that mass has varied but also due to the fact that shape has varied.

2. Assuming gravitational potential is only a function of distance.

⇒ Like gravitational field, gravitational potential also depends upon the shape and geometry. Gravitational field inside the shell is zero but gravitational potential inside the shell is non zero and remains constantly equal to the gravitational potential at the surface.

3. Assuming work done in gravitation is $F \cdot d$

⇒ Work done = $\int F \cdot dx = \Delta PE = PE_{\text{final}} - PE_{\text{initial}}$. Do not apply $W = F \cdot d$ as force is variable.

4. Assuming g varies with height as $g' = g\left(1 - \frac{2h}{R}\right)$

⇒ $g' = g\left(1 - \frac{2h}{R}\right)$ is valid only if $h \le \frac{R}{10}$ otherwise use $g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$.

5. Assuming when the earth is closer to the sun only then summer is experienced.

⇒ Though in principle it appears correct but in case of the earth, the solar radiations are incident oblique. During winter earth is closer to the sun. That is why winter is of short duration.

6. Assuming $g = 0$ at the equator or g to be constant over the surface of the earth.

⇒ g is maximum at poles and minimum at the equator. Note, the variation is small and occurs due to outward radial force because of rotation of the earth.

7. Assuming any star will die as black hole.

⇒ Only those stars whose mass > 5 times the mass of the sun end as black hole.

8. Assuming in the relation of orbital velocity $\upsilon_o = \sqrt{\frac{GM}{r}}$, M is the mass of the satellite.

⇒ M is the mass of the earth/planet. Remember orbital velocity and escape velocity both are independent of the mass of the satellite being put into the orbit or to escape.

9. Assuming gravitational Binding energy is nothing but gravitational PE.

⇒ Binding energy = $KE + PE = -KE = ½\,PE = \frac{-GMm}{2r}$.

10. Assuming Kepler's laws can be applied to planets only.

⇒ Keplers laws can be applied to planets and satellites (artificial or natural).

11. Not remembering relations relating eccentricity e and r (semimajor axis).

⇒ $\frac{r_{\text{aphelion}}}{r_{\text{Perihelion}}} = \frac{1+e}{1-e}$ and $2r = r_{\text{aphelion}} + r_{\text{Perihelion}}$

12. Assuming shielding effect in gravitational force also.

⇒ Gravitational force does not depend upon medium.

Therefore, no medium can shield or block gravitational field.

13. Considering centripetal or centrifugal force cannot be applied on the earth.

⇒ Particles on the poles or on the axis of rotation do not have any such forces. At all other lattitudes apparent weight = $m(g - a_r) = m(g - R\,\omega^2 \cos^2 \lambda)$.

14. Assuming gravitational field inside the shell is zero always irrespective of presence of other masses outside the shell.

⇒ Gravitational field due to the shell is only zero. Refer to fig. 9.14. m is in a shell of mass M and radius R. m_1 is a mass distant x from m then force experienced by m is $\frac{Gmm_1}{x^2}$.

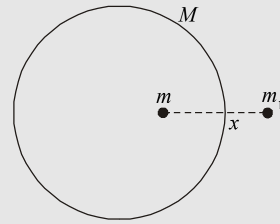


Fig. 9.14

15. Considering that escape velocity depends upon direction.

⇒ Theoretically it is independent of direction. However, practically a little dependence is observed.

16. Considering that gravitational PE is always negative.

⇒ It depends upon the reference used. So far we have assumed PE at infinity is zero. If we assume PE at the surface of the earth is zero then PE elsewhere will be +ve.

SOLVED PROBLEMS

1. For a satellite moving in an orbit around the earth, the ratio of KE to PE is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 2 (d) $\sqrt{2}$

[CBSE PMT 2005]

Solution (a) $KE = -\frac{1}{2}PE$ in a conservative bound system of forces.

2. Imagine a planet having the same density as that of the earth but radius is three times the radius of the earth. If acceleration due to gravity on the surface of the earth is g and that of the said planet is g' then

(a) $g' = \frac{g}{9}$ (b) $g' = 9g$

(c) $g' = \frac{g}{27}$ (d) $g' = 3g$

[CBSE PMT 2005]

Solution (d) $g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3 \rho}{R^2} = G\frac{4}{3}\pi R\rho$

$\because R_{Planet} = 3R$ Hence $g' = 3g$

3. Average density of the earth

(a) does not depend on g.

(b) is a complex function of g.

(c) is directly proportional to g.

(d) is inversely proportional to g.

[AIEEE 2005]

Solution (c) $g = G\frac{4}{3}\pi R\rho$

4. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of the earth. When both h and d are much smaller than the radius of earth, then which one of the following is true?

(a) $a = h/2$ (b) $d = 3h/2$

(c) $d = 2h$ (d) $h = d$

[AIEEE 2005]

Solution (c) $g' = g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$ $\therefore d = 2h$

5. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere.

(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J

(c) 6.67×10^{-9} J (d) 6.67×10^{-10} J

[AIEEE 2005]

Solution (d) $W = \Delta PE = \frac{-GMm}{\infty} - \left(\frac{-GMm}{R}\right)$

$= \frac{6.67\times10^{-11}\times100\times10\times10^{-3}}{.1}$

$= 6.67 \times 10^{-10}$ J

6. The condition for a uniform spherical mass m of radius r to be a black hole is [G = gravitational constant, g = acceleration due to gravity].

(a) $\left[\frac{2GM}{r}\right]^{1/2} \le c$ (b) $\left[\frac{2gm}{r}\right]^{1/2} = c$

(c) $\left[\frac{2GM}{r}\right]^{1/2} \ge c$ (d) $\left[\frac{gm}{r}\right]^{1/2} \ge c$

[AIIMS 2005]

Solution (c) $\left[\frac{2GM}{r}\right]^{1/2} \ge c$

7. Two planets are revolving around the earth with velocities υ_1, υ_2 and in radii r_1 and r_2 ($r_1 > r_2$) respectively. Then

(a) $\upsilon_1 = \upsilon_2$ (b) $\upsilon_1 > \upsilon_2$

(c) $\upsilon_1 < \upsilon_2$ (d) $\frac{\upsilon_1}{r_1} = \frac{\upsilon_2}{r_2}$

Solution (c) $\upsilon_o = \sqrt{\frac{GM}{r}}$ $\therefore \frac{\upsilon_1}{\upsilon_2}$

$= \sqrt{\frac{r_2}{r_1}}$ $\because r_1 > r_2 \therefore \upsilon_2 > \upsilon_1$

8. Earth is revolving around the sun if the distance of the earth from the sun is reduced to $\frac{1}{4}$th of the present distance then the present length of the day is reduced by

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{8}$ (d) $\frac{1}{6}$

Solution (c) $T^2 \propto r^3$ $\frac{T_1}{T_2} = \left(\frac{r/4}{r}\right)^{3/2} = \frac{1}{8}$

9. Helios-B spacecraft had a speed of 71 km/s when it was 4.3×10^7 km from the sun. Its orbit is

(a) circular. (b) helical.

(c) elliptical. (d) parabolic.

Solution (c) $\upsilon_o = \sqrt{\frac{GM}{R}} = \sqrt{\frac{6.67\times10^{-11}\times2\times10^{30}}{4.3\times10^7\times10^3}}$

 56 km/ssince $\upsilon > \upsilon_o$, but $< \upsilon_e$, therefore, orbit is elliptical.

10. Find the weight of an object at neptune which weighs 19.6 N on the earth. Mass of Neptune = 10^{26} kg, radius $R = 2.5 \times 10^4$ km and rotates once around its axis in 16 h.

(a) 19.6 N (b) 20.0 N

(c) 20.4 N (d) 20.8 N

Solution (d) Weight $W = \frac{GMm}{R^2}$

$= \frac{6.67\times10^{-11}\times10^{26}\times2}{(2.5\times10^7)^2} = 20.8$ N.

11. An earth's satellite moves in a circular orbit with an orbital speed 6280 ms^{-1}. Find the time of revolution.

(a) 130 min (b) 145 min

(c) 155 min (d) 175 min

Solution (d) $\upsilon_o = \sqrt{\frac{GM}{r}}$ or $r = \frac{GM}{\upsilon_o^2}$ and

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}} = \frac{2\pi GM}{\upsilon_o^3}.$$

$$T = \frac{2\pi\times6.67\times10^{-11}\times6\times10^{24}}{(6280)^2\times6280\times60} = 175 \text{ min.}$$

12. A mass m_1 is placed at the centre of a shell of radius R, m_2 is placed at a distance R from the surface and is immersed in an oil of dielectric constant 10. Find the force on m_1

(a) zero (b) $\frac{G(M+m_2)m_1}{10R^2}$

(c) $\frac{Gm_1m_2}{4R^2}$ (d) $\frac{Gm_1(4M+m_2)}{4R^2}$

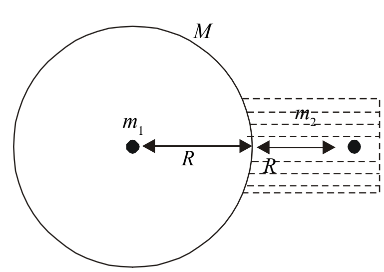


Fig. 9.15

Solution (c) $F = \dfrac{Gm_1m_2}{(2R)^2}$ shell exerts no force as its gravitational field inside the shell is zero. Oil does not play any role.

13. A 75 kg astronaut is repairing Hubble telescope at a height of 600 km above the surface of the earth. Find his weight there.

(a) 740 N (b) 700 N

(c) 650 N (d) 610 N

Solution (d) $mg' = mg\left(1-\dfrac{2h}{R}\right)$

$= 75 \times 10\,(1 - \cdot185) = 610$ N

14. 5 kg and 10 kg spheres are 1 m apart. Where the gravitation field intensity be zero from 5 kg block.

