28 Electromagnetic Waves

BRIEF REVIEW

$E = E_0 \sin(\omega t - kx)$ and $B = B_0 \sin(\omega t - kx)$ are the electric and magnetic fields varying with distance x and time t in YZ plane. Such a combination of electric and magnetic fields in vacuum is known as an electromagnetic wave propagating in vacuum. Maxwell, in 1864, developed the theory of em waves.

Maxwell's equations It is a combination of four equations connecting electric and magnetic fields.

$$\oint E \cdot ds = \frac{Q}{\varepsilon_0} \text{ Gauss law in electrostatics}$$

$$\oint E \cdot dl = \frac{-d\phi_{mag}}{dt} \text{ Faraday's law}$$

$$\oint B \cdot ds = 0 \text{ Gauss law in magnetism}$$

$$\oint B \cdot dl = \mu_0 i_C + \mu_0 i_d \text{ modified Ampere's law}$$

$$= \mu_0 i_c + \mu_0 \varepsilon_0 \frac{\partial \phi_E}{\partial t}$$

In vacuum $i_c = 0 \therefore \oint B \cdot dl = \mu_0 i_d = \mu_0 \varepsilon_0 \frac{d\phi_E}{dE}$ or

$$i_d = \frac{\varepsilon_0 d\phi_E}{dt} = \frac{d(q_{in})}{dt}$$

Properties of Electromagnetic Waves

(a) Though these are generated due to variation of electric or magnetic fields, the waves themselves do not carry any charge.

(b) These waves are not deflected by electric or magnetic fields.

(c) These waves travel with speed of light. c in vacuum.

(d) These waves can pass through vacuum as no medium is required for their propagation.

(e) They are only transverse in nature.

(f) They affect photographic plate (blackening it if wavelength < IR).

(g) Their rest mass is zero but they possess momentum.

(h) They can be polarised.

Relation between E and B

$$E_0 = B_0 c \text{ and } c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

where $\mu_0 = 4\pi \times 10^{-7}\ T\text{–}mA^{-1}$

$\varepsilon_0 = 8.85419 \times 10^{-12}\ C^2N^{-1}m^{-2}$ wavelength $\lambda = \frac{c}{f}$

where f is frequency.

Refractive index $n = \sqrt{\mu_r \varepsilon_r}$ or $v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$

In a travelling electromagnetic wave $E\sqrt{\varepsilon\varepsilon_r} = H\sqrt{\mu_r\mu_0}$, momentum $p = \frac{U}{c}$ where $U = m_0c^2$ is energy.

When the wave reflects, momentum becomes *–ve*

Force $F = \frac{Power}{c}$ (in absorbing bodies) and $F = \frac{2Power}{c}$ (in reflecting bodies like mirror).

Intensity $I = \frac{1}{2}\varepsilon_0 E_0^2 c$ and

energy density $u = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$

$$u = \frac{1}{2}ED + \frac{BH}{2} = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

Flow density of electromagnetic energy or Poynting Vector $\vec{P} = \vec{E}\times\vec{H} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)$ this describes rate of energy flow per unit area in a plane electromagnetic wave. Unit of $\vec{P} = Wm^{-2}$ or $\left|\vec{P}\right| = \frac{E_0B_0}{2\mu_0}$ power per unit area.

Energy flow density of electric dipole radiation in a far field zone varies $\frac{\sin^2\theta}{r^2}$ where r is the distance from the dipole, and θ is the angle between the radius vector r and axis of the dipole.

Radiation power of a dipole with dipole moment $p(t)$ and of a charge q moving with an acceleraton a is

$$P_{\text{radiation}} = \frac{1}{4\pi\varepsilon_0}\frac{2p^2}{3c^3} = \frac{1}{4\pi\varepsilon_0}\frac{2q^2a^2}{3c^3}$$

If h is the height of antenna then program can be received upto a radius of $r = \sqrt{2hR}$ where R is radius of the earth.

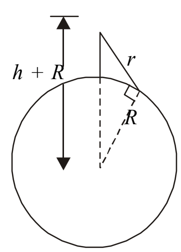

Fig. 28.1 **Radius upto which transmission can be received.**

SHORTCUTS AND POINTS TO NOTE

1. Electromagnetic waves are transverse waves produced due to variation of electric and magnetic fields held perpendicular to one another. Wave propagates perpendicular to both electric and magnetic fields.

   If $E_y = E_0\sin(\omega t - kx)$ varies in y– direction and $B_z = B_0\sin(\omega t - kx)$ varies in z–direction, then wave progresses in x– direction.

2. Maxwell equations in vacuum are

$$\oint E\cdot dS = \frac{Q}{\varepsilon_0};\quad \oint B\cdot dS = 0$$

$$\oint E\cdot dl = \frac{d\phi_{\text{magnetic}}}{dt};\quad \oint B\cdot dl = \mu_0\varepsilon_0\frac{d\phi_E}{dt} = \mu_0 i_d$$

   The electric field so generated is non-conservative. Note that in vacuum conduction current is zero. Therefore only displacement current remains.

3. Force experienced by electomagnetic wave

   $F = \frac{Power}{c}$ for a totally absorbing surface. $F = P_{\text{radiation}}A$ where $P_{\text{radiation}}$ is radiation pressure and A is area.

   $F = \frac{2Power}{c}$ for a perfectly reflecting surface.

4. Momentum $p = \frac{U}{c} = \frac{Energy}{c}$ for totally absorbing surface and $p = \frac{2U}{c}$ for totally reflecting surface

5. Average energy density

$$u = \frac{U_{\text{average}}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

6. Intensity $I = \frac{1}{2}\varepsilon_0 E_0^2 c$ and $E_0 = B_0 c$ and $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

   Note that refractive index $n = \sqrt{\mu_r\varepsilon_r}$ or $\varepsilon_r \propto n^2$

7. Phase velocity

$$v = \frac{c}{\sqrt{\mu_r\varepsilon_r}};\ c = f\lambda = \frac{\omega}{k} = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

8. Impedance of free space $z_0 = \frac{E}{H} = 377\Omega = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

9. In a wave guide $v_{\text{phase}} = \frac{c}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}$

$$v_{\text{group}} = c\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}$$

   Guide wavelength $\lambda g = \frac{v_{phase}}{f} = \frac{v_{phase}\lambda}{c} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}$

   Cut off wavelength $\lambda_c = 2a$

10. Poynting vector $\vec{P} = \frac{1}{\mu_0}\left[\vec{E}\times\vec{B}\right]$. It describes power flowing per unit area and power $= \oiint \vec{P}\cdot dA$

Intensity $I = \langle \vec{P} \rangle = \frac{E_0 B_0}{2\mu_0} = \frac{B_0^2 c}{2\mu_0} = \frac{E_0^2}{2c\mu_0}$. It describes average power flowing per sec or intensity.

