20

Capacitors

It is a device to store charge or electrostatic energy.

Capacitance is the capacity of a capacitor to store charge. In a capacitor $Q \propto V$ or $Q = CV$; C is called the capacitance.
$$C = (M^{-1}L^{-2}T^4A^2)$$

Fig. 20.1 (a) or (b) represent simple capacitor, Fig. 20.1 (c) represents electrolytic and Fig 20.1 (d) represents variable capacitor (tuner or trimmer).
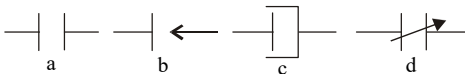

Fig. 20.1

According to shape capacitors are of three types: spherical, parallel plate and cylindrical.

Unit of capacitance is Farad $1F = \dfrac{1C}{1V}$

$1F$ is a very big unit. Therefore smaller units like μF, nF or $\mu\mu F$ (also called pF) are used very commonly.

Spherical Capacitors may be divided into two categories:
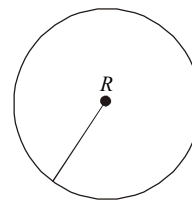**(a) Isolated spherical capacitors**
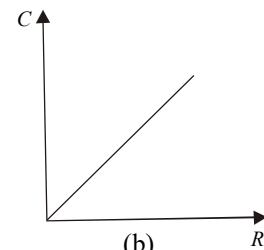
(b) Concentric spherical capacitors

    (a) Isolated spherical capacitor consists of a single sphere. Its capacitance $C = 4\pi\varepsilon_0 R$ i.e. $C \propto R$, where R is radius of the sphere. See Fig. 20.2.

    (b) Two spherical shells (or inner one may be solid) form a concentric spherical capacitor as shown

in Fig. 20.3. Note that normally outer sphere is grounded.

$$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$

(a)
Isolated capacitor

(b)
Capacitance Vs radius

Fig. 20.2

If a dielectric of strength K is introduced between R_1 and R_2

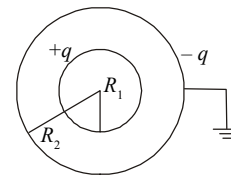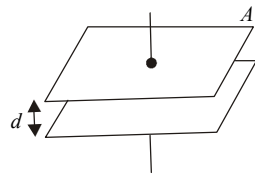$$C = 4\pi\varepsilon_0 K \frac{R_1 R_2}{R_2 - R_1}$$

Fig. 20.3   **Concentric shell capacitor**

Parallel Plate Capacitor If two plates each of area A are separated by a distance d in vacuum as shown in Fig. 20.4
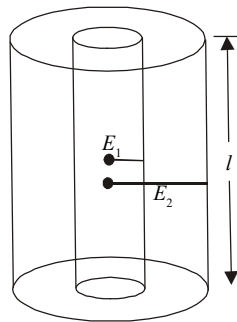
Fig. 20.4 Parallel plate capacitor

then $C = \dfrac{\varepsilon_0 A}{d}$

$C = \dfrac{k\varepsilon_0 A}{d}$ if a dielectric of strength k is completely filled in the gap.

$C = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{k}\right)}$ if the dielectric slab has

thickness $t\,(t < d)$

Capacitance of a cylindrical capacitor shown in Fig. 20.5 is
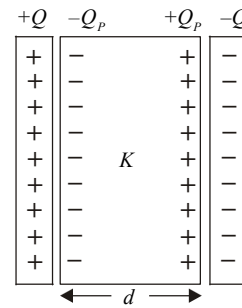

Fig. 20.5 Cylindrical capacitor

$C = \dfrac{2\pi\varepsilon_0 l}{\log_e \dfrac{r_2}{r_1}}$ and capacitance per unit length

$\dfrac{C}{l} = \dfrac{2\pi\varepsilon_0}{\log_e \dfrac{r_2}{r_1}}$

If the space between two cylinders is filled with a dielectric of strength k then $C = \dfrac{2\pi\varepsilon_0 k l}{\log_e \dfrac{r_2}{r_1}}$