(a) 0.4 m (b) 0.3 m

(c) 0.25 m (d) 0.35 m

Solution (a) $\dfrac{GM_1}{x^2} = \dfrac{GM_2}{(1-x)^2}$ or $\dfrac{1-x}{x} = \dfrac{\sqrt{2}}{1}$

or $x = \dfrac{1}{\sqrt{2}+1} = 0.4$ m

15. A Ring has mass M, radius R. A point mass m is placed at a distance x on the axial line as shown. Find x so that force experienced is maximum.

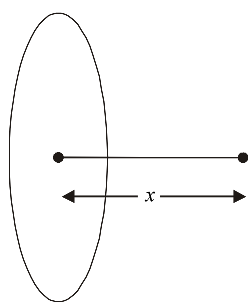


Fig. 9.16

(a) $R/3\in$ (b) $R/2$

(c) $R/\sqrt{2}$ (d) $R/\sqrt{3}$

Solution (c) $\dfrac{d}{dx}\left[\dfrac{GMmx}{\left(x^2+R^2\right)^{3/2}}\right] = 0$ or $x = R/\sqrt{2}$

16. A sattelite is in sufficiently low orbit so that it encounter air drag and if orbit changes from r to $r - \Delta r$. Find the change in orbital velocity and change in PE.

(a) $\dfrac{\Delta r}{2}\sqrt{\dfrac{GM}{r^3}}, \dfrac{GMm\Delta r}{r^2}$ (b) $\dfrac{\Delta r}{2}\sqrt{\dfrac{GM}{r^2}}, \dfrac{GMm}{r}$

(c) $\dfrac{\Delta r}{2}\sqrt{\dfrac{GM}{r^3}}, \dfrac{GMm\Delta r}{2r^2}$ (d) none

Solution (a) $\upsilon_o = \sqrt{\dfrac{GM}{r}}$; $\upsilon_o' = \sqrt{\dfrac{GM}{r-\Delta r}}$

$$= \sqrt{\frac{GM}{r}}\left(1-\frac{\Delta r}{r}\right)^{-1/2} = \sqrt{\frac{GM}{r}}\left(1+\frac{\Delta r}{2r}\right).$$

$$\Delta\upsilon = \upsilon_o' - \upsilon_o = \frac{\Delta r}{2r}\sqrt{\frac{GM}{r}}.$$

$$\Delta U = -2\Delta KE = -2\left(\frac{1}{2}m\right)\left[\upsilon_o'^2 - \upsilon_o^2\right]$$

$$= m\left[\frac{GM}{r-\Delta r} - \frac{GM}{r}\right] = \frac{GMm}{r}\left[\frac{\Delta r}{r}\right].$$

$$= \frac{GMm\Delta r}{r^2}$$

17. When an object is in a circular orbit of radius r, its time period of revolution about the earth is T and orbital velocity is υ when its orbit is $r + \Delta r$. Find the change in time period ΔT and orbital velocity $\Delta\upsilon$.

(a) $\dfrac{3\pi\Delta r}{\upsilon}, \dfrac{\pi\Delta r}{T}$ (b) $\dfrac{2\pi\Delta r}{\upsilon}, \dfrac{\pi\Delta r}{2T}$

(c) $\dfrac{2\pi\Delta r}{3\upsilon}, \dfrac{2\pi\Delta r}{T}$ (d) none

Solution (a) $\upsilon_o = \sqrt{\dfrac{GM}{r}}$, $\upsilon_o - \Delta\upsilon = \sqrt{\dfrac{GM}{r+\Delta r}}$

$$= \sqrt{\frac{GM}{r}}\left(1+\frac{\Delta r}{r}\right)^{-1/2} \text{ or } \Delta\upsilon = \sqrt{\frac{GM}{r}}\,\frac{\Delta r}{2r}$$

$$= \upsilon_o\frac{\Delta r}{2r}$$

$$= \frac{2\pi r}{T}\,\frac{\Delta r}{2r} = \frac{\pi\Delta r}{T}$$

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}, \text{ and, } T' = \frac{2\pi(r+\Delta r)^{3/2}}{\sqrt{GM}}$$

$$= \frac{2\pi r^{3/2}}{\sqrt{GM}}\left(1+\frac{\Delta r}{r}\right)^{3/2}$$

or $$T' = T\left(1+\tfrac{3}{2}\frac{\Delta r}{r}\right)$$

$$\Delta T = \frac{3T\Delta r}{2r} = \frac{3}{2}\frac{(2\pi r)}{\upsilon}\frac{\Delta r}{r} = \frac{3\pi\Delta r}{\upsilon}$$

18. Mass M is distributed uniformly over a rod of length L. Find the gravitational field at P at a distance a on perpendicular bisector.

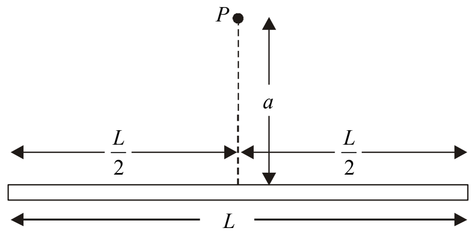


Fig. 9.17 (a)

(a) $\dfrac{2GM}{a(L^2+a^2)}$ (b) $\dfrac{2GM}{a(L^2+4a^2)^{1/2}}$

(c) $\dfrac{GM}{2a(L^2+4a^2)}$ (d) none

Solution (b) $dm = \dfrac{M}{L}\,dx$

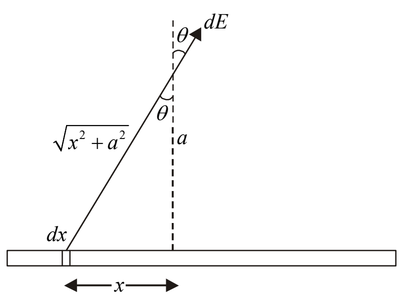


Fig. 9.17 (b)

$$dE = \frac{G\frac{M}{L}dx}{(x^2+a^2)} \quad dE_{net} = dE\cos\theta = \frac{GMa\,dx}{L(x^2+a^2)^{3/2}}$$

$$E = \int dE\cos\theta = \frac{GMa}{L}\int_{-L/2}^{L/2}\frac{dx}{(x^2+a^2)^{3/2}}$$

Put $x = a\tan\theta$ then $dx = a\sec^2\theta\,d\theta$

$$E = \frac{GMa}{L}\int\frac{a\sec^2\theta d\theta}{a^3\sec^3\theta} = \frac{GM}{La}\int\cos\theta d\theta$$

$$= \frac{GM}{La}\sin\theta = \left.\frac{GMx}{La\sqrt{x^2+a^2}}\right|_{-L/2}^{L/2}.$$

$$= \frac{GM(2L)}{aL(L^2+4a^2)^{1/2}} = \frac{2GM}{a(L^2+4a^2)^{1/2}}$$

19. How much work will be done to take a space craft orbiting at 200 km above the surface of earth to an orbit at 4000 km above the surface of the earth. Assume circular orbit. Mass of spacecraft equal to 2000 kg.

(a) 0.83×10^{10} J (b) 1.23×10^{10} J

(c) 1.53×10^{10} J (d) 1.83×10^{10} J

Solution (d) $W = \Delta PE + \Delta KE = PE_{final} - PE_{initial} + KE_{final} - KE_{initial}$

$$= KE_{initial} - KE_{final} = \frac{GMm}{10^3\times}\left[\frac{1}{8400} - \frac{1}{10400}\right]$$

$$= \frac{6.67\times10^{-11}\times6\times10^{24}\times2000}{10^3}$$

$$\left[\frac{2000}{8400\times10400}\right]$$

$$= \frac{40\times10^{14}}{21\times10^4} = 1.83\times10^{10}\text{ J}$$

20. A comet travels around the sun in elliptical orbit. Its mass is 10^8 kg when 2.5×10^{11} m away its speed is 2×10^4 ms^{-1}. Find the change in KE when it has reached 5×10^{10} m away from the sun.

(a) 38×10^8 J (b) 48×10^8 J

(c) 58×10^8 J (d) 56×10^8 J

Solution (b) $\upsilon_1 r_1 = \upsilon_2 r_2$ or

$$\upsilon_2 = \frac{2\times10^4\times2.5\times10^{11}}{5\times10^{10}} = 10^5$$

$$\Delta KE = \frac{1}{2}\times10^8[(10^5)^2 - 4\times10^8]$$

$$= 48\times10^8\text{ J}$$

21. Gravitational field in a region is given by $(3\hat{i} + 2\hat{j})$ N kg^{-1}. Find the work done by the gravitational field when a particle of mass m moves from one point (x_1, y_1) to another (x_2, y_2) on the line $2y + 3x = 5$

(a) zero (b) $9(x_2 - x_1) + 4(y_2 - y_1)$

(c) $9(x_1 - x_1) + 4(y_1 - y_2)$ (d) none

Solution (a) m_1 (slope of the line) $= \dfrac{-3}{2}$

Slope of gravitational field $m_2 = \dfrac{\text{Coeff. of } j}{\text{Coeff. of } i} = \dfrac{2}{3}$

$\because m_1 m_2 = -1$ i.e. line and field are perpendicular.

Hence, work done = 0

22. A body is fired from the surface of the earth. It goes to a maximum height R (radius of the earth) from the surface of the earth. Find the initial velocity given.