11. Another form of Maxwell's equation (differential form)

$$\vec{\nabla}\times\vec{E} = \frac{\partial\vec{B}}{\partial t}\ ;\ \vec{\nabla}\cdot\vec{B} = 0\ ;\ \vec{\nabla}\times\vec{H} = j + \frac{\partial D}{\partial t}\ ;$$

$$\vec{\nabla}\cdot\vec{D} = \rho$$

12. Radius upto which transmission can be received from an antenna of height h $\quad r = \sqrt{2Rh}$

13. Doppler's effect in light $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$ or $\frac{\Delta f}{f} = \frac{v}{c}$

14. Relative velocity $v_{\text{rel}} = \dfrac{u_1 - u_2}{1 - \dfrac{u_1 u_2}{c^2}}$

15. Energy flow density of electric dipole radiation in a far field zone $\sim \frac{\sin^2\theta}{r^2}$

16. Radiation power of an electric dipole with momentum $p(t)$ and charge q moving with an acceleration a is

$$P_{\text{radiation}} = \frac{1}{4\pi\varepsilon_0}\frac{2\ddot{p}^2}{3c^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2q^2a^2}{3c^3}$$

17. Radiation Pressure

$P_{\text{radiation}} = \frac{I}{c}$ if surface is absorbing

$P_{\text{radiation}} = \frac{2I}{c}$ if surface is totally reflecting, where I is intensity.

18. In standing em wave, nodal planes occur at $x = 0$, $\frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$,..... for electric field ($E\infty$) and antinodal planes for magnetic field (B = max). Antinodal planes of electric field (E= max) occur at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ... and are nodal planes for magnetic field ($B = 0$)

Note that standing em waves are formed in wave guides or cavities.

19. Visible spectrum

400–440 nm violet	440–480 nm blue
480–560 nm green	560–590 nm yellow
590–630 m orange	630–700 nm red.

em wave spectrum

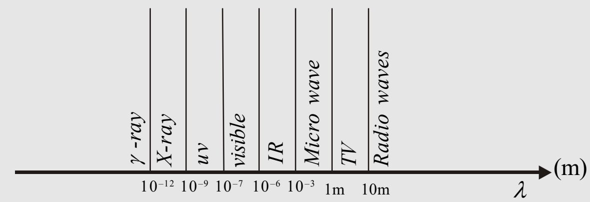

Fig. 28.2

CAUTION

1. Assuming that since rest mass is zero, Therefore em wave cannot have any momentum.

⇒ Momentam $= \frac{Energy}{c} = \frac{U}{c}$ for absorbing surface and $\frac{2U}{c}$ for reflecting surface.

2. Applying ordinary laws of relative velocity for photons or for particles moving with speed close to c.

⇒ Apply special theory of relativity in such cases.

3. Considering electromagnetic waves do not exert any force as they do not possess mass.

⇒ $F = \frac{Power}{c}$ for totally absorbing surface and $F = \frac{2Power}{c}$ for perfectly reflecting surface.

4. Assuming that even in a wave guide em wave travels with c.

⇒ In a wave guide they travel with group or phase velocities.

5. Not remembering the modification made by Maxwell in Ampere's Law

⇒ $\oint B.dl = \mu_0 i_C + \mu_0\varepsilon_0\frac{d\phi_E}{dt} = \mu_0 i_c + \mu_0 i_d$. The second term in the equation is a modification. It gives displacement current (in vaccum).

6. Confusing energy density and intensity.

⇒ Energy density is average energy per unit volume

$$u = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

while intensity is rate of flow of average energy per unit area. $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

7. Confusing Poynting vector with intensity.

⇒ Poynting vector gives instantaneous rate of flow of energy per unit area carried by em waves. Since the frequency of em waves is quite large, it is very difficult to notice the variation with time. We therefore define intenstiy. Intensity gives rate of flow of average energy per unit area.

SOLVED PROBLEMS

1. A cube of edge *a* has its edges parallel to x,y and z-axis of rectangular co-oridnate system. A uniform electric field E is parallel to y-axis and a unifrom magnetic field is parallel to x axis. The rate at which energy flows through each face of the cube is

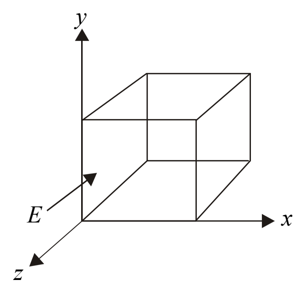

Fig. 28.3

(a) zero in all faces

(b) $\frac{a^2 EB}{2\mu_0}$ parallel to xy plane face and zero in others

(c) $\frac{a^2 EB}{\mu_0}$ parallel to xy plane face and zero in others

(d) $\frac{a^2 EB}{2\mu_0}$ all faces

Solution (c) $\vec{P} = \frac{1}{\mu_0}\left[\vec{E}\times\vec{B}\right]$ gives the clue that energy flowing per second $= \frac{a^2 EB}{\mu_0}$ in faces parallel to *xy* plane and zero in all others.

2. The amplitude of electric field in a parallel light beam of intensity 4 Wm^{-2} is

(a) 35.5 NC^{-1} (b) 45.5 NC^{-1}
(c) 49.5 NC^{-1} (d) 55.5 NC^{-1}

Solution (d) $I = \frac{1}{2}\varepsilon_0 E_0^2 c$ or $E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$

$$= \sqrt{\frac{2\times 4}{8.85\times 10^{-12}\times 3\times 10^{8}}}$$

3. **(A) Assertion:** The energy E and momentum *p* of a photon are related as $p = \frac{E}{c}$.

(R) Reason: The photon behaves like a particle.

(a) Both *A* and *R* are correct and *R* is correct explanation of *A*.

(b) Both *A* and *R* are correct but *R* is not correct explanation of *A*.

(c) *A* is correct but *R* is wrong.

(d) Neither is correct.

