18 Electrostatics

BRIEF REVIEW

Charge It is the fundamental property of matter with which it exerts coulomb force. Charge is of two types, **Positive** and **Negative**. Like charges repel and unlike charges attract. A charged particle can even attract an uncharged particle. Charge can be detected using GLE (Gold Leaf Electroscope) and is measured using electrometers. The unit of charge is **Coulomb**. The charge on electron or proton is termed as natural charge and is the minimum unit of charge which can be transferred from one body to another. $\frac{e}{m}$ is called specific charge.

Charge is conserved (a) In an isolated system charge can neither be created nor destroyed (b) Total charge of the universe is constant (c) Charge can be created or destroyed but in equal and opposite pairs, for instance, a γ-ray can be converted to an electron and a positron, i.e.,

$$E_\gamma\,(\geq 1.02\text{ MeV}) \rightarrow e^- + e^+$$

This process is called pair production. The electron and positron combine to form γ-ray again. This process is termed as pair annihilation

$$e^+ + e^- \longrightarrow \gamma(E_\gamma = 1.02\text{ MeV})$$

Charge is quantised A charged body can have charge which is integral multiple of electronic charge. i.e. $Q = \pm\, ne$. If a body gains electrons, it is said to be negatively charged and if it loses electrons, it is said to be positively charged. Though there are particles called quarks which may have charge $\frac{e}{3}$ or $\frac{2e}{3}$, since these are generated during disintegration of nucleus (neutron, proton and so on) these cannot be transferred. Charge on an electron is 1.6×10^{-19} C.

$$1\ esu = \frac{1}{3\times10^9}\text{ C}$$

and $1\,emu = 10$ C

A body can be charged by **rubbing**. For example, when glass rod and silk cloth are rubbed against each other, glass rod acquires positive charge and silk cloth, negative charge. We can also charge a body by **induction** and, by **physical contact** of an uncharged body with a charged body. A capacitor may be charged with a battery.

Coulomb's Law If two point charges q_1 and q_2 are distance r apart then force between two charges (see Fig. 18.1)

$$F \propto q_1 q_2$$

and $$F \propto \frac{1}{r^2}$$

i.e. $|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ (in free space) where ε_0 is permittivity of free space.

$$|F| = \frac{q_1 q_2}{4\pi\varepsilon_0\varepsilon_r r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 k r^2}$$ (in a medium) where

$$\varepsilon_r = k = \frac{\varepsilon_m}{\varepsilon_0} \text{ is relative permittivity of the medium}$$

or dielectric constant of the medium.

Vector form of Coulomb law

$$\vec{F} = \frac{q_1 q_2 \vec{r}}{4\pi\varepsilon_0 r^3}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{ C}^{-2}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

Note that ε_r *is dimensionless.*

Coulomb's law is valid if (i) charges are point charges or spherical charges (ii) distance r between the two charges $\geq 10^{-15}$ m.

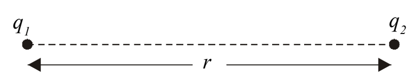

Fig. 18.1 **Coulomb force**

Dielectric is an insulator. It is of two types, polar and nonpolar. Polar dielectrics are those which have permanent dipole moment like water,

$$\varepsilon_r = 80\,(H_2O)$$

and $$\varepsilon_r = \infty \text{ (metals)}$$

Electric field intensity or electric field strength is the force experienced by a unit positive charge at that point when placed in an electric field of the given charge. Its unit is N/C or Vm^{-1}.

$$|E| = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{|F|}{q}$$

In vector for m^2 $$\vec{E} = \frac{Q\vec{r}}{4\pi\varepsilon_0 r^3} = \frac{\vec{F}}{q}$$

Electric field vectors are of three types namely $\vec{E}$, $\vec{P}$ and $\vec{D}$

where $$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

D is called electric displacement vector

$\vec{P} = \varepsilon_0 (K-1)\, \vec{E}$ is polarising vector. For vacuum $K = 1$ and $P = 0$

Electrets are the substances which do not follow

$$\vec{P} = \varepsilon_0 (K-1)\, \vec{E}$$

Further $E = \dfrac{-dv}{dx} = -\vec{\nabla} V$ (in 3-dimension)

where V is electric potential.

For equipotential surface $\boldsymbol{E = 0}$

If charge is not a point charge then linear charge density $\lambda = \dfrac{Q}{l}$, surface charge density $\sigma = \dfrac{Q}{\text{Area}}$ or volume charge density $\rho = \dfrac{Q}{\text{Volume}}$ is determined. A small length dx (for linear charge density), a small area ds (for surface charge density) or a small volume dv (for volume charge density) is considered to find a point charge. Write the equation of electric field/force using the small element and integrate. Electric field and electric force obey superposition principle. Electric field/force is conservative.

Electric field lines or electric lines of force are imaginary lines originating from positive charge and terminating at negative charge, such that tangent at any point gives the direction of force. No two electric lines of force can intersect each other.

Electric Flux The lines of force passing through a given area in an electric field is called electric flux.

$\phi_E = \int \vec{E}.d\vec{s}$. If E and S are mutually perpendicular then $\phi_E = 0$. The unit of electric flux is Nm^2C^{-1} and dimensional formula is $[ML^3T^{-3}A^{-1}]$. It is a scalar quantity.

Electric Potential The amount of work done to bring unit positive charge from infinity to that point against the electric field of a given charge without changing its kinetic energy or velocity.

$$V = \int_{\infty}^{r} -E.dx = \frac{Q}{4\pi\varepsilon_0 r}$$

It is a scalar quantity and its unit is volt. 1 volt $= \dfrac{1J}{1C}$

Its dimensional formula is $[ML^3T^{-3}A^{-1}]$.

Potential Difference

$$\Delta V = V_2 - V_1$$

$$= \int_{r_1}^{r_2} -E.dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

Potential Energy

$$U = qV = \frac{Qq}{4\pi\varepsilon_0 r}$$

Equipotential Surface is the surface, where potential is equal at every point. For a point charge, a sphere will be equipotential surface with point charge at the centre of the sphere. Equipotential surface for a long line charge is a cylinder with line charge along its axis. Equipotential surface for a dipole is shown in Fig. 18.2

The work done in carrying a charge from one point to another along an equipotential surface is zero.

The electric field lines are always perpendicular to the equipotential surface.

Every conductor (metal) is an equipotential surface and hence electric field lines will emerge perpendicular to it.

Electric field and surface charge density are maximum at pointed ends of a conductor.

Note $\oint \vec{E}.d\vec{l} = 0$

and $\int_a^b \vec{E}.d\vec{l} = V_a - V_b$

$$\vec{E} = -\vec{\nabla}V$$

$$= -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

$$= -\left(\hat{i}\frac{\partial}{\partial X} + \hat{j}\frac{\partial}{\partial Y} + \hat{k}\frac{\partial}{\partial Z}\right)V$$

i.e. $\vec{\nabla} = \hat{i}\frac{\partial}{\partial X} + \hat{j}\frac{\partial}{\partial Y} + \hat{k}\frac{\partial}{\partial Z}$ is called gradient operator and is written as grad or del.

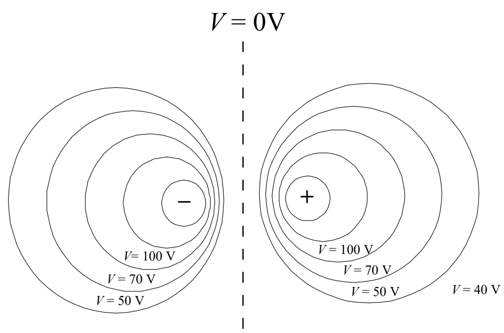

Fig. 18.2 Equipotential surface illustration for dipole

Electric field intensity due to a shell (spherical) having charge Q and radius R

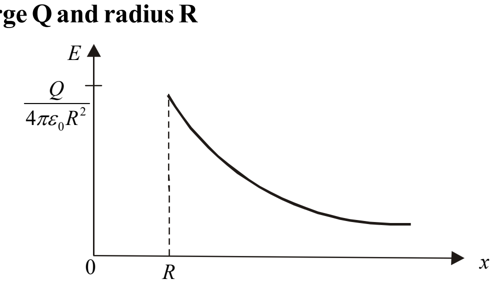

Fig. 18.3 Electric field due to shell

$$E_{\text{inside}} = 0 \qquad x < R$$

$$E_{\text{surface}} = \frac{Q}{4\pi\varepsilon_0 R^2} \qquad x = R$$

$$E_{\text{outside}} = \frac{Q}{4\pi\varepsilon_0 x^2} \qquad x > R$$

Electric potential due to a spherical shell (radius *R*, charge *Q*)

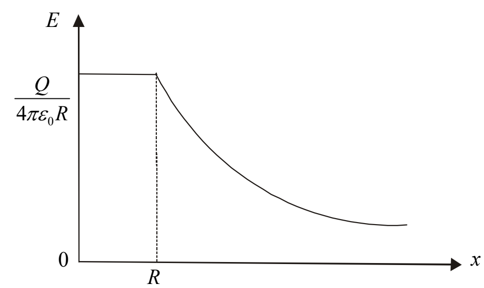

Fig. 18.4 Potential due to shell

$$V_{\text{inside}} = \frac{Q}{4\pi\varepsilon_0 R} = V_{\text{surface}} \qquad x \le R$$

$$V_{\text{out side}} = \frac{Q}{4\pi\varepsilon_0 R} \qquad x > R$$

Electric field due to a finite line charge on perpendicular bisector

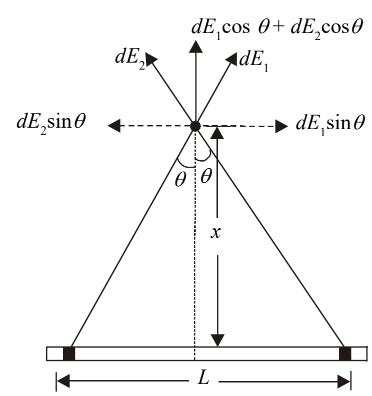

Fig. 18.5 Electric field due to a line charge along equatorial line

$$E = \frac{Q}{2\pi\varepsilon_0 x\sqrt{L^2 + 4a^2}}$$

Electric field intensity due to a ring of radius *R* at a distance *X* on the axial line

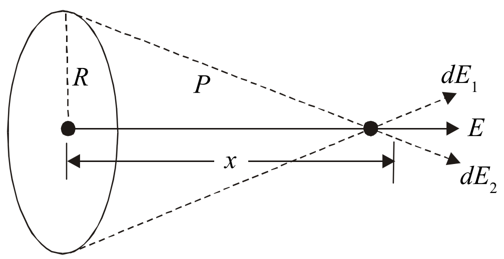

Fig. 18.6 Electric field due to ring

$$E = \frac{Qx}{4\pi\varepsilon_0\left(x^2+R^2\right)^{3/2}}$$

At the centre of the ring $E = 0$

Electric field is maximum at

$$x = \frac{R}{\sqrt{2}}$$

Electric Potential at any point P due a to ring on axial line

$$V = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

$$V\text{(centre of the ring)} = \frac{Q}{4\pi\varepsilon_0 R}$$

Electric field due to a disc of radius R having surface charge density σ at a point P, distant x on the axial line

$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

If $x \to 0$, i.e., at the centre of the disc, $E = \dfrac{\sigma}{2\varepsilon_0}$

Also $E = \dfrac{\sigma}{2\varepsilon_0}$ if $R \to \infty$, i.e., due to a long disc.

Electric potential V at any point P due to the disc along axial line

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+R^2}-x\right]$$

Dipole Moment $\vec{p} = q(2l)$. The direction of electric dipole moment $\vec{p}$ is from negative towards positive charge as shown in Fig. 18.7.

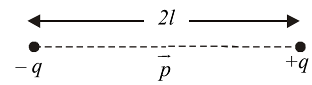

Fig. 18.7 Dipole

Electric field intensity due to a dipole

(a) Along axial line

$$E_{\text{axial}} = \frac{2px}{4\pi\varepsilon_0(x^2-l^2)^2} \quad \text{(See Fig 18.8)}$$

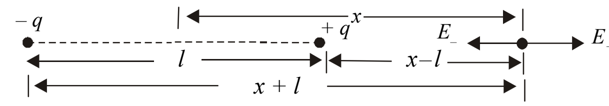

Fig. 18.8 Electric field due to a dipole along axial line

for a short dipole $x >> l$

$$E_{\text{axial}} = \frac{2p}{4\pi\varepsilon_0 x^3}$$

Note the direction of electric field is parallel to electric dipole moment.

Electric potential along axial line

$$V_{\text{axial}} = \frac{p}{4\pi\varepsilon_0(x^2-l^2)}$$

$$V_{\text{axial}} = \frac{p}{4\pi\varepsilon_0 x^2} \qquad \text{due to a short dipole.}$$

(b) Electric field along equatorial line

$$E_{\text{equatorial}} = \frac{p}{4\pi\varepsilon_0(x^2+l^2)^{3/2}}$$

Note that the direction of electric field is antiparallel to dipole movement as shown in Fig. 18.9

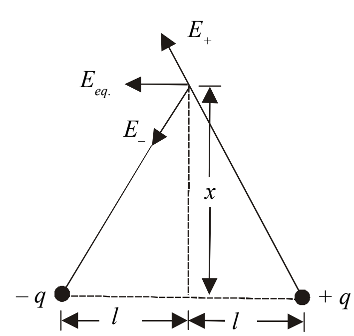

Fig. 18.9 Electric field along equational line

$$E_{\text{equatorial}} = \frac{p}{4\pi\varepsilon_0 x^3} \quad \text{due to a short dipole}$$

Electric potential at any point along equatorial line

$$V_{\text{equatorial}} = 0$$

(c) Electric field due to a short dipole at any point P

$$E_{\text{any point}} = \frac{p}{4\pi\varepsilon_0 x^3}\sqrt{3\cos^2\theta + 1} = \sqrt{E_x^2 + E_y^2}$$

and $\tan\beta = \dfrac{\tan\theta}{2}$

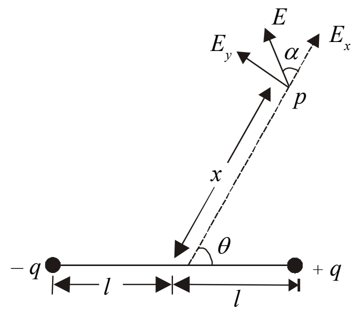

Fig.18.10 **Electric field due to a dipole at any point**

and $E_x = \dfrac{-\partial V}{\partial x}$

and $E_y = \dfrac{-\partial V}{\partial y} = \dfrac{-\partial V}{x\partial\theta}$

Special cases If $\theta = 0$, i.e., along axial line

$$E_{\text{axial}} = \frac{2p}{4\pi\varepsilon_0 x^3} \text{ due to a short dipole}$$

If $\theta = 90°$, i.e., along equaterial line $\cos 90 = 0$,

then $E_{\text{equaterial}} = \dfrac{p}{4\pi\varepsilon_0 x^3}$, due to a short dipole.

Electric potential due to a dipole at any point

$$V_{\text{any point}} = \frac{p\cos\theta}{4\pi\varepsilon_0 (x^2 - l^2\cos^2)}$$

$$V_{\text{any point}} = \frac{p\cos\theta}{4\pi\varepsilon_0 x^2}, \text{ due to a short dipole.}$$

Torque experienced by a dipole when placed in a uniform electric field *E*

$\sum F = 0$, i.e., no linear motion is possible

$$\vec{\tau} = \vec{p} \times \vec{E} = pE\sin\theta$$

as illustrated in Fig. 18.11

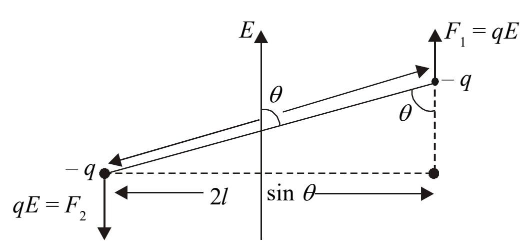

Fig. 18.11 **Torque due to a dipole**

If $\theta = 0$, $\tau = 0$, equilibrium is stable.