Magnitude of induced charge in a dielectric of strength

k is $Q_p = Q\left(1 - \dfrac{1}{k}\right)$

Force between the plates of a capacitor is attractive

and its magnitude is $F = \dfrac{Q^2}{2A\varepsilon_0}$

Fig. 20.6 Polarization illustration

Energy stored in a capacitor

$$U = \frac{1}{2}CV^2 = \frac{QV}{2} = \frac{Q^2}{2C}$$

If the charge is uniformly distributed throughout the

volume then energy stored is $U = \dfrac{1}{2}\int V\rho\,dv$ where dv is

volume element and V is potential difference . Volume density of electric field energy

$$u = \frac{ED}{2} = \frac{\varepsilon_0 E^2}{2} \text{ in free space}$$

volume density of electric field energy in a medium

$$u_{med.} = \frac{\varepsilon_0 K E^2}{2} = \frac{\varepsilon_0 \varepsilon_r E^2}{2}$$

The maximum capacitance of a tuner capacitor (used

for tuning in radio) is $C = \dfrac{\varepsilon_0 A(n-1)}{d}$ where A is the area of

each plate, n is total no. of plates and d is separation between two successive plates. Normally a 11-plate tuner capacitor is available whose ratio of maximum to minimum capacity is $10:1$.

Capacitor in series See Fig 20.7. In series, magnitude of the charge on each plate is equal but voltage across each capacitor is different.
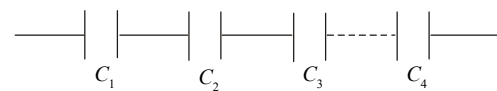
Fig. 20.7 Capacitors in Series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots + \frac{1}{C_n}$$

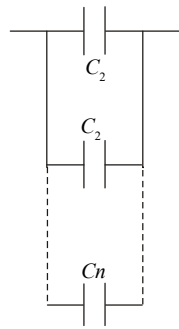If n equal capacitors are in series then $C_{eq} = \dfrac{C}{n}$

Fig. 20.8   **Capacitors in Parallel**

Capacitors in parallel If $C_1, C_2, C_3, ...C_n$ are connected in parallel as shown in Fig. 20.8 then

$$C_{eq} = C_1 + C_2 + C_3 .... + C_n$$

Note that in parallel, charge on each capacitor is different while potential drop or voltage across each capacitor is equal.

If n equal capacitors are in parallel then $C_{eq} = nC$.

There are four methods to simplify capacitance networks

(a)   Series parallel method

(b)   Wheatstone bridge method

(c)   Charge distribution method

(d)   Star/delta method.

Wheatstone bridge cases Fig. 20.9 illustrates some common representations of Wheatstone bridge.

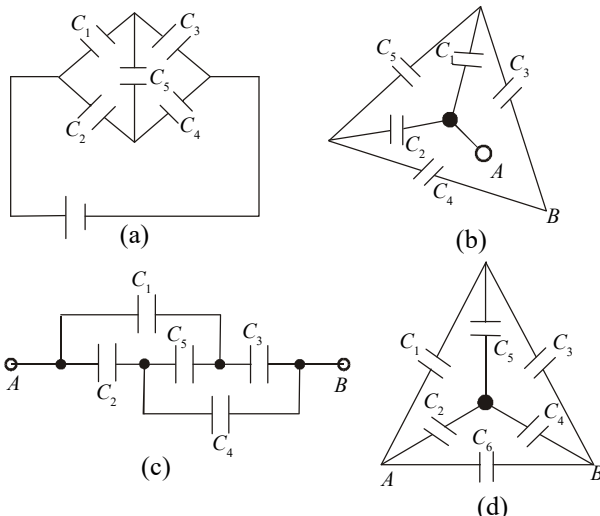If $\dfrac{C_1}{C_2} = \dfrac{C_3}{C_4}$ then remove C_5 and simplify.

Fig. 20.9   **Common representations of Wheatstone bridge**

If in a Wheatstone bridge each capacitor is C then $C_{eq} = C$

Charge distribution method   It can be applied in principle anywhere in tune with Kirchhoff's law but in symmetrical circuits it makes the problem very simple. In symmetrical

circuits charge entering a branch = charge leaving the branch (identical) or mirror image branch.

