

19

Gauss's Law

BRIEF REVIEW

Gauss's Law is an alternative to Coulomb's Law. Electric flux $\oint \vec{E} \cdot d\vec{s}$. Electric flux is independent of the radius R of the sphere. It only depends upon the charge q enclosed in the sphere. According to Gauss's law the closed integral of electric field intensity is equal to $\frac{q}{\epsilon_0}$ where q is charge enclosed in the closed surface. In other words, total flux through a closed surface enclosing a charge q is given by

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}.$$

- If E is at right angle to the surface area A at all points and has same magnitude at all points of the surface then $E_{\perp} = E$ and $\int E_{\perp} \cdot ds = EA$.
- If E is parallel to the surface on all points then $E_{\perp} = 0$ and hence $\int E_{\perp} \cdot ds = 0$.
- If $E = 0$ at all points on a surface then $\phi_E = 0$.
- The surface need not be a real surface, it could be a hypothetical one.
- Electric field in $\oint \vec{E} \cdot d\vec{s}$ is complete electric field, it may be partly due to charge outside the surface and partly

due to charge inside the surface. However, if there is no charge enclosed in the Gaussian surface E_{\perp} will be zero and hence $\oint \vec{E} \cdot d\vec{s} = 0$

- While evaluating $\oint E \cdot ds$, the field should lie on the surface and there should be enough symmetry to evaluate the integral.

Various forms of Gauss's Law

$$\phi_E = \oint E \cos \phi ds = \oint \vec{E} \cdot d\vec{s} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Note that net number of field lines leaving a closed surface is proportional to the total charge enclosed by that surface.

The excess charge (other than the ions and free electrons that make up the neutral conductor) resides entirely on the surface and not in the interior of the material.

Electric field due to a long thread (Line charge) having

linear charge density λ is $E = \frac{\lambda}{2\pi\epsilon_0 y} = \frac{18 \times 10^9 \lambda}{y}$

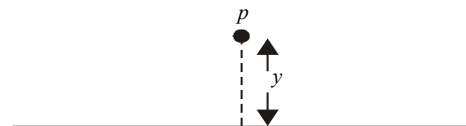


Fig. 19.1 Electric field due to a long line charge

Electric field due to a shell having radius R and charge Q

$$E_{\text{inside}} = 0, E_{\text{surface}} = \frac{Q}{4\pi\epsilon_0 R^2}; E_{\text{out}} = \frac{Q}{4\pi\epsilon_0 x^2} \quad x > R$$

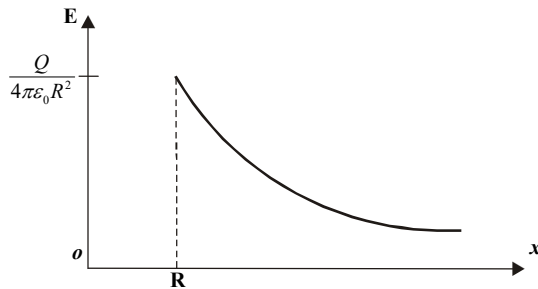


Fig. 19.2 Electric field due to a shell

Potential due to a shell

$$V_{\text{in}} = \frac{Q}{4\pi\epsilon_0 R} = V_{\text{surface}}$$

$$V_{\text{out}} = \frac{Q}{4\pi\epsilon_0 x} \quad x > R$$

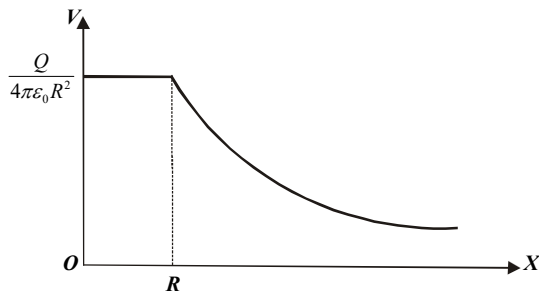


Fig. 19.3 Electric potential due to a shell

Electric field due to a sphere charged uniformly with charge Q

$$E_{\text{inside}} = \frac{Qx}{4\pi\epsilon_0 R^3} \quad x < R$$

$$E_{\text{surface}} = \frac{Q}{4\pi\epsilon_0 R^2} \quad x = R$$

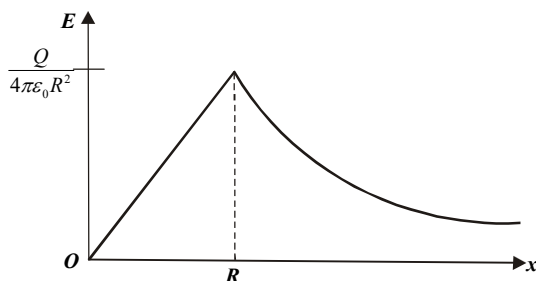


Fig. 19.4 Electric potential due to a sphere charged uniformly

$$E_{\text{outside}} = \frac{Q}{4\pi\epsilon_0 x^2} \quad x > R$$

$$V_{\text{out}} = \frac{Q}{4\pi\epsilon_0 x} \quad x > R$$

$$V_{\text{inside}} = \int_R^x \frac{-Qx^2 dx}{4\pi\epsilon_0 R^3} + \frac{Q}{4\pi\epsilon_0 R} \quad x < R$$

$$V_{\text{surface}} = \frac{Q}{4\pi\epsilon_0 R} \quad x = R.$$

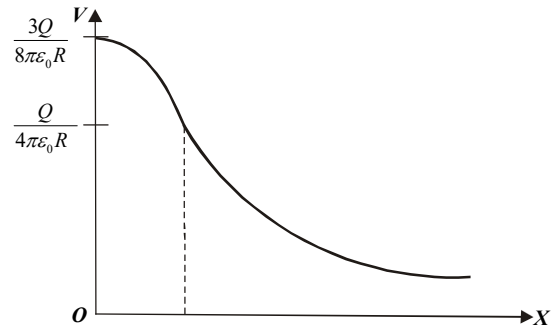


Fig. 19.5 Potential due to a uniformly charged sphere

Electric Field due to a thin plane Sheet (long) of charge

$$\text{density } \sigma \quad E = \frac{\sigma}{2\epsilon_0}.$$

Electric field due to a charged surface having surface

$$\text{charge density } \sigma \quad E = \frac{\sigma}{\epsilon_0}.$$

$$\text{Electric field due to conducting plate } E = \frac{\sigma}{2\epsilon_0}.$$

Electric field between two oppositely charged sheets

at any point is $E_{\text{in}} = \frac{\sigma}{\epsilon_0}$ ($= E_1 + E_2$). Assuming equal surface

charge density (for example in a capacitor) $E = \frac{\sigma}{\epsilon_0}$. Electric

field intensity is zero at any point outside the plates as $E_{\text{net}} = E_1 - E_2 = 0$, as shown in Fig. 19.6.

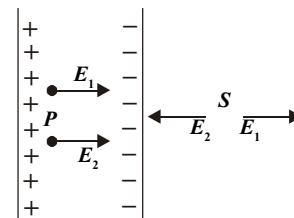


Fig. 19.6 Electric field due to charged plates

SHORT CUTS AND POINTS TO NOTE

1. Electric flux through symmetrical surfaces placed in a uniform electric field is zero. For example, for a cylinder (solid or hollow) placed in a uniform electric field $\phi_E = 0$. Hence no charge is stored.