If $\theta = 90°$, $\tau = pE$ and is maximum.

If $\theta = 180°$, $\tau = 0$, equilibrium is unstable.

Work done $W = \int_{\theta_1}^{\theta_2} \tau \cdot d\theta = pE(\cos\theta_1 - \cos\theta_2)$

If $\theta_1 = 0$, $\theta_2 = 180°$ (i.e., dipole is reversed) when $W = 2pE$

If $\theta_1 = 0$, $\theta_2 = 90°$ then $W = pE$

Potential energy due to a dipole $U = -pE\cos\theta$

If electric field is non-uniform then $\sum F \neq 0$ and $\tau \neq 0$

$$\vec{F} = \vec{p} \times \frac{d\vec{E}}{dx}$$

Potential Energy (PE) It is the amount of work done to bring a charge q from infinity to that point against the electric field of a given charge Q without changing its KE.

$$\text{PE } U = \frac{qQ}{4\pi\varepsilon_0 r} = qV$$

Since the electrostatic force is conservative, therefore work done $W = \Delta\text{PE}$

or $W = U_f - U_i = \dfrac{Qq}{4\pi\varepsilon_0}\left[\dfrac{1}{r_{final}} - \dfrac{1}{r_{initial}}\right]$

$$= q\left[V_{final} - V_{initial}\right]$$

Force on a charged surface The repulsive force acting on an element due to rest of the charged surface is called electric force on a charged conducting surface as illustrated in Fig. 18.12

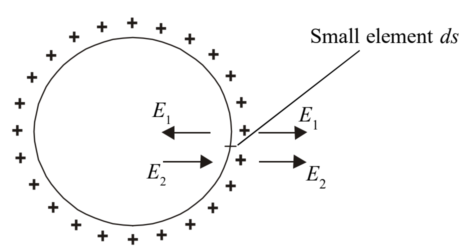

Fig. 18.12 **Force on a charged surface**

Outside $E = E_1 + E_2 = \dfrac{\sigma}{\varepsilon_0}$ (super position principle)

Inside $E = E_l - E_2 = 0$

Note that electric field intensity due to a small element is equal to electric field intensity due to rest of the surface.

Hence $E = \dfrac{\sigma}{2\varepsilon_0}$ near a charged surface and force $dF = \dfrac{\sigma^2}{2\varepsilon_0} ds$ or

$$F = \int \frac{\sigma^2}{2\varepsilon_0} ds$$

Electric pressure $P = \dfrac{\text{Force}}{\text{Area}} = \dfrac{dF}{ds} = \dfrac{\sigma^2}{2\varepsilon_0}$

In case of a soap bubble

$$P_{in} - P_{out} = P_{excess} = P_{ST} - P_{elect}$$

$$= \frac{4T}{r} - \frac{q^2}{2A^2\varepsilon_0}$$

$$= \frac{4T}{r} - \frac{q^2}{2(4\pi r^2)^2 \varepsilon_0} = \frac{4T}{r} - \frac{q^2}{32\pi^2 r^4 \varepsilon_0}$$

In case of equilibrium $P_{in} = P_{out} \Rightarrow \dfrac{4T}{r} = \dfrac{q^2}{32\pi^2 r^4 \varepsilon_0}$

Electric field intensity on soap bubble to maintain equilibrium

$$E = \sqrt{\frac{8T}{r\varepsilon_0}}$$

and electric potential to maintain equilibrium

$$V = \sqrt{\frac{8Tr}{\varepsilon_0}}$$

Energy associated with electric field

$$U = \frac{1}{2\varepsilon_0} \int \sigma^2 dV$$

$= \dfrac{\varepsilon_0}{2} \int E^2 dV$ = where V is volume of the whole field

Energy density $u = \dfrac{U}{V} = \dfrac{\varepsilon_0 E^2}{2} = \dfrac{\sigma^2}{2\varepsilon_0}$

Charged liquid drop If n identical drops each of radius r and charge q join to form a big drop of radius R and charge Q then

$$R = n^{1/3} r \; ; \; Q_{big} = nq_{small}$$

$$E_{big} = n^{1/3} E_{small} \; ; \; V_{big} = n^{2/3} \, V_{small}$$

$$\sigma_{big} = \sigma_{small} \, n^{1/3}$$

If a charged drop is in equilibrium in a given electric field then $qE = mg$ as shown in Fig. 18.13

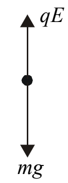

Fig.18.13 **Equilibrium of charged particle**

or $\qquad E = \dfrac{mg}{q}$

Equilibrium is said to be stable if $\sum F = 0$ and PE U = minimum. This is feasible if at extreme ends charges are similar and in between (where equilibrium is found) charge is opposite in nature as shown in Fig. 18.14

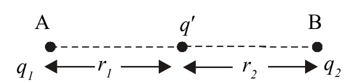

Fig. 18.14 **Stable equilibrium**

Thus for stable equilibrium $\sum F = 0$ at A, B or C

For charge q′ to be in equilibrium $\dfrac{q_1}{q_2} = \dfrac{r_1^2}{r_2^2}$ or $\dfrac{r_1}{r_2} = \sqrt{\dfrac{q_1}{q_2}}$

For q_2 to be in equilibrium $\dfrac{q'}{q_1} = \dfrac{r_2^2}{(r_1 + r_2)^2}$ or $\sqrt{\dfrac{q'}{q_1}} = \dfrac{r_2^2}{(r_1 + r_2)^2}$

A particle in stable equilibrium will execute SHM if disturbed slightly along x or y direction. However, if disturbance x is large, motion is oscillatory but not SHM.

For unstable equilibrium $\sum F = 0$ and PE U = maximum.

This is possible if all charges are similar. Thus for q′ to be in equilibrium

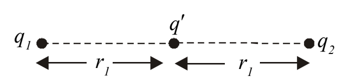

Fig. 18.15

$$\frac{q_1}{q_2} = \frac{r_1^2}{r_2^2} \text{ or } \sqrt{\frac{q_1}{q_2}} = \frac{r_1}{r_2}$$

Note in this case equilibrium cannot occur at q_1 and q_2. Moreover, particle will not execute *SHM* if slightly disturbed from its equilibrium position. Rather, it may move linearly.

Charged particle in motion

Force $F = qE$

$\therefore \qquad ma = qE$ or

$$a = \frac{qE}{m}$$

Velocity v after travelling a distance d using $v^2 = 2ad$ is

$$v = \sqrt{\frac{2qEd}{m}}$$

Velocity after time t if it starts from rest $v = at = \frac{qEt}{m}$

Remember $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$

and $\oint \vec{D} \cdot d\vec{s} = q$

If a point/shell is grounded it means potential $V = 0$ but q may not be zero.

$$\frac{F_E}{F_G} = \frac{F_{electrostatic}}{F_{gravitational}} = 10^{39}$$

Unit of dipole moment is Debye in atomic scale

1 Debye $= 3.3 \times 10^{-33}$ C – m

For a single charge $E \propto r^{-2}$, $V \propto r^{-1}$

For a dipole $E \propto r^{-3}$; $V \propto r^{-2}$

For a quadrupole $E \propto r^{-4}$, $V \propto r^{-3}$

SHORT CUTS AND POINTS TO NOTE

1. Coulomb force $\vec{F} = \frac{q_1 q_2 \vec{r}}{4\pi\varepsilon_0 r^3}$ or $|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ is applicable in free space or vacuum only if (a) charges are point charges or spherical charges (b) separation between the charges > 10^{-15}m. If the charge is distributed, make a point charge by considering a small element and linear charge density λ (if charge is linear), surface charge density σ (if charge is spread on area) and volume charge density ρ (if charge is distributed throughout the volume).

2. Normally force is mutual i.e. $F_{12} = -F_{21}$. In certain cases Newton's 3rd law may not be valid. For example, if a charge q_1 is placed in the shell and q_2 lies outside at a distance r from q_1 as shown in Fig. 18.16

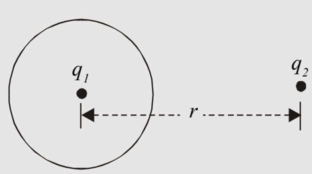

Fig. 18.16 Illustration of Newton's third law failure

then force due to q_1 on q_2 is non zero while force due to q_2 and q_1 is zero.

3. In a medium $F = \frac{q_1 q_2}{4\pi\varepsilon_0\varepsilon_r r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 k r^2}$

where $\varepsilon_r = k$ is dielectric constant.

4. If there is more than one medium as shown in Fig.18.17 where a dielectric slab of thickness t and dielectric constant k has been added in between two charges q_1 and q_2 separated by r. To solve such problems, find equivalent distance in vacuum. In the given problem equivalent distance in vacuum is $t\sqrt{k}$. Thus, net distance between the charges will be $r - t + t\sqrt{k}$ or

Force $F = \frac{q_1 q_2}{4\pi\varepsilon_0 (r - t + t\sqrt{k})^2}$

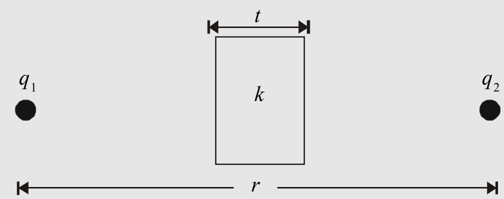

Fig. 18.17 Finding force between charges in more than one medium

Note that effective distance in vacuum for a dielectric of thickness t and dielectric constant k is $t\sqrt{k}$ *i.e.* $t_{eff} = t\sqrt{k}$

5. The electric field intensity or electric force is a vector quantity. Therefore exploit vector algebra to solve the problems.

6. Electric field intensity due to a point charge Q at a distance r form it is $E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}$

7. Electric field intensity inside a hollow conducting body is zero irrespective of its shape.

Gauss's Law $\oint E.ds = \frac{q}{\varepsilon_0}$

Gauss's Law in differential form $\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0}$

8. Electric field intensity due to a shell (spherical) is

$E_{inside} = 0$, $E_{surface} = \frac{Q}{4\pi\varepsilon_0 R^2}$ and

$E_{outside} = \frac{Q}{4\pi\varepsilon_0 x^2}$

9. Electric field intensity due to a dipole $E \propto \frac{1}{x^3}$

$E_{axial} = \frac{2px}{4\pi\varepsilon_0 (x^2 - l^2)^2}$ and

$E_{axial} = \frac{2p}{4\pi\varepsilon_0 x^3}$ due to a short dipole.

E_{axial} is parallel to dipole moment.

$E_{equatorial} = \dfrac{p}{4\pi\varepsilon_0(x^2+l^2)^{\frac{3}{2}}}$ and for a short dipole

$E_{equartorial} = \dfrac{2p}{4\pi\varepsilon_0 x^3}$. $E_{equartorial}$ is antiparallel to dipole moment.

$E_{any\ point} = \dfrac{p}{4\pi\varepsilon_0 x^3}\sqrt{3\cos^2\theta+1}$ and $\tan\alpha = \dfrac{\tan\theta}{2}$ gives the direction.

10. Electric field intensity due to a ring at any point on axial line.

$E_{ring} = \dfrac{Qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$. It is maximum when $x = R/\sqrt{2}$

$E_{ring} = 0$ at the centre of the ring.

11. Electric field due to a disc of radius R, along axial line is

$E_{disc} = \dfrac{\sigma}{2\varepsilon_0}\left[1-\dfrac{x}{\sqrt{x^2+R^2}}\right]$, $E_{centre} = \dfrac{\sigma}{2\varepsilon_0}$, $E = \dfrac{\sigma}{2\varepsilon_0}$ if $R \to \infty$

12. Electric field due to a finite line charge at any point on its perpendicular bisector $E = \dfrac{Q}{2\pi\varepsilon_0 x\sqrt{L^2+4x^2}}$

13. If a dipole is suspended in a uniform Electric field then torque experienced by the dipole $\vec{\tau} = \vec{p}\times\vec{E}$ and $\sum F = 0$. Torque is maximum if $\theta = 90°$.

It is in stable equilibrium if $\theta = 0°$, and, it is in unstable equilibrium if $\theta = 180°$.

14. Work done by the dipole

$W = \int_{\theta_1}^{\theta_2}\tau.d\theta = pE(\cos\theta_1 - \cos\theta_2)$. Work done is maximum if angle of twist is 180°.

$pE(U) = -pE\cos\theta = -\vec{p}.\vec{E}$

15. If electric field is nonuniform then both torque and force act and force is given by $\vec{F} = \vec{p}\times\dfrac{d\vec{E}}{dx}$. Note that to balance a torque, a torque is needed and to balance a force, force is required. Hence force and torque are required to balance a dipole in a nonuniform field.

16. $V = \int -E.dl$ and $\oint E.dl = 0$ because electrostatic force is conservative .

17. If $|E_1| = |E_2|$ or $|F_1| = |F_2|$ then resolve the vector. We get magnitude and direction simultaneously. From Fig. 18.18

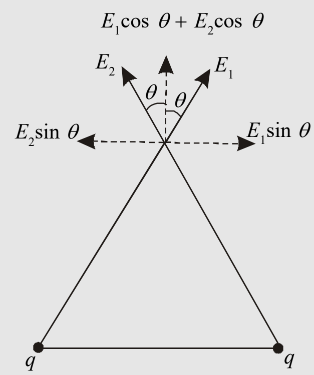

Fig. 18.18

$E = 2E_1\cos\theta$

18. $E = \dfrac{-dV}{dx}$ suggests E = 0, if V = maximum, V is minimum or V is constant.

19. It is possible to have $E = 0$ but $V \neq 0$ or vice versa. $E = 0$, $V \neq 0$ in a shell; E $\neq$ 0, $V = 0$ along the equatorial line of a dipole. Moreover, if $Q = 0$ then $E = 0$ and $V = 0$.

20. A moving charge in a dielectric generates both electric and magnetic field. But current in a conductor generates only magnetic field. In a conductor $E_{inside} = 0$.

21. Electric potential $V = \int_{r_1}^{r_2} -E.dx = \dfrac{q}{4\pi\varepsilon_0 r}$

For a point charge potential difference

$$\Delta V = \int_{r_1}^{r_2} -E.dx = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

For three dimensional electric field

$$V = -\left[\int_\infty^x E_x.\partial x + \int_\infty^y E_y.\partial y + \int_\infty^z E_z.\partial z\right]$$

22. Electric potential due to a shell

$V_{in} = V_{surface} = \dfrac{Q}{4\pi\varepsilon_0 R}$, $V_{out} = \dfrac{Q}{4\pi\varepsilon_0 x}$ $\quad x > R$

23. Electric potential due to a dipole

$V_{axial} = \dfrac{p}{4\pi\varepsilon_0(x^2-l^2)}$;

$V_{equatorial} = 0$;

$V_{\text{any point}} = \dfrac{p\cos\theta}{4\pi\varepsilon_0 x^2}$ due to a short dipole.

24. Potential Energy $U = -\int_{\infty}^{r} F \cdot dx = \dfrac{Qq}{4\pi\varepsilon_0 r} = qV$

Work done = change in potential energy $W = \Delta PE$

$$= -\int_{r_1}^{r_2} F.dx = q\Delta V = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = q\,(V_2 - V_1)$$

25. For equipotential surface, work done $W = \Delta PE = 0$

In linear motion, however, gain in PE = loss in KE or vice versa.