If two capacitors C_1 and C_2 charged to V_1 and V_2 are joined together then common potential is

$$V_{common} = \frac{V_1 C_1 + V_2 C_2}{C_1 + C_2} = \frac{Q_1 + Q_2}{C_1 + C_2} \quad \text{(Fig. 20.10)}$$

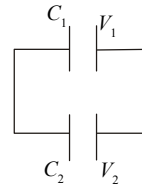Charge on capacitors after joining $\dfrac{Q_1'}{Q_2'} = \dfrac{C_1}{C_2}$

Fig. 20.10   **Common potential**

Loss in energy when two capacitors C_1 and C_2 charged to V_1 and V_2 are joined together as shown in Fig. 20.10 is

$$\Delta E = \frac{C_1 C_2}{(C_1 + C_2)}(V_1 - V_2)$$

If dielectrics are added in the manner shown in Fig. 20.11 (a) then net capacitance is in a parallel combination of C_1, C_2 and C_3 as illustrated in Fig. 20.11 (b)

$$C_1 = \frac{\varepsilon_0 k_1 A/3}{d}, \qquad C_2 = \frac{\varepsilon_0 k_2 A/3}{d},$$

$$C_3 = \frac{\varepsilon_0 k_3 A/3}{d} \quad \text{and} \qquad C = C_1 + C_2 + C_3$$
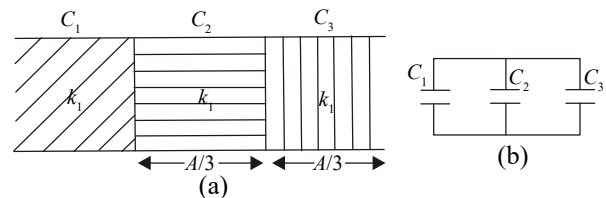

Fig. 20.11   **Effect of dielectrics on capacitor**

If dielectrics are arranged as shown in Fig. 20.12 (a) then C_{eq} is series combination of C_1, C_2 and C_3 as illustrated in Fig 20.12 (b)

$$C_1 = \frac{\varepsilon_0 K_1 A}{\dfrac{d}{3}}$$

$$C_2 = \frac{\varepsilon_0 K_2 A}{\dfrac{d}{3}}$$

$$C_3 = \frac{\varepsilon_0 K_3 A}{\dfrac{d}{3}}$$

$$C_{eq} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d}{3\varepsilon_0 A}\left[\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right]$$
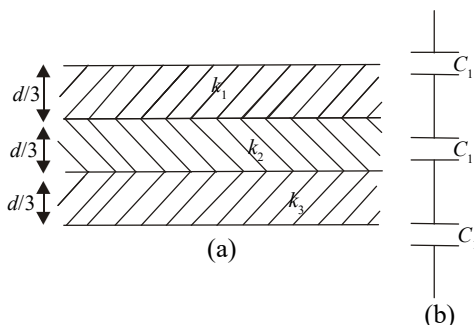

Fig. 20.12   **Effect of dielectrics in a capacitor**

If a dielectric slab in a capacitor is being introduced in the rigidly held plates connected across a battery of *emf* V_0 then the force required to insert the slab is

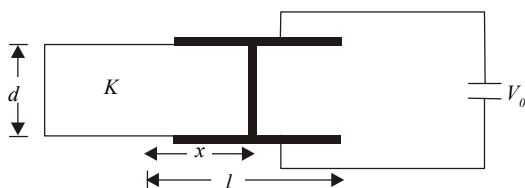$$F = \frac{1}{2}V^2\frac{dC}{dx}$$

Fig. 20.13   **Force required during Introduction of dielectric in a capacitor**

Charging of a capacitor or growth transient

When the switch is made ON at $t = 0$, current passes through capacitor for a very short time during its charging spree. The variation of charge/voltage across the capacitor is called charging transient. See Fig 20.14 (a) and (b)
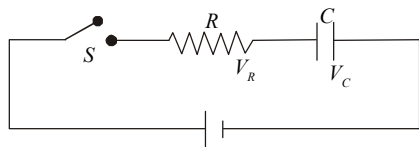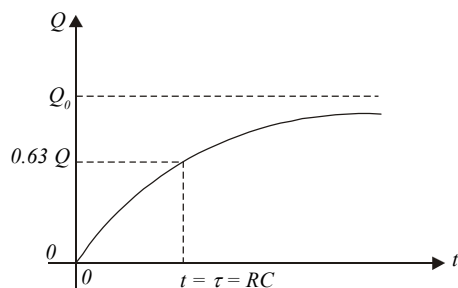