[AIIMS 2005]

Solution (a)

4. **(A)** TV signals are received through sky wave propagation

(R) Ionosphere reflects em waves of frequencies > a critical frequency

(a) A and R are correct and R explains A.

(b) A and R are correct but R does not explain A.

(c) A is correct but R is wrong

(d) Neither is correct.

[AIIMS 2005]

Solution (d)

5. If λ_v, λ_x, λ_m represent the wavelengths of vision, *x*-ray and microwaves respectively then

(a) $\lambda_m > \lambda_x > \lambda_v$ (b) $\lambda_m > \lambda_v > \lambda_x$
(c) $\lambda_v > \lambda_x > \lambda_m$ (d) $\lambda_v > \lambda_m > \lambda_x$

Solution (b)

6. Electrical conductivity of a semiconductor increases when em radiation of $\lambda < 2480$ nm is incident on it. The band gap in (ev) for the semiconductor is

(a) 1.1eV (b) 2.5 eV
(c) 0.7 eV (d) 0.5 eV

[AIEEE 2005]

Solution (d) $E_g = \frac{1240}{\lambda(nm)}$

$$= \frac{1240}{2480} = 0.5 \text{ eV.}$$

7. Infrared radiation was discovered in 1860 by

(a) William Wallaston (b) William Herschel
(c) William Roentgen (d) Thomas Young

[CET Karnataka 2005]

Solution (b)

8. A radio wave of frequency 90 MHz (FM) enters a ferrite rod. In $\varepsilon_r = 10^3$ and $\mu_r = 10$ then the velocity and wavelength of ferrite are

(a) 3×10^6 ms^1, 3.33×10^{-2} m

(b) 3×10^6 ms^{-1}, 3.33×10^{-1} m

(c) 3×10^6 ms^{-1}, 3.33×10^{-3} m

(d) none of these

Solution (a) $v_{\text{ferrite}} = \dfrac{c}{\sqrt{\varepsilon_r \mu_r}} = \dfrac{3\times10^8}{\sqrt{10^3\times10}} = 3 \times 10^6$ ms^1

$$\lambda_{\text{ferrite}} = \frac{v_{\text{ferrite}}}{f} = \frac{3\times10^6}{90\times10^6} = 3.33 \times 10^{-2}\text{m}$$

9. In a wave $E_0 = 100$ Vm^{-1}. Find the Poynting vector magnitude.

(a) 13.25 Wm^{-2} (b) 26.5 Wm^{-2}

(c) 18.25 Wm^{-2} (d) 19.7Wm^{-2}

Solution (b) $B = \dfrac{E}{c}$ $|\vec{p}| = \dfrac{EB}{\mu_0} = \dfrac{E^2}{c\mu_0}$

$$= \frac{10^4}{3\times10^8\times4\pi\times10^{-7}}$$

$= 26.5$ Wm^{-2}

10. A radio station on the surface of the earth radiates 50kW. If transmitter radiates equally in all directions above the surface of earth find the amplitude of electric field detected 100 km away.

(a) 2.45 Vm^{-1} (b) 2.45×10^{-1} Vm^{-1}

(c) 2.45×10^{-2} Vm^{-1} (d) 2.45×10^{-3} Vm^{-1}

Solution (c) $I = \dfrac{P}{2\pi r^2}$ because emission is hemispherical and $I = \dfrac{\varepsilon_0 E_0^2 c}{2} = \dfrac{P}{2\pi r^2}$

$$E_0 = \sqrt{\frac{P}{\varepsilon_0 c\pi r^2}} = \sqrt{\frac{50\times10^3}{8.85\times10^{-12}\times3\times10^8\times10^{10}}}$$

$= 2.45 \times 10^{-2}$ Vm^{-1}

11. Find the radiation pressure of solar radiation on the surface of earth. Solar constant is 1.4 kW m^{-2}

(a) 4.7×10^{-5} Pa (b) 4.7×10^{-6} Pa

(c) 2.37×10^{-6} Pa (d) 9.4×10^{-6} Pa

Solution (b) $P_{\text{rad}} = \dfrac{I}{c} = \dfrac{1.4\times10^3}{3\times10^8} = 4.7 \times 10^{-6}$ Pa

12. An earth-orbiting satellite has solar energy collecting panel with total area 5 m^2. If solar radiations are perpendicular and completely absorbed find the average force associated with the radiation pressure.

(a) 2.33×10^{-5} N (b) 2.33×10^{-6} N

(c) 2.33×10^{-4} N (d) 2.33×10^{-7} N

Solution (a) Power = I Area = $1.4 \times 10^3 \times 5$

and $F = \dfrac{\text{Power}}{c} = \dfrac{1.4\times10^3\times5}{3\times10^8} = 2.33 \times 10^{-5}$ N

13. Find the cut off wavelength in a waveguide of two parallel walls 1.5 cm apart. Where is $E = 0$ and where is E = maximum?

Solution $\lambda c = 2a = 2 \times 1.5 = 3$cm

$E = 0$ at walls as half wavelength points occur there at wall. $E = max$ in the middle of the walls as $\lambda/4$ planes occur there.

14. A radio transmitter transmits at 830 k Hz. At a certain distance from the transmitter magnetic field has amplitude 4.82×10^{-11}T. Find electric field and wavelength.

(a) 14.46×10^{-3} NC^{-1}, 35 m

(b) 14.46×10^{-3} NC^{-1}, 350 m

(c) 1.45×10^{-3} NC^{-1}, 35 m

(d) none of these

Solution (b) $E = Bc = 4.82 \times 10^{-11} \times 3 \times 10^8$

$= 14.46 \times 10^{-3}$ NC^{-1}

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{8.3\times10^5} = 3.5 \times 10^2 \text{ m.}$$

15. An intense light source radiates uniformly in all directions. At a distance 5 m from the source, the radiation pressure on absorbing surface is 9×10^{-6} Pa. Find the total average power output.

(a) 8.5×10^5 W (b) 6.5×10^5 W

(c) 8.5×10^3 W (c) 6.5 c$\times 10^3$ W

Solution (a) $p_{\text{rad}} = \dfrac{I}{c}$ and Power = $I\,4\pi r^2$

$= p_{\text{rad}} \times C\,4\pi r^2$

$= 9 \times 10^{-6} \times 3 \times 10^8 \times 12.56 \times 25$

$= 8.5 \times 10^5$ W

16. A bank of overhead arc lamps can produce a light intensity of 2500 Wm^{-2} in the 25 ft space stimulator facility at NASA. Find the average momentum density of a total aborbing surface.