26. Acceleration of charged particle in an electric field E is $a = \dfrac{qE}{m}$. Apply equations

$v = v + at$, $v^2 - u^2 = 2\,as$ etc., if a is uniform.

If $a = \dfrac{qE}{m}$ is uniform and along y direction

Then $v_y = at = \dfrac{qEt}{m}$ and $u_{\text{net}} = \sqrt{u_x^2 + \left(\dfrac{qEt}{m}\right)^2}$ and

$$\tan\theta = \frac{v_y}{v_x} = \frac{qEt}{mv_x}$$

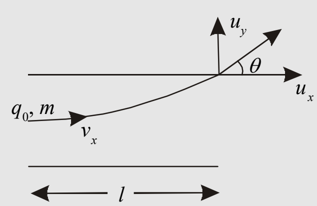

Fig. 18.19 **Motion of a charged particle**

For a charged particle projected in a limited electric field region $t = \dfrac{l}{v_x}$

27. If an opposite charge lies between two similar charges, equilibrium could be stable. If, however, all the charges are similar, equilibrium will be unstable.

28. If n drops, each of radius r and charge q, coalesce to form a big drop then $R_{\text{big}} = n^{1/3}r$, $Q_{\text{big}} = nq$,

$V_{\text{big}} = n^{2/3}V_{\text{small}}$,

$C_{\text{big}} = n^{1/3}C_{\text{small}}$ and $E_{\text{big}} = n^{1/3}E_{\text{small}}$

29. eV or electron volt is energy while volt is potential.

30. Two dipoles taken together having same charge and same separation between two charges as shown in Fig. 18.20 from a quadrupole. In a quadrupole $E \propto$

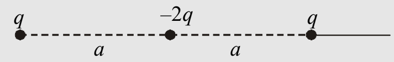

Fig. 18.20

$\dfrac{1}{r^4}$ and $V \propto \dfrac{1}{r^3}$

CAUTION

1. Adding electric field intensity or force algebraically.

$\Rightarrow$ They are vectors and hence vector algebra be applied.

2. Considering potential is also a vector quantity.

$\Rightarrow$ $V = -\int \vec{E} \cdot d\vec{x}$ is a scalar. Therefore potential and PE be added alegebraically.

3. Not knowing how to get vector from scalars.

$E = -\dfrac{dV}{dr}$ as V is a scalar while E is electric field.

$\Rightarrow$ In one dimension $\vec{E} = -\dfrac{dV}{dx}\hat{i}$

In three dimension $\vec{E} = -\vec{\nabla}V = -\left(\hat{i}\dfrac{\partial}{\partial x} + \hat{j}\dfrac{\partial}{\partial y} + \hat{k}\dfrac{\partial}{\partial z}\right)V$

4. Assuming electric field of one charge should affect the electric field of other charge when we have group of plates or group of charges.

$\Rightarrow$ We use superposition theorem. The effect of electric field of each plate or charge is considered individually on the given charge/test charge.

5. Ignoring the directions of velocities and accelerations. For instance, a charged particle is initially moving in x-direction with a velocity u but due to electric field, acceleration is developed in y-direction. Applying equation

$v = u + at$ or $s = ut + \dfrac{1}{2}at^2$ etc. is not correct.

$\Rightarrow$ Use $v = u\hat{i} + a_y t\hat{j}$ so that $|v| = \sqrt{u^2 + (a_y t)^2}$

and $\tan\beta = \left(\dfrac{a_y t}{u}\right)$

6. Assuming $E_{\text{inside}} = 0$ in all kinds of bodies.

$\Rightarrow$ $E_{\text{inside}} = 0$ in a cavity or hollow bodies.

$E_{\text{inside}} = \dfrac{Qx}{4\pi\varepsilon_0 R^3}$ in a uniformly charged sphere.

7. Assuming $V = 0$, if $E = 0$ because $V = -\int E.dx$

$\Rightarrow$ $V = 0$ if $E = 0$ and $Q = 0$, $V_{\text{inside}} = V_{\text{surface}} = \dfrac{Q}{4\pi\varepsilon_0 R}$ in a shell, though $E_{\text{inside}} = 0$. Interpreting $V = 0$ if $E = 0$ is superfluous. We come across cases when $V = 0$ but $E \neq 0$, for example, along equatorial line in a dipole.

8. Not recalling that work done on an equipotential surface is zero.

$\Rightarrow$ Since electrostatic force is conservative, $W = q\ (V_1 - V_2) = 0$ on equipotential surface. Moreover, $\oint E.dl = 0$. However work may be done if charge moves from one equipotential surface to another equipotential surface.

9. Considering a small sphere or an end of a pin can hold a large charge.

$\Rightarrow$ Smaller the radius more is the surface charge density and hence very high electric field such that it surpasses the dielectric breakdown strength and hence charge leaks by means of carona discharge.

10. Considering equipotential surfaces can intersect.

$\Rightarrow$ Equipotential surfaces cannot intersect.

11. Considering a charged metal plate has uniformly distributed charge.

$\Rightarrow$ It has maximum charge density at the corners and minimum at flat portion.

12. Considering that a positively charged body has always positive potential.

$\Rightarrow$ It may have negative potential if placed in the electric field generated by strong negative charge.

13. The notion that similar charges only repel.

$\Rightarrow$ Though in principle it is correct but if one charge is very large as compared to other charge and they are place close to one another then they will attract. The reason being that there will be an induced charge (of opposite nature) in the body having a small charge.

14. Assuming work done is dependent on path followed.

$\Rightarrow$ Work done is independent of path followed as electrostatic force is conservative.

15. Considering that a charged particle must move along the electric field line.

$\Rightarrow$ Though $F = qE$ is the force present and acceleration is tangent to the field line, if particle was already in motion along some other direction then it will follow curved path.

16. Not knowing the directions of field lines and equipotential surfaces.

$\Rightarrow$ Field lines are always perpendicular to equipotential surfaces.

SOLVED PROBLEMS

1. An α-particle is travelling to its right with 1.5 kms^{-1}. What uniform magnetic field be applied so that it starts moving with same speed to its left after 2.65 μs?

(a) 2.35 N/C toward left (b) .235 NC^{-1} toward left
(c) 23.5 NC^{-1} toward left (d) 2.35 NC^{-1} toward left

Solution (c) $\nu = u + at$

$$-1.5_{\text{km}}\hat{i} = -1.5_{\text{km}}\hat{i} + a\,(2.65\times10^{-6}) \text{ or}$$

$$a = \frac{-3\times10^{6}\times10^{3}}{2.65}\hat{i} \text{ using } qE = ma$$

$$E = \frac{ma}{q} = -\frac{6.64\times10^{-27}\times3\times10^{6}\times10^{3}}{2\times1.6\times10^{-19}\times2.65}$$

$$= -23.5\,\hat{i}\ N/C$$

i.e. 23.5 N/C towards left.

2. A charge $+q$ is placed $(a, 0, 0)$ and another $+q$ charge is placed at $(-a, 0, 0)$

A charge $-q_1$ is placed at the origin. If it is slightly displaced along y axis. then

(a) it will move away

(b) it will oscillate but not SHM

(c) it will execute SHM

(d) it will stand at the displaced position.

Solution (c)

3. Assuming mass m of the charged particle, find time period of oscillation in Question 2.

(a) $2\pi\sqrt{\dfrac{q_1 q}{4\pi\varepsilon_0 a^3 m}}$ (b) $2\pi\sqrt{\dfrac{4\pi\varepsilon_0 a^3 m}{q_1 q}}$

(c) $\dfrac{\pi}{2}\sqrt{\dfrac{q_1 q}{4\pi\varepsilon_0 q^3 m}}$ (d) none of these

Solution (c) $E_0 = 2E_1\cos\theta = \dfrac{q}{4\pi\varepsilon_0(x^2+a^2)}\dfrac{x}{\sqrt{x^2+a^2}}$

$$-q_1E = F = mf = \frac{-q_1qx}{4\pi\varepsilon_0(x^2+a^2)^{3/2}}$$ as $x << a$, neglecting x^2 as compared to acceleration, $f = \frac{-q_1q_2x}{4\pi\varepsilon_0a^3m}$

Comparing it with $f = \omega^2x$

$$\omega = \sqrt{\frac{q_1q}{4\pi\varepsilon_0a^3m}} \text{ or } T = 2\pi\sqrt{\frac{4\pi\varepsilon_0a^3m}{q_1q}}$$

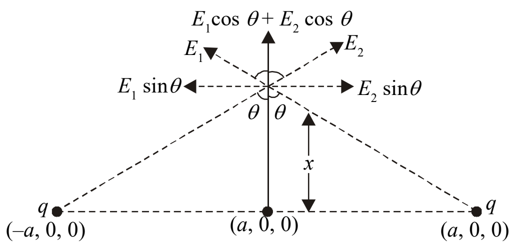

Fig. 18.21

4. As per diagram, a charge q is placed at the origin O. Work done by a charge $-Q$ in taking it from $A(0, a)$ to $B(a, 0)$ along the path AB **(CBSE 2005)**

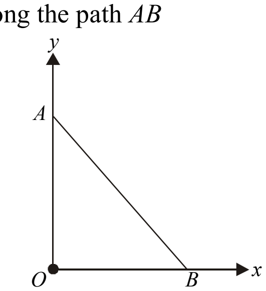

Fig. 18.22

(a) zero (b) $\sqrt{2}a\left(\frac{qQ}{4\pi\varepsilon_0a^2}\right)$

(c) $\left(\frac{-qQ}{4\pi\varepsilon_0a^2}\right)\sqrt{2}a$ (d) $\left(\frac{qQ}{4\pi\varepsilon_0a^2}\right)\frac{a}{\sqrt{2}}$

Solution (a) A & B are at same potential

$\therefore W = 0$

5. Two charges q_1 and q_2 are placed 30 cm apart as shown in Fig. 18.23. Third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D. The change in potential energy of the system is $\frac{q_3k}{4\pi\varepsilon_0}$ where k is **(CBSE 2005)**

(a) $8q_1$ (b) $6q_1$

(c) $8q_2$ (d) $6q_2$

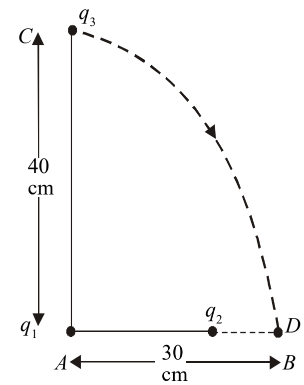

Fig. 18.23

Solution (c) $PE = \frac{q_2q_3}{4\pi\varepsilon_0}\left[\frac{1}{.1} - \frac{1}{.5}\right] = \frac{8q_2q_3}{4\pi\varepsilon_0}$

6. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x-axis at which the net electric field due to these two point charges is zero

[AIEEE 2005]

(a) $2L$ (b) $\frac{L}{4}$

(c) $8L$ (d) $4L$

Solution (a) $-\frac{2q}{4\pi\varepsilon_0(x-L)^2} + \frac{8q}{4\pi\varepsilon_0x^2} = 0$ or $x = 2L$

7. Two thin wire rings each having a radius R are placed at distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the rings are

[AIEEE 2005]

(a) $\frac{QR}{4\pi\varepsilon_0d^2}$ (b) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

(c) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$ (d) zero

Solution (b) $V_1 = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$ and

$$V_2 = \frac{-Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$$

$$\Delta V = V_1 - V_2$$

8. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then

[AIEEE 2005]

(a) its velocity decreases

(b) its velocity increases

(c) it will turn towards right of its motion

(d) it will turn towards left of direction of motion.

Solution (a) $F = -eE + e(\vec{v} \times \vec{B})$

$$= -eE \text{ and } v = v_0 - \frac{eE}{m}t$$

9. Four point positive charges of same magnitude Q are placed at the four corners of a rigid square frame as shown in Fig. 18.24

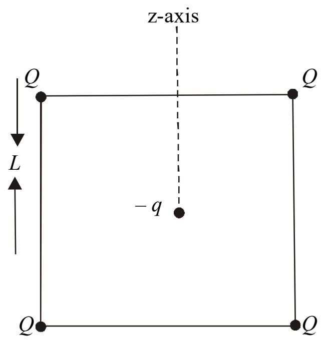

Fig. 18.24

The plane of the frame is perpendicular to z-axis. If a negative charge $-q$ is placed at a distance z away from the above frame ($z << L$) then

(a) negative charge oscillates along the z-axis

(b) it moves away from the frame

(c) it moves slowly towards the frame and stays in the plane of the frame.

(d) it passes through the frame only once.

[AIIMS 2005]

Solution (a) Because the resultant force acts as restoring force.

10. The work done in carrying a charge q once round a circle of radius r with a charge Q at the centre is

(a) $\dfrac{qQ}{4\pi\varepsilon_0 r}$ (b) $\dfrac{qQ}{4\pi\varepsilon_0^2 r^2}$

(c) $\dfrac{qQ}{4\pi\varepsilon_0 r^2}$ (d) none of these

(CET Karnataka 2005)

Solution (d) $W = 0$ $\because$ Electrostatic force is conservative

11. Two small spheres each of mass m and charge q are tied from the same rigid support with the help of silk threads of length L. They make angle θ with the vertical as shown in the Fig. 18.25

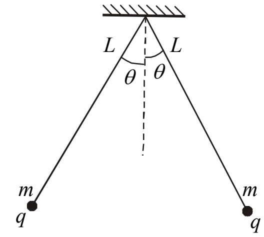

Fig. 18.25

If length L is decreased then angle θ with the vertical

(a) increases (b) decreases

(c) unaffected (d) cannot say

Solution (a) θ is related inversely to length L.

12. Two small spheres each of radius 1mm are kept 10 cm apart. Assuming each proton has a charge $+e$ and each electron has a charge 0.1% less than the $+e$ then find the force between the two spheres. Density of copper is 8.9 gcm^{-3} and atomic mass number is 63.5.

(a) 1.2×10^2N (b) 1.2×10^{-2}N

(c) 1.2×10^8N (d) 1.2×10^{14}N

Solution (d) mass of 1 mm radius sphere

$$= 8.9 \times \frac{4}{3}\pi(.1)^3$$

$$= 3.7 \times 10^{-2}\text{g}$$

Charge on the sphere

$$\frac{3.7\times10^{-2}}{63.5} \times 6.023 \times 10^{23} \times 29 \times \frac{.1}{100} \times 1.6 \times 10^{-19}$$

$$= 1.61\,C$$

$$F = \frac{1.61\times1.61\times9\times10^9}{(0.1)^2}$$

$$= 2.34 \times 10^{12}\,N$$

13. Two thin rods of length L lie along x-axis, one between

$$x = \frac{a}{2} \text{ to } x = \frac{a}{2} + L$$

and the other between $x = -\dfrac{a}{2}$ to $x = -\dfrac{a}{2} - L$.

Each rod has positive charge Q distributed uniformly along the length. Find the magnitude of the force which one rod exerts on the other.