(a) 6.9 km s^{-1} (b) 7.4 km s^{-1}

(c) 7.9 km s^{-1} (d) 8.4 km s^{-1}

Solution (c) $\frac{1}{2}\, m\upsilon^2 = GMm\left[\frac{1}{R}-\frac{1}{2R}\right]$

or $\upsilon = \sqrt{\frac{GM}{R}}$

$= \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{6400\times10^{3}}} = 7.9\text{ km s}^{-1}.$

23. Find the height above the surface of the earth where weight becomes half.

(a) $\frac{R}{2}$ (b) $(\sqrt{2}-1)\ R$

(c) $\frac{R}{(\sqrt{2}+1)}$ (d) $\frac{R}{\sqrt{2}}$

Solution (b) $\frac{1}{2} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$ or $h = (\sqrt{2}-1)\ R$

24. A pendulum clock which keeps correct time at the surface of the earth is taken into a mine then

(a) it keeps correct time. (b) it gains time.

(c) it loses time. (d) none of these.

Solution (c) $T = 2\pi\sqrt{\frac{l}{g}}$ as g decreases, T increases. $\therefore$ it loses time.

25. Find the velocity of the earth at which it should rotate so that weight of a body becomes zero at the equator.

(a) 1.25 rad s^{-1} (b) 1.25×10^{-1} rad s^{-1}

(c) 1.25×10^{-2} rad s^{-1} (d) 1.25×10^{-3} rad s^{-1}

Solution (d) $g' = g\left(1-\frac{R\omega^2}{g}\right) = 0$ or $\omega = \sqrt{\frac{g}{R}}$

$= \sqrt{\frac{10}{6400\times10^3}} = \frac{1}{800} = 1.25\times10^{-3}\text{ rad s}^{-1}$

26. The radius of a planet is R_1 and a satellite revolves around it in a radius R_2 Time period of revolution is T. Find the acceleration due to gravity.

(a) $\frac{4\pi^2R_2^3}{R_1^2T^2}$ (b) $\frac{4\pi^2R_2^2}{R_1T^2}$

(c) $\frac{2\pi^2R_2^3}{R_1T^2}$ (d) $\frac{4\pi^2R_2}{T^2}$

Solution (a) $T = \frac{2\pi R_2^{3/2}}{\sqrt{GM}}$ or $GM = \frac{4\pi^2R_2^3}{T^2}$

and $g = \frac{GM}{R_1^2} = \frac{4\pi^2R_2^3}{R_1^2T^2}.$

27. A pendulum having a bob of mass m is hanging in a ship sailing along the equator from east to west. When the ship is stationary with respect to water, the tension in the string is T_o. Find the difference between tensions when the ship is sailing with a velocity υ.

(a) $m\upsilon\omega$ (b) $2\,m\upsilon\omega$

(c) $\frac{m\upsilon\omega}{2}$ (d) $\sqrt{2}\ m\upsilon\omega$

Solution (b) $2\ m\upsilon\omega$ Additional force is coroilis force which acts perpendicular to the plane of motion.

28. A solid sphere of mass m and radius r is placed inside a hollow thin spherical shell of mass M and radius R as shown in Fig 9.18. A particle of mass m' is placed on the line joining the two centres at a distance x from the point of contact of the sphere and shell. Find the magnitude of force (gravitational) on this particle when $2r < x < 2R$.

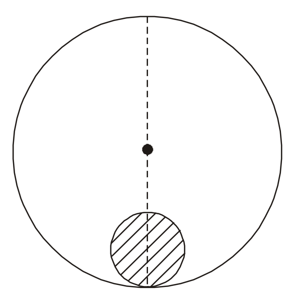


Fig. 9.18

(a) $\frac{Gmm'}{x^2}$ (b) $\frac{Gmm'}{(x-r)^2}$

(c) $\frac{Gmm'}{x^2}+\frac{GmM}{x^2}$ (d) $\frac{Gmm'}{(x-r)^2}+\frac{GMm'}{(x-R)^2}$

Solution (b) $\frac{Gmm'}{(x-r)^2}$

29. A satellite is in a circular orbit of radius r. At some point it is given an impulse along its direction of motion causing its velocity to increase n times. It now goes into an elliptical orbit with the planet at the centre of the ellipse. The maximum possible value of n could be

(a) $\sqrt{2}$ (b) 2

(c) $\sqrt{2}+1$ (d) $\dfrac{1}{\sqrt{2}-1}$

Solution (a) For satellite to revolve around the planet $\upsilon_o \le \upsilon_e$

and $\upsilon_e = \sqrt{2}\,\upsilon_o \quad \therefore n \le \sqrt{2}$

30. A dense sphere of mass M is placed at the centre of a circle of radius R. Find the work done when a particle of mass m is brought from A to B along a circle as shown in Fig 9.19.

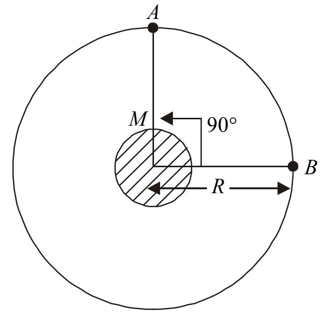


Fig. 9.19

(a) zero (b) $\dfrac{GMm}{R}$

(c) $-\dfrac{GMm}{R}$ (d) $\dfrac{2GMm}{R}$

Solution (a) $W = \Delta PE = 0$

TYPICAL PROBLEMS

31. A, B, C and D are four masses each of mass m lying on the vertices of a square of side 'a'. They always move along a common circle with velocity υ. Find υ so that they always remain on the vertices of the square.

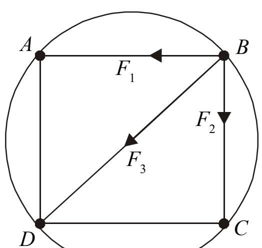


Fig. 9.20

(a) $\sqrt{\dfrac{GM(2\sqrt{2}+1)}{2\sqrt{2}a}}$ (b) $\sqrt{\dfrac{GM(\sqrt{2}+1)}{2a}}$

(c) $\sqrt{\dfrac{GM\sqrt{2}(2+1)}{a}}$ (d) none

$|F_1| = |F_2| \; \therefore \; |\vec{F}_1 + \vec{F}_2| = \sqrt{2}\,F_1$ (See Fig Q.3)

Solution (a) $F_{net} = \sqrt{2}\,F_1 + F_3 = \dfrac{m\upsilon^2}{R}$

$$\sqrt{2}\,a = 2R \text{ or } R = \frac{a}{\sqrt{2}}$$

$$\sqrt{2}\,\frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2} = \frac{m\upsilon^2}{a}\,\sqrt{2}$$

or $$\upsilon = \sqrt{\frac{GM(2\sqrt{2}+1)}{2\sqrt{2}a}}$$

32. A planet of mass m moves along an ellipse so that perihelion and aphelion distances are r_1 and r_2. Find the angular momentum of the planet.

(a) $m\sqrt{\dfrac{GM_S r_1 r_2}{r_1+r_2}}$ (b) $m\sqrt{\dfrac{2GM_S r_1 r_2}{r_1+r_2}}$

(c) $m\sqrt{GM_S(r_1+r_2)}$ (d) $m\sqrt{2GM_S(r_1+r_2)}$

Solution (b) $m\upsilon_1 r_1 = m\upsilon_2 r_2$ or $\upsilon_1^2 = \upsilon_2^2\left(\dfrac{r_2}{r_1}\right)^2$

Using energy conservation $\dfrac{-GmM_S}{r_1} + \dfrac{m\upsilon_1^2}{2}$

$$= \frac{-GmM_S}{r_2} + \frac{m\upsilon_2^2}{2}$$

or $$\frac{-GM_S}{r_1} + \frac{-GM_S}{r_1} = \frac{\upsilon_2^2}{2} - \frac{\upsilon_2^2}{2}\left(\frac{r_2}{r_1}\right)^2$$

or $$\upsilon_2 = \sqrt{\frac{2GM_S r_1}{r_2(r_1+r_2)}}$$

and $$L = m\upsilon_2 r_2 = m\sqrt{\frac{2GM_S r_1 r_2}{r_1 + r_2}}$$

33. At what height over the earth's pole the freefall acceleration decreases by 1%

(a) 64 km (b) 16 km
(c) 8 km (d) 32 km

Solution (d) $g' = g\left(1 - \frac{2h}{R}\right)$ $\frac{2h}{R} = \frac{1}{100}$ or $h = 32$ km

34. A satellite of moon revolves around it in a radius n times the radius of moon (R). Due to cosmic dust it experiences a resistance $F = \alpha\upsilon^2$. Find how long it will stay in the orbit.

(a) $\frac{m}{\alpha\sqrt{\frac{GM}{R}}}\sqrt{n}$ (b) $\frac{m}{\alpha}\sqrt{\frac{R}{am}}\,(\sqrt{n} - 1)$

(c) $\frac{m}{\alpha}\frac{(\sqrt{n}-1)}{\upsilon}$ (d) $\frac{m}{\alpha}\frac{\upsilon_i}{\upsilon_f^2}$

Solution (b) $m\frac{dv}{dt} = \alpha v^2\, dt$ or $\frac{dv}{v^2} = \frac{\alpha dt}{m}$

and $$\upsilon = \sqrt{\frac{GM}{r}}.$$

We know $\upsilon_i = \sqrt{\frac{GM}{nR}}$ and $\upsilon_f = \sqrt{\frac{GM}{R}}$

$$\therefore \int_{v_i}^{v_f}\frac{dV}{V^2} = \int\frac{\alpha dt}{m}$$

or $$t = \frac{m}{\alpha}\left[\frac{1}{\upsilon_i} - \frac{1}{\upsilon_f}\right] = \frac{m}{\alpha\sqrt{\frac{GM}{R}}}[\sqrt{n} - 1]$$

35. A particle is projected with a velocity 15 km s^{-1}. Find its velocity in the space far off from the earth.

(a) 3.8 km s^{-1} (b) 7.6 km s^{-1}
(c) 10 km s^{-1} (d) 11.2 km s^{-1}

Solution (c) $\upsilon_f^2 = \upsilon_i^2 - \upsilon_e^2 = 15^2 - (11.2)^2$ or $\upsilon_f = 10$ km s^{-1}

36. Find the minimum velocity to be imparted to a body so that it escapes the solar system

(a) $\sqrt{\frac{2GM_E}{R_E} + \frac{2GM_S}{R_{SE}}}$

(b) $\sqrt{\frac{2GM_E}{R_E} + (\sqrt{2}-1)^2\frac{GM_S}{R_{SE}}}$

(c) $\sqrt{(\sqrt{2}-1)^2\left[\frac{GM_E}{R_E} + \frac{GM_S}{R_{SE}}\right]}$ (d) none

Solution (b) $\frac{1}{2}m\,\upsilon_3^2 = \frac{1}{2}m\,\upsilon_1^2 + \frac{1}{2}m\,(\Delta\upsilon)^2$

or $$\upsilon_3 = \sqrt{\upsilon_1^2 + (\sqrt{2}-1)^2\upsilon_2^2}$$

where $\upsilon_1 = \sqrt{\frac{2GM_E}{R_E}}$ and $\upsilon_2 = \sqrt{\frac{2GM_S}{R_{SE}}}$

R_{SE} is the orbital distance of earth from the sun.
R_E is the radius of the earth.