(a) 8.33×10^{-6}kgm^{-2}s^{-1} (b) 8.33×10^{-14}kgm^{-2}s–^{1}

(c) 2.78 kgm^{-2}s^{-1} (d) 2.78×10^{-14}kgm^{-2}s^{-1}

Solution (d) $I = \dfrac{1}{2}\varepsilon_0 E_0^2 c$ and energy density

$= \frac{I}{c}$, momentum density $= \frac{I}{c^2}$

$(\because \frac{E}{c} = p)\ \frac{2500}{9 \times 10^{16}} = 2.78 \times 10^{-14}\ \text{kgm}^{-2}\text{s}^{-1}$

17. A standing em wave frequency 2.2×10^{10} Hz is produced in a certian material and nodal planes of magnetic field are 3.5 mm apart. Find wavelenth and speed of the wave in this material.

(a) $2.81 \times 10^8 \text{ms}^{-1}$ (b) $1.79 \times 10^8 \text{ms}^{-1}$
(c) $3.08 \times 10^8 \text{ms}^{-1}$ (d) $1.54 \times 10^8 \text{ms}^{-1}$

Solution (d) $\frac{\lambda}{2} = 3.5$ mm

$\lambda = 7.0$ mm

$v = f\lambda = 2.2 \times 10^{10} \times 0.7 \times 10^{-2}$

$= 1.54 \times 10^8 \text{ ms}^{-1}$

18. Which of the following rays is emitted by a human body?

(a) x-rays (b) visible rays
(c) uv rays (d) IR rays
(e) none of these

Solution (d) IR because temperature of the body is 37°C and it emits heat radiation which falls in IR and microwave region.

19. Which of the following waves is used in Raman spectroscopy?

(a) uv (b) x-rays
(c) γ-rays (d) IR

Solution (d)

20. Which of the following relations is correct for em waves

(a) $\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$ (b) $\frac{\partial^2 E}{\partial x^2} = \left(\frac{\partial E}{\partial t}\right)^2$

(c) $\frac{\partial^2 E}{\partial x^2} = \frac{c^2 \partial^2 E}{\partial t^2}$ (d) $\frac{\partial^2 E}{\partial t^2} = c\frac{\partial^2 E}{\partial t^2}$

Solution (c)

TYPICAL PROBLEMS

21. A radiation of 200 W is incident on a surface which is 60% reflecting and 40% absorbing.

(a) 1.3×10^{-6} N (b) 1.07×10^{-6} N
(c) 1.07×10^{-7} N (d) 1.3×10^{-7} N

Solution (b) $F_{\text{Tot}} = F_{\text{ref}} + F_{\text{abs}} = \frac{1.2p}{c} + \frac{.4p}{c} = \frac{1.6p}{c}$

$= \frac{1.6 \times 200}{3 \times 10^8} = 1.07 \times 10^{-6}$

22. The < $\vec{P}$ > for a standing wave is

(a) zero (b) $\frac{E_0 B_0}{2\mu_0}$

(c) $\frac{\varepsilon_0 E_0^2}{2}$ (d) $\frac{E_0 B_0}{\mu_0}$

Solution (a)

23. A proton with KE 6 MeV is travelling in a particle accelerator of circular orbit 0.75m. What fraction of energy does it radiate per sec?

Solution $r = \frac{mv}{qB} = \frac{\sqrt{2(KE)m}}{qB}$

or $qB = \frac{\sqrt{2(KE)m}}{r}$

and $F = qvB = \frac{\sqrt{(2KE)^2}}{r}$

$a = \frac{F}{m} = \frac{(2KE)}{mr}$

$$\frac{\frac{dE}{dt}}{6MeV} = \frac{\frac{-q^2 2a^z}{6\pi\varepsilon_0 c^3}}{6MeV} = \frac{\frac{(1.6\times10^{-19})^2(2KE)^2}{1.5\times m^2 r^2}\times 9\times10^9}{6MeV}$$

$$\frac{\frac{dE}{dt}}{6MeV} = \frac{(1.6\times10^{-19})^2\times6\times4\times6\times1.6\times10^{-13}\times9\times10^9}{(1.6\times10^{-27})^2\times\left(\frac{3}{4}\right)^2\times6\times1.5\times27\times10^{24}}$$

$$= \frac{4\times6\times1.6\times16\times10^{-12}}{1.5\times27} = 1.4\times10^{-11}\ \text{s}^{-1}$$

24. What amplitude of electric/magnetic field is required to be transmitted in a beam of cross-section area 100 m^2 so that it is comparable to electric power of 500 kV and 10^3 A?

Solution $\frac{1}{2}\ \varepsilon_0 E_0^2\, c\, A = 5 \times 10^8$ or $\frac{1}{2}\ \varepsilon_0 E_0^2$

$= \frac{5\times10^8}{3\times10^8\times100} = \frac{5}{3} \times 10^{-2}$

$$E_0 = \sqrt{\frac{10^{-1}}{3\times 8.85\times 10^{-2}}} = \frac{10^6}{16} = 6.125\times 10^4 \text{ N/C}$$

$$B_0 = \frac{E_0}{c} = \frac{6.125\times 10^4}{3\times 10^8} = 2.08\times 10^{-4}\text{ T}$$

25. The mean power radiated by a dipole is P_0. Find the mean space density of energy of em waves in vacuum in the far field zone at a point distant r along the dipole axis.

Solution $P_0 = S_0 \times 4\pi r^2 \times \frac{2}{3}$ ($\frac{2}{3}$ is the average value of $\sin^2\theta$ over the whole sphere).

Thus $I_0 = \frac{3p_0}{8\pi r^2}$ where I_0 is intensity.

Average energy density $= \frac{I_0}{c} = \frac{3P_0}{8\pi r^2 c}$

26. Find the mean radiation power of an electron performing SHM of amplitude 0.1 nm and frequency $\omega = 6.5\times 10^{14}\text{s}^{-1}$.