Fig. 20.14 (a)   **Charging of a capacitor**

Fig. 20.14 (b)   **Charging transient**

$$Q = Q_0(1 - e^{-t/RC}) \qquad \text{where } Q_0 = CV_0$$

$$I = \frac{dQ}{dt} = \frac{Q_0}{RC}e^{-t/RC} \qquad \text{and } V_R = IR$$

Time constant $\tau = RC$ is the time in which capacitor charges to 63% of its maximum value of charge.

Discharging of a capacitor (or decay transient)

when the capacitor has been charged for a long time. It is connected to a resistance R through a switch S as shown in Fig 20.15 (a). At $t = 0$, switch is closed and the capacitor starts discharging according to the equation.

$Q = Q_0\,e^{-t/RC}$ Fig 20.15 (b) shows discharging transient.

Time constant $\tau = RC$ is the time in which a capacitor discharges to 36% of its maximum value.
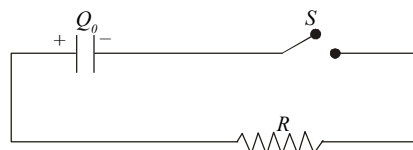
Fig. 20.15 (a)   **Discharging of a capacitor**
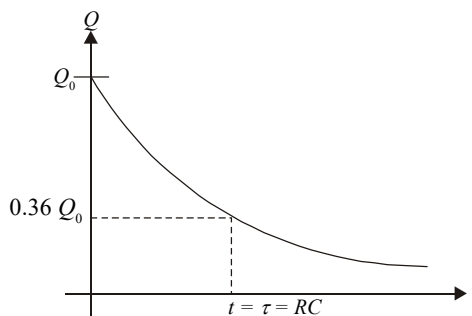
Fig. 20.15 (b)   **Discharging transient**

Important functions of a capacitor

   (i)   In a timer (time setting in almost all automatic devices)

  (ii)   time base circuit in CRO (sawtooth generator).

 (iii)   filter circuits (low pass, high pass, band pass, band reject)

 (iv)   oscillators (*LC* oscillator $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$) and *RC* oscillators

  (v)   tuner circuit in radio

 (vi)   as a trimmer in frequency setting with quartz oscillator

 (vii)   integrating and differentiating circuits.

(viii)   voltage multiplier

Physics by Saurabh Maurya (IIT-BHU)

(ix)   peak detector

(x)   demodulator or detection

(xi)   clamping circuits.

(xii) $0 - 90°$ phase shift producer in one $-RC$ section and $0 -180°$ phase shift in 3-RC sections.

(xiii) in AC motor to enhance torque

(xiv) converts active power into wattless or passive power.

If n drops each of radius r and charge q combine to form a big drop of radius R then charge on big drop is

$$Q_{big} = nq$$
$$C_{big} = n^{1/3} C_{small};$$
$$V_{big} = n^{2/3} V_{small} \text{ and } R = n^{1/3}r$$

Capacitance of a transmission line as shown in Fig 20.16

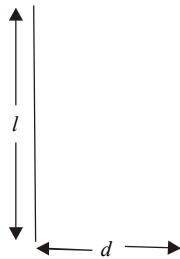is given by $C = \dfrac{\pi\varepsilon_0 Kl}{\log_e \dfrac{d}{r}}$

Fig. 20.16   **Capacitance of transmission line**

where K is dielectric constant of the material between two wires, r is radius of either wire.

Connecting wires offer stray capacitance and conducting wires or conducting points of a device offer parasitic capacitance.