37. A tunnel is dug along a chord of the earth at a perpendicular distance $\frac{R}{3}$ from the earth's centre. Assume wall of the tunnel is frictionless. Find the force exerted by the wall on mass m at a distance x from the centre of the tunnel.

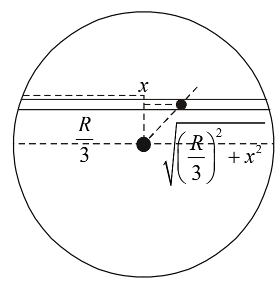


Fig. 9.21

(a) $\frac{mg\sqrt{\frac{R^2}{9} + x^2}}{R}$ (b) $\frac{mg\ x}{\sqrt{R^2/a + x^2}}$

(c) $\frac{mg}{3}$ (d) $\frac{mg\,x}{R}$

Solution (c) $F = mg' = mg\left[1 - \frac{\sqrt{\frac{R^3}{9} + x^2}}{9}\right]$

$$= \frac{mg\sqrt{R^2/9 + x^2}}{R}\cdot\frac{R/3}{\sqrt{R^2/9 + x^2}} = \frac{mg}{3}$$

38. A body weighs 1 kg by a spring balance at the north pole. What will it weigh at the equator?

(a) 0.977 kg (b) 0.967 kg
(c) 0.987 kg (d) 0.997 kg

Solution (d) $mg' = mg\left[1-\frac{R\omega^2}{g}\right]$

$$= 1\left[1-\frac{6400\times10^3\times(2\pi)^2}{10\times(3600\times24)^2}\right] = 0.997\text{ kg}$$

40. Let V_G and E_G denote gravitational potential and field respectively, then it is possible to have

(a) $V_G = 0, E_G = 0$ (b) $V_G \neq 0, E_g = 0$

(c) $V_G = 0, E_g \neq 0$ (d) $V_G \neq 0, E_G \neq 0$

Solution (a, b, c, d). all

41. Which of the following quantities remain constant in a planetary system when seen from the surface of the sun.

(a) KE (b) angular speed

(c) speed (d) angular momentum

(e) binding energy

Solution (d) & (e)

42. The gravitational potential (V_G) and gravitational field (E_g) are plotted against distance r from the centre of a uniform spherical shell. Consider the following statement

(A) The plot of V_G against r is discontinuous.

(B) The plot of E_G against r is discontinuous.

(a) both A and B are correct.

(b) both A and B are wrong.

(c) A is correct but B is wrong.

(d) B is correct but A is wrong.

Solution (d)

43. Two satellites X and Y move round the earth in the same orbit. The mass of B is twice that of A. Then

(a) $\upsilon_A = \upsilon_B$ (b) $KE_A = KE_B$

(c) $(KE+PE)_A = (KE+PE)_B$ (d) $PE_A = PE_B$

Solution (a)

44. Find the work done to take a particle of mass m from surface of the earth to a height equal to $2R$.

(a) $2\,mg\,R$ (b) $\frac{mgR}{2}$

(c) $3\,mg\,R$ (d) $\frac{2mgR}{3}$

Solution (d) $W = \Delta PE = G\,M\,m\left[\frac{1}{R}-\frac{1}{3R}\right]$

$$= \frac{2GMm}{3R} = \frac{2}{3}\,gm\,R.$$

45. Find the height at which the weight will be same as at the same depth from the surface of the earth.

(a) $\frac{R}{2}$ (b) $\sqrt{5}\ R-R$

(c) $\frac{\sqrt{5}R-R}{2}$ (d) $\frac{\sqrt{3}R-R}{2}$

Solution (c) $\frac{g}{\left(1+\frac{x}{R}\right)^2} = g\left(1-\frac{x}{R}\right)$

or $\left(1-\frac{x}{R}\right)\left(1+\frac{x^2}{R^2}+\frac{2x}{R}\right) = 1$

or $\frac{x^3}{R^3}+\frac{x^2}{R^2}-\frac{x}{R} = 0 = \frac{x}{R}\left(\frac{x^2}{R^2}+\frac{x}{R}-1\right)$

or $\frac{x}{R} = \frac{-1\pm\sqrt{1+4}}{2}$ or $x = \frac{\sqrt{5}R-R}{2}$

46. Velocity of a satellite is $\upsilon_o < \upsilon < \upsilon_e$ then

(a) its orbit is open.

(b) its orbit is closed and circular.

(c) its orbit is closed and parabolic.

(d) its orbit is closed and elliptical.

(e) none.

Solution (d)

PASSAGE 1

Read the following passage and answer the questions given at the end.

In 1783, John Mitchell noted that if a body having same density as that of the sun but radius 500 times that of the sun, magnitude of its escape velocity will be greater than c, the speed of light. All the light emitted by such a body will return to it. He, thus, suggested the existence of a black hole.

$\upsilon = c = \sqrt{\frac{2GM}{R}}$ suggests that a body of mass M will act as a black hole if its radius R is less than or equal to a certain critical radius. Karl Schwarzchild, in 1926, derived the expression for the critical radius R_S called Schwarzchild radius. The surface of the sphere with radius R_S surrounding a black hole is called event horizon.

1. Find R_S, Schwarzchild radius.

(a) $> \frac{2GM}{c^2}$ (b) $= \frac{GM}{c^2}$

(c) $< \frac{2GM}{c^2}$ (d) $= \frac{2GM}{c^2}$

2. What is density of the sun?

(a) 14.1 kg m^{-3} (b) 141.1 kg m^{-3}

(c) 1410 kg m^{-3} (d) none

3. To make black hole with density of the sun, the radius of object should be ________ Radius of the sun.

(a) 5 times (b) 50 times

(c) 500 times (d) 2.5 times

4. What is event horizon?

(a) where events can be seen.

(b) where events can not be seen.

(c) where events do not occur.

(d) none.

Solution 1. (d) $R_S = \frac{2GM}{C^2}$

Solution 2. (c) $r = \frac{M_S}{\frac{4}{3}\pi R^3} = \frac{2\times10^{30}}{\frac{4}{3}\pi(6.96\times10^8)^3}$

$= 1410 \text{ kg m}^{-3}$

Solution 3. (c) 500 times radius of sun.

Solution 4. (d) The surface of the sphere with radius R_S surround a black hole is called event horizon. Since, light cannot escape events in this region cannot be seen.

PASSAGE 2

Read the following passage and answer the questions given at the end.

We can learn a lot about planetary motion by considering the special case of circular orbits. We shall neglect the forces between the planets, considering only interaction between the sun and a given planet. These consideration apply equally well to the motion of a satellite (natural or artificial) about a planet.

Consider two spherical bodies of mass M and m moving in circular orbits under the influence of each others gravitational attraction. The COM of this system lies along the line joining them at point C such that $mr = MR$. If no external force acts on the system then COM has no acceleration. Let us choose C as the origin of our reference frame. The large body of mass M moves in an orbit of constant radius R and the small body of mass m moves in an orbit of constant radius r both having same angular velocity ω. In order for this to happen, the gravitational force acting on each body must provide the necessary centripetal acceleration.

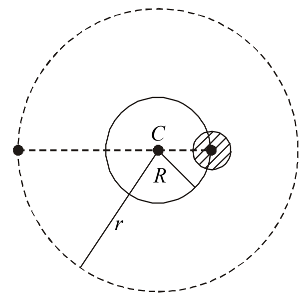


Fig. 9.22

1. COM of the system described above is

(a) invariant when seen in its own frame of reference with origin fixed at C.

(b) invariant when seen through any frame of reference.

(c) variant as circular motion require an external force (centripetal force)

(d) variant as both the masses are moving in their orbits with nothing at the common centre.

Solution (b) As net force is zero

2. The centripetal forces in the above system

(a) cannot exist

(b) can exist as action reaction pair

(c) can be complementary to one another

(d) should be controlled by an external agency

Solution (b)

3. Kepler's third law __________ in the binary system mentioned in the paragraph

(a) is valid

(b) is invalid as at the centre no mass like the sun exists.

(c) may or may not be applied in a binary system of this kind.

(d) No information about Kepler law is mentioned.

Solution (a)

PASSAGE 3

Read the following passage and answer the questions given at the end.

Satellites are natural or man-made objects which revolve around a huge object, such as, Earth or any other planet. Moon is a natural satellite of Earth. Several artificial satellites have been launched by various countries to orbit the Earth.