(a) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2} \log_e \dfrac{L+a}{L-a}$

(b) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2} \log_e \dfrac{(L+a)^2}{a(L-a)}$

(c) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2}\, log_e \dfrac{(L+a)^2}{a(2L+a)}$

(d) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2}\, log_e \dfrac{(L+a)^2}{L(2a+L)}$

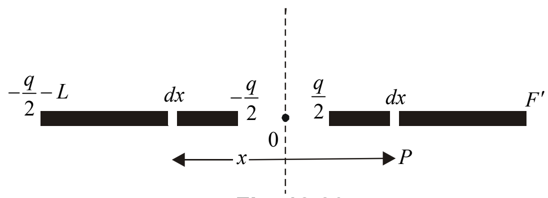

Fig. 18.26

Solution (c) Electric field at point P due to a small element dx of the rod on left side is $\int dE = \int \dfrac{Qdx}{L4\pi\varepsilon_0 x^2}$

$$= \frac{Q}{4\pi\varepsilon_0 L} \times \left[\frac{1}{x+\frac{a}{2}} - \frac{1}{x+\frac{a}{2}+L}\right]$$

Force exerted on a small element dx is

$$dF = \frac{QQ}{4\pi\varepsilon_0 L^2}\left[\frac{1}{x+\frac{a}{2}} - \frac{1}{x+\frac{a}{2}+L}\right]dx \quad \text{or}$$

$$F = \frac{Q^2}{4\pi\varepsilon_0 L^2}\left[\log_e x+\frac{a}{2}\Bigg|_{a/2}^{L+a/2} - \log_e x+\frac{a}{2}+L\Bigg|_{a/2}^{L+a/2}\right]$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\left[\log_e \frac{L+a}{a} - \log\frac{2L+a}{L+a}\right]$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\log_e\left[\frac{(L+a)^2}{a(2L+a)}\right]$$

14. An annular disc has inner and outer radius R_1 and R_2 respectively. Charge is uniformly distributed. Surface charge density is σ. Find the electric field at any point distant y along the axis of the disc.

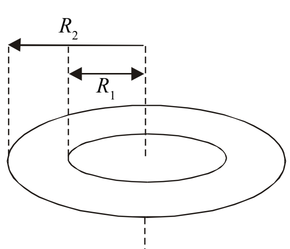

Fig. 18.27

(a) $\dfrac{\sigma}{2\varepsilon_0}$ (b) $\dfrac{\sigma y}{2\varepsilon_0(R_2-R_1)}$

(c) $\dfrac{\sigma y}{2\varepsilon_0}\left[\dfrac{1}{\sqrt{R_1^2+y^2}} - \dfrac{1}{\sqrt{R_2^2+y^2}}\right]$

(d) $\dfrac{\sigma}{2\varepsilon_0}\log\dfrac{R_2+y}{R_1+y}$

Solution (c) Assume a hypothetical ring of radius x and thickness dx. Charge on the ring $dq = \sigma 2\pi x dx$. Electric field due to the ring at a point P distance y form the centre is

$$dE = \frac{dqy}{4\pi\varepsilon_0(x^2+y^2)^{3/2}} \text{ or } dE = \frac{2\pi xdxy}{4\pi\varepsilon_0(x^2+y^2)^{3/2}}$$

$$E = \frac{\sigma y 2\pi}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{xdx}{(x^2+y^2)^{3/2}} = \frac{\sigma y}{2\varepsilon_0}\left[\frac{-1}{\sqrt{x^2+y^2}}\Bigg|_{R_1}^{R_2}\right]$$

$$= \frac{\sigma y}{2\varepsilon_0}\left[\frac{1}{\sqrt{R_1^{\,2}+y^2}} - \frac{1}{\sqrt{R_2^{\,2}+y^2}}\right]$$

15. Find the minimum force between the two electrons of He nucleus. Assume radius of He nucleus = 6.8 A°.

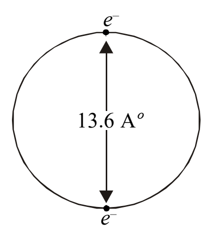

Fig. 18.28

(a) 12×10^{-10} N (b) 1.2×10^{-10} N
(c) 0.12×10^{-10} N (d) 0.012 N

Solution (b) Force will be minimum when electrons are diametrically opposite.

$$\therefore \quad F_{\min} = \frac{1.6\times1.6\times10^{-38}\times9\times10^9}{(13.6)^2\times10^{-20}}$$

$$= 1.2 \times 10^{-10}\text{ N}$$

16. A line charge of length l and charge Q uniformly distributed over the whole length is placed a distance r from one edge from a point charge q as shown. Find the force on the point charge

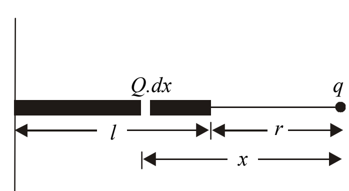

Fig. 18.29

(a) $\dfrac{qQ}{4\pi\varepsilon_0(r+l)}$ (b) $\dfrac{qQ}{4\pi\varepsilon_0\left(r+\dfrac{l}{2}\right)^2}$

(c) $\dfrac{qQ}{4\pi\varepsilon_0}\left[\dfrac{1}{r^2}-\dfrac{1}{(r+l)^2}\right]$ (d) none of these

Solution (a) Consider a small element dx of the line charge at a distance x from the point charge.

Forced $F = q\dfrac{Qdx}{L4\pi\varepsilon_0 x^2}$

$$F = \frac{qQ}{4\pi\varepsilon_0 l}\int_r^{r+l}\frac{dx}{x^2} = \frac{qQ}{4\pi\varepsilon_0 l}\left[\frac{1}{r}-\frac{1}{r+l}\right]$$

$$= \frac{qQ}{4\pi\varepsilon_0 r(r+l)}$$

17. Two charges Q_1 and Q_2 are distance d apart. Two dielectrics of thickness t_1 and t_2 and dielectric constant k_1 and k_2 are introduced as shown. Find the force between the charges.

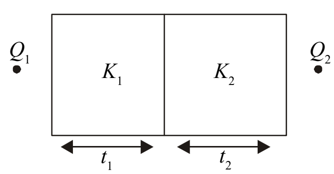

Fig. 18.30

(a) $\dfrac{Q_1Q_2}{4\pi\varepsilon_0\left[d-(t_1+t_2)+k_1t_1+k_2t_2\right)^2}$ (b) zero

(c) $\dfrac{Q_1Q_2}{4\pi\varepsilon_0\left[d+\sqrt{k_1}t_1+\sqrt{k_2}t_2\right)^2}$

(d) $\dfrac{Q_1Q_2}{4\pi\varepsilon_0\left[\sqrt{k_1}t_1+\sqrt{k_2}t_2+d-(t_1+t_2)\right]^2}$

Solution (d) effective distance in vacuum

$$= \sqrt{k_1}\,t_1 + \sqrt{k_2}\,t_2 + \mathrm{d} - (t_1+t_2)$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0\left(\sqrt{k_1}t_1+\sqrt{k_2}t_2+d-(t_1+t_2)\right]^2}$$

18. When two charges are equal q each, force they exert on each other is F. When one of the charge is doubled, the $2q$ charge exerts a force $2F$ on charge q. The force exerted by q on $2q$ is

(a) F (b) $\dfrac{F}{2}$

(c) $\dfrac{F}{4}$ (d) $2F$

(e) $4F$

Solution (d) Force is mutual.

19. ABCD is a square frame of side l. The force at B if charges as shown in Fig. 18.31 (a) are placed at the corners of the square

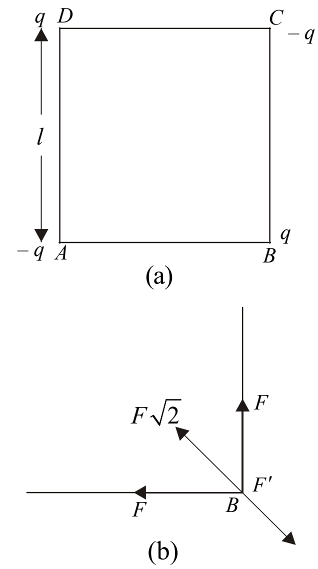

Fig. 18.31

(a) $\dfrac{q^2(2\sqrt{2}-1)}{4\pi\varepsilon_0 2l^2}$ (b) $\dfrac{q^2(2\sqrt{2}+1)}{4\pi\varepsilon_0 l^2}$

(c) $\dfrac{q^2(2\sqrt{2}-1)}{4\pi\varepsilon_0 l^2}$ (d) $\dfrac{q^2(2\sqrt{2}+1)}{4\pi\varepsilon_0 2l^2}$

Solution (a) As illustrated in Fig. 18.31 (b) the net force at B is

$$F\sqrt{2}-F' = \frac{q^2\sqrt{2}}{4\pi\varepsilon_0 l^2}-\frac{q2}{4\pi\varepsilon_0\left(\sqrt{2}l\right)^2} = \frac{q^2\left(2\sqrt{2}-1\right)}{4\pi\varepsilon_0 2l^2}$$

20. Charge Q is distributed uniformly on length l of a wire. It is bent in the form of a ring. Find the electric field at the centre of the ring.

(a) $\dfrac{Q\pi}{4\varepsilon_0 l^2}$ (b) $\dfrac{Q}{4\pi\varepsilon_0 l^2}$

(c) $\dfrac{Q}{2\pi\varepsilon_0 l^2}$ (d) $\dfrac{Q}{2\varepsilon_0 l^2}$

Solution (d) Consider two small elements of length dl

each charge dq on each element. $dq = \frac{Qdl}{l} = \frac{Q(rd\theta)}{\pi r}$

$= \frac{Qd\theta}{\pi}$

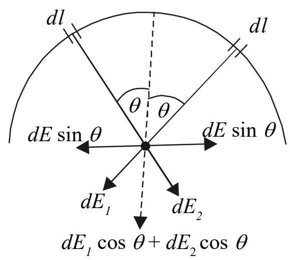

Fig. 18.32

$\because$ $|dE_1| = |dE_2|$, resolve dE_1 and dE_2, their sin components cancel out. $dE = 2dE_1 \cos\theta = \frac{2Qd\theta}{\pi 4\pi\varepsilon_0 r^2}\cos\theta$ and

$$E = \frac{2Q}{4\pi^2\varepsilon_0 r^2}\int_0^{1/2}\cos\theta d\theta \text{ or}$$

$$E = \frac{2Q}{4\pi^2\varepsilon_0\left(\frac{R}{\pi}\right)^2}[\sin 90 - \sin 0] = \frac{Q}{2\varepsilon_0 l^2}$$

21. Two charged particles each of mass 5g and charge q are suspended as shown in Fig. 18.33. The system is taken in a satellite. The force between the charges is

(a) 23×10^{-3} N (b) 2.3×10^{-3} N

(c) 0.23×10^{-3} N (d) none of these

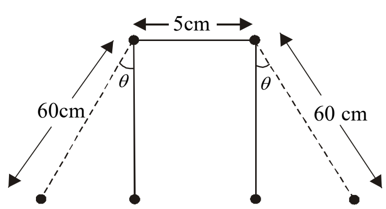

Fig. 18.33

Solution (a) $F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{2 \times 2 \times 10^{-12} \times 9 \times 10^9}{(1.25)^2}$

$= 23 \times 10^{-3}$ N

{Here $d = (2l + 5)$ cm = 125 cm as there is no gravity. Therefore electrostatic force will push them away.}

22. A child stands inside a large charged metal sphere. Will her hair stand on end ?

(a) Yes (b) No

(d) Incomplete information

(d) No guess about her hair style

Solution (b) As electric field inside the shell is zero

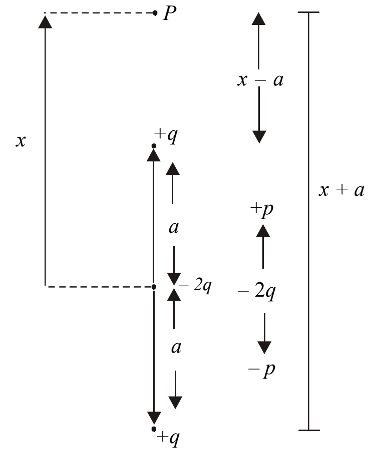

Fig. 18.34

23. Fig. 18.34 shows a quadrupole. Assuming $x >> a$ find the electric field at P where $p = qa$.

(a) $\frac{6pa^2}{4\pi\varepsilon_0 x^4}$ (b) $\frac{6pa}{4\pi\varepsilon_0 x^4}$

(c) $\frac{2pa}{4\pi\varepsilon_0 x^4}$ (d) $\frac{3pa}{4\pi\varepsilon_0 x^4}$

(e) $\frac{5pa}{4\pi\varepsilon_0 x^4}$

Solution (b) $E = \frac{q}{4\pi\varepsilon_0(x-a)^2} - \frac{2q}{4\pi\varepsilon_0 x^2} + \frac{q}{4\pi\varepsilon_0(x+a)^2}$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2x^2 + 2a^2}{(x^2 - a^2)^2} - \frac{2}{x^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{6x^2a^2}{x^2(x^2 - a^2)^2}\right]$$

$$= \frac{6pa}{4\pi\varepsilon_0 x^4}$$

24. An electron is projected with a velocity V_0 at an angle θ in the presence of an electric field E as shown in Fig. 18.35.

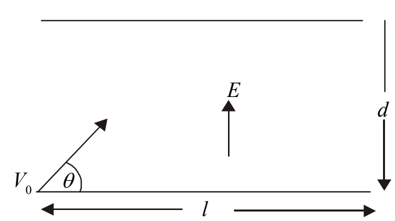

Fig. 18.35

Find minimum value of d so that electron does not hit the plate.

(a) $d \geq \frac{mv_0^2}{2eE}$ (b) $\frac{mv_0^2 \cos^2\theta}{2eE}$

(c) $\frac{mv_0^2 \sin^2\theta}{2eE}$ (d) $\frac{mv_0^2 \tan^2\theta}{2eE}$

Solution (c) $a_y = \frac{eE}{m}$, $u_y = v_0 \sin\theta$

For the particle to fail to hit the plate $2a_y d \geq U_y^2$

or $v_0^2 \sin^2\theta = \frac{2eE}{m} d$

or $d = \frac{mv_0^2 \sin^2\theta}{2eE}$

25. Uniformly charged long cylinder has volume charge density ρ. Find the electric field at a distance $x < R$ from the axis of the cylinder.

(a) $\frac{\rho x}{\varepsilon_0}$ (b) $\frac{\rho x}{2\varepsilon_0}$

(c) $\frac{\rho x}{3\varepsilon_0}$ (d) $\frac{\rho x}{4\varepsilon_0}$

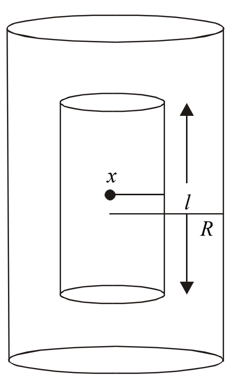

Fig. 18.36

Solution (b) Assume a hypothetical cylinder of radius x and length l. Apply Gauss's law $\oint E.ds = \frac{q_{in}}{\varepsilon_0}$ or

$$\oint E.ds = \frac{\pi x^2 l\rho}{\varepsilon_0}$$

$$E(2\pi x l) = \frac{\pi x^2 l \rho}{\varepsilon_0} \Rightarrow E = \frac{\rho x}{2\varepsilon_0}.$$

26. Two concentric shells carry charges q and Q. Their radius are r and R. The potential difference between the two is

(a) $\frac{q}{4\pi\varepsilon_0 R^2} - \frac{Q}{4\pi\varepsilon_0 R}$ (b) $\frac{R-q}{4\pi\varepsilon_0 R}$

(c) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{R}\right]$ (d) $\frac{(Q-q)}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{R}\right]$

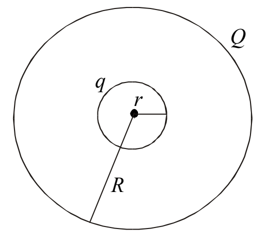

Fig. 18.37

Solution (c) $V_R = \frac{1}{4\pi\varepsilon_0}\left[\frac{R}{R}-\frac{q}{R}\right]$ and

$$V_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}-\frac{Q}{R}\right]$$

$$\Delta V = V_r - V_R$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{R}\right]$$

27. A sample of HCl is placed in an electric field of 2.5×10^4 NC^{-1}. The dipole moment of HCl is 3.4×10^{-30} C–m. Find the maximum torque that can act on a molecule.