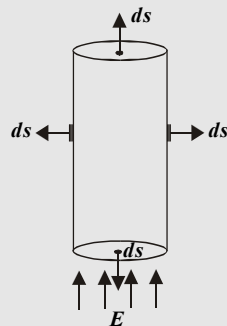


Fig. 19.7

2. $\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$ where Q is charge enclosed in the surface. The electric field is perpendicular to the surface so that it is parallel to surface vector $d\vec{s}$. If \vec{E} and $d\vec{s}$ are not parallel take the dot product. i.e. $EA \cos \phi$ as the flux. Gauss's law in differential form $\frac{\partial E}{\partial r} = \frac{\rho}{\epsilon_0}$.

3. Electric field intensity due to a long line charge at a distance y from one end as shown in Fig. 19.8 at P

from end A is $\frac{\lambda\sqrt{2}}{4\pi\epsilon_0 y}$ and is directed at 45° with the vertical.

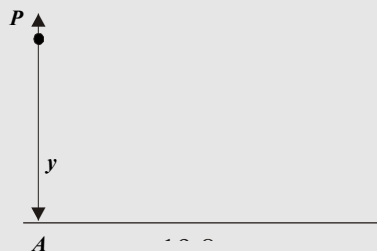


Fig. 19.8

4. Electric field due to a uniformly charged sphere of

radius R and charge Q is $E_{\text{inside}} = \frac{Qx}{4\pi\epsilon_0 R^3}$ $x < R$

$$E_{\text{surface}} = \frac{Q}{4\pi\epsilon_0 R^2} \quad x = R;$$

$$E_{\text{outside}} = \frac{Q}{4\pi\epsilon_0 x^2} \quad x > R \text{ as shown in Fig. 19.9.}$$

Note that $E_{\text{centre}} = 0$.

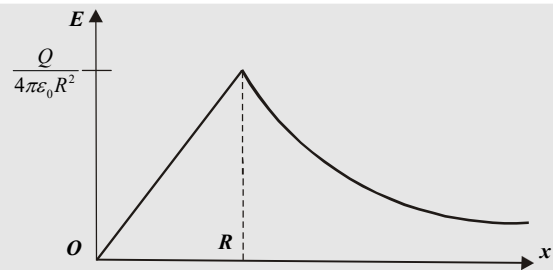


Fig. 19.9

5. Electric potential due to a uniformly charged sphere of radius R and charge Q

$$V_{\text{inside}} = \frac{Q}{4\pi\epsilon_0 R} + \int_R^x -\frac{Qx^2}{4\pi\epsilon_0 R^3} dx \quad x < R$$

$$V_{\text{surface}} = \frac{Q}{4\pi\epsilon_0 R} \quad x = R.$$

Note that $V_{\text{centre}} = \frac{3Q}{8\pi\epsilon_0 R}$ as shown in Fig 19.10.

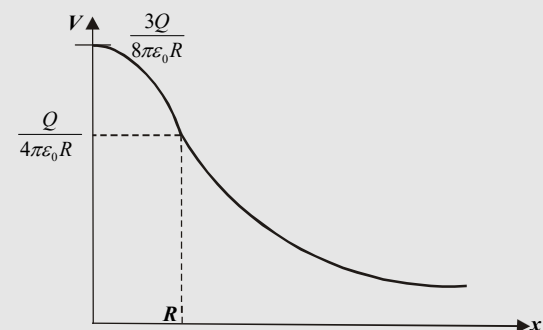


Fig. 19.10

6. Electrical shielding is achieved if a body is kept in a metallic shell or metallic enclosure irrespective of the shape of the enclosure. That is $E_{\text{in}} = 0$.
7. Electric field due to a thin sheet (thin sheet ≤ 200 Å) having linear charge density λ . The charge is

distributed on both sides. Therefore $E = \frac{\sigma}{2\epsilon_0}$.

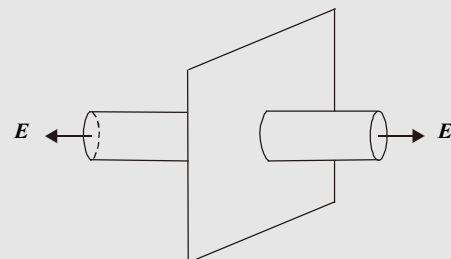


Fig. 19.11

8. Electric field due to a thick conducting sheet $E = \frac{\sigma}{\epsilon_0}$

9. Work done in assembling the charged sphere of

$$\text{radius } R = \frac{3Q^2}{20\pi\epsilon_0 R} = \text{PE of the charged sphere.}$$

10. Work done in assembling the charge on a shell

$$= \frac{Q^2}{8\pi\epsilon_0 R} = \text{PE of the charged shell spherical.}$$

11. The electric field inside the capacitor sheets

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \text{ as shown in Fig 19.12.}$$

However electric field due to a single plate is $\frac{\sigma}{2\epsilon_0}$.

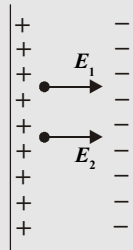


Fig. 19.12

12. Electric field in a long cylinder of radius R having charge per unit length λ

$$E_{\text{outside}} = \frac{\lambda}{2\pi\epsilon_0 r} \text{ for } r > R$$

$$E_{\text{inside}} = 0 \text{ for } r < R.$$

13. Electric field due to long charged plates is uniform.

CAUTION

1. Considering any electric field in $\oint E \cdot ds$ will form flux.

$\Rightarrow E_{\perp}$ which is parallel to surface vector \vec{ds} will form flux. E_{\parallel} , which is parallel to the surface as shown in Fig 19.13, does not form any flux.

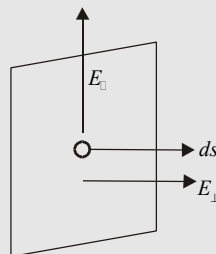


Fig. 19.13

2. Considering like a shell, electric field inside a charged sphere is also zero.

\Rightarrow If the sphere has charge distributed only on its surface then $E_{\text{inside}} = 0$. If the charge is distributed throughout the volume then

$$E_{\text{inside}} = \frac{Qx}{4\pi\epsilon_0 R^3} \text{ for } x < R.$$

3. Considering $E = 0$ if $V = 0$ or vice versa.

\Rightarrow Inside a shell $E = 0$ but $V \neq 0$ Rather

$$V_{\text{inside}} = \frac{Q}{4\pi\epsilon_0 R} = V_{\text{surface}} \text{ and along the equatorial}$$

line of a dipole $V = 0$ but $E \neq 0$.

Note: $E = -\frac{dV}{dx}$ represents $E = 0$ if V is max, V is min or $V = \text{constant}$.

4. Considering $E_{\text{in}} = 0$ only in a shell (spherical).

$\Rightarrow E_{\text{in}} = 0$ in any type of hollow metallic body. $E_{\text{in}} = 0$ even in a long metallic cylinder.

5. Considering $V_{\text{in}} = \frac{Q}{4\pi\epsilon_0 R}$ in a charged sphere (charge Q , radius R).

$\Rightarrow V_{\text{in}} = \frac{Q}{4\pi\epsilon_0 R}$ inside a shell. If the charge is uniformly distributed throughout the volume then

$$V_{\text{in}} = \frac{Q}{4\pi\epsilon_0 R} + \int_R^x \frac{-Qx}{4\pi\epsilon_0 R^3} dx.$$

6. Not knowing the electric field lines direction in a metallic charged body.

\Rightarrow Electric field lines are perpendicular to the surface because a metal body acts as an equipotential surface.