India has also launched various satellites, the latest one being the INSAT—1B satellite which is being used to beam television signals across the country, for accurate meteorological predictions and several other useful purposes. INSAT—1B is a geo-stationary satellite.

1. The satellite INSAT—1B was launched
(a) from the SHAR in Nellore.
(b) from a cosmodrome in U.S.S.R.
(c) from French Guayana.
(d) a board the U.S. space shuttle challenger.

Solution (d)

2. Which country launched the first artificial satellite in space ?
(a) U.S.S.R. (b) U.S.A.
(c) U.K. (d) France

Solution (a)

3. The velocity of an earth satellite is
(a) constant.
(b) maximum at perigee.
(c) maximum at apogee.

Solution (b)

QUESTIONS FOR PRACTICE

1. The rotation of the earth about its axis speeds up such that a man on the equator becomes weightless. In such a situation, what would be the duration of one day?

(a) $2\pi\sqrt{R/g}$ (b) $\frac{1}{2\pi}\sqrt{R/g}$

(c) $2\pi\sqrt{Rg}$ (d) $\frac{1}{2\pi}\sqrt{Rg}$

2. Two identical trains A and B move with equal speeds on parallel tracks along the equator. A moves from east to west and B, from west to east. Which train will exert greater force on the tracks?
(a) A (b) B
(c) They will exert equal force.
(d) The mass and the speed of each train must be known to reach a conclusion.

3. An object is weighed at the North Pole by a beam balance and a spring balance, giving readings of W_B and W_S respectively. It is again weighed in the same manner at the equator, giving readings of W_B' and W_S' respectively. Assume that the acceleration due to gravity is the same everywhere and that the balances are quite sensitive.
(a) $W_B = W_S$ (b) $W_B' = W_S'$
(c) $W_B = W_B'$ (d) $W_S' = W_S$

4. Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles. An object weighed by a spring balance gives the same reading at the equator as at a height h above the poles ($h << R$). The value of h is

(a) $\frac{\omega^2 R^2}{g}$ (b) $\frac{\omega^2 R^2}{2g}$

(c) $\frac{2\omega^2 R^2}{g}$ (d) $\frac{\sqrt{Rg}}{\omega}$

5. Use the assumptions of the previous question. An object weighed by a spring balance at the equator gives the same reading as a reading taken at a depth d below the earth's surface at a pole ($d << R$). The value of d is

(a) $\frac{\omega^2 R^2}{g}$ (b) $\frac{\omega^2 R^2}{2g}$

(c) $\frac{2\omega^2 R^2}{g}$ (d) $\frac{\sqrt{Rg}}{\omega}$

6. A binary star is a system of two stars rotating about their centre of mass only under their mutual gravitational attraction. Let the stars have masses m and $2m$ and let their separation be l. Their time period of rotation about their centre of mass will be proportional to
(a) $l^{3/2}$ (b) l
(c) $m^{1/2}$ (d) $m^{-1/2}$

7. Three point masses, m each are at the corners of an equilateral triangle of side a. Their separations do not change when the system rotates about the centre of the triangle. For this, the time period of rotation must be proportional to
(a) $a^{3/2}$ (b) a
(c) m (d) $m^{-1/2}$

8. For a planet moving around the sun in an elliptical orbit, which of the following quantities remain constant?
(a) The total energy of the 'sun plus planet' system.
(b) The angular momentum of the planet about the sun.
(c) The force of attraction between the two.
(d) The linear momentum of the planet.

9. The escape velocity for a planet is v_e. A particle starts from rest at a large distance from the planet, reaches the planet only under gravitational attraction, and passes through a smooth tunnel through its centre. Its speed at the centre of the planet will be

(a) v_e (b) $1.5\ v_e$

(c) $\sqrt{1.5}\ v_e$ (d) $2\ v_e$

10. The escape velocity for a planet is v_e. A particle is projected from its surface with a speed v. For this particle to move as a satellite around the planet,

(a) $\frac{v_e}{2} < v < v_e$ (b) $\frac{v_e}{\sqrt{2}} < v < v_e$

(c) $v_e < v < \sqrt{2}\ v_e$ (d) $\frac{v_e}{\sqrt{2}} < v < \frac{v_e}{2}$

11. If a satellite orbits as close to the earth's surface as possible,

(a) its speed is maximum.

(b) time period of its rotation is minimum.

(c) the total energy of the 'earth plus satellite' system is minimum.

(d) the total energy of the 'earth plus satellite' system is maximum.

12. For a satellite to orbit around the earth, which of the following must be true?

(a) It must be above the equator at some time.

(b) It cannot pass over the poles at any time.

(c) Its height above the surface cannot exceed 36,000 km.

(d) Its period of rotation must be $> 2\pi\sqrt{R/g}$.

13. A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time

(a) 1.5 hours.

(b) 1.6 hours if it is rotating from west to east.

(c) 24/17 hours if it is rotating from west to east.

(d) 24/17 hours if it is rotating from east to west.

14. For a satellite to be geostationary, which of the following are essential conditions?

(a) It must always be stationed above the equator.

(b) It must rotate from west to east.

(c) It must be about 36,000 km above the earth.

(d) Its orbit must be circular, and not elliptical.

15. Two small satellites move in circular orbits around the earth, at distances r and $r + \Delta r$ from the centre of the earth. Their time period of rotation are T and $T + \Delta T$. ($\Delta r << r, \Delta T << T$)

(a) $\Delta T = \frac{3}{2} T \frac{\Delta r}{r}$ (b) $\Delta T = -\frac{3}{2} T \frac{\Delta r}{r}$

(c) $\Delta T = \frac{2}{3} T \frac{\Delta r}{r}$ (d) $\Delta T = T \frac{\Delta r}{r}$

16. Let S be an imaginary closed surface enclosing mass m. Let $d\vec{S}$ be an element of area on S, the direction of $d\vec{S}$ being outward from S. Let $\vec{E}$ be the gravitational intensity at $d\vec{S}$. We define $\phi = \oint_S \vec{E} \cdot d\vec{S}$, the integration being carried out over the entire surface S.

(a) $\phi = -Gm$ (b) $\phi = -4\pi Gm$

(c) $\phi = -\frac{Gm}{4\pi}$

(d) No relation of the type (a), (b) or (c) can exist.

17. A small mass m is moved slowly from the surface of the earth to a height h above the surface. The work done (by an external agent) in doing this is

(a) mgh, for all values of h (b) mgh, for $h << R$

(c) $\frac{1}{2}\ mgR$, for $h = R$ (d) $-\frac{1}{2}\ mgR$, for $h = R$

18. A solid sphere of uniform density and radius 4 units is located with its centre at the origin of coordinates, O. Two spheres of equal radii of 1 unit, with their centres at A (–2, 0, 0) and B (2, 0, 0) respectively, are taken out of the solid sphere, leaving behind spherical cavities as shown in the figure.

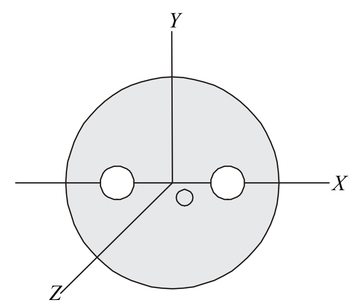


Fig. 9.23

(a) The gravitational force due to this object at the origin is zero.

(b) The gravitational force at the point B (2, 0, 0) is zero.

(c) The gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$.

(d) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

19. The magnitudes of the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then

(a) $F_1/F_2 = r_1/r_2$, if $r_1 < R$ and $r_2 < R$

(b) $F_1/F_2 = r_2^2/r_1^2$, if $r_1 > R$ and $r_2 > R$

(c) $F_1/F_2 = r_1/r_2$, if $r_1 > R$ and $r_2 > R$

(d) $F_1/F_2 = r_1^2/r_2^2$, if $r_1 < R$ and $r_2 < R$

20. Take the effect of bulging of earth and its rotation in account. Consider the following statements.

(A) There are points outside the earth where the value of g is equal to its value at the equator.

(B) There are points outside the earth where the value of g is equal to its value at the poles.

(a) Both A and B are correct.

(b) A is correct but B is wrong.

(c) B is correct but A is wrong.

(d) Both A and B are wrong.

21. The time period of an earth-satellite in circular orbit is independent of

(a) the mass of the satellite.

(b) radius of the orbit.

(c) none of them.

(d) both of them.

22. The magnitude of gravitational potential energy of the moon-earth system is U with zero potential energy at infinite separation. The kinetic energy of the moon with respect to the earth is K.

(a) $U < K$ (b) $U > K$

(c) $U = K$ (d) none

23. Figure 9.24 shows the elliptical path of a planet about the sun. The two shaded parts have equal area. If t_1 and t_2 be the time taken by the planet to go from a to b and from c to d respectively,

(a) $t_1 > t_2$ (b) $t_1 = t_2$

(c) $t_1 > t_2$

(d) insufficient information to deduce the relation between t_1 and t_2.

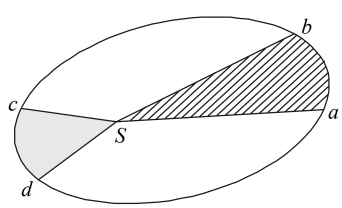


Fig. 9.24

24. A person sitting in a chair in a satellite feels weightless because

(a) the earth does not attract the objects in a satellite.

(b) the normal force by the chair on the person balances the earth's attraction.

(c) the normal force is zero.

(d) the person in satellite is not accelerated.

25. A body is suspended from a spring balance kept in a satellite. The reading of the balance is W_1 then the satellite goes in an orbit of radius R and is W_2 when it goes in an orbit of radius 2 R.

