

Electromagnetic Induction

BRIEF REVIEW

In 1830's, Michael Faraday showed that if a magnet is moved in or out of a coil then emf is induced across the coil. If the circuit is complete a current is induced such a current is called **Induced current** and the corresponding emf is called induced emf. Faraday formulated two laws.

First Law The emf/current is induced as long as there is a variation of magnetic flux.

Second Law emf induced $\varepsilon = -\frac{d\phi_s}{dt}$ where flux

$$\phi_s = \int \vec{B} \cdot d\vec{s}. \text{ Unit of flux is weber or Tm}^2.$$

The current in the loop $= \frac{\varepsilon}{R}$ where R is resistance of the loop.

Lenz's Law The direction of induced current is such that it opposes the change that has induced it. Thus $\varepsilon = \frac{d\phi_B}{dt}$.

The law is based on conservation of energy.

The emf may be induced in two different basic processes: (a) motional emf and (b) induced electric field. In motional emf coil or conductor is varied with time but magnetic field remains fixed. In induced electric field coil remains fixed and magnetic field varies with time. There could be combination of the two also.

emf $\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$ Note that to have an induced electric field the presence of conducting loop is not necessary. As long as \vec{B} keeps varying, the induced electric field is present. If the loop is present free electrons start drifting and induced current results.

Note that $\oint \vec{E} \cdot d\vec{l} \neq 0$, Therefore electric field so generated is **nonconservative** and is different from electric field studied in electrostatics. Such an electric field is called **non electrostatic field**. The electric field lines so generated make closed loop like magnetic field lines. Also note that, however, like electrostatic field it gives force $\vec{F} = q\vec{E}$. The current so generated has a similarity to displacement current.

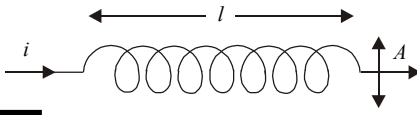
$$\therefore \oint \vec{E} \cdot d\vec{l} = E(2\pi r). \text{ Thus } E = \frac{1}{2\pi r} \left| \frac{d\phi}{dt} \right|$$

Self induction $\phi_B \propto i$ or $\phi_B = Li$ or $\varepsilon = -\frac{d\phi_B}{dt} = -L \frac{di}{dt}$.

If a coil has n turns, the flux through each turn is $\int \vec{B} \cdot d\vec{s}$. If

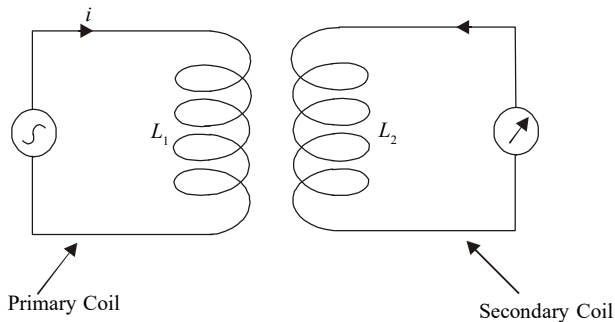
this flux varies then $\varepsilon = -N \frac{d}{dt} \int \vec{B} \cdot d\vec{s}$

$L = \mu_0 n^2 Al = \frac{\mu_0 N^2 A}{l}$ where n is number of turns per unit length and N total number of turns, l length of the coil and A its area of cross section as shown in Fig. 26.1.

**Fig. 26.1** Self inductance illustration

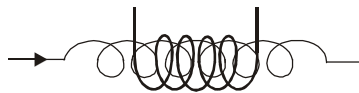
$L = \mu_r \mu_0 n^2 Al$ if a core of relative permeability μ_r is introduced. A coil or a solenoid of thick wire having negligible resistance may be considered as an **ideal inductor**. Unit of self induction is Henry (H).

Mutual induction $\mathcal{E} = -\frac{d\phi_B}{dt} = -M \frac{di}{dt}$. If two coils are placed close to each other and time varying current is passed through one (primary coil) then current is induced in the other such a phenomenon is called mutual induction. M is mutual inductance of two coils having self inductance L_1 and L_2 (as illustrated in Fig. 26.2),

**Fig. 26.2** Mutual inductance

then $M = k \sqrt{L_1 L_2}$ where k is coupling factor and $k \leq 1$. $k = 1$ if coils are wound one over the other.

If N_1 are number of turns in primary coil and N_2 are total number of turns in secondary, then in Fig. 26.3

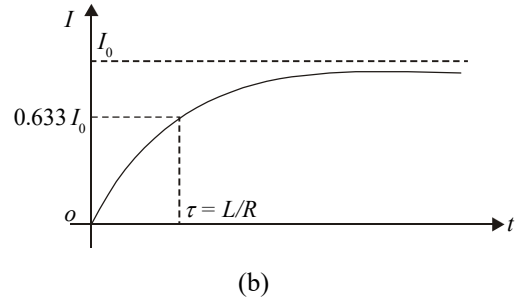
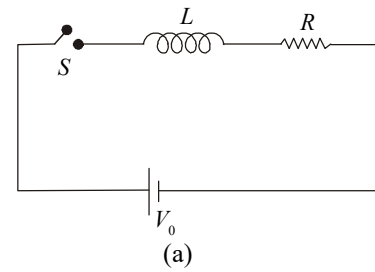
**Fig. 26.3** Mutual inductance

$M = \mu_0 n_1 N_2 A = \mu_0 n_1 n_2 Al$ where l is length of secondary coil. If a core of relative permeability μ_r is introduced then $M = \mu_0 \mu_r n_1 n_2 Al$. Here $n_2 = \frac{N_2}{l}$

Energy stored in an inductor $U = \frac{Li^2}{2}$ and energy is in the form of magnetic energy.

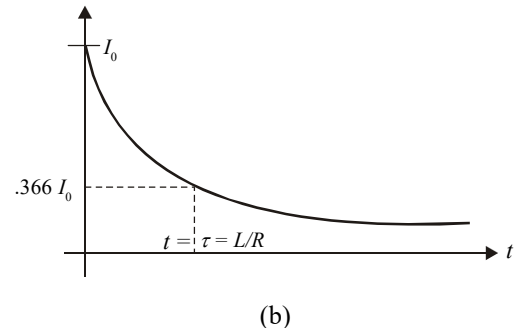
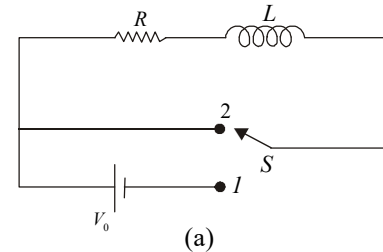
Growth of current in an R-L circuit $I(t) = I_0 [1 - e^{-t/\tau}]$ where $\tau = \frac{L}{R}$ is the time constant of the circuit $I_0 = \frac{V_0}{R}$.

Time constant $\tau = \frac{L}{R}$ is the time in which the current rises to 63.3% of maximum current I_0 as illustrated in Fig. 26.4

**Fig. 26.4** Growth of current in an inductor

Decay transient After a long time at $t = 0$ the switch S is shifted from position 1 to position 2. Then $I(t) = I_0 e^{-t/\tau}$

In one time constant current decays to 36.6% of I_0 (maximum current).

**Fig. 26.5** Decay of current in an inductor

Energy density $= \frac{B^2}{2\mu_0}$ is the magnetic energy per unit volume.

Eddy current Assume a solid plate of metal entering a magnetic field. Consider a loop drawn on the plate, a part of which is in the magnetic field as shown in Fig. 26.6 (a). As the plate moves the magnetic flux through the area bounded by the loop changes and hence a current is induced. There may be number of such loops on the plate and hence currents are induced in **random directions**. Such currents are called **eddy currents**. Note that we do not have a definite conducting

loop to guide the induced current. Because of eddy currents in the metal plate, thermal energy is produced. This energy comes at the cost of KE of the plate, i.e., plates slow down.

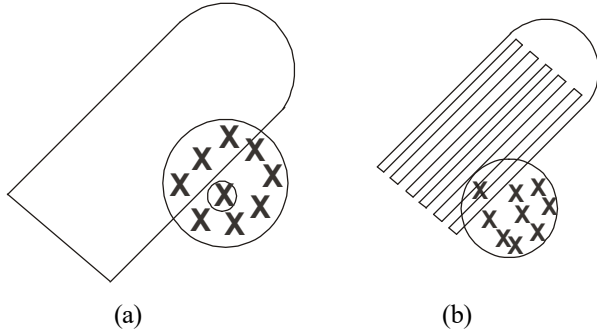


Fig. 26.6 Eddy current illustration

This effect is called **electromagnetic damping**. To reduce electromagnetic damping one can cut slots in the plate. This reduces the possible paths of the eddy current considerably.

AC generator $emf \varepsilon = N\omega B A_0 \sin \alpha$ where N is number of turns and A_0 is maximum area and ω is angular frequency. Note $V_p = N\omega B A_0$ is peak voltage. In AC generator slip rings are used.

In DC generator the scheme is same, however, in place of slip rings, split rings are used so that after each half cycle the direction of emf reverses as illustrated in Fig. 26.7 (b).

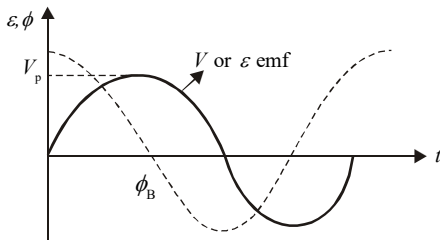


Fig. 26.7(a) Magnetic flux ϕ and voltage V in a AC generator

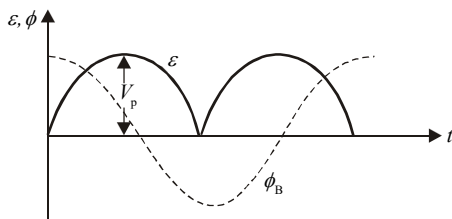


Fig. 26.7(b) Magnetic flux ϕ and voltage V in a DC

Displacement current $i_d = \frac{Ed\phi_E}{dt} = -\frac{1}{R} \frac{d\phi_B}{dt}$

Slide wire generator See Fig. 26.8. Let R be the resistance of circuit (slide wire + U shaped conductor).

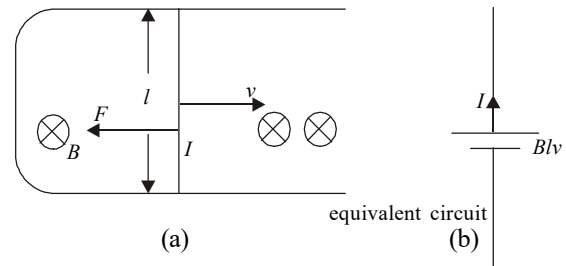


Fig. 26.8 Slide wire generator

$$emf \varepsilon = -Blv$$

$$\text{Current } |I| = \frac{Blv}{R}$$

$$\text{Power dissipated } P = I^2 R = \frac{B^2 l^2 v^2}{R}$$

$$F = IlB = \frac{B^2 l^2 v}{R} \text{ We may also write power } P = F \cdot v = \frac{B^2 l^2 v^2}{R}$$

Inductances are added in series or parallel like resistances i.e. $L_{\text{series}} = L_1 + L_2 + \dots$

$$\frac{1}{L_{\text{parallel}}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots$$

Hall Effect If i is the current in a strip of metal or semiconductor in the direction shown in Fig. 26.9 and B is the magnetic field then a Hall emf is developed in the transverse direction xy . The sign of emf will decide the nature of charge (positive or negative)

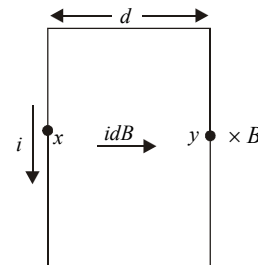


Fig. 26.9 Hall effect illustration

$$E_H = \frac{V_{xy}}{d}, E_H = -v_d \times B; E_H = \frac{JB}{ne}$$

$$\therefore v_d = \frac{J}{ne}$$

Poles of a coil can be found. If the current is clockwise the pole will be south, If the current is anti-clockwise, its pole will be north as shown in Fig. 26.10.

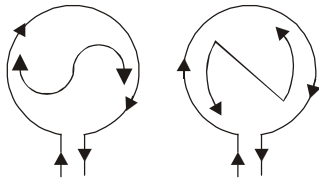


Fig. 26.10 Generator of magnetic pole

SHORT CUTS AND POINTS TO NOTE

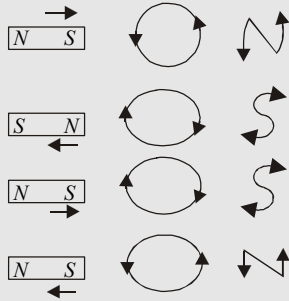


Fig. 26.11 Induced current in different cases

1. Time varying current or *emf* or voltage is *AC*. *AC* voltage and currents are phasors. Phasors are added like vectors. Therefore apply vector laws.
2. Note the directions of currents generated in the coil ring and magnetic pole when magnet moves in or out as shown in Fig. 26.11
3. If magnetic field changes with time and distance

then the *emf* generated $= A \left[\frac{\partial B}{\partial t} + v \frac{\partial B}{\partial z} \right]$ where *A* is area and *v* is velocity.

4. When a rod conducting / non conducting of length *l* moves in a uniform magnetic field *emf* generated is *Blv* if *B*, *l* and *v* are mutually perpendicular (See Fig. 26.12)

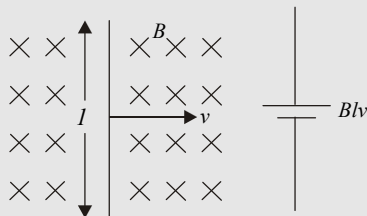


Fig. 26.12 Motional *emf*

If the velocity vector makes an angle θ with length or with magnetic field then *emf* induced $= Blv \sin \theta$ as illustrated in Fig. 26.13

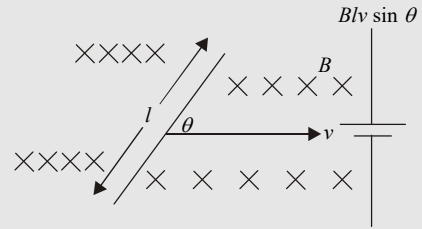


Fig. 26.13 Motional *emf*

If rod is conducting and a loop is made with conducting wire then current will also be induced and the direction of current will be given by Fleming's Right hand Rule.

Note: *emf* can be generated in conducting or non conducting rod. For current to be induced conductor is a must and loop be completed.