SHORT CUTS AND POINTS TO NOTE

1. If the outer shell is grounded as in Fig 20.17 (a)

    then $C = \dfrac{\varepsilon_0 k r_1 r_2}{r_2 - r_1}$

    If inner sphere is grounded as in Fig 20.17 (b) then

    $C = \dfrac{\varepsilon_0 k r_1 r_2}{(r_2 - r_1)} + 4\pi\varepsilon_0 r_2$ because $\dfrac{\varepsilon_0 k r_1 r_2}{r_2 - r_1}$ and

    capacitance of isolated sphere (outer) $4\pi\varepsilon_0 r_2$ are in parallel.
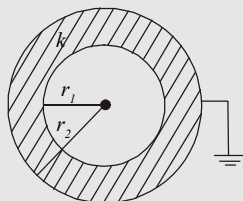

Fig. 20.17 (a)
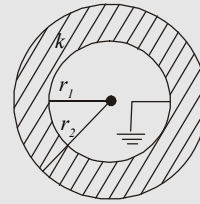
Fig. 20.17 (b)

2. The potential drops V_1 and V_2 across capacitors C_1 and C_2 are in the inverse ratio of their capacitances.

    i.e. $V_1 = \dfrac{V_0 C_2}{C_1 + C_2}$ and $V_2 = \dfrac{V_0 C_1}{C_1 + C_2}$
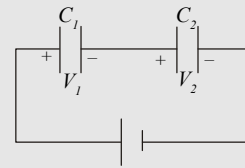
Fig. 20.18

3. If two capacitors are in series then $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$. If n

    identical capacitors are in series $C_{eq} = \dfrac{C}{n}$ and if n

    identical capacitors are in parallel $C_{eq} = nc$.

4. In series, charge remains same and potential across the capacitors may be different. In parallel, potential difference across each capacitor is equal and charge on the capacitors may be different.

5. It two spheres of radius r_1 and r_2 are joined by a conducting wire or directly then common potential V

    $= \dfrac{Q_1 + Q_2}{4\pi\varepsilon_0 (r_1 + r_2)}$ (See Fig 20.19.).

    and charge after joining $Q'_1 = (Q_1 + Q_2)\dfrac{r_1}{r_1 + r_2}$

    $Q'_2 = (Q_1 + Q_2)\dfrac{r_2}{r_1 + r_2}$
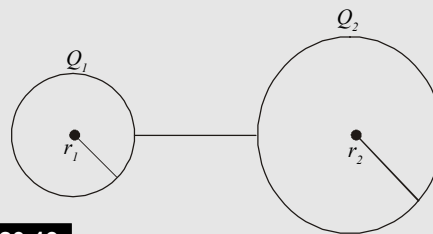
Fig. 20.19

6. To find potential drop across the capacitors in questions as shown is Fig 20.20 (a), convert it to equivalent circuit of Fig 20.20 (b) and then solve it using concept of point 2.

If negative terminal of one battery is connected to the positive terminal of other battery then $V_{net} = V_{01} + V_{02}$ as batteries are in series. It negative terminal of one battery is connected to negative of the other, then $V_{net} = V_{01} - V_{02}$ or $V_{02} - V_{01}$ depending upon which is greater. The net emf has direction of greater emf battery.

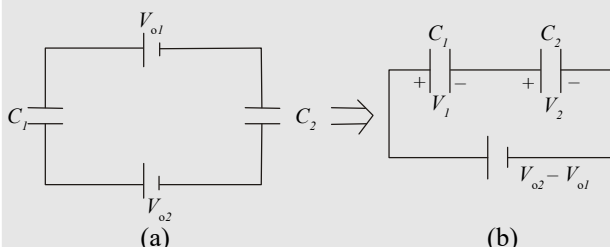From Fig 20.20 (b) $V_1 = \dfrac{(V_{02} - V_{01})C_2}{C_1 + C_2}$ and $V_2 = \dfrac{(V_{02} - V_{01})C_1}{C_1 + C_2}$

(a)   (b)

Fig. 20.20

7. It n identical plates each of area A are connected alternately and separation between two consecutive plates is d, then $C_{eq} = (n-1)\dfrac{A\varepsilon_0}{d}$. For example $C_{eq} = \dfrac{3A\varepsilon_0}{d}$ in Fig 20.21
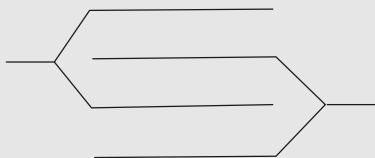
Fig. 20.21