Solution $P = \frac{1}{4\pi\varepsilon_0}\,\frac{2\left(\ddot{\vec{P}}\right)^2}{3c^3}$ $|\ddot{p}|^2 = (e\omega^2 a)^2\cos^2\omega t$

$$\langle P\rangle = \frac{1}{4\pi\varepsilon_0}\times\frac{2}{3c^3}(e\omega^2 a)^2\times\frac{1}{2} = \frac{e^2\omega^4 a^2}{12\pi\varepsilon_0 c^3}$$

$$= \frac{\left(1.6\times 10^{-19}\right)^2\times\left(6.5\times 10^{14}\right)^4\left(10^{-10}\right)^2\times 9\times 10^9}{3\times\left(3\times 10^8\right)^3}$$

$$= \frac{2.56\times 1800\times 9\times 10^7\times 10^{-24}}{3\times 27} \approx 5.1\ 10^{-15}\text{ W}$$

27. A straight coaxial cable of negligible active resistance is receiving energy from a constant voltage source V. Current consumed is I. Find energy flux across the cross-section. Assume conductive sheath to be thin.

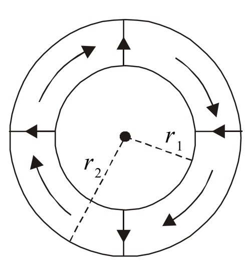

Fig. 28.4

Solution Let electric field from Gauss's law be $E_r = \frac{K}{r}$

and $V = \int_{r_2}^{r_1} -E_r dr = K\log_e\frac{r_2}{r_1}$

or $K = \frac{V}{\log_e\frac{r_2}{r_1}}$

and $E_r = \frac{V}{r\log_e\frac{r_2}{r_1}}$

$H_0 = \frac{I}{2\pi r}$ using Ampere's circuital law

Poynting vector $\vec{p}$ acts along z-axis and is non zero in the region ($r_1<r<r_2$). The total power flux is

$$= \int_{r_1}^{r_2}\frac{IV}{2\pi r^2\log\frac{r_2}{r_1}}2\pi r dr = IV$$

28. Consider that the space between the parallel plates of a capacitor has vacuum. The places are connected to a battery of emf and internal resistance R at $t = 0$. If V is the potential difference between the plates, I_d is displacement current and we define $R_d = \frac{V}{i_d}$ then show that $R_d = R\,(e^{t/Rc} - 1)$.

Solution $R = \frac{V}{i_d} = \frac{V_0\left(1-e^{-t/Rc}\right)}{\frac{Q_0}{RC}\left(e^{-t/Rc}\right)} = R\,(e^{t/Rc} - 1)$ using

$$Q = Q_0(1 - e^{t/Rc})$$

$$\frac{dQ}{dt} = i = \frac{Q_0}{RC}\,e^{t/Rc}$$

$$i_d = i_c = \frac{Q_0}{RC}\,e^{t/Rc}$$

$$i_d = \frac{V_0}{R}\,e^{-t/Rc} \because \frac{Q_0}{C} = V_0$$

PASSAGE 1

Read the following passage and answer the questions given at the end.

Magnetic fields within a sun spot can be as strong as $0.4T$ which is 10^4 times the magnetic field of the earth. Large sun spots could be as large as 25000 km in radius. The material in the sun spot has density 3×10^{-4} kgm^{-3}. If 100% of magnetic energy stored in the sun spot is utilised to eject sun spot material from the sun's surface then,

1. What will be the speed of the sun spot material?

(a) 2×10^4 ms^{-1} (b) 6×10^4 ms^{-1}

(c) 6×10^5 ms^{-1} (d) none of these.

2. Will the sun spot material leave the sun?

(a) Yes (b) No

(c) Data not available to answer.

Solution 29. (a) $\frac{B^2}{2\mu_0} = \frac{1}{2}\rho v^2$ comparing energy density.

$$v = \sqrt{\frac{B^2}{\mu_0 \rho}} = \sqrt{\frac{.4\times.4}{4\pi\times10^{-7}\times3\times10^{-4}}}$$

$$= 2\times10^4 \text{ ms}^{-1}$$

Solution 30. (b) $\because\ v < v_{escape}$ $(6\times10^5 \text{ ms}^{-1})$ of the sun.

PASSAGE 2

Read the following passage and answer the questions given at the end.

Mr. X is the sole crew member of the interplanetary spaceship T: 1339 Vorga which makes regular cargo runs between the earth and the mining colonies in the asteroid belt. He is working outside the ship. One day while at a distance of 2 AU from the sun [$1AU =$ distance between the sun and the earth $= 1.5\times10^{11}$ m] unfortunately Mr. X loses contact with the ship's hull and begins to drift away into space. He uses his space-suit's rockets to try to push himself back towards the ship, but they run out of the fuel and stop working before he can return to the ship. He finds himself in the awkward position of floating 16 m from the space ship with zero velocity relative to the spaceship. Fortunately he carries a 200W flash light. He turns flash light on and uses its beam as a light rocket to push himself back towards the spaceship.

1. Mr. X and his space suit and accessaries have a combined mass of 150 kg. How long will it take him to reach space ship?

(a) 23. 6 h (b) 2.36 h

(c) 2.36 min (d) 23.6 min

2. Is there another way he could reach the spaceship?

(a) No

(b) Yes, he should throw the flash light in opposite direction to the space ship

(c) Yes, he should swim

(d) Send flash light signal to the earth

Solution 1. (a) $a = \frac{F}{M} = \frac{P/c}{M} = \frac{P}{Mc}$

$$= \frac{200}{150\times3\times10^8} = \frac{2\times10^{-8}}{4.5}$$

$$16 = \frac{1}{2}at^2 \text{ or } t = \sqrt{\frac{2\times16}{a}}$$

$$= \sqrt{\frac{2\times16\times4.5\times10^8}{2}} = 6\sqrt{2}\times10^4_5$$

$$= \frac{6\times1.414\times10^4}{3600} = 23.6 \text{ h}$$

Solution 32. (b)

PASSAGE 3

Read the following passage and answer the questions given at the end.

In 19th century inventor Nikola Tesla proposed to transmit electric power via sinusoidal electromagnetic waves. Suppose power is to be transmitted in a beam of cross-sectional area 100 m^2. The power to be transmitted is comparable to the one carried by modern transmission lines i.e., 500 kV voltage and 1000 A current.