5. If a rod *OA* clamped at *O* is rotated about *O* with an angular velocity ω in a uniform magnetic field of strength *B* then *emf* induced is

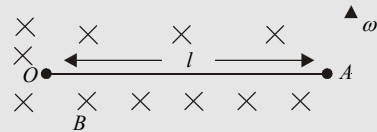


Fig. 26.14

$$\varepsilon = \frac{Bl^2\omega}{2} = B\pi f^2 \text{ where } f \text{ is linear frequency. (See Fig. 26.14)}$$

6. If *R* is the resistance of the circuit. then power consumed in moving the conductor in slide wire

$$\text{generator is } \frac{B^2 l^2 v^2}{R} \text{ on the lines } P = \vec{F} \cdot \vec{v} \text{ and}$$

$$\vec{F} = I\vec{B} = \frac{B^2 l^2 v}{R}$$

7. Fig. 26.15 (a), (b), (c) and (d) illustrate the effect of increasing and decreasing magnetic flux ϕ_B on induced *emf* and current

$$(a) \phi_B > 0, \frac{d\phi_B}{dt} > 0$$

\therefore *emf* ε and hence *I* are negative

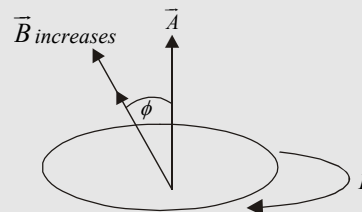


Fig. 26.15(a)

(b) $\phi_B > 0, \frac{d\phi_B}{dt} < 0$

$\therefore \varepsilon$ and hence I are positive

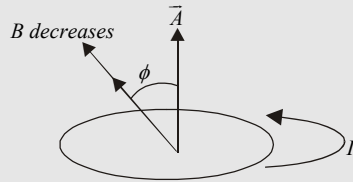


Fig. 26.15(b)

(c) $\phi_B < 0, \frac{d\phi_B}{dt} < 0$

$\therefore \varepsilon$ and hence I are positive

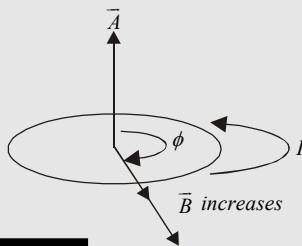


Fig. 26.15(c)

(d) \vec{B} decreasing $\phi_B < 0, \frac{d\phi_B}{dt} > 0$

$\therefore \varepsilon$ and hence I are negative

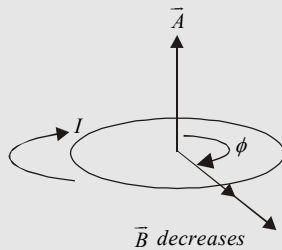


Fig. 26.15(d)

8. Motional emf $d\varepsilon = (\vec{v} \times \vec{B}) \cdot d\vec{l}$ and

$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$ If \vec{B} , \vec{l} and \vec{v} are mutually perpendicular then $\varepsilon = Blv$

9. If a disc of radius R rotates in a magnetic field B perpendicular to the plane of the rod with an angular

velocity ω then emf $\varepsilon = \frac{BR^2\omega}{2}$

10. If in a ring magnetic flux is varying then electric field (non-electrostatic) is given by

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\phi_B}{dt} \quad \text{or}$$

$$E = \frac{1}{2\pi r} \left| \frac{d\phi_B}{dt} \right| \text{ where } r \text{ is radius of the ring.}$$

11. Metal detectors work on the principle of eddy currents. The metal detector generates varying magnetic field. This induces eddy current in a conducting object carried, through the detector. The eddy current in turn produces a varying magnetic field B' . The detector's receiver coil receives this varying field and induces a current.

Eddy currents in action is Jupiter's moon Io . Io moves rapidly through Jupiter's intense magnetic field and this sets up strong eddy currents within the interior of Io . These currents dissipate energy at a rate of 10^{12} W ($= 1 \text{ Kiloton}$) in Io in every 4 s. This energy keeps interior of Io hot and causes volcanic eruption.

12. If time varying current is passed in the inner coil. $B_{\text{out}} = 0$. However, magnetic flux per turn through the outer coil is $B \pi r^2$. If N are the number of turns in secondary (outer) coil then total flux (see Fig. 26.16)

$$\phi_B = NB \pi r^2 \text{ and } \frac{d\phi_B}{dt} = N \pi r^2 \frac{dB}{dt} = N \pi r^2 \mu_0 n \frac{di}{dt}$$



Fig. 26.16

Note that r is the radius of inner coil. Note that flux exists outside the solenoid and is equal to $B \pi r^2$ while magnetic field outside is zero.

13. Self inductance of a solenoid $L = \mu_0 \mu_r n^2 Al = \mu_0 \mu_r n^2 V$ where V is volume of the coil $V = Al$ and n is number of turns per unit length, l is length of the solenoid and A is area of cross-section.

14. Intrinsic energy of a current in a solenoid

$$U = \frac{1}{2} L I^2. \text{ The energy stored is in the form of magnetic energy.}$$

15. If two coils have mutual inductance M and the currents in them are I_1 and I_2 then interaction energy of the two coils (see Fig. 26.17) is given by

$$U = M I_1 I_2$$

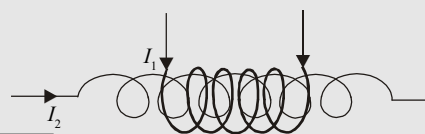


Fig. 26.17

16. Displacement current density $J_{\text{displacement}} = \frac{1}{R} \frac{\partial B}{\partial t}$

17. Volume density of magnetic field energy

$$= \frac{B^2}{2\mu_0\mu_r} = \frac{BH}{2}$$

18. Self inductance of a toroid $L = \frac{\mu_0 N^2 A}{2\pi r} = \frac{\mu_0 N^2 r}{2}$ 19. Coupling factor $K = \frac{M}{\sqrt{L_1 L_2}}$

$$= \frac{\text{flux linked to secondary coil}}{\text{flux linked to primary coil}}$$

20. Self inductance of two co-axial cylinders per meter (Fig. 26.18) is

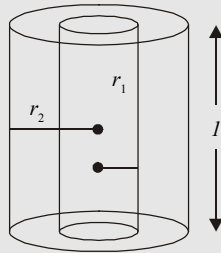


Fig. 26.18

$$L = \frac{\mu_0}{2\pi} \log_e \frac{r_2}{r_1} = \frac{2.303\mu_0}{2\pi} \log_{10} \frac{r_2}{r_1}$$

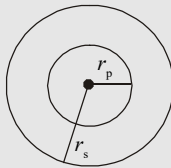
21. Mutual inductance between two concentric coils having radii r_p and r_s for primary and secondary coils as shown in Fig. 26.19 is

Fig. 26.19

$$M = \frac{\pi\mu_0 N_p N_s r_s^2}{2r_p} \text{ where } N_s \text{ and } N_p \text{ are number of turns in secondary and primary coils respectively.}$$

22. Inductance in series if mutual inductance is also present $L_{\text{eff}} = L_1 + L_2 + 2M$

23. Inductance in parallel if mutual inductance of the two coils is taken into account

$$L_{\text{eff}} = \frac{L_1 L_2 + M^2}{L_1 + L_2 + 2M}$$

24. In RL transients time for the current to grow 63% = τ (one time constant = L/R)

$$t = 2.303 \tau \log \frac{I_0}{I_0 - I} \text{ for growth of current}$$

Time for the current to grow 90% of $I_{\text{max}} = 2.303 \tau$

Time for the current to grow 95% of $I_{\text{max}} = 3 \tau$

Time for the current to grow 99% of $I_{\text{max}} = 5 \tau$

The same times are valid for decay also

$$t = 2.303 \tau \log \frac{I}{I_0}$$

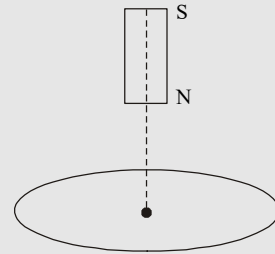
25. When a magnet falls along the axis of a closed metal ring as shown in Fig. 26.20 its acceleration decreases (due to Lenz law) as it approaches the ring. If the ring is not complete, making open circuit, then acceleration will remain = g throughout as induced current will be absent. However emf is induced for decay transient.

Fig. 26.20

26. Induced charge between time interval Δt is

$$\int i dt = \frac{-1}{R} \int d\phi = \frac{\phi_1 - \phi_2}{R}$$

27. In a generator $\text{emf } \varepsilon = NA\omega B \sin \omega t$ and peak voltage $V_p = NA\omega B$. Generator may also be called an Alternator.

CAUTION

1. Considering AC voltage and currents are scalar like their DC counter parts.
 $\Rightarrow AC$ voltage and currents are phasors. In EMI only AC voltage and currents are generated. Note phasors are added like vectors.
2. Assuming electric field generated in EMI (produced due to varying magnetic field) is conservative like electric field in electrostatics.
 \Rightarrow The induced electric field lines make a complete loop and $\oint E \cdot dl = \varepsilon \neq 0$. In electrostatics $\oint E \cdot dl = 0$. Hence field is not conservative.
3. Considering there is no difference between current induced due to EMI and drift current.

- ⇒ Current induced due to varying magnetic field or varying magnetic flux is displacement current.
4. Assuming that angle between the coils plays no role in determining mutual Inductance.
- ⇒ $M \propto \cos \theta$. If two coils are perpendicular as shown in Fig. 26.21 then $M = 0$. Therefore coupling factor $K = 0$ also.

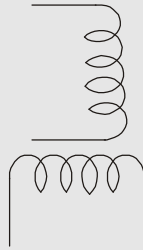


Fig. 26.21

5. While defining flux $B \cdot ds = d\phi$ considering area vector along the plane.
- ⇒ Area vector is perpendicular to the plane and angle between B and area vector ds be taken as illustrated in Fig. 26.22.

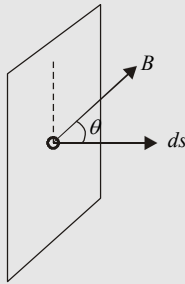


Fig. 26.22

6. Considering magnetic flux produces emf.
- ⇒ Change in magnetic flux produces emf. It may be produced in 3 ways (a) Area is fixed, B varies

$$\text{i.e. } \varepsilon = -A \frac{dB}{dt}$$

- (b) B is fixed and area varies i.e. $\varepsilon = -B \frac{dA}{dt}$
- (c) Both area and magnetic fields vary, i.e.

$$\varepsilon = - \frac{(B_2 - B_1)(A_2 - A_1)}{t}$$

7. Considering a wire or a cylinder does not possess any self inductance.
- ⇒ A wire or a cylinder has a very small self inductance. This property is used in the communication systems and rods / cylinders / wires act like antenna

or tuner circuit. However $L = \frac{\mu_0 m r}{4\pi \rho l}$ where m is mass, r radius, ρ is density and l is length.

8. Not remembering which law be used to find direction of current.

- ⇒ Flemming's Right hand rule which is mirror image of Flemming's left hand rule is used to find direction of current. Flemming's Left hand rule is used to find direction of force in a given magnetic field.

9. Considering no emf will be generated in an incomplete ring as no current is induced in the ring when a magnet is falling along the axis of the ring or conductor is moving in the magnetic field.

- ⇒ emf will be induced. Since induced current is zero, no opposition is caused by the ring to the falling magnet. Hence acceleration = g throughout the motion.

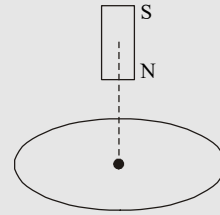


Fig. 26.23

10. If a magnet is falling along the axis of a long Cu cylinder then assuming that acceleration of the magnet $< g$ as in case of a solenoid.

- ⇒ As the Cu cylinder has nearly zero resistance, it opposes the magnet fully and $a = g - g = 0$

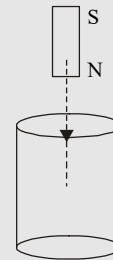


Fig. 26.24

11. Considering there is no effect of temperature when the magnet is falling along the axis of a metal ring.

- ⇒ If temperature increases, resistance increases and current falls. Hence opposition to the motion of magnet decreases. Magnet falls faster.

12. Considering when a rod is rotating in a magnetic field emf $\varepsilon = Blv$ where $v = l\omega$

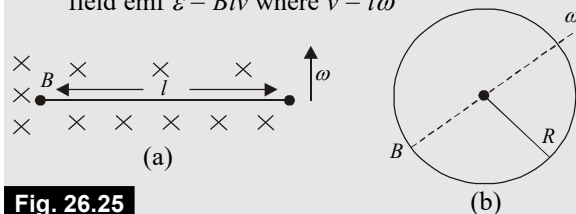


Fig. 26.25

i.e. $\varepsilon = Bl^2 \omega$ similarly for a rotating disc $\varepsilon = BR^2 \omega$

- ⇒ Note the velocity at each point is different, therefore,

for a rod $\varepsilon = \frac{Bl^2 \omega}{2}$ and for a disc

$$\varepsilon = \int_0^R \omega Br dr = \frac{\omega BR^2}{2} = \int_0^l B \omega x dx$$

SOLVED PROBLEMS

1. A rectangular loop with a slide wire of length l is kept in a uniform magnetic field as shown in Fig. 26.26 (a). The resistance of slider is R . Neglecting self inductance of the loop find the current in the connector during its motion with a velocity v .

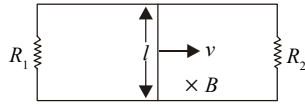


Fig. 26.26(a)

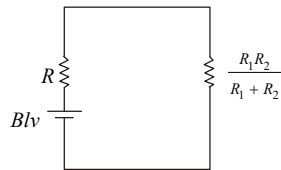


Fig. 26.26(b)

- (a) $\frac{Blv}{R_1 + R_2 + R}$ (b) $\frac{Blv(R_1 + R_2)}{R + (R_1 + R_2)}$
- (c) $\frac{Blv(R_1 + R_2)}{RR_1 + RR_2 + R_1R_2}$ (d) $Blv \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$

Solution (c) The equivalent circuit is shown in Fig. 26.26 (b)

$$\text{obviously } I = \frac{Blv}{R + \frac{R_1R_2}{R_1 + R_2}} = \frac{Blv(R_1 + R_2)}{RR_1 + RR_2 + R_1R_2}$$

2. A square wire frame of side a is placed a distance b away from a long straight conductor carrying current I . The frame has resistance R and self inductance L . The frame is rotated by 180° about OO' as shown in Fig. 26.27. Find the electric charge flown through the frame.

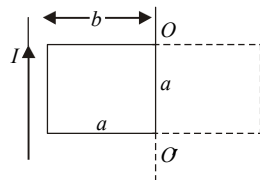


Fig. 26.27

- (a) $\frac{2\mu_0 I a^2}{2\pi R b}$ (b) $\frac{\mu_0 I}{2\pi R} \log_e \frac{b+a}{b-a}$
- (c) $\frac{\mu_0 I a}{2\pi R} \log_e \frac{b+a}{b-a}$ (d) none of these

Solution (c) $i = \frac{1}{R} \left[\frac{d\phi}{dt} + L \frac{di}{dt} \right]$

$$q = \int i dt = \frac{1}{R} [\Delta\phi + 0] = \frac{\Delta\phi}{R} = \frac{1}{R} \int_{b-a}^{b+a} B a dx$$

$$= \frac{1}{R} \int_{b-a}^{b+a} \frac{\mu_0 I a}{2\pi x} dx = \frac{\mu_0 I a}{2\pi R} \log_e \frac{b+a}{b-a}$$

3. One conducting U tube can slide inside the other as shown in Fig. 26.28 maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the Fig. 26.28. If each tube moves towards the other at a constant speed v then the induced emf in terms of B , l and v where l is the width of each tube, will be [AIEEE 2005]

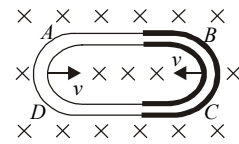


Fig. 26.28

- (a) Blv (b) $-Blv$
- (c) $2Blv$ (d) zero.