8. If a metal plate of thickness t is introduced in between the plates of a capacitor separated by d then $C_{eq} = \dfrac{A\varepsilon_0}{(d-t)}$ (See fig 20.22)
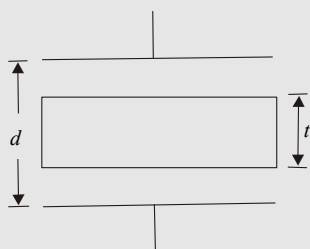
Fig. 20.22

9. If large number of identical capacitors of rating C/V are available and capacitor C^1/nV is to be designed then n capacitors are to be connected in series. Each

row of n capacitors has $C_{eq} = \dfrac{C}{n}$. To make C^1, then $m = \dfrac{C^1}{C/n}$ rows of n capacitors in series will be required.

10. For the network shown in Fig 20.23

$$C_{eq} = C_{AB} = \dfrac{2C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2 + 2C_3}$$
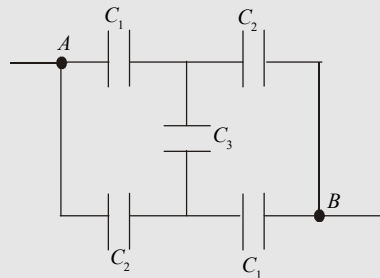
Fig. 20.23

11. If capacitor C is connected along each side of a skeleton cube then equivalent capacitance along the longest diagonal is $\dfrac{6C}{5}$. Equivalent capacitance along face diagonal is $\dfrac{4}{3}C$ and along one side is $\dfrac{12}{7}C$.

12. If one side of skeleton cube is open as shown in Fig 20.24 then $C_{AB} = \dfrac{5}{7}C$.
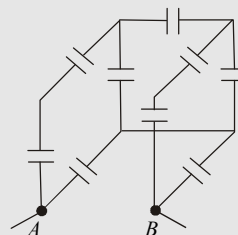
Fig. 20.24

13. If one or more shells of concentric shell system is/are grounded then net potential corresponding to grounded shells is zero. For example in Fig 20.25 $V_b = 0$
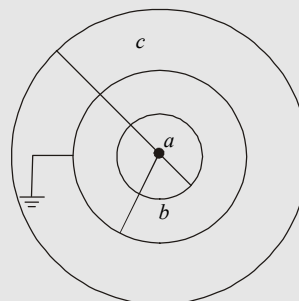
Fig. 20.25

14. If in a charged capacitor (battery not connected) a dielectric is added then charge remains the same, C increases, V decreases, electric field decreases and energy stored decreases each by a factor of dielectric constant.

15. If in a capacitor (connected to a battery) a dielectric is added, then V remains unchanged, C increases K–fold, electric field remains unchanged, energy stored in the capacitor increases K– fold, where K is dielectric constant.

16. If the battery is disconnected after the capacitor is charged and plates of capacitors are moved away then C decreases, Q remains unchanged, V increases, electric field remains unchanged and energy stored increases.

17. To find time of charging at a particular instant t, use $t = \tau \log_e \dfrac{Q_0}{Q_0 - Q}$ or $t = 2.303\, RC \log_{10} \dfrac{Q_0}{Q_0 - Q}$. Similarly discharge time is $t = 2.303\, RC \log_{10} \dfrac{Q_0}{Q}$.

18. A capacitor charges to 63.3% in one RC, 90% in 2.303 RC, 95% in 3RC, and, 99% in 5 RC.

19. DC current does not pass through the capacitor except during charging or discharging for a short while. $\dfrac{dQ}{dt} = i = \dfrac{C\,dV}{dt}$. If V is constant, $i = 0$. i.e. DC current passes during transients only.

20. AC current passes through the capacitor. Displacement current and conduction current have a phase shift of $90°$.

21. Since the plates of a capacitor are thin, if a charge Q is placed $Q/2$ appears on one side, $Q/2$ appears on other side and $-Q/2$ charge is induced on the inner side of plate as shown in Fig. 20.26 (b)
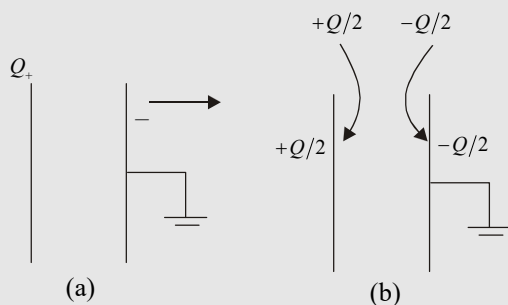

(a)   (b)

Fig. 20.26

22. When a thin metal sheet is introduced in between the space in a parallel plate capacitor then capacitance remains unchanged.