1. Find the magnitude of electric field.

(a) 6.1×10^4 Nc^{-1} (b) 3.2×10^4 Nc^{-1}

(c) 5.7×10^3 Nc^{-1} (d) 7×10^4 Nc^{-1}

2. What is the magnitude of magnetic field?

(a) $2.34\times10^{-4}\,T$ (b) $2.84\times10^{-4}\,T$

(c) $3.2\times10^{-4}\,T$ (d) $2.03\times10^{-4}\,T$

Solution 1. (a) $\frac{1}{2}\varepsilon_0 E_0^2 cA = P$

or $500\times10^3\times10^3 = \frac{1}{2}\times8.85\times10^{-12}\,E_0^2\times3\times10^8\times100$

$$E_0^2 = \frac{10^{11}}{26.55} = 3.7\times10^9$$

or $E = 6.1\times10^4$ N/C

Solution 2. (d) $B = \frac{E_0}{c} = \frac{6.1\times10^4}{3\times10^8} = 2.034\times10^{-4}\,T$

PASSAGE 4

Read the following passage and answer the questions given at the end.

Electromagnetic radiation is emitted by accelerating charges. The rate at which energy is emitted from an accelerating charge that has charge q and acceleration a is given by $\frac{dE}{dt} = \frac{q^2a^2}{6\pi\varepsilon_0 c^3}$ where c is speed of light. A proton and an electron of kinetic energy 6 Mev is travelling in a particle accelarator in a circular orbit of radius 0.75 m.

1. What fraction of its energy does a proton emit per second?

(a) 1.1×10^{-11} s^{-1} (b) 2.1×10^{-11} s^{-1}

(c) $\sqrt{3}\times10^{-11}$ s^{-1} (d) 1.4×10^{-11} s^{-1}

2. What fraction of its energy does an electron radiate per sec?

(a) $2.8 \times 10^{-8}\ s^{-1}$ (b) $3.2 \times 10^{-8}\ s^{-1}$

(c) $6 \times 10\ s^{-1}$ (d) none of these

Solution 1. (d) $\dfrac{d\varepsilon}{dt} = \dfrac{q^2\left(\dfrac{v^2}{r}\right)^2}{6\pi\varepsilon_0 C^3}$

$$\Rightarrow \frac{\frac{d\varepsilon}{dt}}{\varepsilon} = \frac{q^2v^2 \times 2}{6 \times 4\pi\varepsilon_0 r^2 C^3 m^2}$$

$$= \frac{\left(1.6\times10^{-19}\right)^2 \times 6\times10^{-13} \times 1.6\times2\times9\times10^9\times16}{6\times9\times27\times10^{24}\times1.6^2\times10^{-54}}$$

$$= 1.4 \times 10^{-11}\ s^{-1}$$

Solution 2. (a) $m_0c^2 + KE = m^*c^2$

$$.5 + 6 = (.5)\frac{m^*}{m_0}$$

$$m^* = 13\,m_0$$

$$13m_0 = \frac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

$$\frac{v^2}{c^2} = \frac{168}{169}$$

$$v = .99\,c$$

$$\frac{\frac{d\varepsilon}{dt}}{\varepsilon} = \frac{\left(6\times10^{-19}\right)^2\times6\times10^{-13}\times1.6\times2\times9\times10^4\times16}{6\times9\times27\times10^{24}\times9^2\times10^{-12}}$$

$$= 2.8 \times 10^{-8}\ s^{-1}.$$

QUESTIONS FOR PRACTICE

1. An electric field $\vec{E}$ and a magnetic field $\vec{B}$ exist in a region. The fields are not perpendicular to each other.
   (a) This is not possible.
   (b) No electromagnetic wave is passing through the region.
   (c) An electromagnetic wave may be passing through the region.
   (d) An electromagnetic wave is certainly passing through the region.
2. Consider the following two statements regarding a linearly polarised, plane electromagnetic wave:
   (A) The electric field and the magnetic field have equal average values.
   (B) The electric energy and the magnetic energy have equal average values.
   (a) Both A and B are true.
   (b) A is false but B is true.
   (c) B is false but A is true.
   (d) Both A and B are false.
3. A free electron is placed in the path of a plane electromagnetic wave. The electron will start moving
   (a) along the electric field
   (b) along the magnetic field
   (c) along the direction of propagation of the wave
   (d) in a plane containing the magnetic field and the direction of propagation
4. A plane electromagnetic wave is incident on a material surface. The wave delivers momentum p and energy E.
   (a) $p = 0, E \neq 0.$ (b) $p \neq 0, E = 0.$
   (c) $p \neq 0, E \neq 0.$ (d) $p = 0, E = 0.$
5. An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$.
   Which of the following is/are independent of the wavelength ?
   (a) k (b) ω
   (c) $\dfrac{\omega}{k}$ (d) $k\omega$.
6. Displacement current goes through the gap between the plates of a capacitor when the charge of the capacitor
   (a) increases (b) decreases
   (c) does not change (d) is zero
7. Speed of electromagenetic waves is the same
   (a) for all wavelengths (b) in all media
   (c) for all intensities (d) for all frequencies
8. Which of the following have zero average value in a plane electromagnetic wave ?
   (a) electric field (b) magnetic field

(c) electric energy (d) magnetic energy

9. The energy contained in a small volume through which an electromagnetic wave is passing oscillates with

(a) zero frequency

(b) the frequency of the wave

(c) half the frequency of the wave

(d) double the frequency of the wave

10. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic field vectors of electromagnetic waves then the direction of propagation of electromagnetic wave is along the direction of

(a) $\vec{E}$ (b) $\vec{B}$

(c) $\vec{E} \times \vec{B}$ (d) none of these

11. The charge on a parallel plate capacitor is varying as $q = q_0 \sin 2\eth nt$. The plates are very large and close together. Neglecting the edge effects, the displacement current through the capacitor is

(a) $\frac{q}{\varepsilon_0 A}$ (b) $\frac{q_0}{\varepsilon_0} \sin 2p\, nt$

(c) $2\, p\, nq_0 \cos 2p\, nt$ (d) $\frac{2\pi nq_0}{\varepsilon_0} \cos 2p\, nt$

12. The value of magnetic field between plates of capacitor, at distance of 1 m from centre where electric field varies by 10^3 V/m/s will be

(a) 5.56 T (b) 5.56 μT

(c) 5.56 mT (d) 55.6 nT

13. Electromagnetic waves do not transport

(a) energy (b) charge

(c) momentum (d) information

14. A capacitor is connected in an electric circuit. When key is pressed, the current in the circuit is

(a) zero (b) maximum

(c) any transient value

(d) depends on capacitor used

15. Displacement current is continuous

(a) when electric field is changing in the circuit

(b) when magnetic field is changing in the circuit

(c) in both types of fields

(d) through wires and resistance only

16. Instantaneous displacement current $1A$ in the space between the parallel plates of $1\ \mu F$ capacitor can be established by changing the potential difference at the rate of