(a) 7.6×10^{-26}Nm (b) 4.3×10^{-26}Nm

(c) 6.5×10^{-26}Nm (d) 8.5×10^{-26}Nm

Solution (d) $\tau_{max} = pE$

$= 3.4 \times 10^{-30} \times 2.5 \times 10^4$

$= 8.5 \times 10^{-26}$Nm.

28. 12 J of work is to be done against an existing electric field to take a charge of 0.01C from A to B. Find The potential difference between B and A.

(a) 120 V (b) 1200 V

(c) 1.2 V (d) 12 V

Solution (b) $W = q\,\Delta V$

or $\Delta V = \frac{W}{q} = \frac{12}{0.01} = 1200$V

29. $E = 20\hat{i} + 30\hat{j}$ exists in space. If the potential at the origin is taken to be zero, find the potential at P (3, 2).

(a) –150V (b) –100V

(c) +150V (d) –120V

(e) 120V

Solution (d) $V = V_x + V_y$

$$= \int_0^3 -E_x dx + \int_0^2 -E_y dy$$

$$= \int_0^3 -20dx + \int_0^2 -30dx = -60 - 60 = -120\ V$$

30. A ring of radius R has charge Q. It is cut by dl. Find the electric field at the centre.

(a) zero (b) $\dfrac{Qdl}{2\pi r^2 \varepsilon_0}$

(c) $\dfrac{Qdl}{2\pi r^3 \varepsilon_0}$ (d) $\dfrac{Qdl}{8\pi^2 \varepsilon_0 r^3}$

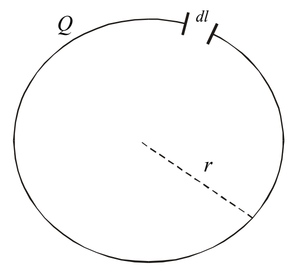

Fig. 18.38

Solution (d) $E = \dfrac{dq}{4\pi\varepsilon_0 r^2}$ and $dq = \dfrac{Qdl}{2\pi r}$.

Thus $E = \dfrac{Qdl}{8\pi^2 \varepsilon_0 r^3}$

TYPICAL PROBLEMS

31. Electric potential existing in space is $V = K(x^2y + y^2z + xyz)$. Find the expression of electric field.

(a) zero

(b) $-K[(2xy + yz)\hat{i} + (x^2+2yz+xz)\hat{j} + (y^2+xy)\hat{k}]$

(c) $-K\left[\left(\dfrac{x^3y}{3} + \dfrac{x^2yz}{2}\right)\hat{i} + \left(\dfrac{y^2z^2}{2} + \dfrac{x^2y^2z}{2}\right)\hat{k}\right]$

(d) $K[(2xy + yz)\hat{i} + (x^2 + 2yz + xz)\hat{j} + (y^2+xy)\hat{k}]$

Solution (b)

$$E = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial x} + \hat{K}\frac{\partial}{\partial x}\right][K(x^2y + y^2z + xyz)]$$

$$E = -K[(2xy + yz)\hat{i} + (x^2 + 2yz + xz)\hat{j}\ (y^2 + xy)\hat{K}]$$

32. Charges $-q$ and $+q$ are fixed at the ends of a light rod of length l. The rod is clamped at one end with axis of the dipole along the electric field. The rod is slightly displaced and then released, Neglecting gravity find the time period of small oscillations. Mass of charges is m each.

(a) $2\pi\sqrt{\dfrac{ml}{qE}}$ (b) $2\pi\sqrt{\dfrac{ml}{3qE}}$

(c) $2\pi\sqrt{\dfrac{ml}{2qE}}$ (d) $2\pi\sqrt{\dfrac{2ml}{qE}}$

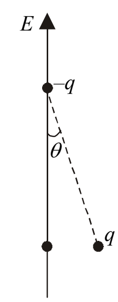

Fig. 18.39

Solution (a) Torque $p \times E = I\alpha$

Assume θ to be small $\therefore \sin\theta = \theta$

or $-pE\theta = ma^2x$

or $\alpha = -\dfrac{(ql)E}{ml^2}\theta$

$\omega = \sqrt{\dfrac{qE}{ml}}$

or $T = 2\pi\sqrt{\dfrac{ml}{qE}}$

33. Three charges are arranged as shown in Fig 18.40 (a). Find the net dipole moment.

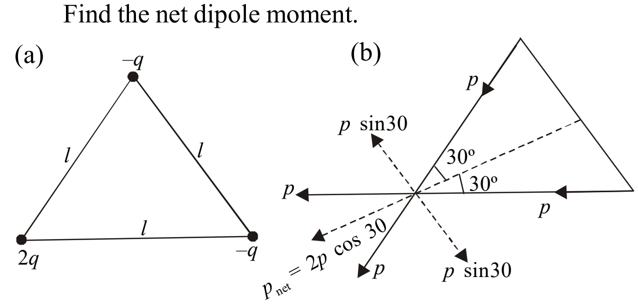

Fig. 18.40

(a) $2ql$ (b) $\sqrt{3}ql$

(c) $\frac{\sqrt{3}ql}{2}$ (d) $\frac{ql}{2}$

Solution (b) $p_{\text{net}} = 2p\cos 30 = ql\sqrt{3}$ [see Fig 18.40 (b)]

34. A block of mass m and charge q is placed on a smooth horizontal table which terminates in a vertical wall at a distance d from the block . A horizontal electric field E towards right is switched on. Assuming elastic collision, find time period of the resulting oscillatory motion.

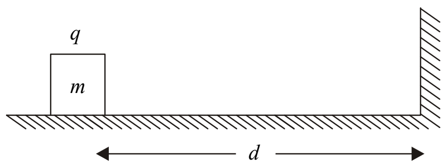

Fig. 18.41

(a) $\sqrt{\frac{8dm}{qE}}$ (b) $\sqrt{\frac{4dm}{qE}}$

(c) $\sqrt{\frac{2dm}{qE}}$ (d) none of these

Solution (a) $a = \frac{qE}{m}$ and $d = \frac{1}{2}at^2$

or $t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2dm}{qE}}$

Time period $T = 2t = 2\sqrt{\frac{2dm}{qE}}$

$= \sqrt{\frac{8dm}{qE}}$

35. A thin nonconducting ring of radius R has linear charge density $\lambda = \lambda_0 \cos\theta$ where λ_0 is a constant, θ is azimuthal angle. Find the electric field at the centre of the ring.

Solution Consider two small elements of length $dl = Rd\theta$ symmetrically at angle θ on both sides as shown in Fig. 18.42

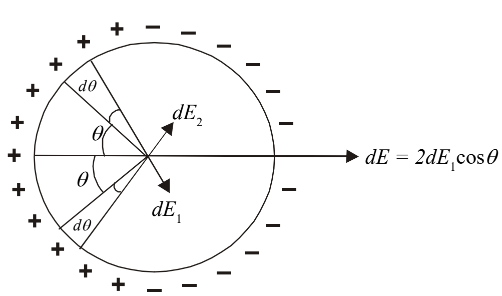

Fig. 18.42

Then $dE = 2dE_1\cos\theta = \frac{2dq}{4\pi\varepsilon_0 R^2}\cos\theta$

$$E = \int_0^{\pi} \frac{2\lambda_0 \cos\theta R d\theta \cos\theta}{4\pi\varepsilon_0 R^2} = \frac{2\lambda_0 R}{4\pi\varepsilon_0 R^2}\int_0^{\pi}\frac{(1+\cos 2\theta)}{2}d\theta$$

$$= \frac{2\lambda_0 R}{4\pi\varepsilon_0 R^2} \times \frac{\pi}{2} = \frac{\lambda_0}{4\varepsilon_0 R}$$

36. Two long parallel threads carry charge per unit length. The threads are separated by a distance l. Find the maximum field strength in the symmetry plane between them.

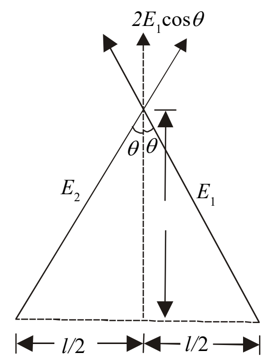

Fig. 18.43

Solution $E = 2E_1 \cos\theta = \frac{2\lambda\cos\theta}{2\pi\varepsilon_0\left(x^2 + \frac{l^2}{4}\right)^{1/2}}$

$$= \frac{\lambda x}{\pi\varepsilon_0\left(x^2 + \frac{l^2}{4}\right)} \qquad \text{..........(1)}$$

For maximum E, $\frac{dE}{dx} = 0$

$$= \frac{\lambda}{\pi\varepsilon_0}\left[\frac{1}{x^2 + \frac{l^2}{4}} - \frac{x(2x)}{\left(x^2 + \frac{l^2}{4}\right)^2}\right] = 0$$

or $x^2 = \frac{l^2}{4}$ or $x = \frac{l}{2}$

Substituting $x = \frac{l}{2}$ in eq. (1), $E = \frac{\lambda}{\pi\varepsilon_0 l}$

37. The electric field potential at a point in the space region depends only on x coordinates as $V = -ax^3 + b$. Find the space charge density $\rho(x)$.

Solution $E = \dfrac{-dV}{dx} = 3ax^2$. Using Gauss's law in differential form $\dfrac{\partial E}{\partial x} = \dfrac{\rho(x)}{\varepsilon_0} = 6\,ax$ or $\rho(x) = 6a\varepsilon_0 x$

38. In a Millikan's oil drop method, a charged drop of oil has mass m and charge q. It is made stationary by applying an electric field E. Find the strength of the electric field. If the result of Millikans experiment with different drops is 6.48×10^{-19}C, 12.82×10^{-19}C, 19.3×10^{-19}C, 8.02×10^{-19}C, 25.62×10^{-19}C, 16.04×10^{-19}C, find the elementary charge.

Solution $qE = mg \qquad E = {}^{mg}\!/_{q}$

Fig. 18.44

The HCF of the data is 1.6×10^{-19}C

Hence elementary charge is 1.6×10^{-19}C.

39. A long wire carrying a uniform charge density λ per unit length has the configuration as shown in Fig 18.45. Find the electric field at O.

Solution The field strength at O is $\dfrac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$ due to each straight part and net electric field due to both straight wires is

$$\sqrt{2}\left(\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}\right) = \frac{\lambda}{2\pi\varepsilon_0 R} \text{ in forward direction.}$$

E_{net} due to semicircular part (as found in problem 20) is $\dfrac{\lambda}{2\pi\varepsilon_0 R}$ in the backward direction $\therefore E_{Tot} = 0$.

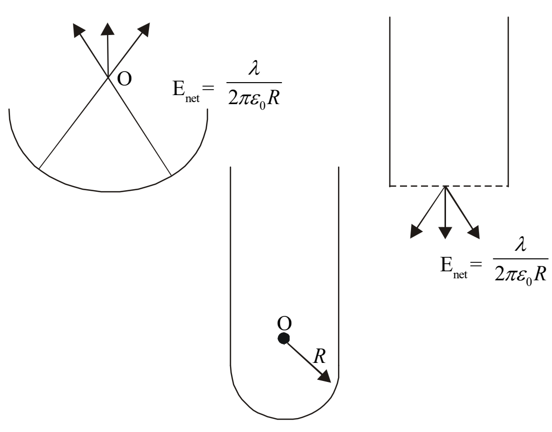

Fig. 18.45

40. A space is filled up with volume charge density $\rho = \rho_0\, e^{-\alpha r^3}$ where ρ_0 and α are positive constants. r is the distance from the centre of this system. Find the magnitude of electric field strength as a function of r.

Solution Differentiating from Gauss's Law is $\dfrac{\partial E}{\partial r} = \dfrac{\rho}{\varepsilon_0}$

$$\text{or} \quad \int \partial E = \int \frac{\rho}{\varepsilon_0} dr$$

$$E = \int_0^r \frac{\rho_0 e^{-\alpha r^3}}{\varepsilon_0} dr$$

$$= \frac{\rho_0}{3\varepsilon_0 \alpha r^2}\left(1 - e^{-\alpha r^3}\right).$$

PASSAGE 1

Read the following passage and answer the questions given at the end.

The earth has a net electric charge that causes a field at points near its surface. The charge on the earth is supposed to be a result of an atmospheric battery created between ionosphere and the earth. The electric field near the earth's surface is believed to be 150 N C^{-1} and directed towards the centre of the earth. A man suggested that this electric field may be used in flying.

1. What magnitude and sign of charge would a 60 kg human have to acquire to overcome his or her weight?

(a) 6 C (b) 4 C

(c) –6 C (d) – 4 C

(e) none

2. What would be the force of repulsion between two people with the said charge when they are 100 m apart?

(a) 1.44×10^5 C (b) 14.4×10^5 C

(c) 1.44×10^6 C (d) 1.44×10^7 C

3. Can electric field of the earth be practically used as a means of flight?

(a) yes (b) no

(c) only on solar flare days

Solution **1.** (d) $60 \times 10 = qE$

$$\text{or} \quad q = \frac{60 \times 10}{150} \;\square\; 4\,C$$

Solution **2.** (d) $F = \dfrac{4 \times 4 \times 9 \times 10^9}{(100)^2}$

$= 144 \times 10^5\, N$

Solution **3.** (b) Though theoretically it appears as flying may be achieved with electric field of earth but the electric field is so small that a very big charge is required on the other body. The streamlining condition would demand making edges or sharp corners from where charge will leak by means of carona discharge.

PASSAGE 2

Read the following passage and answer the questions given at the end.

The surface of a polar liquid such as water, can be viewed as a series of dipoles strung together in the stable arrangement in which dipole moment vectors are parallel to the surface and all point in the same direction. Now suppose that something presses downwards (inwards) on the surface, distorting the dipoles as shown in Figure 18.46. Then

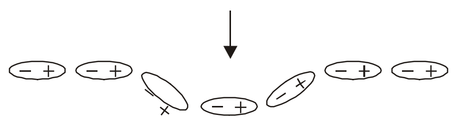

Fig. 18.46

1. The two slanted dipoles exert a net force

(a) downwards (b) upwards

(c) horizontal (d) none of these

2. (A) The surface tension may be assumed to be a result of force between dipoles which opposes penetration of the liquid.

(B) The surface tension is a result of cohesive force which is mechanical, not electrical.

(a) both A and B are correct

(b) A is correct but B is wrong

(c) A is wrong but B is correct

(d) both A and B are wrong.

Solution 1. (b)

Solution 2. (b)

PASSAGE 3

Read the following passage and answer the questions given at the end.

The imaging drum of a photocopier is positively charged to attract negatively charged particles of toner. Near the surface of the drum, its electric field has magnitude 1.4×10^5 NC^{-1}. A toner particle is to be attached to the drum with a force that is 10 times the weight of the particle. Assume toner particles are made of carbon $^{12}_{6}C$.

1. Find charge to mass ratio of the charged toner particle.

(a) 7.0×10^{-4} Ckg^{-1} (b) 7.0×10^{-3} Ckg^{-1}

(c) 7.0×10^{-5} Ckg^{-1} (d) 7.0×10^{-4} C g^{-1}

2. Find the number of carbon atoms that for each excess electron on a toner particle.

(a) 1.15×10^8 (b) 1.15×10^7

(c) 1.15×10^9 (d) 1.15×10^{10}

Solution 1. (a) $qE = 10\,mg$.

$$\frac{q}{m} = \frac{\log}{E} = \frac{9.8\times10}{1.4\times10^5} = 7.0 \times 10^{-4}\text{ C kg}^{-1}$$

Solution 2. (d) $q = \dfrac{10\,mg}{E} = \dfrac{12\times10^{-3}\times9.8\times10}{1.4\times10^5}$

Number of electrons $n = \dfrac{84\times10^{-7}}{1.6\times10^{-19}}$

$$n = \frac{84\times10^{12}}{1.6}$$

Number of carbon atoms $= \dfrac{6.023\times10^{23}\times1.6}{84\times10^{12}}$

$= 1.15 \times 10^{10}$ per excess e

QUESTIONS FOR PRACTICE

1. Three charged particles are in equilibrium under their electrostatics forces only.

(a) The particles must be collinear.