CAUTION

1. Treating spherical charges of unequal radius like point charges when they are joined.

$\Rightarrow$ Charged spheres behave as capacitors and hence charge is distributed in accordance to capacitive laws. i.e. charge after joining the two spheres is proportional to the radius i.e. $Q'_1 = \dfrac{(Q_1 + Q_2)r_1}{r_1 + r_2}$ and $Q'_2 = \dfrac{(Q_1 + Q_2)r_2}{r_1 + r_2}$

2. Confusing whether on increasing or decreasing the distance between the plates of a capacitor (the battery removed after charging it) voltage remains constant or not.

$\Rightarrow$ When the battery is removed charge is conserved, i.e. charge remains constant and voltage increases or decreases depending upon the fact that separation between the plates is increased or decreased.

3. Confusing whether or not current passes through a capacitor.

$\Rightarrow$ DC Current does not pass through capacitor except during growth and decay transient. AC current passes through the capacitor.

4. Confusing that capacitors are added in series and parallel like resistors.

$\Rightarrow$ Capacitors in series are added according to the law $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + ...$ and in parallel $C_{eq} = C_1 + C_2 + ...$

5. Confusion in series and parallel cases

$\Rightarrow$ Note that in series only one end of a capacitor is connected to one end of the other and in parallel both ends of the capacitors are joined with two ends of other capacitors as shown in Fig 20.27 (a) and 20.27 (b) respectively.
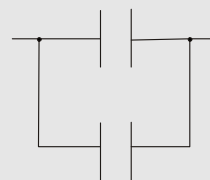

Fig. 20.27 (a)

Fig. 20.27 (b)

6. Confusing in cases as shown in Fig 20.28 whether capacitors are in series or parallel.

$\Rightarrow$ Capacitors are in series in this case as battery is another element present in between and hence capacitors are not connected end to end.
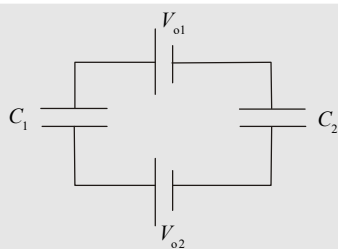
Fig. 20.28

7. Confusing whether new capacitors formed due to addition of dielectrics are is series or parallel.

⇒ Look into Fig 20.29 (a) and Fig 20.29 (b) carefully If dielectric divides the capacitors horizontally, they are in series and if the space between the capacitors is divided vertically then capacitors so formed are in parallel.
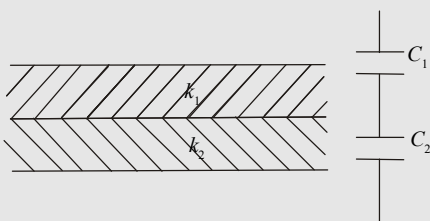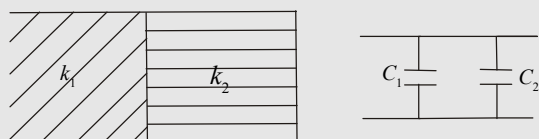

Fig. 20.29 (a)

Fig. 20.29 (b)

8. Not understanding the effect of rating of the capacitor.

⇒ If a capacitor is marked $10\ \mu F/250V$ then it cannot hold a charge $>2500\ \mu c$. If two capacitors of different rating are joined in series, then we cannot supply a charge

greater than the rating value of smaller charge as charge remains same in series.

9. If more than two plates are connected at a point then difficulty in recognising series or parallel case.

⇒ Mark the plates 1, 2, 3,... etc. and reconstruct a simplified circuit so that you can easily recognise series and parallel case.