Solution (c) $\left| \frac{d\phi}{dt} \right| = 2Blv$

4. A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2V . The current reaches half of its steady state value in

- (a) 0.1 s (b) 0.3 s
- (c) 0.05 s (d) 0.05 s

[AIEEE 2005]

Solution (a) $I = I_0 \left(1 - e^{-\frac{Rt}{L}} \right)$

$$t = \tau \log_e \left(\frac{I_0}{I_0 - I} \right) = \frac{L}{R} \log_e 2 = 0.693 \frac{L}{R}$$

$$= 0.693 \times \frac{0.3}{2} \approx 0.1 \text{ s}$$

5. As a result of change in magnetic flux linked to the closed loop shown in Fig. 26.29, an emf V volt is induced in the loop. The work done in taking a charge Q coulomb once along the loop is [CBSE PMT 2005]

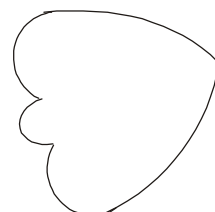


Fig. 26.29

- (a) QV (b) $2QV$
 (c) $QV/2$ (d) zero

Solution (a) QV because induced electric field so generated is non conservative i.e. $\oint E \cdot dl = V$

6. A conducting ring of radius 1 m is placed in a uniform magnetic field of 0.01 T oscillating with frequency 100 Hz with its plane at right angle to B . What will be the induced electric field?

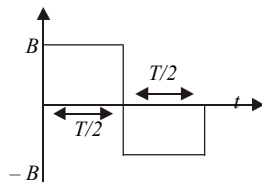


Fig. 26.30

- (a) $\pi\text{ V/m}$ (b) 2 V/m
 (c) 10 Vm^{-1} (d) 62 Vm^{-1}

[AIIMS 2005]

Solution (b) After every $T/2$ the field will change from B to $-B$ as illustrated in Fig. 26.30. $\varepsilon = 2BAf$

$$\oint E \cdot dl = \varepsilon$$

or
$$E = \frac{\varepsilon}{2\pi R} = \frac{2B\pi R^2 f}{2\pi R}$$

$$= BRf = .01 \times 1 \times 200 = 2\text{ Vm}^{-1}$$

7. A magnet is made to oscillate with a particular frequency passing through a coil as shown in Fig. 26.31. The time variation of the magnitude of emf generated across the coil during one cycle is

[CET Karnataka 2005]

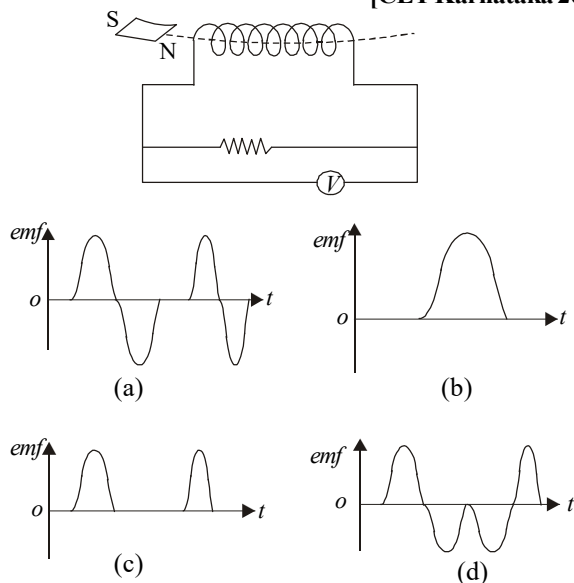


Fig. 26.31

Solution (a)

8. The induction coil works on the principle of

- (a) self induction (b) mutual induction
 (c) Amperes rule (d) Fleming's Right hand rule.

Solution (b)

9. The coil is wound on an iron core and looped back on itself so that core has two sets of closely wound coils carrying current in opposite directions. The self inductance is

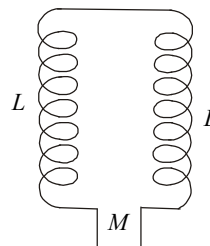


Fig. 26.32

- (a) zero (b) $2L$
 (c) $2L + M$ (d) $L + 2M$

Solution (a) Left $= L_1 + L_2 - 2M = L + L - 2\sqrt{LL} = 0$

10. The magnetic flux in a coil is $\phi = 12t^2 + 5t + 1$. The emf induced in 5 s is (ϕ is in milliweber and t in s)

- (a) 0 (b) 12.5 V
 (c) 0.15 V (d) 0.125 V

[CEE 1996]

Solution (d) $\frac{d\phi}{dt}$

$$= (24t + 5) \times 10^{-3} \Big|_{t=5}$$

$$= (24 \times 5 + 5) \times 10^{-3} = 0.125\text{ V}$$

11. A magnet falls with its S-pole along the axis of a ring. The current generated is and acceleration is

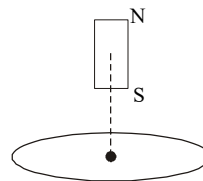


Fig. 26.33

- (a) clockwise, $> g$ (b) clockwise, $< g$
 (c) anti-clockwise, $> g$ (d) anti-clockwise, $< g$
 (e) clockwise, $= g$

Solution (b) South pole should be formed by the current in the ring. \therefore current is clockwise and south will repel south, hence a $< g$

12. A long wire carries a current 5 A . The energy stored in the magnetic field inside a volume 1 mm^3 at a distance 10 cm from the wire is

- (a) $\frac{\pi}{4} \times 10^{-13} \text{ J}$ (b) $\frac{\pi}{2} \times 10^{-13} \text{ J}$
 (c) $\pi \times 10^{-13} \text{ J}$ (d) $\frac{\pi}{8} \times 10^{-13} \text{ J}$

Solution (d) u (energy per unit volume) = $\frac{B^2}{2\mu_0}$ and

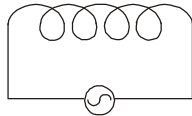
$$\text{energy } U = \frac{B^2}{2\mu_0} \times \text{vol.}$$

$$U = \left(\frac{\mu_0 I}{2\pi d} \right)^2 \times \frac{1}{2\mu_0} \times \text{vol.}$$

$$= \frac{\mu_0 I^2}{8\pi^2 d^2} \times \text{vol.}$$

$$= \frac{4\pi \times 10^{-7} \times 25 \times 10^{-9}}{8 \times 10 (10^{-2})} = \frac{\pi}{8} \times 10^{-13} \text{ J}$$

13. If a Bismuth rod is introduced in the air coil as shown then current in the coil



$$V = 10 \sin \omega t$$

Fig. 26.34

- (a) increases (b) remains unchanged
 (c) decreases (d) none of these

Solution (a) L will decrease as Bi is diamagnetic

$$\therefore I = \frac{V}{X_L} \text{ will increase}$$

14. The voltmeter reading in the Fig. 26.35 is

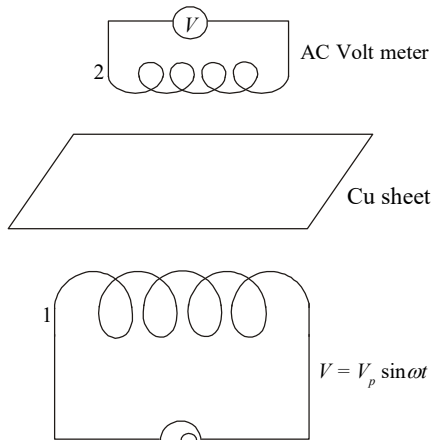


Fig. 26.35

- (a) zero (b) $\frac{V_p}{\sqrt{2}}$
 (c) $\frac{V_p}{2}$ (d) none of these

Solution (a) because Cu is diamagnetic, no magnetic flux will link to coil 2.

15. A satellite orbiting the earth at 400 km above the surface of the earth has a 2 m long antenna oriented perpendicular to the earth's surface. At the equator the earth's magnetic field is $8 \times 10^{-5} \text{ T}$ and is horizontal. Assuming the orbit to be circular, find emf induced across the ends of the antenna.

- (a) 1.3 V (b) 1.2 V
 (c) 1.0 V (d) 0.12 V
 (e) 0.13 V

Solution (b) $v_0 = \sqrt{(R+h)g}$

$$\text{emf} = Blv_0 = 8 \times 10^{-5} \times 2 \times 7.2 \times 10^3 = 1.2 \text{ V}$$

16. Assume a long solenoid is wound with 500 turns m^{-1} and current is increasing at 100 As^{-1} , the cross-section of the coil has area 4 cm^2 . Find the induced electric field within the loop of radius 2 cm

- (a) $2 \times 10^{-4} \text{ Vm}^{-1}$ (b) $4 \times 10^{-4} \text{ Vm}^{-1}$
 (c) $3 \times 10^{-4} \text{ Vm}^{-1}$ (d) none of these

Solution (a) emf $\mathcal{E} = -\mu_0 n A \frac{dI}{dt}$

$$= 4\pi \times 10^{-7} \times 500 \times 4 \times 10^{-4} \times 100 = 25 \times 10^{-6} \text{ V}$$

$$\oint E \cdot dl = \mathcal{E} \text{ or } E = \frac{\mathcal{E}}{2\pi r}$$

$$= \frac{25 \times 10^{-6}}{2\pi \times 2 \times 10^{-2}} = 2 \times 10^{-4} \text{ V}$$

17. A long solenoid of radius 2 cm has 100 turns/cm and is surrounded by a 100 turn coil of radius 4 cm having a total resistance 20Ω . If current changes from 5 A to -5 A , find the charge through galvanometer.

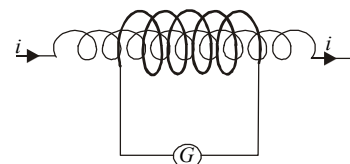


Fig. 26.36

- (a) zero (b) $800 \mu\text{C}$
 (c) $400 \mu\text{C}$ (d) $600 \mu\text{C}$

Solution (b) $\phi = B\pi r^2 \mathcal{E} = \frac{d\phi}{dt} = N\pi r^2 \frac{dB}{dt}$

$$= N\pi^2\mu_0 n \frac{di}{dt}$$

$$I = \frac{\mathcal{E}}{R} \text{ and } \Delta Q = I\Delta t = \frac{N\pi r^2\mu_0 n}{R} \Delta t$$

$$\Delta Q = \frac{100 \times \pi \times (2 \times 10^{-2})^2 \times 10^4 \times 4\pi \times 10^{-7} \times 10}{20}$$

$$= 8 \times 10^{-4} \text{ C} = 800 \mu\text{C}$$

18. A rod of length l is moved with a velocity v in a magnetic field B as shown in Fig. 26.37. Sketch the equivalent electrical circuit.

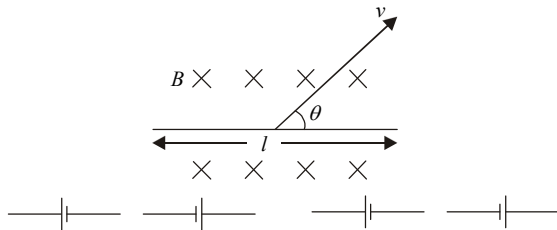


Fig. 26.37

- (a) Blv (b) Blv
(c) $Blv \sin \theta$ (d) $Blv \sin \theta$

Solution (c) The positive charge of the rod shifts towards left due to $F = q(\vec{v} \times \vec{B})$.

19. Two conducting circular loops of radii R_1 and R_2 ($R_1 > R_2$) are placed in the same plane with their centres coinciding. Find the mutual inductance between them.

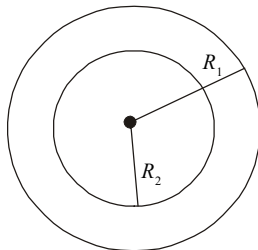


Fig. 26.38

- (a) $\frac{\mu_0 \pi R_1^2}{R_2}$ (b) $\frac{\mu_0 \pi R_2^2}{R_1}$
(c) $\frac{\mu_0 \pi R_1^2}{2R_2}$ (d) $\frac{\mu_0 \pi R_2^2}{2R_1}$

Solution (d) Assume current i passes through outer loop. Then $B = \frac{\mu_0 i}{2R_1}$ and

$$\phi = \frac{\mu_0 i}{2R_1} \pi R_2^2 \text{ using } \phi = Mi, M = \frac{\mu_0 \pi R_2^2}{2R_1}$$

20. In a closed ring A and in an open ring B magnets are falling along the axis of the ring. The current generated in A and B have directions

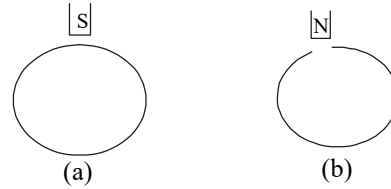


Fig. 18.39

- (a) clockwise, anti-clockwise
(b) anti-clockwise, clockwise
(c) clockwise, zero
(d) anti-clockwise, zero
(e) zero, zero

Solution (c) According to Lenz's law the current generated in A shall develop S pole to oppose the cause producing it. Therefore current is clockwise. In B the circuit is open. Therefore no current will flow.

21. A metallic wire bent into a right Δabc moves with a uniform velocity v as shown in Fig. 26.40. B is the strength of uniform magnetic field perpendicular outwards the plane of triangle. The net emf is and emf along ab is

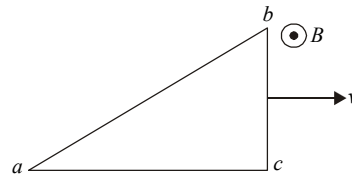


Fig. 26.40

- (a) zero, zero
(b) zero, $Bv(bc)$ with b positive
(c) zero, $Bv(bc)$ with a positive
(d) $Bv(bc)$ with c positive, zero
(e) $Bv(bc)$ with b positive, zero.

Solution (c) Net emf and hence net current in a loop moved with uniform velocity is zero because $\phi = \text{constant}$ and

$$\frac{d\phi}{dt} = 0.$$

22. Two rail tracks are 1m apart and insulated from each other and insulated from ground. A millivoltmeter is connected across the railtracks. When a train travelling at 180 km/h passes through what will be the reading in millivoltmeter? Given: horizontal component of earth's field $\sqrt{3} \times 10^{-4} \text{ T}$ and dip at the place 60° .

- (a) 1.5 mV (b) 15 mV
 (c) $\frac{15}{\sqrt{3}}$ mV (d) $\frac{1.5}{\sqrt{3}}$ mV
 (e) none of these

Solution (b) Vertical component of the magnetic field will be cut. $\varepsilon = B_v l v$ and B_v

$$= B_H \tan \delta = 3 \times 10^{-4} T$$

$$= 3 \times 10^{-4} \times 1 \times 50 = 15 \text{ mV}$$

23. A copper wire of length l is bent into a semicircle. It is moved with a velocity v in a region where magnetic field is uniform and perpendicular to the plane of the wire. If the strength of the field is B then emf induced is

- (a) Blv (b) $B \frac{l}{\pi} v$
 (c) $B \frac{2l}{\pi} v$ (d) none of these

Solution (c) $\pi r = l$

or $r = \frac{l}{\pi} \varepsilon = B(2r)v = B \left(\frac{2l}{\pi} \right) v.$

24. A small circular ring is kept inside a larger loop connected to a switch and a battery as shown. The direction of induced current when the switch is made (i) ON (ii) OFF after it was ON for a long time is
 (a) clockwise, anti-clockwise
 (b) clockwise, clockwise
 (c) anti-clockwise, clockwise
 (d) anti-clockwise, anti-clockwise

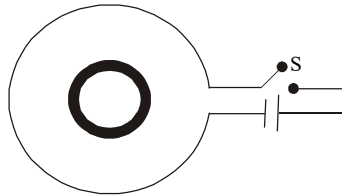


Fig. 26.41

Solution (a) Apply Lenz's law.