(b) All the charges cannot have the same magnitude.

(c) All the charges cannot have the same sign.

(d) The equilibrium is unstable.

2. Four charges, all of the same magnitude, are placed at the four corners of a square. At the centre of the square, the potential is V and the field is E. By suitable choices of the signs of the four charges, which of the following can be obtained ?

(a) $V = 0, E = 0$ (b) $V = 0, E \neq 0$

(c) $V \neq 0, E = 0$ (d) $V \neq 0, E \neq 0$

3. A deuteron and an α-particle are placed in an electric field. The forces acting on them are F_1 and F_2, and their accelerations are a_1 and a_2 respectively.

(a) $F_1 = F_2$ (b) $F_1 \neq F_2$

(c) $a_1 = a_2$ (d) $a_1 \neq a_2$

4. Four identical charges are placed at the points (1, 0, 0), (0, 1, 0), (–1, 0, 0) and (0, –1, 0).

(a) The potential at the origin is zero.

(b) The field at the origin is zero.

(c) The potential at all points on the z-axis, other than the origin, is zero.

(d) The field at all points on the z-axis, other than the origin, acts along the z-axis.

5. Two identical charges $+Q$ are kept fixed some distance apart. A small particle P with charge q is placed midway between them. If P is given a small displacement Δ, it will undergo simple harmonic motion if

(a) q is positive and Δ is along the line joining the charges.

(b) q is positive and Δ is perpendicular to the line joining the charges.

(c) q is negative and Δ is perpendicular to the line joining the charges.

(d) q is positive and Δ is along the line joining the charges.

6. A positively charged thin metal ring of radius R is fixed in the xy plane, with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$, where $z_0 > 0$. Then the motion of P is

(a) periodic, for all values of z_0 satisfying $0 < z_0 < \infty$

(b) simple harmonic, for all values of z_0 satisfying $0 < z_0 \leq R$

(c) approximately simple harmonic, provided $z_0 << R$

(d) such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$

7. A ring with a uniform charge Q and radius R, is placed in the yz plane with its centre at the origin.

(a) The field at the origin is zero.

(b) The potential at the origin is $k\dfrac{Q}{R}$.

(c) The field at the point $(x, 0, 0)$ is $k\dfrac{Q}{x^2}$.

(d) The field at the point $(x, 0, 0)$ is $k\dfrac{Q}{R^2 + x^2}$.

8. Two equal positive charges are kept at points A and B. The electric potential at the points between A and B (excluding these points) is studied while moving from A to B. The potential

(a) continuously increases

(b) continuously decreases

(c) increases then decreases

(d) decreases then increases

9. The electric field at the origin is along the positive X-axis. A small circle is drawn with the centre at the origin cutting the axes at points A, B, C and D having coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$, $(0, -a)$ respectively. Out of the points on the periphery of the circle, the potential is minimum at

(a) A (b) B

(c) C (d) D

10. Consider the situation of Fig. 18.47. The work done in taking a point charge from P to A is W_A from P to B is W_B and from P to C is W_C.

(a) $W_A < W_B < W_C$ (b) $W_A > W_B > W_C$

(c) $W_A = W_B = W_C$ (d) None of these.

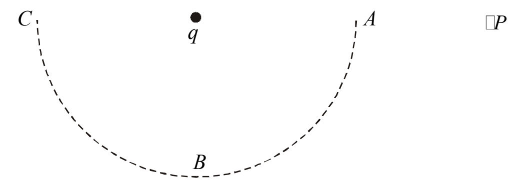

Fig. 18.47

11. If a body is charged by rubbing it, its weight

(a) remains precisely constant

(b) increases slightly

(c) decreases slightly

(d) may increase slightly or may decrease slightly

12. A point charge q is rotated along a circle in the electric field generated by another point charge Q. The work done by the electric field on the rotating charge in one complete revolution is

(a) zero (b) positive

(c) negative

(d) zero if the charge Q is at the centre and nonzero otherwise.

13. An electric dipole is placed in a uniform electric field. The net electric force on the dipole

(a) is always zero

(b) depends on the orientation of the dipole

14. A particle A of mass m and charge Q moves directly towards a fixed particle B, which has charge Q. The speed of A is υ when it is far away from B. The minimum seperation between the particles is proportional to

(a) Q^2 (b) $\dfrac{1}{\upsilon^2}$

(c) $\dfrac{1}{\upsilon}$ (d) $\dfrac{1}{m}$

15. Charges Q_1 and Q_2 are placed inside and outside respectively of an uncharged conducting shell. Their separation is r.

(a) The force on Q_1 is zero.

(b) The force on Q_1 is $k\dfrac{Q_1Q_2}{r^2}$.

(c) The force on Q_2 is $k\dfrac{Q_1Q_2}{r^2}$.

(d) The force on Q_2 is zero.

16. In a uniform electric field,

(a) all points are at the same potential

(b) no two points can have the same potential

(c) pairs of points seperated by the same distance must have the same difference in potential

(d) none of the above.

17. Let V and E be the potential and the field respectively at a point. Which of the following assertions are correct ?

(a) If $V = 0$, E must be zero.

(b) If $V \neq 0$, E cannot be zero.

(c) If $E \neq 0$, V cannot be zero.

(d) None of these.

18. Charges Q_1 and Q_2 lie inside and outside respectively of a closed surface S. Let E be the field at any point on S and ϕ be the flux of E over S.

(a) If Q_1 changes, both E and ϕ will change.

(b) If Q_2 changes, E will change but ϕ will not change.

(c) If $Q_1 = 0$ and $Q_2 \neq 0$ then $E \neq 0$ but $\phi = 0$.

(d) If $Q_1 \neq 0$ and $Q_2 = 0$ then $E = 0$ but $\phi \neq 0$.

19. P is a point on an equipotential surface S. The field at P is E.

(a) E must be perpendicular to S in all cases.

(b) E will be perpendicular to S only if S is a plane surface.

(c) E cannot have a component along a tangent to S.

(d) E may have a nonzero component along a tangent to S if S is a curved surface.

20. A spherical conductor A lies inside a hollow spherical conductor B. Charges Q_1 and Q_2 are given to A and B respectively.

(a) Charge Q_1 will appear on the outer surface of A.

(b) Charge $-Q_1$ will appear on the inner surface of B.

(c) Charge Q_2 will appear on the outer surface of B.

(d) Charge $Q_1 + Q_2$ will appear on the outer surface of B.

21. Two large, identical and parallel conducting plates have surfaces X and Y, facing each other. The charge per unit area on X is σ_1, and on Y it is σ_2.

(a) $\sigma_1 = -\sigma_2$ in all cases.

(b) $\sigma_1 = -\sigma_2$ only if a charge is given to one plate only.

(c) $\sigma_1 = \sigma_2 = 0$ if equal charges are given to both the plates.

(d) $\sigma_1 > \sigma_2$ if X is given more charge than Y.

22. A, B and C are three concentric metallic shells. Shell A is the innermost and shell C is the outermost. A is given some charge.

(a) The inner surfaces of B and C will have the same charge.

(b) The inner surfaces of B and C will have the same charge density.

(c) The outer surfaces of A, B and C will have the same charge.

(d) The outer surfaces of A, B and C will have the same charge density.

23. Mark out the correct options.

(a) The total charge of the universe is constant.

(b) The total positive charge of the universe is constant.

(c) The total negative charge of the universe is constant.

(d) The total number of charged particles in the universe is constant.

24. The electric field and the electric potential at a point are E and V respectively.

(a) If $E = 0$, V must be zero.

(b) If $V = 0$, E must be zero.

(c) If $E \neq 0$, V cannot be zero.

(d) If $V \neq 0$, E cannot be zero.

25. An electric dipole is placed in an electric field generated by a point charge.

(a) The net electric force on the dipole must be zero.

(b) The net electric force on the dipole may be zero.

(c) The torque on the dipole due to the field must be zero.

(d) The torque on the dipole due to the field may be zero.

26. A point charge is brought in an electric field. The electric field at a nearby point

(a) will increase if the charge is positive

(b) will decrease if the charge is negative

(c) may increase if the charge is positive

(d) may decrease if the charge is negative.

27. The electric potential decreases uniformly from 120 V to 80 V as one moves on the X-axis from $x = -1$ cm to $x = +1$ cm. The electric field at the origin

(a) must be equal to 20 V/cm

(b) may be equal to 20 V/cm

(c) may be greater than 20 V/cm

(d) may be less than 20 V/cm.

28. A proton and an electron are placed in a uniform electric field.

(a) The electric forces acting on them will be equal.

(b) The magnitudes of the forces will be equal.

(c) Their accelerations will be equal.

(d) The magnitudes of their accelerations will be equal.

29. Which of the following quantities do not depend on the choice of zero potential or zero potential energy ?

(a) potential at a point

(b) potential difference between two points

(c) potential energy of a two-charge system

(d) change in potential energy of a two-charge system

30. The electric field in a region is directed outward and is proportional to the distance r from the origin. Taking the electric potential at the origin to be zero,

(a) it is uniform in the region

(b) it is proportional to r

(c) it is proportional to r^2

(d) it increases as one goes away from the origin

31. S_1 and S_2 are two equipotential surfaces on which the potentials are not equal.

(a) S_1 and S_2 cannot intersect.

(b) S_1 and S_2 cannot both be plane surfaces.

(c) In the region between S_1 and S_2, the field is maximum where they are closest to each other.

(d) A line of force from S_1 to S_2 must be perpendicular to both.

32. A solid metallic sphere is placed in a uniform electric field. Which of the curves shown in Fig. 18.48 represent the lines of force correctly ?

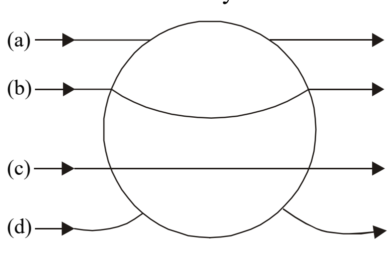

Fig. 18.48

33. The field at a distance r from a long string of charge per unit length λ is

(a) $k\dfrac{\lambda}{r^2}$ (b) $k\dfrac{\lambda}{r}$

(c) $k\dfrac{\lambda}{2r}$ (d) $k\dfrac{2\lambda}{r}$

34. A dipole of moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. The force on the dipole is $\vec{F}$ and the torque is $\vec{\tau}$.

(a) $\vec{F} = 0$ (b) $\vec{F} = |\vec{p}|\vec{E}$

(c) $|\vec{\tau}| = \vec{p}.\vec{E}$ (d) $\vec{\tau} = \vec{p} \times \vec{E}$

35. In a uniform electric field, equipotential surfaces must

(a) be plane surfaces

(b) be normal to the direction of the field

(c) be spaced such that surfaces having equal differences in potential are separated by equal distances

(d) have decreasing potentials in the direction of the field

36. Two points are at distances a and b ($a < b$) from a long string of charge per unit length λ. The potential differnece between the points is proportional to

(a) $\dfrac{b}{a}$ (b) $\dfrac{b^2}{a^2}$

(c) $\sqrt{\dfrac{b}{a}}$ (d) In $\left(\dfrac{b}{a}\right)$

37. In the previous question, if the conductor has a charge per unit length λ, the particle has mass m and charge q then

(a) $\upsilon \propto \sqrt{q}$ (b) $\upsilon \propto \sqrt{\lambda}$

(c) $\upsilon \propto \sqrt{m}$ (d) $\upsilon \propto \dfrac{1}{\sqrt{m}}$

38. A charged particle moves with a speed υ in a circular path of radius r around a long uniformly charged conductor.

(a) $\upsilon \propto r$ (b) $\upsilon \propto \dfrac{1}{r}$

(c) $\upsilon \propto \dfrac{1}{\sqrt{r}}$ (d) υ is independent of r

39. A simple pendulum of length l has a bob of mass m, with a charge q on it. A vertical sheet of charge, with charge σ per unit area, passes through the point of suspension of the pendulum. At equilibrium, the string makes an angle θ with the vertical. Its time period of oscillations is T in this position.

(a) $\tan\theta = \dfrac{\sigma q}{2\varepsilon_0 mg}$ (b) $\tan\theta = \dfrac{\sigma q}{\varepsilon_0 mg}$

(c) $T < 2\pi\sqrt{\dfrac{l}{g}}$ (d) $T > 2\pi\sqrt{\dfrac{l}{g}}$

40.

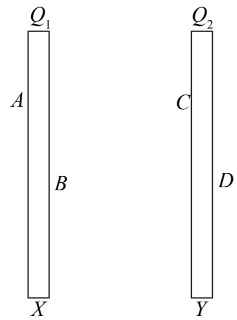

Fig. 18.49

Two large, parallel conducting plates X and Y, kept close to each other, are given charges Q_1 and Q_2 ($Q_1 > Q_2$). The four surfaces of the plates are A, B, C and D, as shown in Fig. 18.49.

(a) The charge on A is $\frac{1}{2}(Q_1 + Q_2)$.

(b) The charge on B is $\frac{1}{2}(Q_1 - Q_2)$.

(c) The charge on C is $\frac{1}{2}(Q_2 - Q_1)$.

(d) The charge on D is $\frac{1}{2}(Q_1 + Q_2)$.

41. A dipole is placed in an electric field whose direction is fixed but whose magnitude varies with distance. It is possible that the dipole experiences

(a) no net force and no torque

(b) a net force but no torque

(c) a net force and a torque

(d) no net force but a torque

42. Two large, parallel conducting plates are placed close to each other. The inner surfaces of the two plates have surface charge densities $+\sigma$ and $-\sigma$. The other surfaces are without charge. The electric field has a magnitude of

(a) $\frac{2\sigma}{\varepsilon_0}$ in the region between the plates

(b) $\frac{\sigma}{\varepsilon_0}$ in the region between the plates

(c) $\frac{\sigma}{\varepsilon_0}$ in the region outside the plates

(d) zero in the region outside the plates

43. Two identical rings of radii 0.1 m are placed co–axially at a distance 0.5 m apart. The charges on the rings are $2\mu C$ and $4\mu C$ respectively. The work done in transferring $5\mu C$ charge from the centre of one ring to that of the other will be nearest to

(a) 0.50 J (b) 0.75 J

(c) 1.00 J (d) 1.50 J

44. The electric field strength due to a ring of radius R at a distance x from its centre on the axis of ring carrying charge Q is given by $E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}}$.

At what distance from the centre will the electric field be maximum ?

(a) $x = R$ (b) $x = \frac{R}{2}$

(c) $x = \frac{R}{\sqrt{2}}$ (d) $x = \frac{R}{\sqrt{2}}$

45. Positive charges of $2\mu C$ and $8\mu C$ are placed 15cm apart. At what distance from the smaller charge will the electric field due to them be zero ?

(a) 3 cm (b) 5 cm

(c) 7 cm (d) 10 cm.

46. A charge is distributed over two concentric hollow spheres of radii R and r, where $R > r$, such that the surface densities of charges are equal (σ). What is the potential at their common centre ?

(a) $\frac{\sigma}{\varepsilon_0}(R+r)$ (b) $\frac{\sigma}{\varepsilon_0}(R+r)$

(c) $\frac{\sigma}{\varepsilon_0}R$ (d) $\frac{\sigma}{\varepsilon_0}r$.

47. A charge q is distributed over two spheres of radii R and r such that their surface densities are equal. What is the ratio of the charges on the spheres ?