7. Considering equipotential surface has electric field intensity also equal at all points of the body.

\Rightarrow Electric field is very large at pointed ends, sharp corners. i.e. $E \rightarrow \infty$ if $R \rightarrow 0$. In Fig 19.14 $E_B > E_C > E_A$. E is minimum at D.

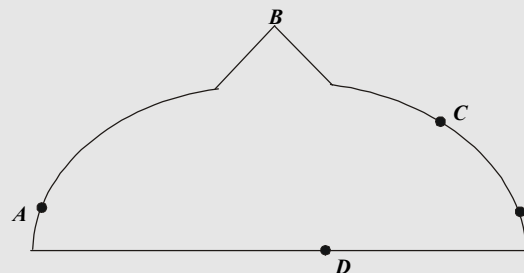


Fig. 19.14

SOLVED PROBLEMS

1. Three infinitely charged sheets are kept parallel to $x-y$ plane having charge densities as shown in Fig. 19.15. The electric field at P is.

(IIT Screening 2005)

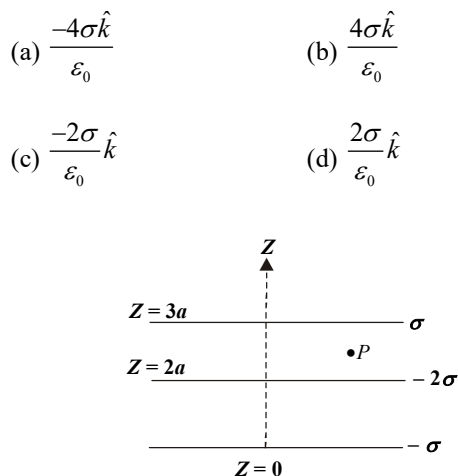


Fig. 19.15

Solution (c) $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$$= \frac{\sigma}{2\epsilon_0}(-\hat{k}) + \left(\frac{2\sigma}{2\epsilon_0}\right)(-\hat{k}) + \left(\frac{-\sigma}{2\epsilon_0}\right)\hat{k} = \frac{-2\sigma\hat{k}}{\epsilon_0}$$

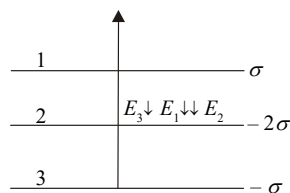


Fig. 19.16

2. A charged ball B hangs from a silk thread S which makes an angle θ with a large charged conducting sheet P as shown in Fig. 19.17. The surface charge density σ of the sheet is proportional to

- (a) $\cos \theta$ (b) $\cot \theta$
(c) $\sin \theta$ (d) $\tan \theta$

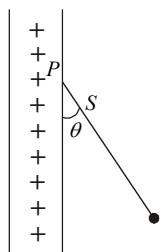


Fig. 19.17

Solution (d) Resolve T ; $T \cos \theta = mg$ (1)

$$T \sin \theta = qE = q \frac{\sigma}{2\epsilon_0} \quad \dots(2)$$

Divide Eq (2) by (1)

$$\tan \theta = \frac{q\sigma}{2\epsilon_0 g} \text{ i.e., } \sigma \propto \tan \theta$$

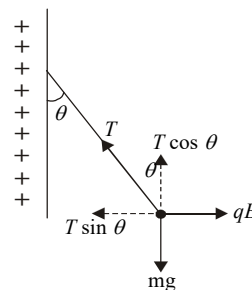


Fig. 19.18

(AIIMS 2005)

3. Two infinitely long parallel conducting plates having surface charge densities $+\sigma$ and $-\sigma$ respectively are separated by a small distance (Fig 19.19). The medium between the plates is vacuum. If ϵ_0 is the dielectric permittivity of vacuum then the electric field in the region between the plates is

(AIIMS 2005)

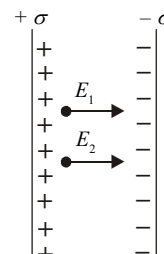


Fig. 19.19

- (a) 0 Vm^{-1} (b) $\frac{\sigma}{2\epsilon_0} \text{ Vm}^{-1}$
(c) $\frac{\sigma}{\epsilon_0} \text{ Vm}^{-1}$ (d) $\frac{2\sigma}{\epsilon_0} \text{ Vm}^{-1}$

Solution (c) $E = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$

4. A charge Q is distributed uniformly in a sphere (solid). Then the electric field at any point r where $r < R$ (R is radius of the sphere) varies as

(BHU 2005)

- (a) $r^{1/2}$ (b) r^{-1}
(c) r (d) r^{-2}

Solution (c) $E = \frac{Qr}{4\pi\epsilon_0 R^3}$ i.e. $E \propto r$

5. An elliptical cavity is carved out in a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are shown in Fig. 19.20. Then

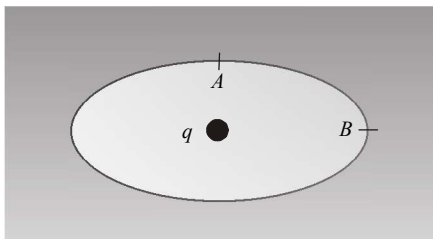


Fig. 19.20

- (a) electric field near A in the cavity = electric field near B in the cavity.
 (b) charge density at A = charge density at B
 (c) potential at A = potential at B
 (d) total electric field flux through the surface of the

cavity = $\frac{q}{\epsilon_0}$ [IIT 1999]

Solution (c) and (d) Because A and B lie on the same conductor, potential at each point is equal. For (d) Gauss law.

6. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V . If the shell is now given a charge $-3Q$, the new potential difference between the same two surfaces is

- (a) V (b) $2V$
 (c) $4V$ (d) $-2V$

[IIT 1989, DCE 1995]

Solution (a) Because the potential difference between solid sphere and hollow shell depends on the radii of two spheres and charge on the inner sphere. Since the two values have not changed, potential difference does not change.

7. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in Fig. 19.21 as

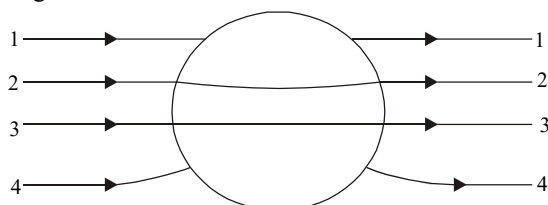


Fig. 19.21

(a) 1

(b) 2

(c) 3

(d) 4

[IIT 1999]

Solution (d) $E_{in} = 0$ and electric field lines are perpendicular to the equipotential surface.