(a) 0.1 V/s (b) 1 V/s

(c) 106 V/s (c) 10^{-6} V/s

17. The magnetic field between the plates of a capacitor when $r > R$ is given by

(a) $\frac{\mu 0 I_D r}{2\pi R^2}$ (b) $\frac{\mu 0 I_D}{2\pi R}$

(c) $\frac{\mu 0 I_D}{2\pi r}$ (d) zero

18. The magnetic field between the plates of a capacitor is $u_0 Ir$ given by $B = \frac{\mu_0 Ir}{2\pi R^2}$ when

(a) $r > R$ (b) $r < R$

(c) $r < R$ (d) $r = R$

19. The conduction current is the same as displacement current when the source is

(a) AC only (b) DC only

(c) both AC and DC

(d) neither for AC nor for DC

20. The wave function (in SI units) for an electromagnetic wave is given as

$\psi(x, t) = 10^3 \sin p\,(3 \times 10^6 x - 9 \times 10^{14} t)$. The speed of the wave is

(a) 9×10^{14} m/s (b) 3×10^8 m/s

(c) 3×10^6 m/s (d) 3×10^7 m/s

21. In the above problem, wavelength of the wave is

(a) 666 nm (b) 666 ?

(c) 666 μm (d) 6.66 nm

22. Maxwell's four equations are written as

(i) $\oint \vec{E}.\vec{ds} = \frac{q_0}{\varepsilon_0}$

(ii) $\oint \vec{B}.\vec{ds} = 0$

(iii) $\oint \vec{E}.\vec{dl} = \frac{d}{dt} \oint \vec{B}.\vec{ds}$

(iv) $\oint \vec{B}.\vec{ds} = \mu_0 \varepsilon_0 \frac{d}{dt} \oint \vec{E}.\vec{ds}$

The equations which have sources of field are

(a) (i), (ii), (iii) (b) (i), (ii)

(c) (i) and (iii) only (d) (i) and (iv) only

23. Out of the above four equations, the equations which do not contain source field are

(a) (i) and (ii) (b) (ii) only

(c) all of four (d) (iii) only

24. Out of the four Maxwell's equations above, which one shows non-existence of monopoles ?

(a) (i) and (iv) (b) (ii) only

(c) (iii) only (d) none of these

25. Which of the above Maxwell's equations shows that electric field lines do not form closed loops ?

(a) (i) only (b) (ii) only

(c) (iii) only (d) (iv) only

26. In an electromagnetic wave the average energy density is associated with

(a) electric field only

(b) magnetic field only

(c) equally with electric and magnetic fields

(d) average energy density zero

27. In an electromagnetic wave the average energy density associated with magnetic field will be

(a) $\frac{1}{2}LI^2$ (b) $\frac{B^2}{2\mu_0}$

(c) $\frac{1}{2}\mu_0 B^2$ (d) $\frac{1}{2}\frac{\mu_0}{B^2}$

28. In the above problem, the energy density associated with the electric field will be

(a) $\frac{1}{2}CV^2$ (b) $\frac{1}{2}\frac{q^2}{C}$

(c) $\frac{1}{2}\frac{\varepsilon^2}{E}$ (d) $\frac{1}{2}\varepsilon_0 E^2$

29. If there were no atmosphere, the average temperature on earth surface would be

(a) lower (b) higher

(c) same (d) 0°C

30. In which part of earth's atmosphere is the ozone layer present?

(a) Troposphere (b) Stratosphere

(c) Ionosphere (d) Mesosphere

31. Kenneley's Heaviside layer lies between

(a) 50 Km to 80 Km (b) 80 Km to 400 Km

(c) beyond 110 Km (d) beyond 250 Km

32. The ozone layer in earth's atmosphere is crucial for human survival because it

(a) has ions

(b) reflects radio signals

(c) reflects ultraviolet rays

(d) reflects infrared rays

33. The frequency from 3×10^9 Hz to 3×10^{10} Hz is

(a) high frequency band

(b) super high frequency band

(c) ultra high frequency band

(d) very high frequency band

34. The frequency from 3 to 30 MHz is known as

(a) audio band

(b) medium frequency band

(c) very high frequency band

(d) high frequency band

35. The AM range of radiowaves has frequency

(a) less than 30 MHz (b) more than 30 MHz

(c) less than 20000 Hz (d) more than 20000 Hz

36. The displacement current flows in the dielectric of a capacitor when the potential difference across its plates

(a) becomes zero

(b) has assumed a constant value

(c) is increasing with time

(d) is decreasing with time

37. Select wrong statement from the following: Electromagnetic waves

(a) are transverse

(b) travel with same speed in all media

(c) travel with the speed of light

(d) are produced by accelerating charge

38. The waves related to tele-communication are

(a) infrared (b) visible light

(c) microwaves (d) ultraviolet rays

39. Electromagnetic waves do not transport

(a) energy (b) charge

(c) momentum (d) information

40. The nature of electromagnetic wave is

(a) longitudinal (b) longitudinal stationary

(c) transverse (d) transverse stationary

41. Greenhouse effect keeps the earth surface

(a) cold at night (b) dusty and cold

(c) warm at night (d) moist

42. A parallel plate capacitor consists of two circular plates each of radius 12 cm and separated by 5.0 mm. The capacitor is being charged by an external source. The charging current is constant and is equal to 0.15 *A*. The rate of change of potential difference between the plates will be

(a) 8.173×10^7 V/s (b) 7.817×10^8 V/s
(c) 1.873×10^9 V/s (d) 3.781×10^{10} V/s

43. In the above problem, the displacement current is
(a) 15 A (b) 1.5 A
(c) 0.15 A (d) 0.015 A

44. The wave emitted by any atom or molecule must have some finite total length which is known as the coherence length. For sodium light, this length is 2.4 cm. The number of oscillations in this length will be
(Given: $\lambda = 5900$)
(a) 4.068×10^5 (b) 4.068×10^6
(c) 4.068×10^7 (d) 4.068×10^8

45. In the above problem, the coherence time will be
(a) 8×10^{-8} s (b) 8×10^{-9} s
(c) 8×10^{-10} s (d) 8×10^{-11} s

46. A parallel plate capacitor made of circular plates each of radius R = 6 cm has capacitance C = 100 pF. The capacitance is connected to a 230 V AC supply with an angular frequency of 300 rad/s. The *rms* value of conduction current will be
(a) 5.7 μA (b) 6.3 μA
(c) 9.6 μA (d) 6.9 μA