25. A square loop of Cu of side a enters a magnetic field spread from $-a$ to $+a$ as shown in Fig. 26.39. Plot induced emf as a function of x .

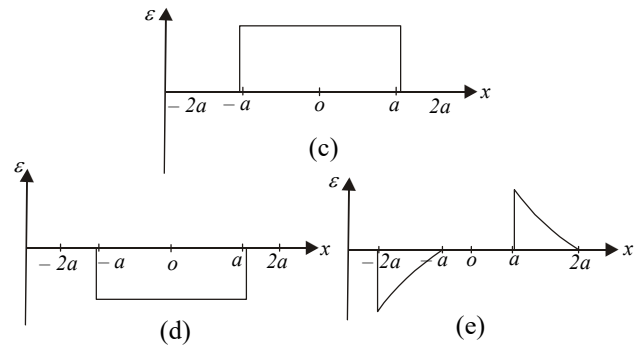
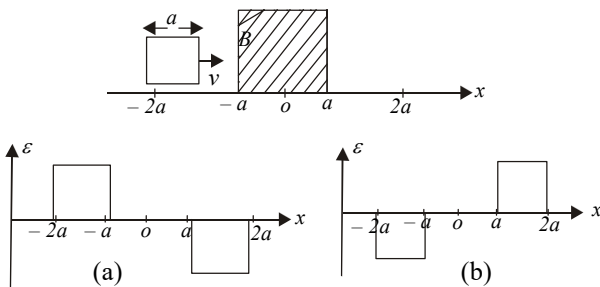


Fig. 26.42

Solution (b) $\therefore \varepsilon = -\frac{d\phi}{dx}$

26. The armature of a demonstrator generator consists of a flat square coil of side 4 cm and 200 turns. The coil rotates in a magnetic field of 0.75 T. The angular speed so that a maximum emf of 1.6 V is generated is

- (a) $\frac{20}{3}$ rad/s (b) $\frac{20}{3}$ rotations/s
 (c) $\frac{20}{3}$ rpm (d) none of these

Solution (a) $\varepsilon_{\max} = NA_0 B \omega$ or

$$\omega = \frac{\varepsilon_{\max}}{NA_0 B}$$

$$= \frac{1.6}{200 \times 16 \times 10^{-4} (0.75)}$$

$$= \frac{20}{3} \text{ rad/s}$$

27. The electric flux through a certain area of dielectric is $8.76 \times 10^3 t^4$. The displacement current through the area is 12.9 pA at $t = 26.1$ ms. Find the dielectric constant of the material.

- (a) 2×10^{-8} (b) 4×10^{-8}
 (c) 8×10^{-8} (d) 2×10^{-7}

Solution (a) $i_D = \varepsilon \frac{d\phi_E}{dt}$ or

$$\varepsilon = \frac{i_D}{\frac{d\phi_E}{dt}}$$

$$\varepsilon = \frac{12.9 \times 10^{-9}}{4(8.76) \times 10^3 \times (26.1 \times 10^{-3})^3}$$

$$\square 2 \times 10^{-8}$$

TYPICAL PROBLEMS

28. Magnetic flux during time interval τ varies through a stationary loop of resistance R as $\phi_B = at(\tau - t)$. Find the amount of heat generated during that time. Neglect the inductance of the loop.

(a) $\frac{a^2 \tau^3}{R}$ (b) $\frac{a^2 \tau^2}{2R}$
 (c) $\frac{a^2 \tau^3}{3R}$ (d) $\frac{a^2 \tau^3}{4R}$

Solution (c) $i = \frac{d\phi}{dt} / R = \frac{a(\tau - 2t)}{R}$

Heat produced $H = \int_0^\tau i^2 R dt$

$$H = \int_0^\tau \frac{a^2 (\tau - 2t)^2}{R} dt = \frac{a^2 \tau^3}{3R}$$

29. N turns are tightly wound to form a spiral plane of outer radius a . If the magnetic induction $B = B_0 \sin \omega t$ varies perpendicular to the plane of spiral then find the emf induced in the spiral.

(a) $\frac{\pi a^2 B_0 \omega N \cos \omega t}{2}$ (b) $\frac{\pi a^2 B_0 N \omega \cos \omega t}{4}$
 (c) $\pi a^2 B_0 N \omega \cos \omega t$ (d) $\frac{\pi a^2 B_0 N \omega \cos \omega t}{3}$

Solution (d) Let the hypothetical elementary loop have

radius x . Then emf induced in this loop $d\varepsilon = \frac{AdB}{dt} =$

$$\pi x^2 B_0 \cos \omega t dN = \pi x^2 B_0 \omega \cos \omega t \left(\frac{N}{a} \right) dx$$

or $\varepsilon = \int_0^a B_0 \pi \omega \frac{N}{a} \cos \omega t x^2 dx = \frac{\pi a^2 B_0 \omega N \cos \omega t}{3}$

30. Find the inductance of a solenoid of length l_o , made of Cu windings of mass m . The winding resistance is equal to R . The diameter of solenoid $\ll l_o$. ρ_o is resistivity of Cu and ρ is density of the Cu .

(a) $\frac{\mu_o R m}{2\pi l_o \rho \rho_o}$ (b) $\frac{\mu_o R m}{4\pi l_o \rho \rho_o}$
 (c) $\frac{\mu_o R m}{3\pi l_o \rho \rho_o}$ (d) $\frac{2\mu_o R m}{3\pi l_o \rho \rho_o}$

Solution (b) Length of the wire $l = \frac{RA}{\rho_o} = \frac{Rm}{\rho_o \rho l}$

or $l = \sqrt{\frac{Rm}{\rho \rho_o}}$ where ρ_o is resistivity of the Cu and ρ is density of the Cu . $l = n l_o 2\pi r$ and $L = \mu_o n^2 l_o \pi r^2$ where l_o is length of the solenoid and r is the radius of the

solenoid. Then $l = l_o 2\pi r \sqrt{\frac{L}{\mu_o l_o \pi r^2}}$

or $l = \sqrt{\frac{4\pi L l_o}{\mu_o}}$

or $l = \sqrt{\frac{Rm}{\rho \rho_o}}$

or $L = \frac{\mu_o R m}{4\pi l_o \rho \rho_o}$

31. How many meters of a thin wire are required to design a solenoid of length 1m and $L = 1mH$? Assume cross-sectional diameter is very small.

(a) 10 m (b) 40 m
 (c) 70 m (d) 100 m
 (e) 140 m

Solution (d) Length of the wire $l = n l_o 2\pi r$ and $L = \mu_o n^2 l_o \pi r^2$

or $n = \sqrt{\frac{L}{\mu_o l_o \pi r^2}}$

Thus $l = \sqrt{\frac{L}{\mu_o l_o \pi r^2}} l_o 2\pi r$

$$2\pi r = \sqrt{\frac{L l_o 4\pi}{\mu_o}}$$

$$= \sqrt{\frac{10^{-3} \times 1 \times 4\pi}{4\pi \times 10^{-7}}} = 100m$$

32. Find the steady state current through L_1 in the Fig. 26.43

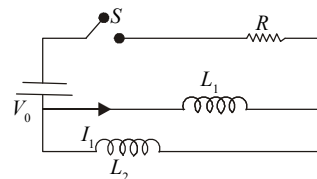


Fig. 26.43

- (a) $\frac{V_0}{R}$ (b) $\frac{V_0 L_1}{R(L_1 + L_2)}$
 (c) $\frac{V_0 L_2}{R(L_1 + L_2)}$ (d) none of these

Solution (c) $I_0 = \frac{V_0}{R}$ divide the current in L_1 and L_2 like resistors $I_1 = I_0 \frac{L_2}{L_1 + L_2}$

33. Two identical galvanometers are joined by connecting wires. One of them is placed on the table and the other is held in the hand. One in the hand is shaken violently so that it shows a deflection of 10 division. The reading in the other galvanometer (on the table) is
 (a) zero (b) 10 division
 (c) 5 division (d) insufficient data to reply.

Solution (b) The one shaken violently acts as a generator and the other reads the emf generated.

34. A wire shaped as a semicircle of radius r rotates about the axis OO' with an angular velocity ω as shown in Fig. 26.44. Resistance of the circuit is R . Find the mean thermal power generated in the loop during a rotation period.

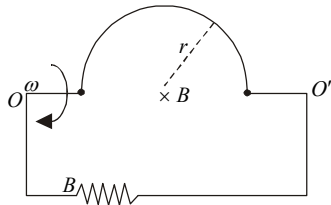


Fig. 26.44

Solution $A = \pi a^2 \cos \omega t$; $\phi = BA = B\pi a^2 \cos \omega t$

$$i = \frac{\varepsilon}{R} = -\frac{d\phi}{dt} / R = \frac{B\pi a^2 \omega \sin \omega t}{R} \quad \text{and}$$

$$\langle P \rangle = \frac{1}{T} \int_0^T i^2 R dt = \frac{(B\pi a^2 \omega)^2}{2R}$$

35. A long wire carries a current i . A rod of length l is moved with a velocity v in a direction parallel to the wire as shown in Fig. 26.45 (a). Find the motional emf induced in the rod.

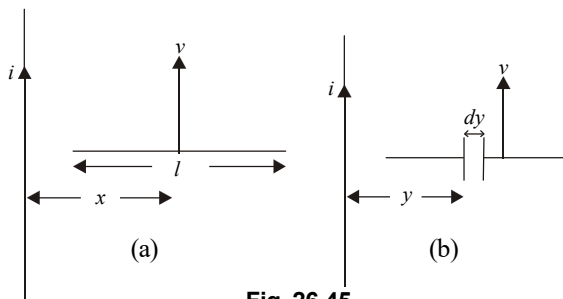


Fig. 26.45

- (a) $\frac{\mu_0 i}{2\pi x} l v$ (b) $\frac{\mu_0 i}{2\pi} v \log_e \frac{x+l/2}{x-l/2}$
 (c) $\frac{\mu_0 i}{2\pi} v \log_e \frac{x-l/2}{x+l/2}$ (d) $\frac{\mu_0 i v}{2\pi} \log_e \frac{l+x}{x}$

Solution (b) Consider small element dy at distance y

from the long wire as shown in Fig 26.42 (b) $B = \frac{\mu_0 i}{2\pi y}$

and emf in element dy is

$$d\varepsilon = Bvdy. \text{ Thus } \varepsilon = \int_{x-l/2}^{x+l/2} \frac{\mu_0 i v}{2\pi} \frac{dy}{y}$$

$$= \frac{\mu_0 i v}{2\pi} \log_e \frac{x+l/2}{x-l/2}$$

36. A current in a 240 turn solenoid varies at 0.8 A/s . Find emf induced if the length of the solenoid is 12 cm and radius 2 cm
 (a) $6.14 \times 10^{-4} \text{ V}$ (b) $6.4 \times 10^{-3} \text{ V}$
 (c) $3.07 \times 10^{-3} \text{ V}$ (d) $3.07 \times 10^{-4} \text{ V}$

Solution (a) $\varepsilon = L \frac{di}{dt}$

$$= \frac{\mu_0 N^2 (\pi r^2)}{l} \frac{di}{dt}$$

$$= \frac{4\pi \times 10^{-7} \times (240)^2 \times (\pi) (2 \times 10^{-2})^2 \times 0.8}{12 \times 10^{-2}}$$

$$= 6.14 \times 10^{-4} \text{ V}$$

37. The magnetic field inside a 2 mH inductor becomes 0.8 of its maximum value in $20 \mu\text{s}$ when the inductor is joined to battery. Find resistance of the circuit.
 (a) 160Ω (b) 80Ω
 (c) 320Ω (d) 240Ω
 (e) none of these

Solution (a) $i \propto B$. \therefore current also becomes 0.8 of its maximum value

$$t = \tau \log_e \frac{i_0}{i_0 - i}$$

$$\text{or } 20 \times 10^{-6} = \frac{2 \times 10^{-3}}{R} \log_e 5$$

$$R = 100 \times 2.303 (.6990)$$

$$= 160 \Omega$$

38. A wire bent as a parabola $y = kx^2$ is located in a uniform magnetic field of induction B , the vector B being perpendicular to the plane xy . At $t = 0$, sliding wire starts sliding from the vertex O with a constant acceleration a linearly as shown in Fig. 26.46. Find the emf induced in the loop.

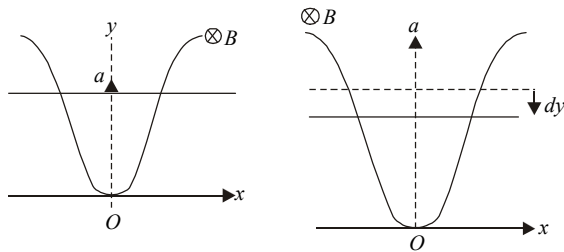


Fig. 26.46

- (a) $B y \sqrt{\frac{2a}{k}}$ (b) $B y \sqrt{\frac{4a}{k}}$
 (c) $B y \sqrt{\frac{8a}{k}}$ (d) $B y \sqrt{\frac{a}{k}}$

Solution (c) $d\phi = B \cdot dA = 2B x \, dy$ and $y = kx^2$

$$\therefore \varepsilon = \sqrt{\frac{y}{k}}$$

$$\therefore \frac{d\phi}{dt} = 2B \sqrt{\frac{y}{k}} \frac{dy}{dt}$$

$$\text{using } v^2 = 2as$$

$$\frac{dy}{dt} = v = \sqrt{2ay}$$

$$\text{or } |\varepsilon| = \frac{d\phi}{dt} = 2B \sqrt{\frac{y}{k}} \sqrt{2ay}$$

$$\text{or } |\varepsilon| = B y \sqrt{\frac{8a}{k}}$$

39. A long wire carrying current i is placed close to a u shaped conductor (of negligible resistance). A wire of length l as shown in Fig. 26.47 slides with a velocity v . Find the current induced in the loop as a function of distance x from the current carrying wire to slider.

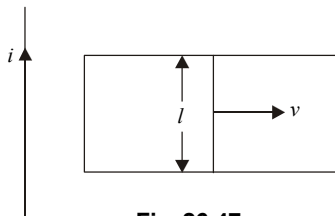


Fig. 26.47

- (a) $\frac{\mu_0 i l u}{R x}$ (b) $\frac{\mu_0 i l u}{2\pi R x}$
 (c) $\frac{\mu_0 i l u}{2R x}$ (d) $\frac{\mu_0 i l u}{4\pi R x}$

Solution (b) $B = \frac{\mu_0 i}{2\pi x}$ and $\varepsilon = B l v = \frac{\mu_0 i l v}{2\pi x}$

$$I = \frac{\varepsilon}{R} = \frac{\mu_0 i l v}{2\pi R x}$$

and

40. A square frame with side a as shown in Fig. 26.48 is moved with a velocity v from a long straight wire carrying current I . Initial separation between straight long wire and square frame is x . Find the emf induced in the frame as a function of distance x .