8. Two conducting plates A and B are parallel. A is given a charge Q_1 and B is given a charge Q_2 . The charge on inner side of B is

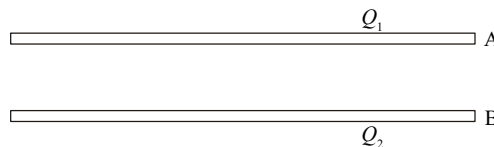


Fig. 19.22 (a)

- (a) $\frac{Q_1 - Q_2}{2}$ (b) $\frac{(Q_1 - Q_2)}{2}$
 (c) $\frac{(Q_1 + Q_2)}{2}$ (d) $\frac{-(Q_1 + Q_2)}{2}$

Solution (b) Electric field inside the conductor at point $P = 0$

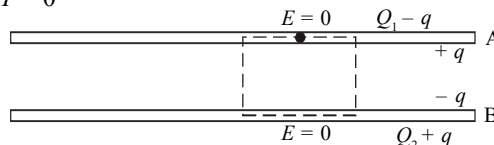


Fig. 19.22 (b)

$$\therefore \frac{(Q_1 - q)}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{(Q_2 + q)}{2A\epsilon_0} = 0$$

or $Q_1 - q - (Q_2 + q) = 0$ or $q = \frac{Q_1 - Q_2}{2}$

9. A hemisphere of radius r is placed in a uniform electric field of strength E . The electric flux through the hemisphere is

- (a) $2E\pi r^2$ (b) $-E\pi r^2$
 (c) $-2E\pi r^2$ (d) zero

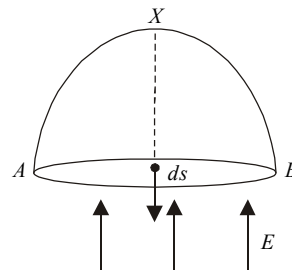


Fig. 19.23

Solution (b) $\phi = \oint E \cdot ds = \int_A^B E \cdot ds + \int_B^X E \cdot ds + \int_X^A E \cdot ds$
 $= E\pi r^2 + E\pi r^2 - E\pi r^2 = E\pi r^2$

Short cut: AXB is symmetrical surface \therefore Electric flux due to this part is zero. However, electric flux due to AB part is $-E\pi r^2$

10. A positively charged sphere suspended with a silk thread is slowly pushed in a metal bucket. After its insertion the lid is closed. What will be the electric field intensity inside when the sphere has touched the bucket? σ is the surface charge density of sphere.

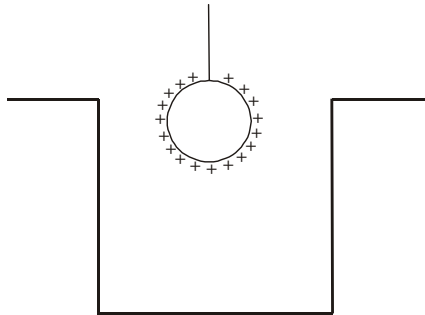


Fig. 19.24

- (a) zero
(b) $\frac{\sigma}{2\epsilon_0}$
(c) $\frac{\sigma}{\epsilon_0}$
(d) none of these

Solution (a) The charge of sphere will be taken by the bucket and appear on its outer surface. $\therefore E_{\text{in}} = 0$

11. An electric field in a region is $800\sqrt{x}\hat{i}$. The charge contained in a cubical volume bounded by the surfaces $x=0, x=a, y=0, y=a, z=0$ and $z=a$ is

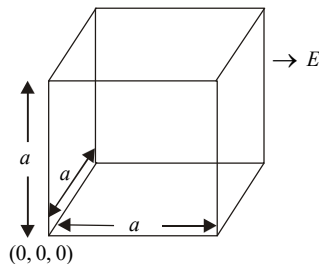


Fig. 19.25

- (a) $\frac{800\sqrt{a}}{\frac{1}{2}}\epsilon_0$
(b) $800 a^{\frac{5}{2}}\epsilon_0$
(c) $800 a^2\epsilon_0$
(d) $800 \sqrt{a}\epsilon_0$

Solution (b) $\phi = Ea^2$ and $Q = \phi\epsilon_0 = Ea^2\epsilon_0 = 800 a^{\frac{5}{2}}\epsilon_0$

12. A dipole is placed in a shell as shown. Find the electric flux emerging out of the shell and in a hypothetical sphere of radius r as shown in Fig. 19.26.

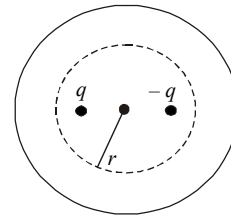


Fig. 19.26

- (a) $\frac{2q}{\epsilon_0}, 0$
(b) $\frac{q}{\epsilon_0}, \frac{-q}{\epsilon_0}$
(c) $\frac{q}{\epsilon_0}, \frac{-q}{\epsilon_0}$
(d) 0, 0

Solution (d)

13. Two concentric shells as shown enclose charge q and $-q$. The electric flux from the shell of radius R is

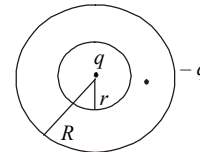


Fig. 19.27

- (a) $\frac{-q}{\epsilon_0}$
(b) $\frac{q}{\epsilon_0}$
(c) $\frac{2q}{\epsilon_0}$
(d) 0

Solution (d)

14. A charge Q is placed in a medium of dielectric constant K . The maximum number of lines of force are

- (a) $\frac{Q}{\epsilon_0}$
(b) $\frac{Q}{K}$
(c) $\frac{Q}{K\epsilon_0}$
(d) 0

Solution (c)

TYPICAL PROBLEMS

15. A region in space has a total charge Q distributed spherically such that the volume charge density $\rho(r)$ is given by

$$\rho(r) = 3\alpha r(2R) \quad \text{for } r \leq \frac{R}{2}$$

$$\rho(r) = \alpha \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad \text{for } \frac{R}{2} \leq r \leq R$$

$$\rho(r) = 0 \quad \text{for } r > R$$

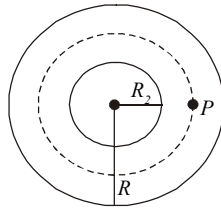


Fig. 19.28

Find the electric field at P ($\frac{R}{2} < x < R$)

Solution Charge on hypothetical sphere of radius x is given by

$$Q_{\text{enclosed}} = \int_0^{R/2} \frac{3\alpha r}{2R} (4\pi r^2 dr) + \int_{R/2}^x \alpha \left[1 - \left(\frac{r}{R} \right)^2 \right] 4\pi r^2 dr$$

$$Q_{\text{enclosed}} = \frac{3\alpha}{2R} (4\pi) \int_0^{R/2} r^3 dr + 4\pi\alpha \int_{R/2}^x \left[1 - \frac{r^2}{R^2} \right] r^2 dr$$

$$= \frac{3\alpha}{2R} (4\pi) \times \frac{\left(\frac{R}{2} \right)^4}{4} - 4\pi\alpha \frac{R^3}{3 \times 8} + \frac{4\pi\alpha x^3}{3} +$$

$$\frac{4\pi\alpha x^5}{5R^2} - \frac{4\pi\alpha R^3}{32 \times 5}$$

$$= \frac{3\alpha \pi R^3}{32} - \frac{4\pi\alpha R^3}{24} + \frac{4\pi\alpha x^3}{3} + \frac{4\pi\alpha x^5}{5R^2}$$

$$- \frac{4\pi\alpha R^3}{160}$$

$$= \alpha \pi \left(\frac{3R^3}{32} - \frac{R^3}{6} + \frac{4x^3}{3} - \frac{4x^5}{5R^2} - \frac{1}{40} R^3 \right)$$

$$\oint E \cdot ds = Q_{\text{enclosed}} E (4\pi x^2 = \alpha \pi \left(\frac{3R^3}{32} - \frac{R^3}{6} + \frac{4x^3}{3} - \right.$$

$$\left. \frac{4x^5}{5R^2} - \frac{R^3}{40} \right)$$

$$E = \left[\frac{3\alpha R^3}{128x^2} + \frac{\alpha R^3}{24x^2} + \frac{\alpha x}{3} - \frac{\alpha x^3}{5R^2} - \frac{R^3}{160\pi x^2} \right]$$

16. An insulating sphere with radius a has a uniform charge density ρ . The sphere is not centered at the origin but at $\vec{r} = \vec{b}$. Find the electric field at any point inside the sphere.