(a) $\frac{r}{R}$ (b) $\frac{r^2}{R^2}$

(c) $\frac{r^2}{R^2}$ (d) $\frac{r^4}{R^4}$

48. A charge q is distributed over two spheres of radii R and r such that their surface densities are equal. What is the ratio of their potentials ?

(a) $\frac{R}{r}$ (b) $\frac{R^2}{r^2}$

(c) $\frac{R^3}{r^3}$ (d) $\frac{R^4}{r^4}$

49. A ring of radius 6 cm is given a charge $10\mu C$. How much work will be done in transporting a charge of $6\mu C$ from its centre to a point 8 cm along its axis ?

(a) 63 m J (b) 84 m J

(c) 105 m J (d) 126 m J

50. Two conducting spheres of radii r_1 and r_2 are at the same potential. What is the ratio of the charges on them ?

(a) $\sqrt{\frac{r_1}{r_2}}$ (b) $\frac{r_1}{r_2}$

(c) $\frac{r_1^2}{r_2^2}$ (d) $\sqrt{r_1 r_2}$

51. Two conducting spheres of radii r_1 and r_2 are charged such that they have the same electric field on their surfaces. The ratio of the electric potential at their centres is

(a) $\sqrt{\frac{r_1}{r_2}}$ (b) $\frac{r_1}{r_2}$

(c) $\frac{r_1^2}{r_2^2}$ (d) none of the above.

52. An uncharged metallic hollow sphere is placed in uniform external electric field (Fig 18.50). The path of the electric field lines in and around the conductor is represented by

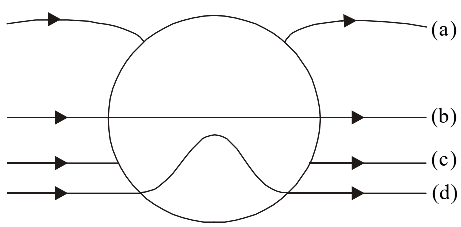

Fig. 18.50

53. How does the electric field (E) between the plates of a charged cylindrical capacitor vary with the distance (r) from the axis of the cylinder ?

(a) $E \propto \frac{1}{r^2}$ (b) $E \propto \frac{1}{r}$

(c) $E \propto r^2$ (d) $E \propto r$

54. The electric field line in the $X-Y$ plane is represented by $x^2 + y = 1$. A test charge q_0 is placed at a distance $x = 1$, $y = 1$. What will be the nature of the path followed by it ?

(a) Circle (b) Straight line

(c) Parabola (d) Cannot be predicted

55. A ring of radius R is carrying uniformly distributed charge $+Q$. A test charge $-q_0$ is placed on its axis at a distance $2R$ from the centre and released. The motion of the particle on the axis will be

(a) periodic (b) non periodic

(c) simple harmonic (d) random.

56. Five equal and similar charges are placed at the corners of a regular hexagon as shown in Fig. 18.51. What is the electric field and potential at the centre of the hexagon ?

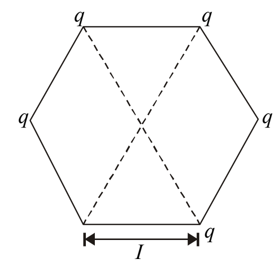

Fig. 18.51

(a) $\frac{5}{4\pi\varepsilon_0}\frac{q}{l}, \frac{5}{4\pi\varepsilon_0}\frac{q}{l^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{l}, \frac{5}{4\pi\varepsilon_0}\frac{q}{l^2}$

(c) $\frac{5}{4\pi\varepsilon_0}\frac{q}{l}, \frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q}{l}, \frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$

57. Which of the following combinations of seven identical capacitors each of $2\mu F$ gives a capacitance of $10/11\mu F$?

(a) 5 in parallel with 2 in series

(b) 4 in parallel with 3 in series

(c) 3 in parallel with 4 in series

(d) 2 in parallel with 5 in series

58. The electric field at the centre of a uniformly charged ring is zero. What is the electric field at the centre of a half ring if the charge on it be Q and its radius be R?

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi R^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{2Q}{\pi R^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{2Q}{R^2}$

59. What is the electric potential at the centre of a hemisphere of radius R and having surface charge density σ?

(a) $\frac{\sigma}{2\varepsilon_0}$ (b) $\frac{\sigma}{\varepsilon_0}$

(c) $\frac{\sigma}{\varepsilon_0}R$ (d) $\frac{\sigma}{2\varepsilon_0}R$

60. Fig. 18.52 shows four capacitors connected across a power supply of 310 V. What is the charge and potential difference across the $4\sigma F$ capacitor ?

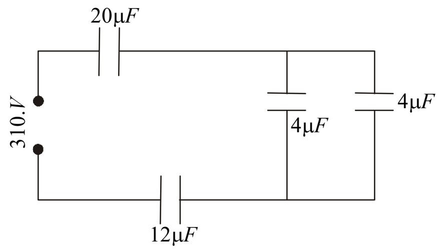

Fig. 18.52

(a) 1200 σF, 310 V (b) 600 σF, 310 V

(c) 600 σF, 150 V (d) 1200 σF, 150 V.

61. Five capacitors are connected to each other as shown in Fig. 18.53. What is potential drop and charge across $4\mu F$ capacitor ?

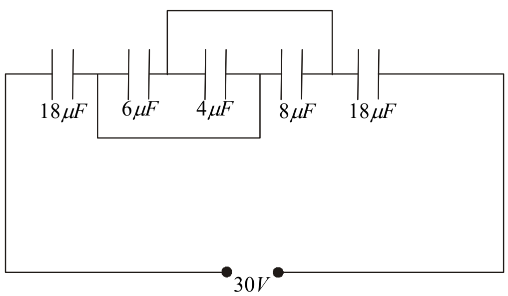

Fig. 18.53

(a) 6 V, 3μC (b) 10 V, 30 μC

(c) 6 V, 40 μC (d) 10 V, 40 μC.

62. Five identical plates are connected across a battery as in Fig. 18.54. If the charge on plate 1 be +q, then the charges on the lates 2, 3, 4 and 5 are

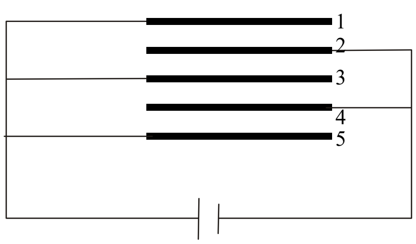

Fig. 18.54

(a) – q, + q, – q, + q (b) – 2q, + 2q, – 2q, + q

(c) – q, + 2q, – 2q, + q (d) none of the above.

63. Three capacitors are connected across a 45V power supply as shown in Fig. 18.55. What is the charge on the 6 μF capacitor?

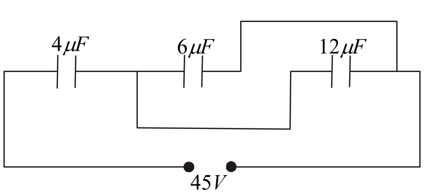

Fig. 18.55

(a) 60μC (b) 90μC

(c) 120μC (d) 180μC

64. Six identical capacitors each of $2\mu F$ are joined in parallel and each is charged to 10 V. They are then disconnected and joined in series so that positive plate of one is joined to the negative plate of the adjacent capacitor. What is the potential difference of the combination ?

(a) 10 V (b) 30 V

(c) 60 V (d) 120 V

65. A capacitor of capacitance $10\mu F$ is charged by connecting through a resistance of 20 Ω and a battery of 20 V. What is the energy supplied by the battery ?

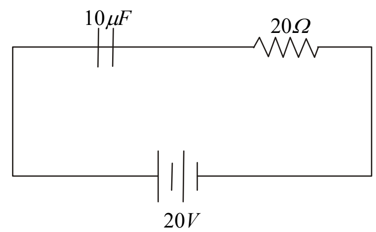

Fig. 18.56

(a) Less than 2 m J (b) 2 m J

(c) More than 2 m J (d) Cannot be predicted

66. Force acting on a test charge between the plates of a parallel capacitor is F. If one of the plates is removed, then the force on the same test charge will be

(a) zero (b) F

(c) $2F$ (d) $\frac{F}{2}$

67. Two point charges Q and $-3Q$ are placed certain distance apart. If the electric field at the location of Q be $\vec{E}$, then that at the location of $-3Q$ will be

(a) $3\vec{E}$ (b) $-3\vec{E}$

(c) $\frac{\vec{E}}{3}$ (d) $-\frac{\vec{E}}{3}$

68. The length of each side of a cubical closed surface is α. If charge q is situated on one of the vertices of the cube then the flux passing through each face of the cube will be

(a) $\frac{q}{6\varepsilon_0}$ (b) $\frac{q}{24\varepsilon_0}$

(c) $\frac{q}{8\varepsilon_0}$ (d) zero

69. A charged conductor has charge on its

(a) outside surface (b) surrounding

(c) middle point (d) inner surface

70. The laws of forces that govern the force between two electric charges were discovered by

(a) Faraday (b) Ampere

(c) Ohm (d) Coulumb

71. A charge Q is placed on to two opposite corners of a square. A charge q is placed at each of other two corners. Given that resultant electric force on Q is zero, then Q is equal to

(a) $\left(2\sqrt{2}\right)/q$ (b) $-q/\left(2\sqrt{2}\right)$

(c) $\left(2\sqrt{2}\right)q$ (d) $\left(-2\sqrt{2}\right)q$

72. Let us suppose that earth (radius 6400 km) had a net charge equivalent to one electron per m^2 of its surface area. Its potential in volts will be

(a) –1.2 (b) – 0.12

(c) 0.12 (d) 1.2

73. A charge Q is divided into two parts. The two charges kept at a distance apart have a maximum columbian repulsion. Then the ratio of Q and one of the parts is given by

(a) 1 : 4 (b) 1 : 2

(c) 2 : 1 (d) 4 : 1

74. In comparison with the electrostatic force between two electrons, the electrostatic force between two protons is

(a) zero (b) smaller

(c) same (d) greater

75. A positively charged ball hangs from a long silk thread. We put a positive test charge q_0 at a point and measure F/q_0, then it can be predicted that field E

(a) $>F/q_0$ (b) $<F/q_0$

(c) is equal to F/q_0 (d) none of these

76. An electron of mass m and charge e is accelerated from rest through a potential difference V in vacuum. Its final speed will be

(a) $\sqrt{\dfrac{2eV}{m}}$ (b) $\sqrt{\dfrac{eV}{m}}$

(c) $\dfrac{eV}{2m}$ (d) $\dfrac{eV}{m}$

77. A helium ion and a hydrogen ion are accelerated from rest through a potential difference of V to velocities v_{He} and v_H respectively. If helium has lost one electron, the ratio of v_{He}/v_H is

(a) 1/4 (b) 1/2

(c) 1 (d) $\sqrt{2}$

78. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be equilibrium if q is equal to

(a) $-(Q/4)$ (b) $-(Q/2)$

(c) $(Q/2)$ (d) $(Q/4)$

[IIT 87]

79. A proton has a mass 1.67×10^{-27} kg and charge $+ 1.6 \times 10^{-19}$ C. If the proton is being accelerated through a potential difference of one millions volts then the $K.E.$ is

(a) 1.6×10^{-25} J (b) 3.2×10^{-13} J

(c) 1.6×10^{-15} J (d) 1.6×10^{-13} J

80. An electron having charge $-e$ located at A, in the presence of point charge $+q$ located at O, is moved to the point B such that OAB forms an equilateral triangle. The work done in the process is equal to

(a) $-eq/AB$ (b) eq/AB

(c) q/AB (d) zero

81. The electric flux ϕ through a hemispherical surface of radius R, placed in a uniform electric field of intesity E parallel to the axis of its circular plane is

(a) $(4/3)\,\pi R^3E$ (b) $2\pi R^2E$

(c) πR^2E (d) $2\pi RE$

82. A charge of 6.76 μC in an electric field is acted upon by a force of 2.5 N. The potential gradient at this point is

(a) 3.71×10^5 Vm^{-1} (b) 3.71×10^{12} Vm^{-1}

(c) 3.71×10^{10} Vm^{-1} (d) 3.71×10^5 Vm^{-1}

83. Two identical thin rings each of radius R are coaxially placed at a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to the other is

(a) $\dfrac{q(Q_1/Q_2)(\sqrt{2}+1)}{\sqrt{2}4\pi\varepsilon_0R}$ (b) $\dfrac{q(Q_1-Q_2)(\sqrt{2}-1)}{\sqrt{2}4\pi\varepsilon_0R}$

(c) $\dfrac{q\sqrt{2}(Q_1+Q_2)}{4\pi\varepsilon_0R}$ (d) zero

84. Two pith balls each of mass 1g and carrying a charge 1μC are attached to the ends of silk threads 1m long, the other ends of which are attached to some fixed point, in a gravity free space. The force between them is

(a) 9.8×10^{-12} N (b) 9.0×10^{-6} N

(c) 4.5×10^{-6} N (d) 2.25×10^{-3} N

85. Two equal metal balls are charged to 10 and 20 units of electricity. They are brought in contact with each other and then again separated to the original distance. The ratio of forces between the two balls before and after contact is

(a) 3 : 2
(b) 1 : 8
(c) 2 : 3
(d) 8 : 9

86. No current flows between two charged bodies when connected if they have same

(a) capacity
(b) charge
(c) potential
(d) none of the above

87. The dielectric strength of air is 3.0×10^6 NC^{-1}. The largest charge that a 0.30 cm radius metal sphere can hold without sparking is

(a) 9 nC
(b) 8.2 nC
(c) 6 nC
(d) 3 nC

88. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge q is placed on the axis of the ring and released from rest. The force experienced by the particle varies with distance from the centre as

(a) $x/(R^2+x^2)^{3/2}$
(b) $1/\sqrt{x}$
(c) $1/x^3$
(d) $1/x^2$

89. An electron is accelerated through a potential difference of 500 volt. The velocity acquired by the electron is

(a) $(2/3) \times 10^7$ ms^{-1}
(b) $(1/6) \times 10^7$ ms^{-1}
(c) $(1/3) \times 10^7$ ms^{-1}
(d) none of the above

90. An electrical charge of 2 μC is placed at the point (1, 2, 3). At the point (2, 3, 4) the electric field and potential will be

(a) 6×10^3 NC^{-1} and 6×10^3 JC^{-1}

(b) 6000 NC^{-1} and $6000\sqrt{3}$ JC^{-1}

(c) 6×10^3 NC^{-1} and $3\sqrt{3}$ JC^{-1}

(d) none of the above

91. The charge per unit length for a very long straight wire is λ. The electric field at points near the wire (but outside it) and far from the ends varies with distance r as

(a) r
(b) $1/r$
(c) $1/r^2$
(d) $1/r^3$

92. A charged particle of mass m and charge q is released from rest in an electric field of constant magnitude E. The $K.E.$ of the particle after time r is

(a) $\dfrac{Eq^2m}{2t^2}$
(b) $\dfrac{Eqm}{2t}$
(c) $\dfrac{2E^2t^2}{mq}$
(d) $\dfrac{E^2q^2t^2}{2m}$

93. A solid conducting sphere having a charge Q is surrounding by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$ the new potential difference between the same two surface is

(a) $-2\,V$
(b) V
(c) $2\,V$
(d) $4\,V$

[IIT 1989]

94. A uniform electric field having a magnitude E_0 and direction along the positive x-axis exists. If the potential V is zero at $x = 0$, then its value at $x = +x$ will be

(a) $V(x) = x^2E_0$
(b) $V(x) = -x^2E_0$
(c) $V(x) = -xE_0$
(d) $V(x) = xE_0$

95. A given charge situated at a certain distance from an electric dipole in the end on position, experiences a force F. If the distance of the charge is doubled, the force acting on the charge will be

(a) $F/8$
(b) $F/4$
(c) $F/2$
(d) $2F$

96. Eight charged water drops each with a radius of 1 mm and a charge of 10^{-10} coulumb merge into a single drop. The potential of this single drop is

(a) 36 V
(b) 1000 V
(c) 3600 V
(d) 8000 V

97. $ABCD$ is a square of 1 metre side of a non-conducting material. Four metallic spheres of 4, 5, 8 and 10 cm diameters are placed at the four corners. All of them are connected by a fine metallic wire and charge of 540 units is imparted to the system. The potential at the centre of the square is

(a) $\dfrac{540\sqrt{2}}{400}$
(b) $\dfrac{540\sqrt{2}}{200}$
(c) $\dfrac{540\sqrt{2}}{100}$
(d) $\dfrac{540\sqrt{2}}{10}$

98. The electric potential at a point situated at a distance r on the axis of a short electric dipole of moment p will be $1/4(\pi\varepsilon_0)$ times

(a) p/r^3
(b) p/r^2
(c) p/r
(d) none of the above

99. Two concentric thin, metallic spheres of radii R_1 and $R_2 (R_1 > R_2)$ bear charges Q_1 and Q_2 respectively. Then the potential at a radius r between R_1 and R_2 will be $1/4(\pi\varepsilon_0)$ times

(a) $\dfrac{Q_1+Q_2}{4}$
(b) $\dfrac{Q_1}{R_1}+\dfrac{Q_2}{r}$
(c) $\dfrac{Q_1}{R_1}+\dfrac{Q_2}{R_2}$
(d) $\dfrac{Q_1}{R_2}+\dfrac{Q_2}{R_1}$

100. An electric dipole of moment p is kept along an electric field E. The work done in rotating it from an equilibrium position by an angle θ is

(a) $PE(1-\cos\theta)$ (b) $PE(1-\sin\theta)$

(c) $PE\cos\theta$ (d) $PE\sin\theta$

101. A body has a charge of one coulumb. The number of excess (or lesser) electrons on it from its normal state will be

(a) ∞ (b) 1.6×10^{-19}

(c) 1.6×10^{19} (d) 6.25×10^{18}

PASSAGE 1

Read the following passage and answer the questions given at the end.