(a) $W_1 = W_2$ (b) $W_1 < W_2$

(c) $W_1 > W_2$ (d) $W_1 \neq W_2$

26. The kinetic energy needed to project a body of mass m from the earth's surface to infinity is

(a) $\frac{1}{4}\ mgR$ (b) $\frac{1}{2}\ mgR$

(c) mgR (d) $2\ mgR$

27. A particle is kept at rest at a distance R (earth's radius) above the earth's surface. The minimum speed with which it should be projected so that it does not return is

(a) $\sqrt{\frac{GM}{4R}}$ (b) $\sqrt{\frac{GM}{2R}}$

(c) $\sqrt{\frac{GM}{R}}$ (d) $\sqrt{\frac{2GM}{R}}$

28. A satellite is orbiting the earth close to its surface. A particle is to be projected from the satellite to just escape from the earth. The escape speed from the earth is v_e. Its speed with respect to the satellite

(a) will be less than v_e.

(b) will be more than v_e.

(c) will be equal to v_e.

(d) will depend on direction of projection.

29. Let V and E denote the gravitational potential and gravitational field at a point. It is possible to have

(a) $V = 0$ and $E = 0$ (b) $V = 0$ and $E \neq 0$

(c) $V \neq 0$ and $E = 0$ (d) $V \neq 0$ and $E \neq 0$.

30. Inside a uniform spherical shell

(a) the gravitational potential is zero.

(b) the gravitational field is zero.

(c) the gravitational potential is same everywhere.

(d) the gravitational field is same everywhere.

31. A uniform spherical shell gradually shrinks maintaining its shape. The gravitational potential at the centre

(a) increases. (b) decreases.

(c) remains constant. (d) oscillates.

32. Consider a planet moving in an elliptical orbit round the sun. The work done on the planet by the gravitational force of the sun

(a) is zero in any small part of the orbit.

(b) is zero in some parts of the orbit.

(c) is zero in one complete revolution.

(d) is zero in no part of the motion.

33. Two satellites A and B move round the earth in the same orbit. The mass of B is twice the mass of A.

(a) Speeds of A and B are equal.

(b) The potential energy of earth + A is same as that of earth + B.

(c) The kinetic energy of A and B are equal.

(d) The total energy of earth + A is same as that of earth + B.

34. Which of the following quantities remain constant in a planetory motion (consider elliptical orbits) as seen from the sun?

(a) Speed (b) Angular speed

(c) Kinetic energy (d) Angular momentum

35. The velocity of a satellite in a parking orbit is—

(a) 8 Km/s (b) 3.1 Km/s

(c) 2.35 Km/s (d) Zero

36. The distances of two satellites P and Q from earth are in the ratio 3 : 1. The ratio of their total energy will be

(a) 3 : 1 (b) 1 : 3

(c) 1 : 1 (d) $\frac{1}{3}$: 1

37. Two satellites P and Q of same mass are revolving near the earth surface in the equatorial plane. The satellite P moves in the direction of rotation of earth whereas Q moves in the opposite direction. The ratio of their kinetic energies with respect to a frame attached to earth will be

(a) $\left(\frac{8363}{7437}\right)^2$ (b) $\left(\frac{7437}{8363}\right)^2$

(c) $\frac{8363}{7437}$ (d) $\frac{7437}{8363}$

38. The semi-major axes of the orbits of Mercury and Mars in the astronomical units are 0.387 and 1.524 respectively.

If the time period of Mercury is 0.241 year, then the time period of Mars will be

(a) 0.9 Year (b) 0.19 Year

(c) 1.9 Year (d) 2.9 Years

39. If the orbital speed of moon is increased by 41.4% then moon will

(a) leave its orbit and will escape out.

(b) fall on earth.

(c) attract all bodies on earth towards it.

(d) have time period equal to 27 days.

40. Two artificial satellites P and Q are revolving round the earth in circular orbits. If the ratio of their radii is 1 : 4 and ratio of their masses is 3 : 1 then the ratio of their time periods will be

(a) $\frac{1}{8}$ (b) 8

(c) 4 (d) 3

41. Three particles of equal mass m are situated at the vertices of an equilateral triangle of side 1. The work done in increasing the side of the triangle to 21 will be

(a) $\frac{3G_2\text{m}}{21}$ (b) $\frac{G\text{m}^2}{21}$

(c) $\frac{3G\text{m}^2}{21}$ (d) $\frac{3G\text{m}^2}{1}$

42. A space ship is released in a circular orbit near earth surface. How much additional velocity will have to be given to the ship in order to escape out of this orbit.

(a) 3.28 m/s (b) 3.28×10^3 m/s

(b) 3.28×10^7 m/s (d) 3.28×10^{-3} m/s

43. The centripetal force acting on a satellite revolving round the earth is F. The gravitational force on that planet is also F. The resultant force on the satellite is

(a) Zero (b) F

(c) 2 F (d) $\frac{F}{2}$

44. If the force inside earth surface varies as rx then the value of x will be

(r = distance of body from centre of earth)

(a) $x = -1$ (b) $x = -2$

(c) $x = 1$ (d) $x = 2$

45. An artificial satellite is revolving round the earth. The radius of its circular orbit is half the orbital radius of moon. The time taken by this satellite in completing one revolution will be

(a) 2 lunar months. (b) $2^{-2/3}$ lunar months.

(c) $2^{-3/2}$ lunar months. (d) 1/2 lunar months.

46. The value of acceleration due to gravity at height h from earth surface will become half its value on the surface if (R = radius of earth)

(a) $h = R$ (b) $h = 2\,R$

(c) $h = \sqrt{2} - 1$ (d) $h = (\sqrt{2} + 1)$

47. If *Ve* is the escape velocity of a body from a planet of mass *M* and radius *R*. Then, the velocity of satellite revolving at height *h* from the surface of planet will be

(a) $v = v_e\sqrt{R/(R+h)}$ (b) $v = v_e\sqrt{2R/(R+h)}$

(c) $v = v_e\sqrt{(R+h)/R}$ (d) $v = v_e\sqrt{R/2(R+h)}$

48. The orbital radius of moon around the earth is 3.8×10^8 meter and its time period is 27.3 days. The centripetal acceleration of moon will be

(a) -2.4×10^{-3} m/s^2 (b) 11.2 m/s^2

(c) 2.7×10^{-3} m/s^2 (d) 9.8 m/s^2

49. The change in the value of acceleration of earth towards sun, when the moon comes from the position of solar eclipse to the position on the other side of earth in line with sun is (mass of moon = 7.36×10^{22} Kg., the orbital radius of moon 3.82×10^8 m)

(a) 6.73×10^{-2} m/s^2 (b) 6.73×10^{-3} m/s^2

(c) 6.73×10^{-4} m/s^2 (d) 6.73×10^{-5} m/s^2

50. A projectile is fired from the surface of earth with initial velocity of 10 Km/sec If the radius of earth is 6400 Km. how high will it go from earth surface

(a) 2500 Km (b) 2500 Km

(c) 2.5×10^4 Km (d) 2.5×10^6

51. A satellite is launched in a circular orbit of radius *R* and another satellite is launched in circular orbit of radius 1.01 R. The time period of second satellite is different from that of the first satellite by

(a) 1.5% increased (b) 1% decreased

(c) 1% increased (d) 1.5% decreased

52. How far must a particle be on the line joining earth to sun, in order that the gravitational pull on it due to sun is counterbalanced by that due to earth. (Given orbital radius of earth is 10^8 *Km* and $Ms = 3.24 \times 10^5\,ME$)

(a) 64×10^5 Km (b) 1.75×10^2 Km

(c) 1.75×10^9 Km (d) 6400 Km

53. A satellite is projected with a velocity $\sqrt{1.5}$ times its orbital velocity just above earth atmosphere. The initial velocity of the satellite is parallel to the surface. The maximum distance of the satellite from earth will be

(a) 2 *R* (b) 8 *R*

(c) 4 *R* (d) 3 *R*

54. The gravitational potential difference between the surface of a planet and a point 20 m above the surface is 2 Joule/Kg. If the gravitational field is uniform then the work done in carrying a 5 Kg body to a height of 4 m above the surface is

(a) 2 Joule (b) 20 Joule

(c) 40 Joule (d) 10 Joule

55. A sky laboratory of mass 2×10^3 Kg is raised from a circular orbit of radius 2 *R* to a circular orbit of radius 3 *R*. The work done is approximately.

(a) 1×10^{16} Joule (b) 2×10^{10}

(c) 1×10^6 Joule (d) 3×10^{10} Joule

56. A planet revolves round the sun. Its velocity at the nearest point, distant d_1 from sun, is v_1. The velocity of the planet at the farthest point distant d_2 from sun will be.

(a) $\frac{d_1^2 v_1}{d_2^2}$ (b) $\frac{d_2 v_1}{d_1}$

(c) $\frac{d_1 v_1}{d_2}$ (d) $\frac{d_2^2 v_1}{d_1^2}$

57. If the change in the value of g at height h above earth surface is the same as that at depth *x* (*x* or $h < Re$), then

(a) $x = h^2$ (b) $x = h$

(c) $x = \frac{h}{2}$ (d) $x = 2\,h$

58. The escape velocity on a planet with radius double that of earth and mean density equal to that of earth will be (escape velocity on earth = 11.2 Km/s).

(a) 11 Km/s (b) 22 Km/s

(c) 5.5 Km/s (d) 15.5Km/s

59. The motion of a planet around sun in an elliptical orbit is shown in the following figure. Sun is situated on one focus. The shaded areas are equal. If the planet takes time t_1 and t_2 in moving from *A* to *B* and from *C* to *D* respectively then

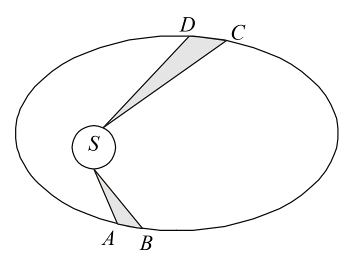


Fig. 9.25

(a) $t_1 > t_2$ (b) $t_1 < t_2$

(c) $t_1 = t_2$

(d) information incomplete

60. The acceleration due to gravity at a place is g m/s^2 *A* lead sphere of density *d* Kg/m^3 is dropped into a liquid column of density *p* Kg/m^3. If $d > p$ then the sphere will fall down with

(a) *g* acceleration.