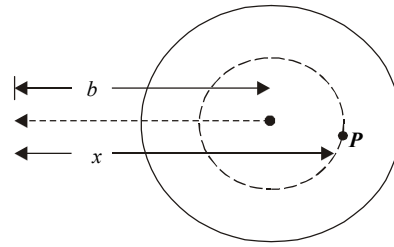


Fig. 19.29

Solution $Q_{\text{enclosed}} = \rho \frac{4}{3} \pi (x-b)^3$

$$\oint E \cdot ds = \frac{\rho \frac{4}{3} \pi (x-b)^3}{\epsilon_0} E 4\pi (x-b)^2$$

$$= \frac{\rho \frac{4}{3} \pi (x-b)^3}{\epsilon_0} \text{ or } \vec{E} = \frac{\rho(\vec{x} - \vec{b})}{3\epsilon_0}$$

17. A charge Q is placed at the outer side of a cube. Find the flux passing through a face adjoining the charge

(a) $\frac{Q}{6\epsilon_0}$

(b) $\frac{Q}{8\epsilon_0}$

(c) $\frac{Q}{24\epsilon_0}$

(d) $\frac{Q}{32\epsilon_0}$

Solution (C) The charge will be completely covered by 8 cubes. 3 faces of each cube will share the flux. Hence 24 faces in all will share the flux

$$\therefore \oint E \cdot ds = \frac{Q_{\text{inside}}}{\epsilon_0} \text{ or}$$

$$E (24 a^2) = \frac{Q}{\epsilon_0} \text{ or } E a^2 = \frac{Q}{24\epsilon_0}$$

18. An electron of 100eV is fired directly towards a metal plate having surface charge density $-2 \times 10^6 \text{ cm}^{-2}$. Find distance from where the electron be projected so that it just fails to strike the plate.

(a) 0.22 mm

(b) 0.44 mm

(c) 0.66 mm

(d) 0.33 mm

Solution (b) Gain in PE = loss in $KE = 100e$

$$\frac{e\sigma}{\epsilon_0} d \text{ or } d = \frac{100 \times 8.85 \times 10^{-12}}{2 \times 10^{-6}} = 0.44 \text{ mm}$$

19. A long conducting cylinder carrying a charge $+q$ is surrounded by a conducting cylindrical shell having charge $-2q$. The charge on the inner and outer side of conducting shell is

(a) $q, -q$

(b) $-q, -3q$

(c) $-q, +3q$

(d) $-q, -q$

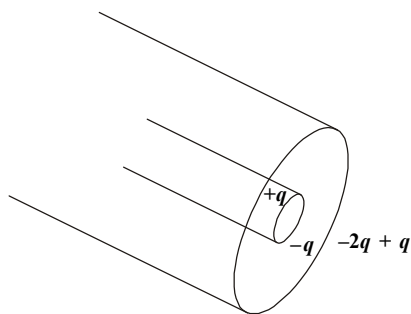


Fig. 19.30

Solution (d) Let $-q$ be the charge induced on the inner side then $-2q + q$ will be charge on outer side of the shell.

20. Two point charges q and $-q$ are $2l$ apart. Find the flux of the electric field strength vector across a circle of radius R . (See Fig. 19.31)

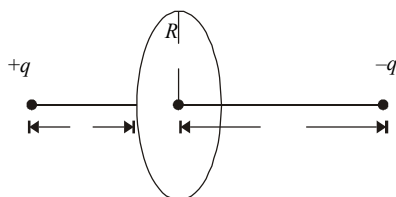


Fig. 19.31

Solution Electric field due to a dipole at equatorial line

$$E = \frac{p}{4\pi\epsilon_0 (\sqrt{l^2 + x^2})^3}$$

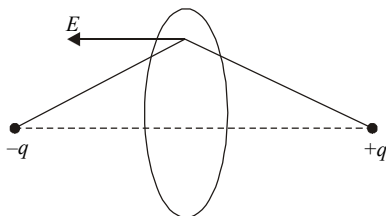


Fig. 19.32

$$E = \frac{2ql}{4\pi\epsilon_0 (l^2 + x^2)^{3/2}}$$

Let elemental surface area be $x d\theta dx$

$$\begin{aligned} \phi &= \int E x d\theta dx = \frac{2ql}{4\pi\epsilon_0} \int_0^R \frac{x dx}{(l^2 + x^2)^{3/2}} \int_0^{2\pi} d\theta \\ &= \frac{ql}{\epsilon_0} \int_0^R \frac{x dx}{(l^2 + x^2)^{3/2}} = \frac{q}{\epsilon_0} \left[1 - \frac{l}{\sqrt{l^2 + R^2}} \right] \end{aligned}$$

PASSAGE 1

Read the following passage and answer the questions given at the end.

Consider the electrical field of a simple positive point charge q shown in Fig. 19.33. By symmetry the field is everywhere radial (there is no reason why it should deviate to one side of a radial direction rather than to another) and its magnitude is the same at all points at the same distance r from the charge (any point at this distance is like any other). Hence if we select as a gaussian surface a spherical surface of radius r , $E_1 = E = \text{constant}$ at all points of the surface.

Then

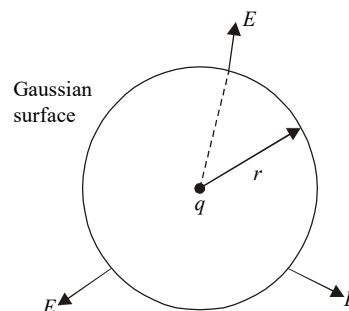


Fig. 19.33

$$\oint E_1 dA = E_1 \oint dA = EA = 4\pi r^2 E$$

From Gauss's law

$$4\pi r^2 E = \frac{q}{\epsilon_0} \text{ and } E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

The force on a point charge q_1 at a distance r from the charge q is then

$$F = q_1 E = \frac{1}{4\pi\epsilon_0} \frac{qq_1}{r^2}$$

which is coulomb's law.

1. A cavity of radius $r/3$ is made in a sphere of radius r , charge Q . The emptied part is placed at a distance $2r$ from the centre as shown. If the charge was uniformly distributed throughout, then force between the two is

- (a) $\frac{11Q^2}{192 \times 27\pi\epsilon_0 r^2}$ (b) $\frac{11Q^2}{4\pi\epsilon_0 r^2}$
(c) $\frac{11Q^2}{192\pi\epsilon_0 r^2}$ (d) none

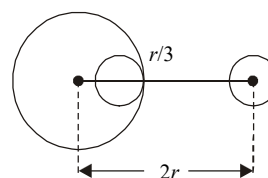


Fig. 19.34

2. A point charge q is placed asymmetrically in a metal cavity as shown in Figure 19.35 (a). Which one of the diagrams correctly represents the field lines?