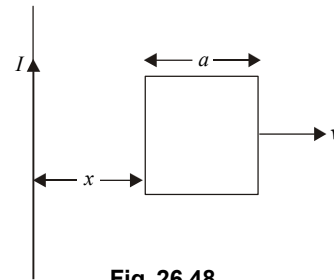


Fig. 26.48

- (a) $\frac{\mu_0 I a^2 v}{2\pi x(x+a)}$ (b) $\frac{\mu_0 I a x v}{2\pi x(x+a)}$
 (c) $\frac{\mu_0 I a^2 v}{4\pi x(x+a)}$ (d) zero

Solution (a) $\varepsilon_1 = \frac{\mu_0 I a v}{2\pi x}$

$$\text{and } \varepsilon_2 = \frac{\mu_0 I a v}{2\pi(x+a)}$$

$$\varepsilon_{\text{net}} = \varepsilon_1 - \varepsilon_2$$

$$= \frac{\mu_0 I a v}{2\pi} \left[\frac{1}{x} - \frac{1}{x+a} \right]$$

$$= \frac{\mu_0 I a^2 v}{2\pi x(x+a)}$$

PASSAGE 1

Read the following passage and answer the questions given at the end.

A metal bar ab with length L , mass m and resistance R is placed on frictionless metal rails that are inclined at an angle ϕ above the horizontal. The rails have negligible resistance. A uniform magnetic field of magnitude B is directed downwards as shown in Figure 26.49. The bar is released from rest and slides down the rails and ultimately acquires terminal velocity.

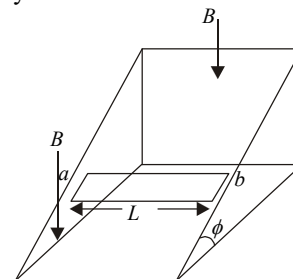


Fig. 26.49

1. In which direction does the current pass through the bar?

(a) along ab (b) along ba
(c) along random direction (d) none

2. What is the terminal speed of the bar?

(a) $\frac{mgR}{B^2 l^2 \sin \phi}$ (b) $\frac{mgR \cos \phi}{B^2 l^2 \sin^2 \phi}$
(c) $\frac{mg \sin \phi R}{B^2 l^2 \cos^2 \phi}$ (d) $\frac{mg \sin \phi R}{B^2 l^2 \cos \phi}$

3. At what rate is electrical energy converted into thermal energy in the resistance of the bar after the terminal velocity has been reached?

(a) $\frac{B^2 l^2 v \cos^2 \phi}{R}$ (b) $\frac{B^2 l^2 v^2 \cos^2 \phi}{R}$
(c) $\frac{B^2 l^2 v^2 \sin^2 \phi}{R}$ (d) $\frac{B^2 l^2 v^2 \sin^2 \phi}{R}$

4. At what rate is work being done by gravity on the bar after the terminal speed has been reached?

(a) $\frac{B^2 l^2 v^2 \cos^2 \phi}{R}$ (b) $\frac{B^2 l^2 v \cos^2 \phi}{R}$
(c) $\frac{B^2 l^2 v \sin^2 \phi}{R}$ (d) $\frac{B^2 l^2 v^2 \sin^2 \phi}{R}$

Solution 1. (a) Along ab so that it opposes the gravitational force which causes the motion.

Solution 2. (c) It will attain terminal velocity when magnetic force is balanced by gravitational force.

$$mg \sin \phi = Il B \cos \phi$$

$$I = \frac{Blv \cos \phi}{R}$$

$$\text{or } mg \sin \phi = \frac{B^2 l^2 v^2 \cos^2 \phi}{R}$$

$$\text{or } v = \frac{mg \sin \phi R}{B^2 l^2 \cos^2 \phi}$$

Solution 3. (b) $P = \frac{B^2 l^2 v^2 \cos^2 \phi}{R}$

Solution 4. (a) $P_{\text{gravity}} = mg \sin \phi \cdot v$

$$= Il B \cos \phi \frac{(mg \sin \phi) R}{B^2 l^2 \cos^2 \phi}$$

$$= \frac{I^2 l^2 B^2 \cos \phi}{B^2 l^2} = \frac{B^2 l^2 v^2 \cos^2 \phi}{R}$$

i.e., rate of doing work against gravity = heat being produced in electrical resistance.

PASSAGE 2

Read the following passage and answer the questions given at the end.

Consider a uniform metallic disc rotating through a perpendicular magnetic field \vec{B} as shown in Fig. 26.50. Mass of the disc is M . Its radius is R and thickness t . It is made of a material of resistivity ρ , and is rotating clockwise as shown in Fig. 26.50 with angular speed ω . Magnetic field is directed into the plane of the disc. Assume the region to which magnetic field is confined is a square of side L ($L \ll R$) centered a distance d from point O , the centre of the disc. The sides of this square are horizontal and vertical

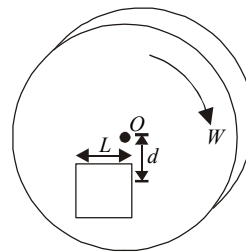


Fig. 26.50

1. Find the approximate value of induced current assuming the resistance to the current is confined to the square.

(a) $\frac{BL\omega dt}{\rho}$ (b) $\frac{BL^2\omega t}{\rho}$
(c) $\frac{BL^2\omega d}{\rho}$ (d) $\frac{BL\omega d^2}{\rho}$
(e) none

2. Find the approximate value of torque obtained through the induced current.

(a) $\frac{B^2 L^2 \omega dt^2}{\rho}$ (b) $\frac{B^2 L^2 \omega d^3}{\rho}$
(c) $\frac{B^2 L^2 \omega d^2 t}{\rho}$ (d) $\frac{B^2 L \omega d^2 t^2}{\rho}$

3. The direction of torque produced due to induced current is

(a) counter-clockwise (b) clockwise
(c) lateral to produce precession
(d) none of these

Solution 1. (a) $\text{emf } \varepsilon = \frac{d\phi}{dt} = BL^2\omega$

$$I = \frac{\varepsilon}{R} = \frac{BL^2\omega}{\frac{\rho L}{dt}} = \frac{BL\omega dt}{\rho}$$

Solution 2. (c) $F = IlB = \frac{B^2 L^2 \omega dt}{\rho}$

$$\tau = F(d) = \frac{B^2 L^2 \omega d^2 t}{\rho}$$

Solution 47. (a) According to Lenz's law the current induced is in such a direction to oppose the cause producing it.

PASSAGE 3

Read the following passage and answer the questions given at the end.

A tank containing a liquid has turns of wire wrapped around it, causing it to act as an inductor. The liquid content of the tank can be measured by using its inductance to determine the height of the liquid level in the tank. The inductance of the tank changes from a value of L_0 corresponding to a relative permeability of 1 when the tank is empty to value L_f corresponding to a relative permeability χ_m (relative permeability of liquid) when the tank is full. The appropriate electronic circuit can determine the inductance correct upto 5 significant figures and thus the effective relative permeability of the combined air and liquid within the rectangular has height D . The height of the liquid level in the tank is d . Ignore the fringing effects. Assume tank is fitted with Hg $\chi_{\text{Hg}} = 2.9 \times 10^5$.

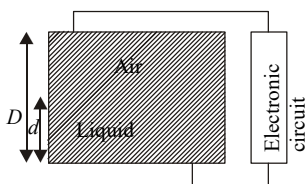


Fig. 26.51

1. Express d as a function of L , inductance corresponding to a certain liquid height L_0 , L_f and D .

$$(a) d = \frac{(L - L_0)D}{L_f - L_0} \quad (b) d = \frac{LD}{L_f - L_0}$$

$$(c) d = \frac{(L - L_0)D}{L_f} \quad (d) \text{ none of these}$$

2. If $L_0 = 0.63000 \text{ H}$ and $\chi_{\text{Hg}} = 2.9 \times 10^5$ find L when tank is 1/4th full.

Solution 1. (a) $L - L_0 = \chi_m \frac{d}{D}$ (1)

$L_f - L_0 = \chi_m$ (2)

or $\frac{L - L_0}{L_f - L_0} = \frac{d}{D}$

or $d = \left(\frac{L - L_0}{L_f - L_0} \right) D$

Solution 2. $L - L_0 = \chi_m \frac{d}{D}$

$$= 0.63000 - \frac{.000029}{4} = .62999 \text{ H}$$

PASSAGE 4

Read the following passage and answer the questions given at the end.

Physics by Saurabh Maurya (IIT-BHU)

A circular loop of radius a and resistance R initially has a magnetic flux through it due to an external magnetic field. The external field then decreases to zero. A current is induced in the loop while the external field is changing. However, this current does not stop at the instant when the external field stops changing. The reason being that the current itself generates a magnetic field, which gives rise to flux through the loop. If the current varies, the flux through the loop varies as well, and an emf appears in the loop to oppose the change. The magnetic field at the centre of the loop produced due to

current I in the loop is given by $B = \frac{\mu_0 I}{2a}$.

1. Use the crude approximation that magnetic field is same within the loop then flux through the loop is

$$(a) \frac{\mu_0 i a}{2} \quad (b) \mu_0 i \pi a$$

$$(c) \frac{\mu_0 i \pi a}{2} \quad (d) \text{ none of these}$$

2. Using $\varepsilon = -\frac{d\phi}{dt}$ and $\varepsilon = iR$ find the current in the loop after the external field has stopped changing.

$$(a) \frac{di}{dt} = -\left(\frac{2R}{\mu_0 \pi a} \right) i \quad (b) \frac{di}{dt} = -\frac{R}{\mu_0 a} i$$

$$(c) \frac{di}{dt} = \frac{-2R}{\mu_0 a} i \quad (d) \frac{di}{dt} = \frac{-2R}{3\mu_0 \pi a} i$$

3. Assuming $i = i_0$ at $t = 0$; The current at any instant in the loop is _____ after the external field has stopped changing.

$$(a) i = i_0 e^{\frac{-2Rt}{\mu_0 \pi a}} \quad (b) i = i_0 e^{\frac{-Rt}{\mu_0 \pi a}}$$

$$(c) i = i_0 e^{\frac{2Rt}{\mu_0 \pi a}} \quad (d) i = i_0 e^{\frac{-Rt}{2\mu_0 \pi a}}$$

Solution 1. (c) $\phi = B.A \chi_a$

$$= \frac{\mu_0 i}{2a} (\pi a^2) = \frac{\mu_0 i \pi a}{2}$$

Solution 2. (a) $\varepsilon = -\frac{d\phi}{dt}$

$$= -\frac{\mu_0 \pi a}{2} \frac{di}{dt} = iR$$

or $\frac{di}{dt} = -\left(\frac{2R}{\mu_0 \pi a} \right) i$

Solution 3. (a) $\int_{i_0}^i \frac{di}{i} = -\int_0^t \frac{2Rdt}{\mu_0 \pi a}$

$$\log_e \frac{i}{i_0} = -\frac{2Rt}{\mu_0 \pi a}$$

or $i = i_0 e^{\frac{-2Rt}{\mu_0 \pi a}}$

QUESTIONS FOR PRACTICE

1. A flat coil carrying a current has a magnetic moment $\vec{\mu}$. It is placed in a magnetic field \vec{B} . The torque on the coil is $\vec{\tau}$.

- (a) $\vec{\tau} = \vec{\mu} \times \vec{B}$ (b) $\vec{\tau} = \vec{B} \times \vec{\mu}$
 (c) $|\vec{\tau}| = \mu \cdot B$
 (d) $\vec{\tau}$ is perpendicular to both $\vec{\mu}$ and \vec{B} .

2.

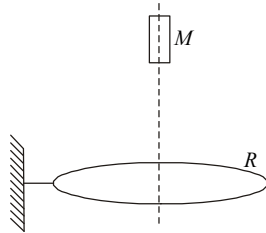


Fig. 26.52

A small magnet M is allowed to fall through a fixed horizontal conducting ring R . Let g be the acceleration due to gravity. The acceleration of M will be

- (a) $< g$ when it is above R and moving towards R
 (b) $> g$ when it is above R and moving towards R
 (c) $< g$ when it is below R and moving away from R
 (d) $> g$ when it is below R and moving away from R
3. In the previous question, the directions of the current flow in the ring, when M is above R and below R will be
- (a) the same in all cases
 (b) opposite in all cases
 (c) the same only if the N -pole of M moves towards R when M is above R
 (d) the same only if the S -pole of M moves towards R when M is above R
4. A flat coil carrying a current has a magnetic moment $\vec{\mu}$. It is placed in a magnetic field \vec{B} such that $\vec{\mu}$ is antiparallel to \vec{B} . The coil is
- (a) not in equilibrium
 (b) in stable equilibrium
 (c) in unstable equilibrium
 (d) in neutral equilibrium
5. The magnetic flux linked with a coil is ϕ and the emf induced in it is ε .

- (a) If $\phi = 0$, ε must be 0.
 (b) If $\phi \neq 0$, ε cannot be 0.
 (c) If ε is not 0, ϕ may or may not be 0.
 (d) None of the above is correct.
6. The magnetic flux (ϕ) linked with a coil depends on time t as $\phi = at^n$, where a and n are constants. The emf induced in the coil is ε .
- (a) If $0 < n < 1$, $\varepsilon = 0$.
 (b) If $0 < n < 1$, $\varepsilon \neq 0$ and $|\varepsilon|$ decrease with time.
 (c) If $n = 1$, ε is constant.
 (d) If $n > 1$, $|\varepsilon|$ increases with time.

7.

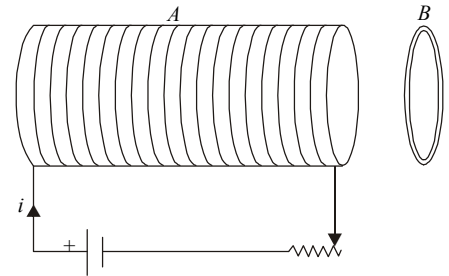


Fig. 26.53

An aluminium ring B faces an electromagnet A . The current i through A can be altered.

- (a) If i increases, A will repel B .
 (b) If i increases, A will attract B .
 (c) If i decreases, A will attract B .
 (d) If i decreases, A will repel B .
8. A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is
- (a) clockwise
 (b) anti-clockwise
 (c) zero
 (d) clockwise or anti-clockwise depending on whether the resistance is increased or decreased
9. A conducting square loop of side l and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnitude field B exists along the perpendicular to the plane of the loop as shown in figure. The current induced in the loop is

- (a) $\frac{Blv}{R}$ clockwise (b) $\frac{Blv}{R}$ anti-clockwise
 (c) $\frac{2Blv}{R}$ anti-clockwise (d) zero

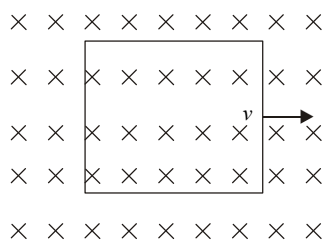


Fig. 26.54

10. A rod AB moves with a uniform velocity v in a uniform magnetic field as shown in figure.

- (a) The rod becomes electrically charged.
 (b) The end A becomes positively charged.
 (c) The end B becomes positively charged.
 (d) The rod becomes hot because of Joule heating.