(b) without acceleration.

(c) acceleration.

(d) with an acceleration $g(p/d)$.

61. A tunnel is dug along a diameter of earth. The force on a particle of mass of distant x from the centre in this tunnel will be

(a) $\frac{GM_e m}{R^3 x}$ (b) $\frac{GM_e mR^3}{x}$

(c) $\frac{GM_e mx}{R^2}$ (d) $\frac{GM_e mx}{R^3}$

62. A balloon filled with hydrogen gas is carried from earth on moon. Then the balloon will

(a) neither fall nor rise.

(b) fall with acceleration less than g.

(c) fall with acceleration g.

(d) rise with acceleration g.

63. An artificial satellite is revolving close to earth. Its orbital velocity mainly depends upon

(a) the mass of earth. (b) the radius of earth.

(c) the orbital radius. (d) the mass of satellite.

64. The potential energy of a rocket of mass 100 kg at height 10^7 m from earth surface is 4×10^9 joule. The weight of the rocket at height 10^9 will be

(a) 4×10^{-2} N (b) 4×10^{-3} N

(c) 8×10^{-2} N (d) 8×10^{-3} N

65. The venus appears more shining than other stars because

(a) it is heavier than other stars.

(b) its density is more than that of other stars.

(c) it is nearer the earth than other stars.

(d) there is no atmosphere on it.

66. A communication satellite is carried from one orbit to another orbit, with radius double that of the first. Its time period in the new orbit will be

(a) $24\sqrt{2}$ Hours (b) $48\sqrt{2}$ Hours

(c) 24 Hours (d) 48 Hours

67. The initial concept about the origin of universe was given by

(a) Bondi (b) Hawllacks

(c) Narlikar (d) Hubli

68. The value of G for two bodies in vacuum is 6.67×10^{-11} N/m²/Kg² Its value in a dense medium of density 10^{10} gm/cm³ will be

(a) 6.67×10^{-11} N/m²/Kg (b) 6.67×10^{-31} N/m²/Kg

(c) 6.67×10^{-21} N/m²/Kg (d) 6.67×10^{-10} N/m²/Kg

69. The length of the day from today when the sun is directly overhead till tomorrow again when the sun is directly overhead can be determined by the

(a) rotation of earth about its own axis.

(b) revolution of earth around sun.

(c) inclination of axis of rotation of earth from the plane of revolution.

(d) rotation of earth about its own axis as well as its revolution around sun.

70. Two satellites of mass m_1 and m_2 ($m_1 > m_2$) revolve round the earth in circular orbits of radii r_1 and r_2 respectively ($r_1 > r_2$) Their speeds are related as

(a) $v_1 = v_2$ (b) $v_1 < v_2$

(c) $v_1 > v_2$ (d) $\frac{v_1}{r_1} = v_2 r_2$

71. Presuming earth to be a uniform sphere, a scientist A goes deep inside a mine and another scientist B goes high in a balloon above earth surface. The intensity of gravitational field

(a) decreases when measured by A and increases when measured by B.

(b) decreases when measured by B and increases when measured by A.

(c) decreases when measured by both.

(d) remains constant when measured by both.

72. A person jumps from the fifth storey of a building with load on his head. The weight experienced by him before reaching the earth will be

(a) Zero (b) g Kg/wt

(c) $m(g+a)$ (d) mg

73. Two artificial satellites of unequal masses are revolving in a circular orbit around the earth with a constant speed. Their time periods

(a) will be different.

(b) will be same.

(c) will depend on their masses.

(d) will depend upon the place of their projection.

74. The mass of earth is 80 times that of moon. Their diameters are 12800 Km and 3200 Km respectively. The value of g on moon will be, if its value on earth is 980 cm/s²

(a) 98 cm/s² (b) 196 cm/s²

(c) 100 cm/s² (d) 294 cm/s²

75. A bomb blasts on moon. Its sound will be heard on earth after

(a) 3.7 minutes (b) 10 minutes

(c) 138 minutes

(d) sound will never be heard

PASSAGE 1

Read the following passage and answer the questions given at the end.

At points inside the earth the statements of gravitational field need to be modified. If one could drill a hole to the center of the earth and measure the force of gravity on a body at various distances from the center, the force would be found to decrease as the center is approached, rather than increasing as $1/r^2$. Qualitatively, it is easy to see why this should be so; as the body enters the interior of the earth (or other spherical body), some of the earth's mass is on the side of the body opposite from the center of the earth and pulls the body in the opposite direction. Exactly at the center of the earth, the gravitational force on the body is zero.

The magnitude of the gravitational constant G can be found experimentally by measuring the force of gravitational attraction between two bodies of known masses m_1 and m_2 at a known separation for bodies of moderate size the force is extremely small, but it can be measured with an instrument called a torsion balance, invented by the Rev. John Michell and first used for this purpose by Sir Henry Cavendish in 1798. The same type of instrument was also used by Coulomb for studying forces of electrical and magnetic attraction and repulsion.

1. (a) The value of G is constant throughout

(b) $G \propto \frac{1}{r^2}$

(c) G is slightly less (by about 1%) when distance < 200 m

(d) G is slightly greater when distance < 200 m.

2. The gravitational field of a sphere of mass M and radius R at a distance $2R$ from the centre of the sphere is ____ if a cavity of radius $R/3$ is made at the periphery as shown in figure

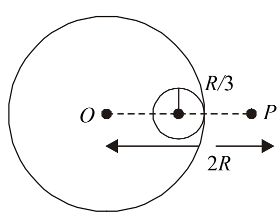


Fig. 9.26

(a) $\frac{11GM}{48R^2}$ (b) $\frac{9GM}{48R^2}$

(c) $\frac{7GM}{48R^2}$ (d) none of these

Solution **76.** (c)

Solution **77.** (a) $E_g = \frac{GM}{(2R)^2} - \frac{GM'}{\left(R+\frac{R}{3}\right)^2}$ where M'

$$= \frac{M\left(\frac{R}{3}\right)^3}{R^3} = \frac{M}{27}$$

$$= \frac{GM}{R^2}\left[\frac{1}{4} - \frac{9}{16\times 27}\right]$$

$$= \frac{GM}{R^2 4}\left[\frac{11}{12}\right] = \frac{11GM}{48R^2}$$

PASSAGE 2

Read the following passage and answer the questions given at the end.

The most efficient way to send a spacecraft from the earth to another planet is by using a Hohmann transfer orbit as shown in Fig 9.27. If the orbits of the departure and destination planets are circular, the Hohmann transfer orbit is an elliptical orbit whose aphelion and perihelion are tangents to the orbits of the two planets. The rockets are fired briefly at the departure planet to put the space craft into the transfer orbit; the spacecraft then coasts until it reaches the destination planet. The rockets are then fired again to put the spacecraft into the same orbit about the sun as the destination planet. To reach Mars from the earth, the launch must be timed so that mars will be at the right spot when the space craft reaches. The orbital radius of Mars is 2.28×10^{11} m.

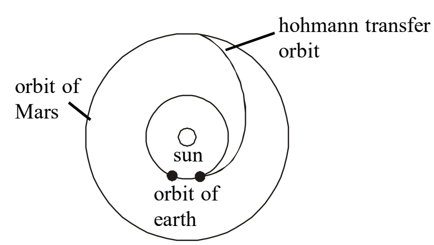


Fig. 9.27

1. How long will it take to reach mars from the Earth?

(a) 8 days (b) 6 days

(c) 60 days (d) 80 days

2. (a) Hohmann orbit is elliptical with perihelian distance at the launch planet to aphelion distance at the destination planet.

(b) Hohmann orbit is elliptical with perihelion and aphelion as tangents to the orbits of two planets.

(c) Hohmann orbit is elliptical with aphelion orbit at destination.

(d) Hohmann orbit is a transfer orbit used from one space ship to another.

3. How much time it will take for the solar radiation to reach Mars?

(a) 10 min (b) 12.6 min

(c) 15 min (d) 14.8 min

Solution **1.** (d) using escape velocity as unit for measuring time then t.

$$= \frac{2.28\times10^{11} - 1.5\times10^{11}}{11.2\times10^{3}}$$

or $$t = \frac{7.8\times10^{10}}{11.2\times10^{3}\times3600\times24} = \frac{7.8\times10^{2}}{9.68} = 80 \text{ days}$$

Solution **2.** (b)

Solution **3.** (b)

$$t = \frac{2.28\times10^{11}}{3\times10^{8}} = 0.76\times10^{3} = 760 \text{ s}$$

$$= \text{About } 12.6 \text{ min}$$

PASSAGE 3

Read the following passage and answer the questions given at the end.

An astronaut inside a spacecraft which protects her from harmful radiations is orbiting a black hole at a distance of 120 km from its centre. The black hole has mass 5 times the mass of the sun and has a Schwarzchild radius 15 km. Astranaut is positioned inside the spacecraft such that one of her ears is 6 cm farther from center of mass of the spacecraft and other 6 cm closer. Since her whole body orbits with same angular speed, one ear is moving too slowly for the radius of the orbit and other ear is moving too fast. Hence, her head must exert forces on her ears to keep them in their orbits. Mass of the ear = 0.03 kg

1. What is the tension between her ears?

(a) 2.1 kN (b) 2.6 kN

(c) 2.9 kN (d) none

2. Would she find it difficult to keep from being torn apart?

(a) yes. (b) no.