47. In the above problem, the displacement current will be
(a) 6.9 μA (b) 9.6 μA
(c) 6.3 μA (d) 5.7 μA

48. In Q. 46, the value of B at a point 3 cm from the axis between the plates will be
(a) 1.63×10^{-8} T (b) 1.63×10^{-9} T
(c) 1.63×10^{-10} T (d) 1.63×10^{-11} T

49. A plane electromagnetic wave of frequency 40 MHz travels in free space in the X-direction. At some point and at some instant, the electric field has its maximum value of 750 N/C in Y-direction. The wavelength of the wave is
(a) 3.5 m (b) 5.5 m
(c) 7.5 m (d) 9.5 m

50. In the above problem, the period of the wave will be
(a) 2.5 μs (b) 0.25 μs
(c) 0.025 μs (d) none of these

51. In Q. 49, the magnitude and direction of magnetic field will be
(a) 2.5 μT in X-direction (b) 2.5 μT in Y-direction
(c) 2.5 μT in Z-direction (d) none of these

52. In Q. 49, the angular frequency of emf wave will be (in rad/s)
(a) $8p \times 10^7$ (b) $4p \times 10^6$
(c) $2p \times 10^5$ (d) $\pi \times 10^4$

53. In Q. 49, the propagation constant of the wave will be
(a) 8.38 m^{-1} (b) 0.838 m^{-1}
(c) 4.19 m^{-1} (d) 0.419 m^{-1}

54. The sun delivers 10^3 W/m^2 of electromagnetic flux to the earth's surface. The total power that is incident on a roof of dimensions 8 m × 20 m, will be
(a) 6.4×10^3 W (b) 3.4×10^4 W
(c) 1.6×10^5 W (d) none of these

55. In the above problem, the radiation force on the roof will be
(a) 3.33×10^{-5} N (b) 5.33×10^{-4} N
(c) 7.33×10^{-3} N (d) 9.33×10^{-2} N

56. In Q. 54, the solar energy incident on the roof in 1 hour will be—
(a) 5.76×10^8 J (b) 5.76×10^7 J
(c) 5.76×10^6 J (d) 5.76×10^5 J

57. The sun radiates electromagnetic energy at the rate of 3.9×10^{26} W. Its radius is 6.96×10^8 m. The intensity of sunlight at the solar surface will be (in W/m^2)
(a) 1.4×10^4 (b) 2.8×10^5
(c) 4.2×10^6 (d) 5.6×10^7

58. In the above problem, if the distance from the sun to the earth is 1.5×10^{11} m, then the intensity of sunlight on earth's surface will be (in W/m^2)
(a) 1.38×10^3 (b) 2.76×10^4
(c) 5.52×10^5 (d) none of these

59. A laser beam can be focussed on an area equal to the square of its wavelength. A He-Ne laser radiates energy at the rate of 1mW and its wavelength is 632.8 nm. The intensity of focussed beam will be
(a) 1.5×10^{13} W/m^2 (b) 2.5×10^9 W/m^2
(c) 3.5×10^{17} W/m^2 (d) none of these

60. A flood light is covered with a filter that transmits red light. The electric field of the emerging beam is represented by a sinusoidal plane wave E_x = 36 sin ($1.20 \times 10^7 z + 6 \times 10^{15} t$) V/m. The average intensity of the beam will be
(a) 0.86 W/m^2 (b) 1.72 W/m^2
(c) 3.44 W/m^2 (d) 6.88 W/m^2

61. An electric field of 300 V/m is confined to a circular area 10 cm in diameter. If the field is increasing at the rate of 20 V/m/s, the magnitude of magnetic field at a point 15 cm from the centre of the circle will be

(a) $1.85 \times 10^{-15} T$ (b) $1.85 \times 10^{-16} T$

(c) $1.85 \times 10^{-17} T$ (d) $1.85 \times 10^{-18} T$

62. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 10 m from the lamp will be

(a) 1.34 V/m (b) 2.68 V/m

(c) 5.36 V/m (d) 9.37 V/m

63. A plane electromagnetic wave of wave intensity 6W/m^2 strikes a small mirror of area 40 cm , held perpendicular to the approaching wave. The momentum transferred by the wave to the mirror each second will be

(a) 6.4×10^{-7} kg/m/s (b) 4.8×10^{-8} kg/m/s

(c) 3.2×10 kg/m/s (d) 1.6×10^{-10} kg/m/s

64. In the above problem, the radiation force on the mirror will be

(a) 6.4×10^{-7} N (b) 4.8×10^{-8} N

(c) 3.2×10^{-9} N (d) 1.6×10^{-10} N

Answers to Questions for Practice

1.	(c)	**2.**	(a)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(a, b)	**7.**	(c)
8.	(a, b)	**9.**	(d)	**10.**	(c)	**11.**	(c)	**12.**	(d)	**13.**	(b)	**14.**	(b)
15.	(a)	**16.**	(c)	**17.**	(c)	**18.**	(c)	**19.**	(b)	**20.**	(b)	**21.**	(a)
22.	(d)	**23.**	(b)	**24.**	(b)	**25.**	(a)	**26.**	(c)	**27.**	(b)	**28.**	(d)
29.	(a)	**30.**	(b)	**31.**	(c)	**32.**	(c)	**33.**	(b)	**34.**	(b)	**35.**	(a)
36.	(c)	**37.**	(b)	**38.**	(c)	**39.**	(a)	**40.**	(c)	**41.**	(c)	**42.**	(c)
43.	(c)	**44.**	(b)	**45.**	(d)	**46.**	(b)	**47.**	(a)	**48.**	(d)	**49.**	(c)
50.	(c)	**51.**	(c)	**52.**	(a)	**53.**	(b)	**54.**	(c)	**55.**	(b)	**56.**	(a)
57.	(d)	**58.**	(a)	**59.**	(b)	**60.**	(b)	**61.**	(d)	**62.**	(a)	**63.**	(d)
64.	(d)												

EXPLANATIONS

12 (d) $\oint P.\,dl = \varepsilon_0 \dfrac{d\phi_E}{dt}$ or

$$B = 8.85 \times 10^{-12} \times 10^3 \times 2\pi$$
$$= 55.6\,\eta T$$