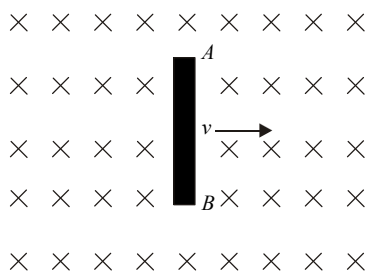


Fig. 26.55

11. A conducting rod is moved with a constant velocity v in a magnetic field. A potential difference appears across the two ends

- (a) if $\vec{v} \parallel \vec{l}$ (b) if $\vec{v} \perp \vec{B}$
 (c) if $\vec{l} \perp \vec{B}$ (d) none of these

12. L , C and R represent the physical quantities inductance, capacitance and resistance respectively. Which of the following combinations have dimensions of frequency?

- (a) $\frac{1}{RC}$ (b) $\frac{R}{L}$
 (c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$

13. An LR circuit with a battery is connected at $t=0$. Which of the following quantities is not zero just after the connection?

- (a) current in the circuit
 (b) magnetic field energy in the inductor

- (c) power delivered by the battery
 (d) emf induced in the inductor

14. A metal sheet is placed in front of a strong magnetic pole. A force is needed to

- (a) hold the sheet there if the metal is magnetic
 (b) hold the sheet there if the metal is nonmagnetic
 (c) move the sheet away from the pole with uniform velocity if the metal is magnetic
 (d) move the sheet away from the pole with uniform velocity if the metal is nonmagnetic.

Neglect any effect of paramagnetism, diamagnetism and gravity.

15. The switches in Fig. 26.56 (a) and (b) are closed at $t=0$ and reopened after a long time at $t=t_0$.

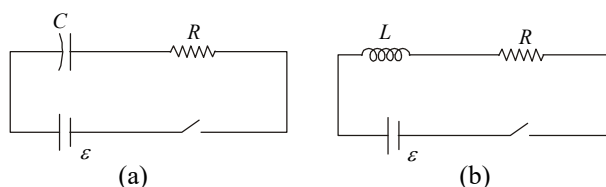


Fig. 26.56

- (a) The charge on C just after $t=0$ is εC .
 (b) The charge on C long after $t=0$ is εC .
 (c) The current in L just before $t=t_0$ is $\frac{\varepsilon}{R}$.
 (d) The current in L long after $t=t_0$ is $\frac{\varepsilon}{R}$.

16. A conducting loop is placed in a uniform magnetic field with its plane perpendicular to the field. An emf is induced in the loop if

- (a) it is translated
 (b) it is rotated about its axis
 (c) it is rotated about a diameter
 (d) it is deformed

17. Two solenoids have identical geometrical construction but one is made of thick wire and the other of thin wire. Which of the following quantities are different for the two solenoids?

- (a) self-inductance
 (b) rate of Joule heating if the same current goes through them
 (c) magnetic field energy if the same current goes through them
 (d) time constant if one solenoid is connected to one battery and the other is connected to another battery

18. A bar magnet is moved along the axis of a copper ring placed far away from the magnet. Looking from the

side of the magnet, an anti-clockwise current is found to be induced in the ring. Which of the following may be true ?

- (a) The south pole faces the ring and the magnet moves towards it.
 - (b) The north pole faces the ring and the magnet moves towards it.
 - (c) The south pole faces the ring and the magnet moves away from it.
 - (d) The north pole faces the ring and the magnet moves away from it.
19. A constant current i is maintained in a solenoid. Which of the following quantities will increase if an iron rod is inserted in the solenoid along its axis ?
- (a) Magnetic field at the centre
 - (b) Magnetic flux linked with the solenoid
 - (c) Self-inductance of the solenoid
 - (d) Rate of Joule heating

20.

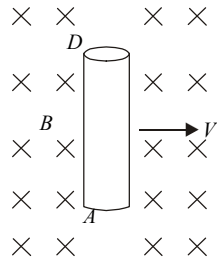


Fig. 26.57

The conductor AD moves to the right in a uniform magnetic field directed into the paper.

- (a) The free electrons in AD will move towards A .
- (b) D will acquire a positive potential with respect to A .
- (c) If D and A are joined by a conductor externally, a current will flow from A to D in AD .
- (d) The current in AD flows from lower to higher potential.

21.

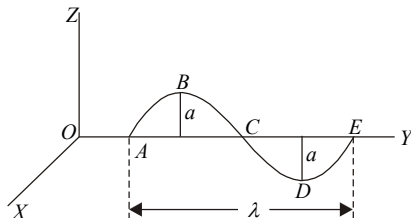


Fig. 26.58

The conductor $ABCDE$ has the shape shown. It lies in the yz plane, with A and E on the y -axis. When it moves with a velocity v in a magnetic field B , an emf ε is induced between A and E .

- (a) $\varepsilon = 0$, if v is in the y -direction and B is in the x -direction.
- (b) $\varepsilon = 2Bav$, if v is in the y -direction and B is in the x -direction.
- (c) $\varepsilon = B\lambda v$, if v is in the z -direction and B is in the x -direction.
- (d) $\varepsilon = B\lambda v$, if v is in the x -direction and B is in the z -direction.

22. The loop shown moves with a velocity v in a uniform magnetic field of magnitude B , directed into the paper. The potential difference between P and Q is ε .

- (a) $\varepsilon = \frac{1}{2}B/v$
- (b) $\varepsilon = B/v$
- (c) P is positive with respect to Q .
- (d) Q is positive with respect to P .

23. The magnitude of the earth's magnetic field at the north pole is B_0 . A horizontal conductor of length l moves with a velocity v . The direction of v is perpendicular to the conductor. The induced emf is

- (a) zero, if v is vertical
- (b) B_0lv , if v is vertical
- (c) zero, if v is horizontal
- (d) B_0lv , if v is horizontal

24.

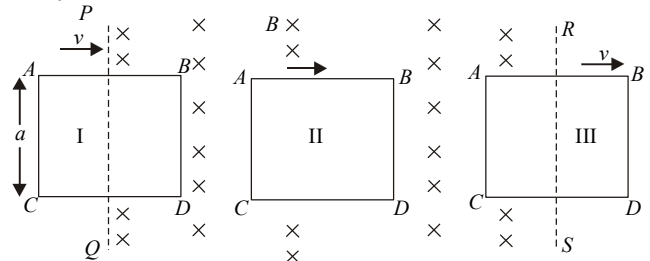


Fig. 26.59

A square loop $ABCD$ of edge a moves to the right with a velocity v , parallel to AB . There is a uniform magnetic field of magnitude B , directed into the paper, in the region between PQ and RS only. I, II and III are three positions of the loop.

- (a) The emf induced in the loop has magnitude Bav in all three positions.
- (b) The induced emf is zero in position II.
- (c) The induced emf is anti-clockwise in position I.
- (d) The induced emf is clockwise in position III.

25.

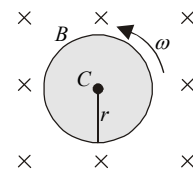


Fig. 26.60

A conducting disc of radius r spins about its axis with an angular velocity ω . There is a uniform magnetic field of magnitude B perpendicular to the plane of the disc. C is the centre of the ring.

- (a) No emf is induced in the disc.
- (b) The potential difference between C and the rim is $\frac{1}{2}Br^2\omega$.
- (c) C is at a higher potential than the rim.
- (d) Current flows between C and the rim.

26. A vertical conducting ring of radius R falls vertically in a horizontal magnetic field of magnitude B . The direction of B is perpendicular to the plane of the ring. When the speed of the ring is v ,

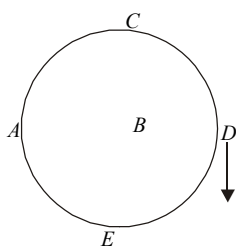


Fig. 26.61

- (a) no current flows in the ring
- (b) A and D are at the same potential
- (c) C and E are at the same potential
- (d) the potential difference between A and D is $2BRv$, with D at a higher potential

27. In the previous question, let E be the electric intensity at any point on the ring.

- (a) E is tangential to the ring everywhere.
- (b) $E = 0$ everywhere on the ring.
- (c) $E = r \frac{dB}{dt}$.
- (d) $E = \frac{1}{2}r \frac{dB}{dt}$.

28.

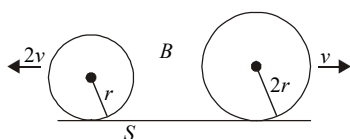


Fig. 26.62

Two conducting rings of radii r and $2r$ move in opposite directions with velocities $2v$ and v respectively on a conducting surface S . There is a uniform magnetic field of magnitude B perpendicular to the plane of the rings. The potential difference between the highest points of the two rings is

- (a) zero
- (b) $2rvB$
- (c) $4rvB$
- (d) $8rvB$

29. A nonconducting ring of radius r has charge Q . A magnetic field perpendicular to the ring changes at the rate $\frac{dB}{dt}$. The torque experienced by the ring is

- (a) zero
- (b) $Qr^2 \frac{dB}{dt}$
- (c) $\frac{1}{2}Qr^2 \frac{dB}{dt}$
- (d) $\pi r^2 Q \frac{dB}{dt}$

30. The magnetic field perpendicular to the plane of a conducting ring of radius r changes at the rate $\frac{dB}{dt}$.

- (a) The emf induced in the ring is $\pi r^2 \frac{dB}{dt}$.
- (b) The emf induced in the ring is $2\pi r \frac{dB}{dt}$.
- (c) The potential difference between diametrically opposite points on the ring is half of the induced emf.
- (d) All points on the ring are at the same potential.

31. The SI unit of inductance, the henry, can be written as

- (a) weber / ampere
- (b) volt second / ampere
- (c) joule / ampere²
- (d) ohm second

32.

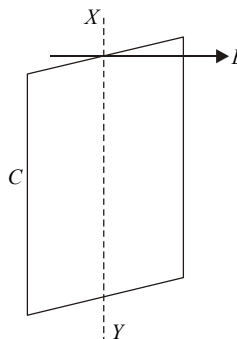


Fig. 26.63

A flat coil, C , of n turns, area A and resistance R is placed in a uniform magnetic field of magnitude B . The plane of the coil is initially perpendicular to B . If the coil is rotated by an angle about the axis XY , charge of amount Q flows through it.

- (a) If $\theta = 90^\circ$, $Q = \frac{BAN}{R}$.
- (b) If $\theta = 180^\circ$, $Q = \frac{BAN}{R}$.
- (c) If $\theta = 180^\circ$, $Q = 0$.
- (d) If $\theta = 360^\circ$, $Q = 0$.

33. In question 32. if the coil rotates about XY with a constant angular velocity ω , the emf induced in it
- is zero
 - changes nonlinearly with time
 - has a constant value $= BAN\omega$
 - has a maximum value $= BAN\omega$
34. A cycle wheel with 64 spokes is rotating with N rotations per second at right angles to horizontal component of magnetic field. The induced emf generated between its axle and rim is E . If the number of spokes is reduced to 32 then the value of induced emf will be
- E
 - $2E$
 - $\frac{E}{2}$
 - $\frac{E}{4}$
35. When a current of 5 A flows in the primary coil then the flux linked with the secondary coil is 200 weber . The value of coefficient of mutual induction will be
- 1000 H
 - 40 H
 - 195 Henry
 - 205 H
36. The direction of induced current in a coil or circuit is such that it opposes the very cause of its production. This law is given by
- Faraday
 - Kirchoff
 - Lenz
 - Ampere
37. The expression for induced charge in a coil is
- $q = \frac{N}{R}(\phi_1 - \phi_2)$
 - $q = R(\phi_1 - \phi_2)$
 - $q = \frac{NR}{(\phi_1 - \phi_2)}$
 - $q = (\phi_1 - \phi_2) / NR$
38. When a conductor is rotated in a perpendicular magnetic field then its free electrons
- move in the field direction
 - move at right angles to field direction
 - remain stationary
 - move opposite to field direction
39. The maximum value of induced emf in a coil rotating in magnetic field does not depend on
- the resistance of coil
 - the number of turns in the coil
 - the area of the coil
 - rotational frequency of the coil
40. The coefficient of mutual induction between two coils is 4 H . If the current in the primary reduces from 5 A to zero in 10^{-3} second then the induced emf in the secondary coil will be

- 10^4 V
- $25 \times 10^3\text{ V}$
- $2 \times 10^4\text{ V}$
- $15 \times 10^3\text{ V}$

41. A transformer is used to
- convert DC into AC
 - convert AC into DC
 - obtain the required DC voltage
 - obtain the required AC voltage
42. When a conducting ring is moved in a magnetic field then the total charge induced in it depends on
- initial magnetic flux
 - final magnetic flux
 - the rate of change of magnetic flux
 - the total change in magnetic flux
43. When the number of turns per unit length in a solenoid is doubled then its coefficient of self induction will become
- half
 - double
 - four times
 - unchanged
44. The number of turns in the primary and secondary coils of a transformer are 100 and 300 respectively. If the input power is 60 watt the output power will be
- $3 \times 10^3\text{ Watt}$
 - 20 Watt
 - 60 Watt
 - 180 Watt
45. A magnetic field is directed normally downwards through a metallic frame as shown in the figure. On increasing the magnetic field

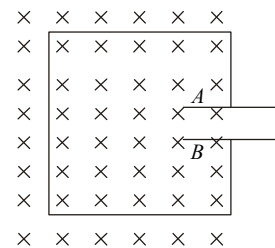


Fig. 26.64

- plate B will be positively charged
 - plate A will be positively charged
 - none of the plates will be positively charged
 - all of the above
46. The coefficient of mutual induction between two coils is 1.25 H . If the rate of fall of current in the primary is 80 As^{-1} , then the induced emf in the secondary coil will be
- 100 V
 - 64 V
 - 12.5 V
 - 0.016 V
47. Same current is flowing in two identical coaxial circular coils. On looking at the coils from a point at the centre between them, the current in one coil appears to be flowing in clockwise direction. If the

two coils are displaced towards each other, then the value of current

- (a) in each coil will decrease
- (b) in each coil will increase
- (c) in each coil will remain unchanged
- (d) nothing can be predicted

48. A 1.2 m wide railway track is parallel to magnetic meridian. The vertical component of earth's magnetic field is 0.5 Gauss. When a train runs on the rails at a speed of 60 Km/hr, then the induced potential difference between the ends of its axle will be

- (a) 10^{-4} V
- (b) 2×10^{-4} V
- (c) 10^{-3} V
- (d) zero

49. The number of turns in an air core solenoid of length 25 cm and radius 4 cm is 100. Its self inductance will be

- (a) 5×10^{-4} H
- (b) 2.5×10^{-4} H
- (c) 5.4×10^{-3} H
- (d) 2.5×10^{-3} H

50. Two coils P and Q are lying a little distance apart coaxially. If a current I is suddenly set up in the coil P then the direction of current induced in coil Q will be—

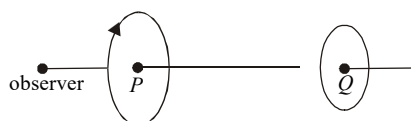


Fig. 26.65

- (a) clockwise
- (b) towards north
- (c) towards south
- (d) anti-clockwise

51. A straight copper wire is moved in a uniform magnetic field such that it cuts the magnetic lines of force. Then

- (a) emf will not be induced
- (b) emf will be induced
- (c) sometimes emf will be induced and sometimes not
- (d) nothing can be predicted