(c) cannot say as her resistance is not given.

3. Is centre of gravity of her head at the same point as centre of mass ?

(a) yes. (b) no.

(c) cannot be determined.

4. What is Schwarzchild radius ?

(a) $\frac{GM}{V^2}$ (b) $\frac{GM}{2V^2}$

(c) $\frac{2GM}{c^2}$ (d) $\frac{GM}{2c^2}$

Solution **1.** (b) $T = \text{GMm}\left[\frac{1}{(R-x)^2} - \frac{1}{(R+x)^2}\right]$

$$= \frac{GMm}{R^2}\left[\left(1+\frac{2x}{R}\right)-\left(1-\frac{2x}{R}\right)\right]$$

$$= \frac{GMm}{R^2}\left(\frac{4x}{R}\right)$$

$$= \frac{6.67\times10^{-11}\times10^{31}\times0.03\times4\times.06}{\left(120\times10^{3}\right)^{3}} = \frac{20\times24\times10}{1.73}$$

$= 2.6$ kN.

Solution **2.** (a)

Solution **3.** (b)

Solution **4.** (c) $\sqrt{\frac{2GM}{r}} \geq c$ or $r = \frac{2GM}{c^2}$

PASSAGE 4

Read the following passage and answer the questions given at the end.

Indeed, the study of planetary motion historically played a pivotal role in the development of physics. Johannes Kepler (1571 to 1630) spent several pains taking years analyzing the motions of the planets, basing his work on precise measurements made by the Danish astronomer Tycho Brahe (1546 to 1601). Kepler discovered that the orbits of the planets are (nearly circular) ellipses, and that the period of a planet in its orbit is proportional to the three-halves power of the orbit radius, as shown by $T = \frac{2\pi r}{v} = 2pr\sqrt{\frac{r}{GM_E}} = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$.

But it remained for Newton (1642 to 1727) to show, with the aid of his law of motion and law of gravitation, that this behavior of the planets could be understood on the basis of the very same physical principles he had developed to analyze terrestrial motion. From our historical perspective three hundred years later, there is absolutely no doubt that this Newtonian synthesis (as it has come to be called) is one of the greatest achievements in the entire history of science, certainly comparable in significance to the development of quantum mechanics, the theory of relativity, and the understanding of genome, in our own century.

1. How orbital and escape velocities are related?

(a) $v_e = 2v_0$ (b) $v_e = \sqrt{3}\, v_0$

(c) $v_e = 1.31\, v_0$ (d) $v_e = 1.41\, v_0$

2. Can Kepler's laws be applied to atomic structure as electrons are also bound to nucleus like planets to the sun.

(a) yes. (b) no.

(c) cannot say.

(d) insufficient data to reply.

3. The term 'Newton synthesis' refers to

(a) Newton's laws of motion (in linear motion).

(b) Newton's gravitational law.

(c) Newton's laws of rotational motion.

(d) (a), (b) and (c) combined.

Solution **1.** (d)

Solution **2.** (b) No, as electrons are spin particles

Solution **3.** (d)

Answers to Questions for Practice

1.	(a)	2.	(a)	3.	(a,c)	4.	(b)	5.	(a)	6.	(a,d)	7.	(a,d)
8.	(a,b)	9.	(c)	10.	(b)	11.	(a,b,c)	12.	(a,d)	13.	(b,d)	14.	(a,b,c,d)
15.	(a)	16.	(b)	17.	(b,c)	18.	(a,c,d)	19.	(a,b)	20.	(b)	21.	(a)
22.	(b)	23.	(b)	24.	(c)	25.	(a)	26.	(c)	27.	(c)	28.	(d)
29.	(a,b,c,d)	30.	(b,c,d)	31.	(b)	32.	(b,c)	33.	(a)	34.	(d)	35.	(b)
36.	(b)	37.	(a)	38.	(c)	39.	(a)	40.	(a)	41.	(c)	42.	(b)
43.	(b)	44.	(c)	45.	(c)	46.	(c)	47.	(d)	48.	(c)	49.	(d)
50.	(c)	51.	(a)	52.	(b)	53.	(d)	54.	(a)	55.	(b)	56.	(c)
57.	(d)	58.	(b)	59.	(c)	60.	(b)	61.	(d)	62.	(b)	63.	(b)
64.	(a)	65.	(c)	66.	(b)	67.	(d)	68.	(a)	69.	(b)	70.	(b)
71.	(c)	72.	(a)	73.	(b)	74.	(b)	75.	(d)				

EXPLANATION

1(a) Let ω = angular velocity of the earth about its axis.

$$mg - N = m\omega^2 R$$

for $N = 0$, $\omega^2 = g/R$.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{R/g}.$$

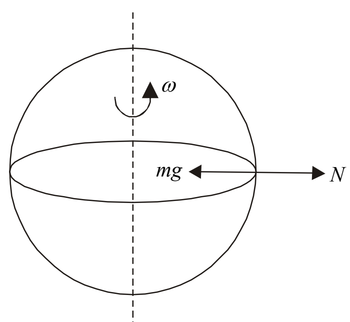


Fig. 9.28

2(a) Let v = speed of each train relative to the earth's surface, v_E = speed of earth's surface relative to the earth's axis, v_A, v_B = speeds of A and B relative to the earth's axis.

Then, $v_A = v_E - v$, $v_B = v_B + v$.

$$N_A = mg - m\left(\frac{v_A^2}{R}\right), N_B = mg - m\left(\frac{v_B^2}{R}\right).$$

$\therefore$ $N_A > N_B$.

3(a, c) Due to the rotation of the earth on its axis, the apparent weight of an object becomes slightly smaller at the equator than at the poles. This difference can be recorded on a spring balance, but not on a beam balance.

4(b) Apparent weight at the equator = $mg - m\omega^2 R$

Weight at a height h above the pole = $mg\left(1 - \frac{2h}{R}\right)$.

Putting $mg - m\omega^2 R = mg\left(1 - \frac{2h}{R}\right)$.

or $\omega^2 R = \frac{2gh}{R}$ or $h = \frac{\omega^2 R^2}{2g}$

6(a,d) $F = G\frac{m(2m)}{l^2} = m\omega^2 (2l/3)$

or $\frac{3mG}{l^3} = \omega^2$. $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l^3}{3mg}}$.

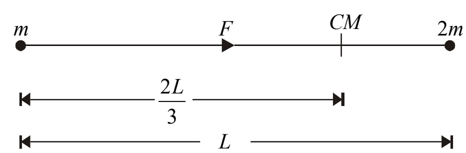


Fig. 9.29

9(c) Taking the potential at a large distance from the planet as zero, the potential at the centre of the planet = $-\frac{3GM}{2R}$.

$\therefore$ $\frac{1}{2}mv^2 = m\left[0 - \left(-\frac{3GM}{2R}\right)\right]$

or $v^2 = \frac{3GM}{R} = 3Rg = \frac{3}{2}(2RG) = \frac{3}{2}v_e^2$

or $v = \sqrt{1.5}\, v_e$.

10(b) For a satellite orbiting very close to the earth's surface, the orbital velocity = $\sqrt{Rg}$ ($\because$ $mg = mv^2/R$). This is equal to the velocity of projection and is the minimum velocity required to go into orbit. Also, the satellite would escape completely and not go into orbit for $v \geq v_e$.

$\therefore$ $v_e/\sqrt{2} < v < v_e$.

13(b, d) Let ω_0 = the angular velocity of the earth about its axis.

$\therefore \quad 24 = \dfrac{2\pi}{\omega_0}$ or $\omega_0 = \dfrac{2\pi}{24}$

Let ω = the angular velocity of the satellite.

$\therefore \quad 1.5 = \dfrac{2\pi}{\omega}$ or $\omega = \dfrac{2\pi}{1.5}$

For a satellite rotating from west to east (the same as the earth), the relative angular velocity, $\omega_1 = \omega - \omega_0$.

Time period of rotation relative to the earth $= \dfrac{2\pi}{\omega_1} =$ 1.6 h.

For a satellite rotating from east to west (opposite to the earth), the relative angular velocity, $\omega_2 = \omega + \omega_0$.

15(a) $T^2 \propto r^3$ or $T^2 = cr^3$

$\therefore \quad 2T\Delta T = 3cr^2 \Delta r$.

Dividing, $\dfrac{2T\Delta T}{T^2} = \dfrac{3cr^2\Delta r}{cr^3}$.

16(b) Follow the method used to prove Gauss's law.

$$E = G \cdot \frac{m}{r^2}$$

$$\vec{E} \cdot d\vec{S} = EdS \cos(180° - \theta) = -EdS\cos\theta$$

$$\phi = \oint_S \vec{E} \cdot d\vec{S} = \oint_S -G\frac{m}{r^2}\, dS\cos\theta$$

$$= -Gm \cdot \oint_S \frac{dS\cos\theta}{r^2} = -Gm \cdot \oint_S d\omega = -4\pi Gm.$$

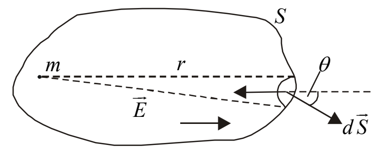


Fig. 9.30

17(b, c) Work done = m × difference in gravitational potential.

18(a, c, d) Use arguments of symmetry as the yz plane divides the object symmetrically.

19(a, b) $F \propto T$ if $r < R$, and

$F \propto \dfrac{1}{r^2}$ if $r > R$.

$\therefore$ for $r_1, r_2 < R$, $F_1/F_2 = r_1/r_2$

for $r_1, r_2 > R$, $F_1/F_2 = r_2^2/r_1^2$.