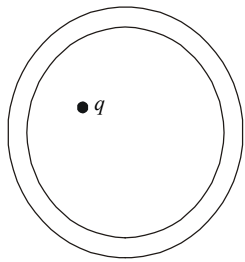


Fig. 19.35 (a)

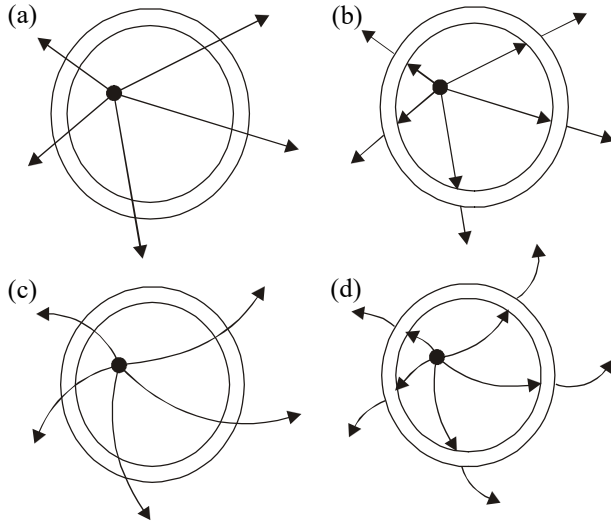


Fig. 19.35 (b)

3. Two point charges q and q_1 are placed distance r apart. A dielectric sheet of dielectric constant k and thickness t is placed in between. Find the force between the two charges

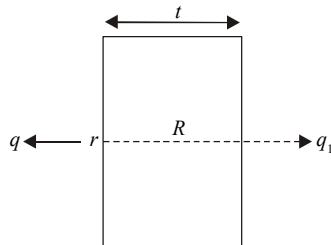


Fig. 19.36

- (a) $\frac{qq_1}{4\pi\epsilon_0 kr^2}$ (b) $\frac{qq_1}{4\pi\epsilon_0 [(r-t)^2 + kt^2]}$
 (c) $\frac{qq_1}{4\pi\epsilon_0 (r-t + \sqrt{kt})^2}$ (d) none

4. A ring has charge Q , radius R and a charge q is placed at the centre. The increase in tension in the ring is

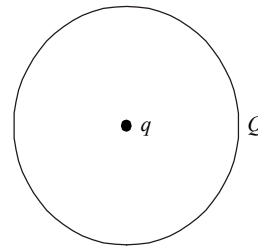


Fig. 19.37

- (a) $\frac{Qq}{4\pi\epsilon_0 R^2}$ (b) $\frac{Qq}{4\pi^2\epsilon_0 R^2}$
 (c) $\frac{Qq}{8\pi^2\epsilon_0 R^2}$ (d) zero

Solution 1. (a) $q = \frac{Q}{r^3} (r/3)^3 = \frac{Q}{27}$

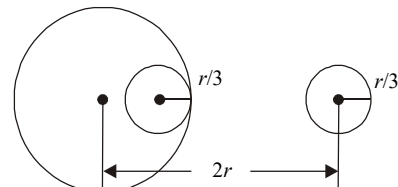


Fig. 19.38

$$F = qE$$

$$= \frac{Q}{27} \left[\frac{Q}{4\pi\epsilon_0 (2r)^2} - \frac{Q/27}{4\pi\epsilon_0 \left(\frac{4r}{3}\right)^2} \right]$$

$$= \frac{Q}{27} \left[\frac{Q}{16\pi\epsilon_0 r^2} - \frac{Q}{48 \times 4\pi\epsilon_0 r^2} \right]$$

$$= \frac{11Q^2}{27 \times 16\pi\epsilon_0 r^2 \times 12}$$

Solution 2. (d)

Solution 3. (c) The equivalent distance between the charges q and q_1 is $(r-t) + \sqrt{kt}$. Therefore

$$F = \frac{qq_1}{4\pi\epsilon_0 (r-t + \sqrt{kt})^2}$$

Solution 4. (c) $2T \sin \theta = \frac{qQ(2\theta R)}{2\pi R}$

$$\frac{4\pi\epsilon_0 R2}{4\pi\epsilon_0 R2}$$

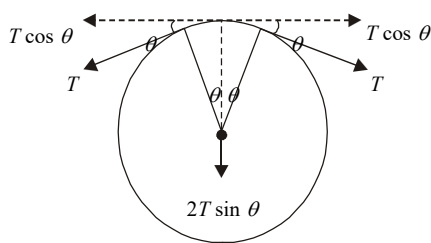


Fig. 19.39

$$2T \theta = \frac{qQ2\theta}{8\pi^2 \epsilon_0 R^2}$$

$$T = \frac{qQ}{8\pi^2 \epsilon_0 R^2}$$

PASSAGE 2

Read the following passage and answer the questions given at the end.

The Gravitational force between two point masses separated by a distance r is proportional to $\frac{1}{r^2}$, just like the electric force between two point charges. Because of this similarity between gravitational and electric interactions, there is also a Gauss's law for gravitation. Let g be the acceleration due

to gravity caused by a point mass m at the origin so that $g = \frac{-Gm}{r^2} \hat{r}$. Consider a spherical Gaussian surface with radius r centered on this point mass.

1. The flux of \vec{g} through this surface is given by

(a) $\oint \vec{g} \cdot d\vec{s} = 4\pi Gm$ (b) $\oint \vec{g} \cdot d\vec{s} = -4\pi Gm$

(c) $\oint \vec{g} \cdot d\vec{s} = -Gm$ (d) $\oint \vec{g} \cdot d\vec{s} = Gm$

2. Using general logic for electric field, the flux of \vec{g} through any closed surface is given by

(a) $\oint \vec{g} \cdot d\vec{s} = 4\pi Gm_{\text{enclosed}}$

(b) $\oint \vec{g} \cdot d\vec{s} = -4\pi Gm_{\text{enclosed}}$

(c) $\oint \vec{g} \cdot d\vec{s} = Gm_{\text{enclosed}}$

(d) $\oint \vec{g} \cdot d\vec{s} = -Gm_{\text{enclosed}}$

Solution 1. (b)

Solution 2. (b)

QUESTIONS FOR PRACTICE

- A charge is kept at the centre of a shell. Shell has charge Q and radius R . The force on the central charge due to the shell is
 - towards left
 - towards right
 - upward
 - zero.
- Fig. 19.40 (a) shows an imaginary cube of edge $L/2$. A uniformly charged rod of length L moves towards left at a small but constant speed v . At $t = 0$, the left end just touches the centre of the face of the cube opposite it. Which of the graphs shown in Fig. 19.40 (b) represents the flux of the electric field through the cube as the rod goes through it?

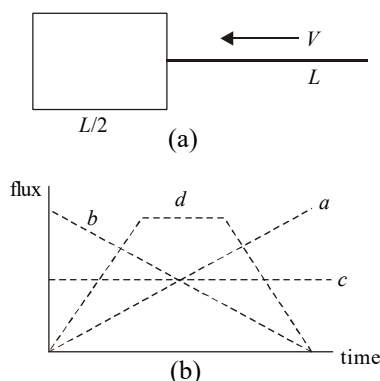


Fig. 19.40

- Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V/m. The flux over a concentric sphere of radius 20 cm will be
 - 25 V-m
 - 50 V-m
 - 100 V-m
 - 200 V-m.
- A charge q is placed at the centre of the open end of a cylindrical vessel (Fig. 19.41). The flux of the electric field through the surface of the vessel is

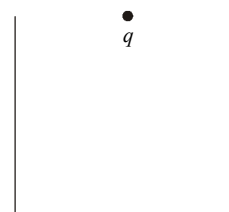


Fig. 19.41

- zero
- $\frac{q}{\epsilon_0}$
- $\frac{q}{2\epsilon_0}$
- $\frac{2q}{\epsilon_0}$

5. Mark the correct options:

- (a) Gauss's law is valid only for symmetrical charge distributions.
- (b) Gauss's law is valid only for charges placed in vacuum.
- (c) The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.
- (d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

6. A large nonconducting sheet M is given a uniform charge density. Two uncharged small metal rods A and B are placed near the sheet as shown in Fig. 19.42.