52. A small straight conductor PQ is lying at right angles to an infinite current carrying conductor XY . If the conductor PQ is displaced on metallic rails parallel to the conductor XY then the direction of induced emf in PQ will be

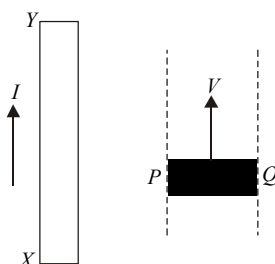


Fig. 26.66

- (a) from Q to P
- (b) from P to Q
- (c) vertically downwards
- (d) vertically upwards

53. A conducting rod PQ is moving parallel to X -axis in a uniform magnetic field directed in positive Y -direction. The end P of the rod will become

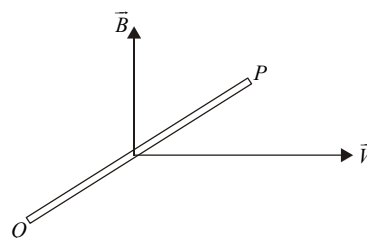


Fig. 26.67

- (a) negative
- (b) positive
- (c) neutral
- (d) sometimes negative

54. Two coils P and Q are lying parallel and very close to each other. Coil P is connected to an AC source whereas Q is connected to a sensitive galvanometer. On pressing key K

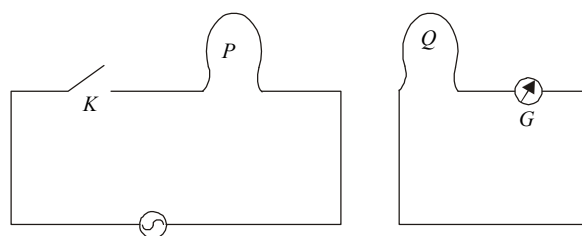


Fig. 26.68

- (a) small variations are observed in the galvanometer for applied 50 Hz voltage
- (b) deflections in the galvanometer can be observed for applied voltage of 1 Hz to 2 Hz.
- (c) no deflection in the galvanometer will be observed
- (d) constant deflection will be observed in the galvanometer for 50 Hz supply voltage

55. The dimensions of RC are

- (a) MLT^{-1}
- (b) $M^0L^0T^1$
- (c) $M^0L^0T^{-1}$
- (d) $M^{-1}L^{-1}T^{-1}$

56. A coil of insulating wire is connected to battery. If it is moved towards a galvanometer then its pointer gets deflected because

- (a) the coil behaves like a magnet
- (b) induced current is produced in the coil
- (c) the number of turns in the galvanometer coil remains constant
- (d) none of the above

57. The coefficient of self induction of a coil is given by

$$(a) L = \left(-\frac{dI}{dt} \right) \quad (b) L = -\frac{edI}{dt}$$

$$(c) L = \frac{dI}{edt} \quad (d) L = \frac{dI}{dt} e^2$$

58. When a coil of cross-sectional area A and number of turns N is rotated in a uniform magnetic field B with angular velocity ω , then the maximum emf induced in the coil will be

$$(a) BNA \quad (b) \frac{Ba\omega}{N}$$

$$(c) BNA\omega \quad (d) \text{zero}$$

59. Two inductance coils, of same self inductance, are connected in parallel and the distance between them is large. The resultant self inductance of the coil will be

$$(a) \frac{L}{4} \quad (b) 2L$$

$$(c) L \quad (d) \frac{L}{2}$$

60. The resistance coils in a resistance box are made of double folded wire so that their

- (a) self induction effect is nullified
(b) self inductance is maximum
(c) induced emf is maximum
(d) induced emf is zero

61. If a spark is produced on removing the load from an AC circuit then the element connected in the circuit is

- (a) high resistance (b) high capacitance
(c) high inductance (d) high impedance.

62. The time constant in an L - R circuit is that time in which the value of current in the circuit becomes

$$(a) I_0 \quad (b) \frac{I_0}{2}$$

$$(c) 63\% I_0 \quad (d) 37\% I_0$$

63. The expression for the induced emf generated in a coil as a result of change in magnetic flux linked with it is

$$(a) e = -A \frac{dB}{dt} \quad (b) e = -B \frac{dA}{dt}$$

$$(c) e = \frac{d}{dt}(\vec{A} \cdot \vec{B}) \quad (d) e = \frac{d}{dt}(\vec{A} \times \vec{B})$$

64. The turns ratio (r) for a step-up transformer is

- (a) $r < 1$ (b) $r = 1$
(c) $r > 1$ (d) $r = 0$

65. Eddy currents can be minimised by

- (a) using a laminated iron core
(b) moving the conductor rapidly
(c) moving the conductor slowly
(d) using a metallic core

66. The cause of production of eddy currents is

- (a) the motion of a conductor in a varying magnetic field
(b) the motion of an insulator in a varying magnetic field
(c) current flowing in a conductor
(d) current flowing in an insulator

67. The correct relation between the impedance of secondary coil with that of primary coil is

$$(a) Z_S = Z_P \quad (b) Z_S = Z_P \frac{N_S}{N_P}$$

$$(c) Z_S = Z_P \left(\frac{N_S}{N_P} \right)^2 \quad (d) Z_S = Z_P \left(\frac{N_P}{N_S} \right)^2$$

68. The long distance transmission of electrical energy is done at

- (a) high potential and low current
(b) low potential and high current
(c) high potential and high current
(d) low potential and low current

69. Starter is used in

- (a) high power electric motors
(b) low power electric motors
(c) transformers
(d) galvanometers

70. If the turns ratio of a transformer is 2 and the impedance of primary coil is 250 Ω then the impedance of secondary coil will be

$$(a) 1000 \Omega \quad (b) 500 \Omega$$

$$(c) 250 \Omega \quad (d) 125 \Omega$$

71. Number of turns in a generator coil is 10. The area of this coil is $4 \times 10^{-2} \text{m}^2$. This coil is rotating at the rate of 20 rotations/sec, about an axis lying in its own plane in a perpendicular magnetic field of 0.3 Tesla. The maximum emf induced in the coil will be

$$(a) 30.2 V \quad (b) \text{Zero}$$

$$(c) 15.1 V \quad (d) 60.4 V$$

72. The efficiency of a transformer is

- (a) $\eta < 1$ (b) $\eta = 1$
(c) $\eta > 1$ (d) $\eta = 0$

73. Which of the remain constant in a transformer?

- (a) current (b) potential
(c) power (d) frequency

74. If the current in the primary coil and number of turns in it are I_p and N_p respectively and the number of turns and current in the secondary are N_s and I_s respectively then the value of $N_s : N_p$ will be

- (a) $I_s : I_p$ (b) $I_p : I_s$
(c) $I_s^2 : I_p^2$ (d) $I_p^2 : I_s^2$

75. A metallic circular ring is suspended by a string and is kept in a vertical plane. When a magnet is approached towards the ring then it will

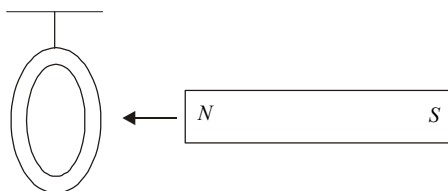


Fig. 26.69

- (a) remain stationary
(b) get displaced away from the magnet
(c) get displaced towards the magnet
(d) nothing can be said

76. A current is flowing in a wire C as shown in the figure. The force on this conducting wire will be

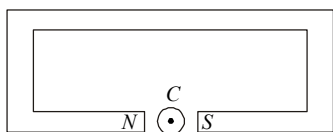


Fig. 26.70

- (a) towards right (b) towards left
(c) downwards (d) upwards

77. The voltage in the primary and the secondary coils of a step up transformer are 200 V and 4 KV respectively. If the current in the primary is 1 ampere then the current in the secondary coil will be

- (a) 50 mA (b) 500 mA
(c) 5 A (d) 5 mA

78. If 2.2 kilowatt power is being transmitted at 44 KV on a 20 Ω line, then power loss will be

- (a) 0.1 watt (b) 1.4 watt
(c) 100 watt (d) 0.05 watt

79. The rate of change of magnetic flux density through a circular coil of area 10 m² and number of turns 100 is 10³ Wb/m²/s. The value of induced emf will be

- (a) 10⁻² V (b) 10⁻³ V
(c) 10 V (d) 10³ V

80. The magnetic fields through two identical rings made of copper and wood are changing at the same rate. The induced electric field in copper ring will be

- (a) more than that in the wooden ring
(b) less than that in the wooden ring
(c) finite and that in the wooden ring will be zero
(d) same as that in the wooden ring

81. The length of side of a square coil is 50 cm and number of turns in it is 100. If it is placed at right angles to such a magnetic field which is changing at the rate of 4 Tesla/s then induced emf in the coil will be

- (a) 0.1 V (b) 1.0 V
(c) 10 V (d) 100 V

82. The number of turns in the primary and secondary coils of a step-down transformer are 200 and 50 respectively. If the power in the input is 100 watt at 1A then the output power and current will respectively be

- (a) 100 W, 2 A (b) 200 W, 2 A
(c) 400 W, 4 A (d) 100 W, 4 A

83. A transformer changes 220 V to 22 V. If the currents in the primary and secondary coils are 10 A and 70 A respectively then its efficiency will be

- (a) 100% (b) 90%
(c) 70% (d) 80%

84. If the magnetic field in the following figure is increased then



Fig. 26.71

- (a) plate A of the condenser will get positively charged
(b) plate B of the condenser will get positively charged
(c) the condenser will not be charged
(d) both the plates will be charged alternately

85. An e.m.f. of 15 volt is applied in a circuit of inductance 5 henry and resistance 10 Ω . The ratio of currents flowing at $t = \infty$ and $t = 1$ second will be

- (a) $\frac{e^{1/2}}{e^{1/2} - 1}$ (b) $\frac{e^2}{e^2 - 1}$
(c) $1 - e^{-1}$ (d) e^{-1}

86. Two circular conducting loops of radii R_1 and R_2 are lying concentrically in the same plane. If $R_1 > R_2$ then the mutual inductance (M) between them will be proportional to

(a) $\frac{R_1}{R_2}$

(b) $\frac{R_2}{R_1}$

(c) $\frac{R_1}{R_2}$

(d) $\frac{R_2^2}{R_1}$

87. A coil of area 80 cm^2 and number of turns 50 is rotating about an axis perpendicular to a magnetic field of 0.05 Tesla at 2000 rotations per minute. The maximum value of emf induced in it will be

(a) $200\pi \text{ volt}$

(b) $\frac{10\pi}{3} \text{ volt}$

(c) $\frac{4\pi}{3} \text{ volt}$

(d) $\frac{2}{3} \text{ volt}$

88. An athlete with 3 m long iron rod in hand runs towards east with a speed of 30 km/hour . The horizontal component of earth's magnetic field is $4 \times 10^{-5} \text{ Wb/m}^2$. If he runs with the rod in horizontal and vertical positions then the induced emf generated in the rod in two cases will be

(a) zero in vertical position and $1 \times 10^{-3} \text{ volt}$ in horizontal position.

(b) $1 \times 10 \text{ volt}$ in vertical position and zero volt in horizontal position.

(c) zero in both positions

(d) $1 \times 10^{-3} \text{ volt}$ in both positions

89. A copper disc of radius 0.1 m rotates about its axis in a uniform magnetic field of 0.1 Tesla at 10 rotations per second. The plane of the disc remains normal to the magnetic field. The induced emf along the radius of the disc will be

(a) $\frac{\pi}{10} \text{ volt}$

(b) $\frac{2\pi}{10} \text{ volt}$

(c) $\frac{\pi}{100} \text{ volt}$

(d) $2\pi \text{ volt}$

90. The distance between the ends of wings of an aeroplane is 50 m . It is flying in a horizontal plane at a speed of 360 Km/hour . The vertical component of earth's magnetic field at that place is $2.0 \times 10^{-4} \text{ Wb/m}^2$, then the potential difference induced between the ends of the wings will be

(a) 0.1 volt

(b) 1.0 volt

(c) 0.2 volt

(d) 0.01 volt

91. The coefficients of self induction of two coils are $L_1 = 8 \text{ mH}$ and $L_2 = 2 \text{ mH}$ respectively. The current rises in the two coils at the same rate. The power given to the two coils at any instant is same. The ratio of currents flowing in the coils will be

(a) $\frac{i_1}{i_2} = \frac{1}{4}$

(b) $\frac{i_1}{i_2} = \frac{4}{1}$

(c) $\frac{i_1}{i_2} = \frac{3}{4}$

(d) $\frac{i_1}{i_2} = \frac{4}{3}$

92. In the above problem, the ratio of induced emf's in the coils will be

(a) $\frac{V_1}{V_2} = 4$

(b) $\frac{V_1}{V_2} = \frac{1}{4}$

(c) $\frac{V_1}{V_2} = \frac{1}{2}$

(d) $\frac{V_1}{V_2} = \frac{1}{2}$

93. In Q. 91, the ratio of energies stored will be

(a) $\frac{W_1}{W_2} = 4$

(b) $\frac{W_1}{W_2} = \frac{1}{4}$

(c) $\frac{W_1}{W_2} = \frac{3}{4}$

(d) $\frac{W_1}{W_2} = \frac{4}{3}$

94. The number of turns in a coil of wire of fixed radius is 600 and its self inductance is 108 mH . The self inductance of a coil of 500 turns will be

(a) 74 mH

(b) 75 mH

(c) 76 mH

(d) 77 mH

95. Two coils X and Y are lying in a circuit. The change in current in X is 2 ampere and change in magnetic flux in Y is 0.4 weber . The coefficient of mutual induction between the coils will be

(a) 0.2 Henry

(b) 5 Henry

(c) 0.8 Henry

(d) 0.4 Henry

96. A millivoltmeter is connected in parallel to an axle of the train running with a speed of 180 Km/hour . If the vertical component of earth's magnetic field is $0.2 \times 10^{-4} \text{ Wb/m}^2$ and the distance between the rails is 1 m , then the reading of voltmeter will

(a) 10^{-2} volt

(b) 10^{-4} volt

(c) 10^{-3} volt

(d) 1 volt

97. The north pole of long horizontal bar magnet is carried towards a vertical conducting plane. The direction of induced current in the conducting plane will be

(a) horizontal

(b) vertical

(c) clockwise

(d) anti-clockwise

98. If the voltage applied to a motor is 200 volt and back emf is 160 volt , then the efficiency of the motor will be

(a) 100%

(b) 80%

(c) 50%

(d) 25%

99. An inductance L and a resistance R are connected to a battery. After sometime the battery is removed but L and R remain connected in the closed circuit. The value of current will reduce to 37% of its maximum value in
- (a) RL second (b) $\frac{R}{L}$ second
(c) $\frac{L}{R}$ second (d) $\frac{1}{LR}$ second
100. The magnetic flux linked with a coil is $\phi \leq 8t^2 + 3t + 5$ Weber. The induced emf in fourth second will be
(a) 16 V (b) 139 V
(c) 67 V (d) 145 V
101. The self inductance of a coil is 5 mH. If a current of 2 A is flowing in it then the magnetic flux produced in the coil will be
(a) 0.01 Weber (b) 10 Weber
(c) zero (d) 1 Weber
102. The magnetic flux in a coil of 100 turns increases by 12×10^3 Maxwell in 0.2 second due to the motion of a magnet. The emf induced in the coil will be—
(a) 6 V (b) 0.6 V
(c) 0.06 V (d) 60 V
103. A proton enters a perpendicular magnetic field of 20 Tesla. If the velocity of proton is 4×10^7 m/s then the force acting on it will be
(a) 1.28 Newton (b) 1.28×10^{-11} Newton
(c) 12.8 Newton (d) 12.8×10^{-11} Newton
104. A cylindrical bar magnet is lying along the axis of a circular coil. If the magnet is rotated about the axis of the coil then
(a) emf will be induced in the coil
(b) only induced current will be generated in the coil
(c) no current will be induced in the coil.
(d) both emf and current will be induced in the coil
105. The quantity in electricity which is equivalent to mass is
(a) L (b) C
(c) R (d) I
106. Eddy currents are not used in
(a) speedometer of vehicles
(b) induction furnace
(c) electromagnetic damping
(d) diffraction of X-rays
107. In the following figure the bulb will start lighting suddenly if