While real crystals are three-dimensional, much can be learned from simple one-dimensional models for which calculations can be carried out much more easily. As a one-dimensional model of an ionic crystal such as sodium chloride (NaCl), consider alternating positive and negative ions of charge $+e$ and $-e$ respectively, which are uniformly spaced along the x-axis with spacing d (Fig. 18.57). The charges may be considered to extend to infinity in both directons.

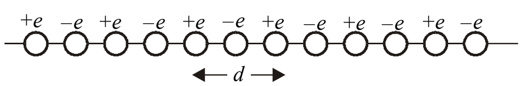

Fig. 18.57

Consider the potential energy of interaction between the positive ion at $x = 0$ and all the other ions. This represents the potential energy per ion in this one dimensional crystal. The potential energy per ion in 3-dimensional crystal for Nacl ion is -8×10^{-19} coulomb per ion when spacing $d = 2.824°$ is taken.

1. The potential energy per ion for positive ion is-----.

(a) $\dfrac{-2e^2}{4\pi\varepsilon_0 d}\log_e 2$ (b) $\dfrac{-e^2}{4\pi\varepsilon_0 d}\log_e 2$

(c) $\dfrac{2e^2}{4\pi\varepsilon_0 d}$ (d) $\dfrac{-2e^2}{4\pi\varepsilon_0 d}$

2. Is the potential energy for negative ion same as for the positive ion?

(a) Yes (b) No

(c) Cannot say

3. Evaluate potential energy per ion for Nacl crystal. Use inter atomic spacing 2.82×10^{-10} m.

(a) 8.12×10^{-19} J (b) 2.67×10^{-19} J

(c) 5.67×10^{-19} J (d) none.

4. How accurate is one-dimesional model?

(a) Less accurate (b) Quite accurate

(c) Cannot say

Solution **1.** (b) $U = \dfrac{-e^2}{4\pi\varepsilon_0 d} + \dfrac{-e^2}{4\pi\varepsilon_0 2d} + \dfrac{-e^2}{4\pi\varepsilon_0 3d} + \dfrac{-e^2}{4\pi\varepsilon_0 4d}$ ------

$$= \frac{-e^2}{4\pi\varepsilon_0 d}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+----\right]$$

$$= \frac{-e^2\log_e 2}{4\pi\varepsilon_0 d}$$

Solution **2.** (a)

Solution **3.** (a)

$$= \frac{1.6\times10^{-14}\times1.6\times10^{-9}\times9\times10^{9}}{2.82\times10^{-10}}$$

$$= 8.12\times10^{-19}\text{ J}$$

Solution **4.** (b) The one-dimensional model is quite accurate.

PASSAGE 2

Read the following passage and answer the questions given at the end.

The charge of an electron was first measured by the American physicist Robert Millikan during (1909-1913). In his experiment, oil was sprayed in very fine drops (around 10^{-4} mm in diameter) into the space between two parallel horizontal plates separated by a distance d. A potential difference V_{AB} is maintained between the parallel plates, causing a downward electric field between them. Some of the oil drops acquire a negative charge because of frictional effects or because of ionisation of the surrounding air by X-rays or radioactivity. The drops are observed through a microscope. ρ is density of oil and r is radius of the drop under investigation.

1. The charge so that oil drops remain at rest is

(a) $\dfrac{4\pi}{3}\dfrac{\rho r^3 gd}{V_{AB}}$ (b) $\dfrac{4\pi}{3}\dfrac{\rho r^3 g}{V_{AB}}$

(c) $\dfrac{3\pi}{4}\dfrac{\rho r^3 gd}{V_{AB}}$ (d) $\dfrac{3\rho r^3 gd}{4\pi V_{AB}}$

2. To measure the radius of the drop Millikan used —— law for freely falling drops.

(a) Poise uille's (b) Ostwald's

(c) Brewester's (d) Stoke's

3. When $V_{AB} = 0$, charged drop was observed to fall 1mm in 39.3 S at a constants speed. The same drop can be held at rest by applying 9.16 V = V_{AB} and plates separated by 1mm. Then charge on the drop is ------.

Given $\eta_{air} = 1.81 \times 10^{-5}$ $N–S–m^{-2}$ and $\rho_{oil} = 824$ kg m^{-3}

(a) e (b) 2e

(c) 3e (d) 4e

Solution **1.** (a) $mg = qE$

or $\frac{4}{3}\pi r^3 \rho g = \frac{qV_{AB}}{d}$

or $q = \frac{4\pi r^3 \rho g d}{3V_{AB}}$

Solution **2.** (d)

Solution **3.** (c) $v = \frac{10^{-3}}{39.3}$ ms^{-1}

using stoke's law $F = 6\pi\eta r v$, terminal velocity is

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

or $$r = \sqrt{\frac{9v\eta}{2(\rho - \sigma)g}}$$

or $$q = 4\pi\left(\sqrt{\frac{9v\eta}{2\rho g}}\right)^3 \frac{\rho g d}{3V_{AB}}$$

$$= 18\pi \frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v^3}{\rho g}}$$

$$q = \frac{18 \times 3.14 \times 10^{-3}\left(1.81 \times 10^{-5}\right)^{3/2} \times \left(\frac{10^{-3}}{39.3}\right)^{3/2}}{9.16 \times \sqrt{8240}}$$

$$= \frac{18 \times 3.14 \times \frac{1}{30\pi} \times 10^{-15}}{9.16 \times 91}$$

$$= \frac{18 \times \pi \times \frac{1}{30\pi} \times \frac{1}{\sqrt{3}} \times 10^{-15}}{9.16 \times 91}$$

$= 4.86 \times 10^{-19}$ C $= 3e$

PASSAGE 3

Read the following passage and answer the questions given at the end.

Some cell walls in the human body have a layer of negative charge on the inside surface and a layer of positive charge of equal magnitude on the outside surface. Suppose that the surface charge densities are $\pm 0.50 \times 10^{-3}$ C-m^{-2}, the cell wall is 5.0×10^{-9} m thick and the cell wall material has a dielectric constant of $k = 5.4$. A typical cell in human body has volume 10^{-16} m^3. Assume the cell to be spherical.

1. The electric field between the inside and outside layers is

(a) 10^6 Vm^{-1} (b) 5×10^6 Vm^{-1}

(c) 0.2×10^7 Vm^{-1} (d) 10^7 Vm^{-1}

2. The potential difference between inside and outside walls is

(a) 0.25 V (b) 0.025 V

(c) 0.05 V (d) 2.5 mV

3. Which wall is at higher potential

(a) Inner (b) Outer

(c) Both at same potential (d) None.

4. Estimate the energy stored in the typical cell.

(a) 1.4×10^{-13} J (b) 1.1×10^{-14} J

(c) 1.4×10^{-19} J (d) 1.1×10^{-15} J

Solution **1.** (d) $E = \frac{\sigma}{\varepsilon_0 k}$

$$= \frac{5 \times 10^{-4}}{8.85 \times 10^{-12} \times 5.4}$$

$= 10^7$ Vm^{-1}

Solution **2.** (c) $V = Ed$

$= 10^7 \times 5 \times 10^{-9} = 0.05$V

Solution **3.** (b)

Solution **4.** (a) $\frac{1}{2}\varepsilon_0 KE^2$(Vol)

= Energy stored

$$= \frac{1}{2}\frac{\sigma^2}{2\varepsilon_0 K}(\text{Vol})$$

$$= \frac{1}{2} \times \frac{\left(0.5 \times 10^{-3}\right)^2 \times 10^{-16}}{2 \times 8.85 \times 10^{-2} \times 5.4}$$

$= 1.4 \times 10^{-13}$ J

Answers to Questions for Practice

1.	(a,b,c,d)	**2.**	(a,b,c,d)	**3.**	(b,c)	**4.**	(b,d)	**5.**	(a,c)	**6.**	(a,c,d)	**7.**	(a,b)
8.	(d)	**9.**	(a)	**10.**	(c)	**11.**	(d)	**12.**	(a)	**13.**	(a)	**14.**	(a,b,d)
15.	(a,c)	**16.**	(d)	**17.**	(d)	**18.**	(a,b,c)	**19.**	(a,c)	**20.**	(a,b,d)	**21.**	(a,c)
22.	(a,c)	**23.**	(a)	**24.**	(none)	**25.**	(d)	**26.**	(c,d)	**27.**	(b,c)	**28.**	(b)
29.	(b,d)	**30.**	(c)	**31.**	(a,c,d)	**32.**	(d)	**33.**	(d)	**34.**	(a,d)	**35.**	(a,b,c,d)
36.	(d)	**37.**	(a,b,d)	**38.**	(d)	**39.**	(a,c)	**40.**	(a,b,c,d)	**41.**	(b,c,d)	**42.**	(b,d)
43.	(b)	**44.**	(c)	**45.**	(b)	**46.**	(a)	**47.**	(b)	**48.**	(a)	**49.**	(d)
50.	(b)	**51.**	(b)	**52.**	(a)	**53.**	(b)	**54.**	(d)	**55.**	(a)	**56.**	(c)
57.	(a)	**58.**	(c)	**59.**	(d)	**60.**	(c)	**61.**	(d)	**62.**	(b)	**63.**	(b)
64.	(c)	**65.**	(c)	**66.**	(d)	**67.**	(d)	**68.**	(b)	**69.**	(a)	**70.**	(d)
71.	(d)	**72.**	(b)	**73.**	(c)	**74.**	(c)	**75.**	(a)	**76.**	(a)	**77.**	(b)
78.	(a)	**79.**	(d)	**80.**	(d)	**81.**	(c)	**82.**	(d)	**83.**	(b)	**84.**	(d)
85.	(d)	**86.**	(c)	**87.**	(d)	**88.**	(a)	**89.**	(d)	**90.**	(b)	**91.**	(b)
92.	(d)	**93.**	(b)	**94.**	(c)	**95.**	(a)	**96.**	(c)	**97.**	(c)	**98.**	(b)
99.	(b)	**100.**	(a)	**101.**	(d)								

EXPLANATION

2. (a,b,c,d)

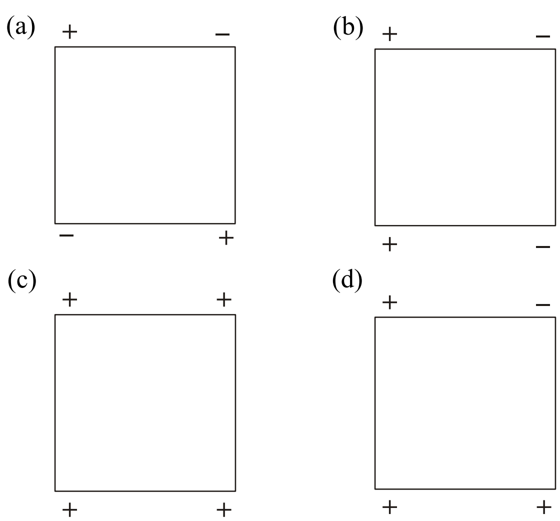

Fig. 18.58

6. (a,c,d)

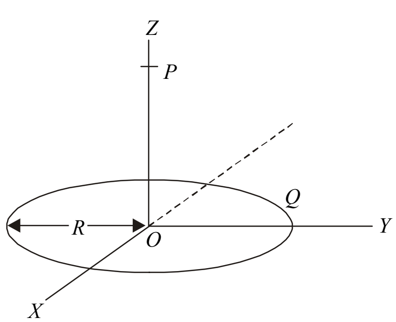

Fig. 18.59

Let $OP = z_0$.

Field at $P = E = kQ\dfrac{z_0}{\left(R^2+z_0^2\right)^{3/2}}$

E is always directed away from O. Hence a negatively charged particle is accelerated towards O and undergoes periodic motion. For $z_0 << R$, the acceleration is proportional to the z-coordinate, and the particle undergoes approximate SHM.

14. (a,b,d) By conservation of energy, $\dfrac{1}{2}mv^2$

$$= k.\frac{Q^2}{x_{\min}},$$

where $x_{\min}$ = minimum seperation.

32. (d) The field is zero inside a conductor and hence lines of force cannot exist inside it. Also, due to induced charges on its surface, the field is distorted close to its surface, and a line of force must deviate near the surface outside the sphere.

33.

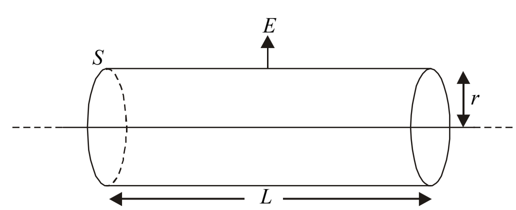

Fig. 18.60

$$\phi_s \vec{E}.d\vec{s} = 2\pi r / E = \frac{1}{\varepsilon_0}(\lambda l)$$

or $$E = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

36. (d) $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$$= -\frac{\partial V}{\partial r}$$

or $\displaystyle\int_{V_a}^{V_b} dV = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0}.\frac{dr}{r}$

or $V_a - V_b = \dfrac{\lambda}{2\pi\varepsilon_0}$. In $\left(\dfrac{b}{a}\right)$.

37. (a,b,d)

38 (d) $F = \dfrac{m\upsilon^2}{r}$

$$= qE = q.\frac{\lambda}{2\pi\varepsilon_0 r}$$

or $\upsilon^2 = \dfrac{q\lambda}{2\pi\varepsilon_0 m}$.

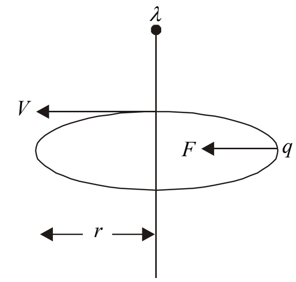

Fig. 18.61