- (a) M attracts A .
- (b) M attracts B .
- (c) A attracts B .
- (d) B attracts A .

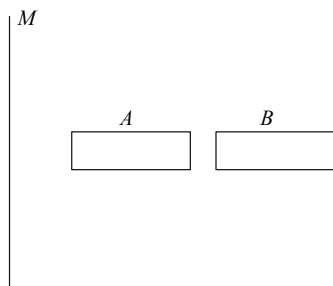


Fig. 19.42

7. An electric dipole is placed at the centre of a sphere. Mark the correct options:

- (a) The flux of the electric field through the sphere is zero.
- (b) The electric field is zero at every point of the sphere.
- (c) The electric field is not zero anywhere on the sphere.
- (d) The electric field is zero on a circle on the sphere.

8. A positive point charge Q is brought near an isolated metal cube.

- (a) The cube becomes negatively charged.
- (b) The cube becomes positively charged.
- (c) The interior becomes positively charged and the surface becomes negatively charged.
- (d) The interior remains charge free and the surface gets nonuniform charge distribution.

9. A closed surface S is constructed around a conducting wire connected to a battery and switch (Fig. 19.43). As the switch is closed, the free electrons in the wire start moving along the wire. In any time interval, the number of electrons entering the closed surface S is equal to the number of electrons leaving it. On closing the switch, the flux of the electric field through the closed surface

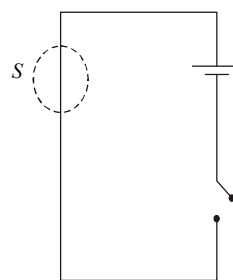


Fig. 19.43

- (a) increases
- (b) decreases
- (c) remains unchanged
- (d) is zero.

10. If the flux of the electric field through a closed surface is zero,

- (a) the electric field must be zero everywhere on the surface
- (b) the electric field may be zero everywhere on the surface
- (c) the charge inside the surface must be zero
- (d) the charge in the vicinity of the surface must be zero.

11. Fig. 19.44 shows a charge q placed at the centre of a hemisphere. A second charge Q is placed at one of the positions A , B , C and D . In which position(s) of this second charge, does the flux of the electric field through the hemisphere remain unchanged?

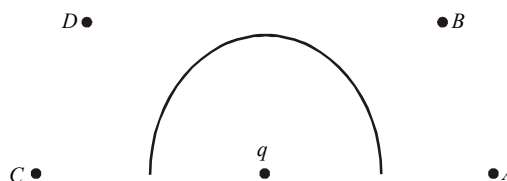


Fig. 19.44

- (a) A
- (b) B
- (c) C
- (d) D .

12. The electric field in a region is radially outwards and has a magnitude $E = Kr$. The charge contained in a sphere of radius a is

- (a) $K4\pi\epsilon_0 a^2$
- (b) $K \frac{4}{3} \pi\epsilon_0 a^3$
- (c) $K4\pi\epsilon_0 a^3$
- (d) none of these

13. A charged particle having a charge $-2 \times 10^{-6} \text{ C}$ is placed close to the non-conducting plate having a surface charge density $4 \times 10^{-6} \text{ cm}^{-2}$. The force of attraction between the particle and the plate is nearly

- (a) 0.9 N
- (b) 0.71 N
- (c) 0.62 N
- (d) 0.45 N

14. A non-conducting sheet of large surface area and thickness d contains uniform charge density ρ . The electric field at a point P inside the plane at a distance x from the central plane $0 < x < d$

- (a) $\frac{\rho x}{2\epsilon_0}$ (b) $\frac{\rho d}{2\epsilon_0}$
 (c) $\frac{\rho x}{\epsilon_0}$ (d) $\frac{\rho x}{2\epsilon_0}$
15. A long cylinder contains uniformly distributed charge density ρ . The electric field at a point P inside the cylinder at a distance x from the axis is
- (a) $\frac{\rho x}{\epsilon_0}$ (b) $\frac{\rho x}{2\epsilon_0}$
 (c) $\frac{\rho x}{4\epsilon_0}$ (d) none of these
16. A long cylindrical wire carries a linear density of $3 \times 10^{-8} \text{ cm}^{-1}$. An electron revolves around it in a circular path under the influence of the attractive force. KE of the electron is
- (a) $1.44 \times 10^{-7} \text{ J}$ (b) $2.88 \times 10^{-17} \text{ J}$
 (c) $4.32 \times 10^{-17} \text{ J}$ (d) $8.64 \times 10^{-17} \text{ J}$
17. The electric field at a point 5 cm from a long line charge of density $2.5 \times 10^{-6} \text{ cm}^{-1}$ is
- (a) $9 \times 10^3 \text{ NC}^{-1}$ (b) $9 \times 10^4 \text{ NC}^{-1}$
 (c) $9 \times 10^5 \text{ NC}^{-1}$ (d) $9 \times 10^6 \text{ NC}^{-1}$
18. A charge q is uniformly distributed in the hollow sphere of radii r_1 and r_2 ($r_2 > r_1$). The electric field at a point P distance x from the centre for $r_1 < x < r_2$ is
- (a) $\frac{Q(x)}{4\pi\epsilon_0(r_2^3 - r_1^3)}$ (b) $\frac{Q(x^3 - r_1^3)}{4\pi\epsilon_0(r_2^3 - r_1^3)}$
 (c) $\frac{Q(x^3 - r_1^3)}{4\pi\epsilon_0 x^2 (r_2^3 - r_1^3)}$ (d) $\frac{Qr_1^3}{4\pi\epsilon_0 x^2 (r_2^3 - r_1^3)}$
19. The radius of gold nucleus is about $7 \times 10^{-15} \text{ m}$ ($Z = 79$). The electric field at the mid-point of the radius assuming charge is uniformly distributed is
- (a) $1.16 \times 10^{19} \text{ NC}^{-1}$ (b) $1.16 \times 10^{21} \text{ NC}^{-1}$
 (c) $2.32 \times 10^{21} \text{ NC}^{-1}$ (d) $2.32 \times 10^{19} \text{ NC}^{-1}$
20. A spherical volume has a uniformly distributed charge density $2 \times 10^{-4} \text{ cm}^{-3}$. The electric field at a point inside the volume at a distance 4.0 cm from the centre is
- (a) $3.15 \times 10^5 \text{ NC}^{-1}$ (b) $2.1 \times 10^5 \text{ NC}^{-1}$
 (c) $6.2 \times 10^5 \text{ NC}^{-1}$ (d) none of these
21. A charge Q is placed at the centre of a cube. The flux through the six surfaces of the cube is
- (a) $\frac{Q}{\epsilon_0}$ (b) $\frac{6Q}{\epsilon_0}$
 (c) $\frac{Q}{6\epsilon_0}$ (d) $\frac{Q}{3\epsilon_0}$

22. The electric field in a region is

$$E = \frac{5 \times 10^3 (\text{NC}^{-1} \text{cm}^{-1}) x}{2} \hat{i}$$

The charge contained inside a cubical volume bounded by the surfaces $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ is (where x, y, z are in cm)