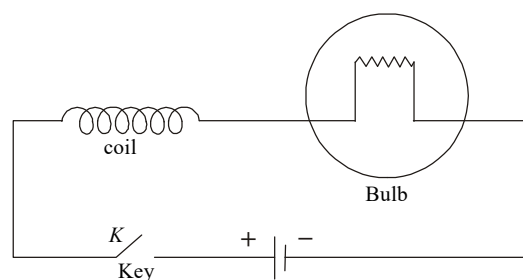


Fig. 26.72

- (a) key is closed
(b) key is opened
(c) key is either closed or opened
(d) nothing is done
108. A coil of area A_0 is lying in such a magnetic field whose value changes from B_0 to $4B_0$ in t seconds. The induced emf in the coil will be
(a) $\frac{4B_0}{A_0 t}$ (b) $\frac{4B_0 A_0}{t}$
(c) $\frac{3B_0 A_0}{t}$ (d) $\frac{3B_0}{A_0 t}$
109. Electric current is flowing in same direction in two coaxial coils. On increasing distance between the two coils the value of current will
(a) decrease (b) increase
(c) remain unchanged (d) nothing can be said.
110. If the number of turns in a coil is N then the value of self inductance of the coil will become
(a) N times (b) N^2 times
(c) N^{-2} times (d) N^0 times
111. The value of mutual inductance can be increased by
(a) decreasing N (b) increasing N
(c) winding the coil on wooden frame
(d) winding the coil on china clay
112. The value of coefficient of mutual induction for the arrangement of two coils shown in the figure will be

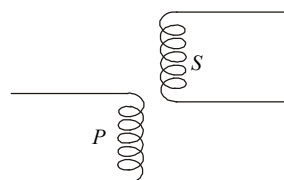


Fig. 26.73

- (a) zero (b) maximum
(c) negative (d) positive
113. A conducting rod of length L is falling with velocity V in a uniform horizontal magnetic field B normal to the

rod. The induced emf between the ends the rod will be

- (a) $2B\ell v$ (b) zero
(c) $B\ell v$ (d) $\frac{B\ell v}{2}$

114. A coil of area 0.01m^2 is lying in a perpendicular magnetic field of 0.1 Tesla . If a current of 10 A is passed in it then the maximum torque acting on the coil will be

- (a) 0.01 N/m (b) 0.001 N/m
(c) 1.1 N/m (d) 0.8 N/m

PASSAGE 1

Read the following passage and answer the questions given at the end.

Suppose an inductor of self-inductance L and a resistor of resistance R are connected in series across the terminals of a capacitor. If the capacitor is initially charged, it starts to discharge at the instant the connections are made but, because of i^2R losses in the resistor, the energy of the inductor when the capacitor is completely discharged is less than the original energy of the capacitor. In the same way the energy of the capacitor when the magnetic field has collapsed is still smaller, and so on.

If the resistance R is relatively small, the circuit oscillates, but with damped harmonic motion, as illustrated in Fig. 26.74 (a). As R is increased the oscillations die out more rapidly. At a sufficiently large value of R , the circuit no longer oscillates and is said to be critically damped, as in Fig. 26.74 (b). For still larger resistances it is over damped as in Fig. 26.74 (c).

With the proper electronic circuitry, energy can be led into an R-L-C circuit at the same rate as that at which it is dissipated by i^2R (radiation losses). In effect, if negative resistance is inserted in the circuit so that its total resistance is zero, the circuit then oscillates with sustained oscillations, as does the idealised circuit with no resistance.

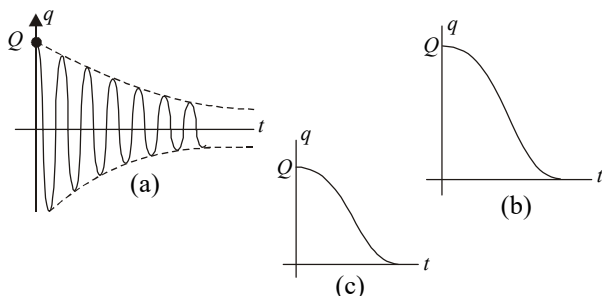


Fig. 26.74

- To obtain sustained oscillations one should use
(a) negative feedback (b) positive feedback
(c) amplifier (d) negative resistance
- Can RC circuit be used to make oscillator? How?

- (a) Yes, using 3 RC sections with a CE amplifier
(b) Yes, using two RC sections and a CE amplifier
(c) Yes, using Wein's bridge
(d) Yes, using op-amp

3. In negative resistance

- (a) power is dissipated (b) power is generated
(c) power is consumed (d) none

4. Negative resistance is equivalent to

- (a) positive feedback (b) negative feedback
(c) mixed feedback (d) any type of feedback

Solution 1. (b, d)

Solution 2. (a, c)

Solution 3. (b)

Solution 4. (a)

PASSAGE 2

Read the following passage and answer the questions given at the end.

The Faraday law states that $\oint E_n \cdot dl = \frac{d\phi}{dt}$ (1)

As an example, suppose the loop in Fig. 26.75 is a circle of radius r . Because of the axis symmetry, the nonelectrostatic field E_n has the same magnitude at all points on the circle and is everywhere tangent to this circle. The line integral in Eq. (1) becomes simply the magnitude E_n times the circumference $2\pi r$ of the circle. Thus, the induced electric field at a distance r from the axis is given by

$$E_n = \frac{1}{2\pi r} \frac{d\phi}{dt}$$

where ϕ is the flux through a circle of radius r .

If it should happen that at a particular instant the B field is uniform across this circle (as is not the case with the solenoid) then

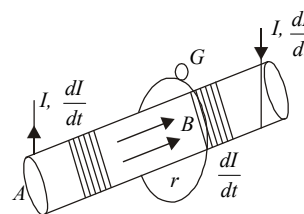


Fig. 26.75

$$\phi = \pi^2 B$$

$$\frac{d\phi}{dt} = \pi^2 \frac{dB}{dt}$$

- The electric field produced in the case mentioned in the passage is

- (a) conservative (b) nonconservative
(c) polar (d) nonpolar
(e) none
2. A magnet with its north pole pointing downwards along the axis of an open ring as illustrated. As the magnet reaches close to the centre of the ring

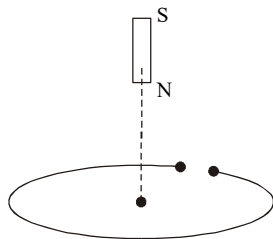


Fig. 26.76

- (a) its acceleration becomes greater than g
(b) its acceleration becomes less than g
(c) its acceleration remains equal to g
(d) none of these
3. A time varying (AC) current is passed through a wire wound resistance. Its conductivity
- (a) is same as DC conductivity
(b) increases as compared to DC conductivity
(c) decreases as compared to DC conductivity
(d) none of these
4. Reciprocal of absorption coefficient in optical frequency range gives
- (a) depth of penetration
(b) strength of absorption
(c) reflectivity
(d) refractive index
5. The induced emf produced due to a varying magnetic flux is V . A charge q is taken around a loop. Then work done is
- (a) zero (b) qV
(c) $\frac{qV}{2}$ (d) none

Solution 1. (b)**Solution** 2. (c)**Solution** 3. (c)**Solution** 4. (a)**Solution** 5. (b)**PASSAGE 3**

Read the following passage and answer the questions given at the end.

You have designed a new type of exercise machine with an extremely simple mechanism. A vertical bar PQ 3 m long is free to move on two bars (rails). The resistance of the bar and rails can be neglected. The entire apparatus is placed on a horizontal magnetic field of strength 0.25 T . When you push the bar to the left or right a current is set up in the circuit including the bar PQ. The magnetic field exerts a force on the bar and opposes its motion. One has to exert force against it. The design is made such that one shall have a rate of doing work of 25 W while moving the rod at 2 ms^{-1} .

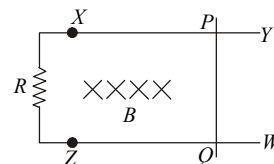


Fig. 26.77

1. What should be the value of R ?
- (a) $0.9\ \Omega$ (b) $0.09\ \Omega$
(c) $9.0\ \Omega$ (d) $9 \times 10^{-4}\ \Omega$
2. What should be R if the power exerted be 50 W ?
- (a) $0.09\ \Omega$ (b) $0.06\ \Omega$
(c) $0.045\ \Omega$ (d) none of these
3. The prototype shows that magnetic field cannot be kept constant in a wide range of 3 m . Therefore you reduce the length of the bar to 0.2 m keeping all other remaining parameters unchanged. What is the power exerted?
- (a) 1 W (b) $1/25\text{ W}$
(c) $1/9\text{ W}$ (d) 5 W

Solution 1. (b) $F = IlB$,

$$P = F \cdot v = IlBv$$

$$P = Blv \frac{(Blv)}{R} = \frac{(Blv)^2}{R} \text{ or } R = \frac{(Blv)^2}{P}$$

$$R = \frac{(0.25 \times 3 \times 2)^2}{25} = 0.09\ \Omega$$

Solution 2. (c)**Solution** 3. (c) $P = \frac{(Blv)^2}{R}$

$$= \frac{(0.25 \times 0.2 \times 2)^2}{.09} = \frac{(.1)^2}{.09} = 1/9\text{ W}$$

Answers to Questions for Practice

1. (a,d)	2. (a,c)	3. (b)	4. (c)	5. (c)	6. (b,c,d)	7. (a,c)
8. (c)	9. (d)	10. (b)	11. (d)	12. (a,b,c)	13. (d)	14. (a,c,d)
15. (b,c)	16. (c,d)	17. (b,d)	18. (b,c)	19. (a,b,c)	20. (a,b,c,d)	21. (a,c,d)
22. (a,c)	23. (a,d)	24. (b,c,d)	25. (b,c)	26. (c,d)	27. (a,d)	28. (d)
29. (c)	30. (a,d)	31. (a,b,c,d)	32. (a,b,d)	33. (a,d)	34. (a)	35. (b)
36. (c)	37. (a)	38. (b)	39. (a)	40. (c)	41. (d)	42. (c)
43. (c)	44. (c)	45. (a)	46. (a)	47. (a)	48. (c)	49. (b)
50. (a)	51. (b)	52. (c)	53. (b)	54. (b)	55. (b)	56. (b)
57. (a)	58. (c)	59. (d)	60. (a)	61. (c)	62. (c)	63. (c)
64. (c)	65. (a)	66. (a)	67. (c)	68. (a)	69. (a)	70. (a)
71. (c)	72. (a)	73. (d)	74. (b)	75. (b)	76. (d)	77. (a)
78. (d)	79. (d)	80. (d)	81. (d)	82. (d)	83. (c)	84. (b)
85. (b)	86. (d)	87. (c)	88. (c)	89. (c)	90. (b)	91. (a)
92. (a)	93. (b)	94. (b)	95. (a)	96. (c)	97. (d)	98. (b)
99. (c)	100. (c)	101. (a)	102. (c)	103. (d)	104. (c)	105. (a)
106. (d)	107. (c)	108. (c)	109. (b)	110. (b)	111. (b)	112. (a)
113. (c)	114. (a)					

EXPLANATION

23. (a,d) At the poles, the earth's magnetic field is vertical.

26. (c,d) Replace the ring by a diameter perpendicular to its direction of motion. The spin of a ring about its axis causes no emf.

27. (a,d) $\oint_L \vec{E} \cdot d\vec{l} = \text{emf in } L$.

By symmetry in this case, \vec{E} is parallel to $d\vec{l}$ everywhere.

$$\oint_L \vec{E} \cdot d\vec{l} = E \oint d\vec{l} = 2\pi r E = e = \pi r^2 \frac{dB}{dt}$$

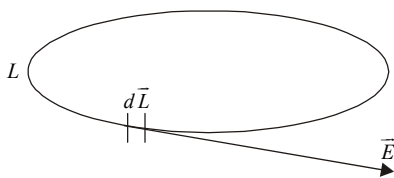


Fig. 26.78

or $E = \frac{1}{2} r \frac{dB}{dt}$.

28. (d)

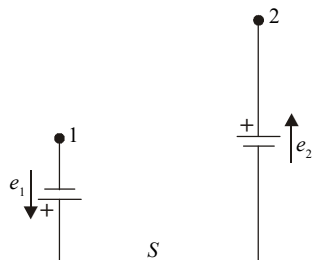


Fig. 26.79

Replace the induced emfs in the rings by cells.

$$\varepsilon_1 = B2r(2v) = 4Brv.$$

$$\varepsilon_2 = B(4r)v = 4Brv.$$

$$V_2 - V_1 = \varepsilon_2 + \varepsilon_1 = 8Brv.$$

29. (c) The same emf is induced in a conducting and a

nonconducting ring, and it is equal to $\pi r^2 \frac{dB}{dt}$.

from Q. 27

At any point on ring, $E = \frac{1}{2} r \frac{dB}{dt}$.

For any charge dQ on the ring, force = $dF = EdQ$, tangential to the ring

Torque about the centre due to $dF = d\tau = r dF = rEdQ$.

$$\text{Total torque} = \sum rEdQ = r \left(\frac{1}{2} r \frac{dB}{dt} \right) Q.$$

30. (a, d) $\phi = \pi r^2 B$

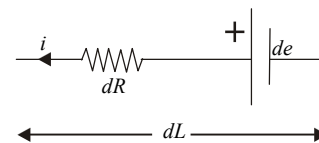


Fig. 26.80

$$e = \phi = \pi r^2 \frac{dB}{dt}.$$

Let R = resistance of the ring

\therefore the current in the ring = $i = e/R$.

Consider a small element dl on the ring.

emf induced in the element $= de$

$$= \left(\frac{e}{2\pi r} \right) dl.$$

Resistance of the element $= dR = \left(\frac{R}{2\pi r} \right) dl.$

p.d. across the element $= -idR + de$

$$= -\left(\frac{e}{R} \right) \left(\frac{R}{2\pi r} \right) dl + \left(\frac{e}{2\pi r} \right) dl = 0.$$

\therefore all points on the ring are at the same potential.

32. (a, b, d) Charge flowing in a circuit $= \frac{\Delta\phi}{R},$

where $\Delta\phi = \text{change in flux} = \phi_{\text{final}} - \phi_{\text{initial}}$

and $R = \text{resistance in circuit}.$