- (a) $2.21 \times 10^{-12} \text{ C}$ (b) $4.42 \times 10^{-12} \text{ C}$
 (c) $2.21 \times 10^{-8} \text{ C}$ (d) $4.42 \times 10^{-8} \text{ C}$
23. A charge Q is uniformly distributed over a rod of length l . Consider a hypothetical cube of edge l with the centre of the cube at one end of the rod. The minimum possible flux of the electric field through the entire surface of the cube is
- (a) $\frac{Q}{\epsilon_0}$ (b) $\frac{Q}{2\epsilon_0}$
 (c) $\frac{Q}{8\epsilon_0}$ (d) $\frac{Q}{6\epsilon_0}$
24. The electric field in a region is given by $\vec{E} = \frac{3}{5} E_0 \hat{j}$ with $E_0 = 2 \times 10^3 \text{ NC}^{-1}$. Find the flux of this field through a rectangular surface of area 0.2 m^2 parallel to the Y - Z plane.
- (a) $320 \text{ Nm}^2 \text{C}^{-1}$ (b) $240 \text{ Nm}^2 \text{C}^{-1}$
 (c) $400 \text{ Nm}^2 \text{C}^{-1}$ (d) none of these
25. A positive point charge Q is brought near an isolated metal cube.
- (a) The cube becomes negatively charged.
 (b) The cube becomes positively charged.
 (c) The interior becomes positively charged and the surface becomes negatively charged.
 (d) The interior remains charge free and the surface gets non-uniform charge distribution.
26. Mark the correct options.
- (a) Gauss's law is valid only for symmetrical charge distribution.
 (b) Gauss's law is valid only for charges placed in vacuum.
 (c) The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.
 (d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.
27. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V-m .

The flux over concentric sphere of radius 20 cm will be

- (a) 25 Vm (b) 50 Vm
(c) 100 Vm (d) 200 Vm

28. A metallic particle having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the particle will be

- (a) towards the plate (b) away from the plate
(c) parallel to the plate (d) zero

PASSAGE 1

Read the following passage and answer the questions given at the end.

A long coaxial cable consists of an inner cylindrical conductor with radius a and outer coaxial cylinder with radius b and outer radius c . The outer cylinder is mounted on insulating supports and has no net charge. The inner cylinder has a uniform positive charge per unit length λ .

1. The electric field at any point between inner and outer cylinders is

(a) $\frac{\lambda}{2\pi\epsilon_0 r}$ radially outward

(b) $\frac{\lambda}{2\pi\epsilon_0 r}$ radially inward

(c) $\frac{\lambda}{2\pi\epsilon_0 r}$ co-axially

(d) none of these

2. The electric field at any point P ($x > b$) outside the outer cylinder is

(a) $\frac{\lambda}{2\pi\epsilon_0 r}$ radially inward

(b) $\frac{\lambda}{2\pi\epsilon_0 r}$ radially outward

(c) $\frac{\lambda}{2\pi\epsilon_0 r}$ co-axially

(d) none of these

3. Find the charge per unit length of inner surface and outer surface of the outer cylinder.

(a) $-\lambda$ on inner, $+\lambda$ on outer surface

(b) $-\lambda$ on outer, $+\lambda$ on inner surface

(c) $-\lambda/2$ on both inner and outer surfaces

(d) $+\lambda/2$ on both inner and outer surfaces

Solution 1. (a) $\oint E \cdot ds = \frac{q_{in}}{\epsilon_0};$

$$q_{in} = \lambda l$$

$$E(2\pi l) = \frac{\lambda l}{2\pi\epsilon_0 r}$$

or $E = \frac{\lambda}{2\pi\epsilon_0 r}$

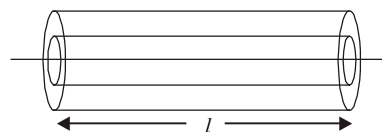


Fig. 19.48

Solution 2. (b) radially outward.

Solution 3. (a) Induced charge is expected on inner and outer surface so that net charge is zero as no charge is given on this cylinder.

PASSAGE 2

Read the following passage and answer the questions given at the end.

In an arrangement an insulating sphere with radius a has uniform charge density ρ . The sphere is not centered at the origin but at $\vec{r} = \vec{b}$ as shown in Fig. 19.49. In another arrangement an insulating sphere of radius R has a spherical hole of radius a located within its volume and centered at b , from the center of the sphere such that $a < b < R$. A cross-section of the sphere is shown in Fig. 19.50. The solid part of the sphere has a uniform volume charge density ρ .

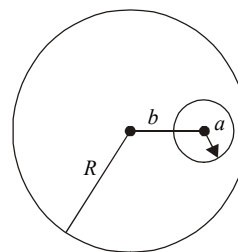


Fig. 19.49

1. Find the electric field in the sphere in first arrangement of Fig. 19.49.

(a) $\vec{E} = \frac{\rho r}{4\pi\epsilon_0}$

(b) $\vec{E} = \frac{\rho(\vec{r}-\vec{b})}{3\epsilon_0}$

(c) $\vec{E} = \frac{\rho \vec{b}}{4\pi\epsilon_0}$

(d) none of these

2. Find the electric field in the hole in the second arrangement of Fig. 19.50.

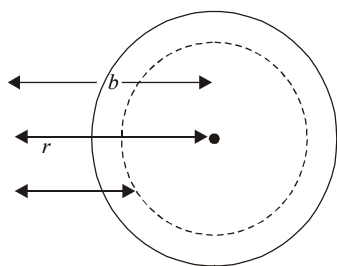


Fig. 19.50

- (a) $\vec{E} = \frac{\rho a}{3\epsilon_0}$ (b) $\vec{E} = \frac{\rho \vec{r}}{3\epsilon_0}$
 (c) $\vec{E} = \frac{\rho b}{3\epsilon_0}$ (d) $\vec{E} = \frac{\rho(\vec{b} - \vec{r})}{3\epsilon_0}$

3. The electric field inside the hole in second arrangement is

- (a) constant (b) varies radially
 (c) varies exponentially (d) none of these

Solution 1. (b) $\oint \vec{E} \cdot d\vec{s} = \frac{Q_{in}}{\epsilon_0}$

$$\Rightarrow E 4\pi(b-r)^2 = \frac{\rho \frac{4}{3}\pi(b-r)^3}{\epsilon_0}$$

or $E = \frac{\rho(b-r)}{3\epsilon_0}$

The direction is radially out ward

$$\therefore \vec{E} = \frac{\rho(\vec{r} - \vec{b})}{3\epsilon_0} = \frac{\rho \vec{b}}{3\epsilon_0}$$

Solution 2. (c) $E = \vec{E}_1 + \vec{E}_2 = \frac{-\rho \vec{r}}{3\epsilon_0} + \frac{\rho(\vec{r} - \vec{b})}{3\epsilon_0}$
 $= \frac{\rho \vec{b}}{3\epsilon_0}$

Solution 3. (a) as E is independent of r .

Answers to Questions for Practice

1. (b)	2. (d)	3. (a)	4. (c)	5. (d)	6. (a, b, c, d)	7. (a, c)
8. (d)	9. (c, d)	10. (b, c)	11. (a, c)	12. (c)	13. (d)	14. (c)
15. (a)	16. (c)	17. (c)	18. (c)	19. (b)	20. (a)	21. (a)
22. (a)	23. (b)	24. (b)	25. (d)	26. (d)	27. (a)	28. (a)