10. Confusion about Wheatstone bridge. Considering in the circuit shown in Fig 20.30 (a) AOPL, APML, AONP, LPBM, PONB, MPNB etc. as wheat stone bridge.

⇒ Note that AOPL, APML, etc. in Fig 20.30 (a) are not Wheatstone bridges. For a circuit to qualify as Wheat-stone bridge $\dfrac{C_1}{C_2} = \dfrac{C_3}{C_4}$ then, remove C_5 as shown in Fig. 20.30 (b).
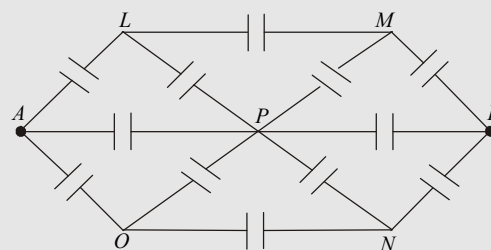
Fig. 20.30 (a)
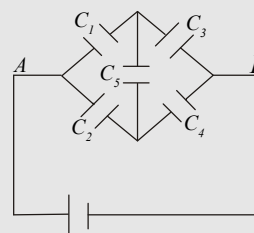
Fig. 20.30 (b)

SOLVED PROBLEMS

1. A capacitor has charge 50 μC. When the gap between the plates is filled with glass wool 120 μC charge flows through the battery. The dielectric constant of glass wool is
   (a) 3.4   (b) 1.4
   (c) 2.4   (d) none of these

   Solution (a) $K = \dfrac{Q'}{Q} = \dfrac{120+50}{50} = 3.4$

2. A charge of 1μC is given to one plate of a capacitor and a charge of 2 μC is given to the other plate of a 0.1 μF capacitor. Find the potential difference across the two plates of the capacitor.

   (a) 5 V (b) 10 V
   (c) 15 V (d) 30 V

   Solution (a) $Q_{net} = 2 - 1 = 1\ \mu$C. charge $= \dfrac{1}{2}\ \mu$C will appear on each side of the plate as illustrated in fig.

   $$\therefore V = \dfrac{Q_{net}/2}{C} = \dfrac{0.5}{0.1} = 5\ V$$

3. A large conducting plane has surface charge density 10^{-4} C/m². Find the electrostatic energy stored in a cubical volume of side 1cm in front of the plane.
   (a) 1.4 J   (b) 2.8 J
   (c) 5.6 J   (d) none of these

Solution (c) $U = \frac{1}{2} \varepsilon_0 E^2 \text{(Vol.)} = \frac{1}{2} \varepsilon_0 \left(\frac{\sigma}{\varepsilon_0} \right)^2 \text{(Volume)} =$

$\frac{\sigma^2}{2\varepsilon_0} \text{(Vol)} = \frac{\left(10^{-4}\right) \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} = \frac{100}{2 \times 8.85} = 5.6 \text{ J}$

4. In the network shown $C_1 = C, C_2 = 2C, C_3 = 3C, C_4 = 4C$ find the ratio of charge C_2 to C_4.

(a) $\frac{4}{7}$ (b) $\frac{22}{3}$

(c) $\frac{7}{4}$ (d) $\frac{3}{22}$

(CBSE, 2005)
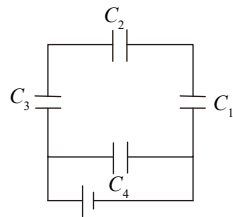
Fig. 20.31

Solution (d) C_2, C_1 and C_3 in series $\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} +$

$\frac{1}{3C} = \frac{11}{6C}$ or $C_{eq} = \frac{6C}{11} = \frac{Q_2}{Q_4} = \frac{C_{eq} V_0}{C_4 V_0} = \frac{6/11}{4} = \frac{3}{22}$

5. A fully charged capacitor has a capacitance C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference V across the capacitance is

(a) $\sqrt{\frac{2mC\Delta T}{s}}$ (b) $\frac{mC\Delta T}{s}$

(c) $\frac{ms\Delta T}{C}$ (d) $\sqrt{\frac{2ms\Delta T}{C}}$

[AIEEE 2005]

Solution (d) $\frac{1}{2} CV^2 = ms\Delta T$ or $V = \sqrt{\frac{2ms\Delta T}{C}}$

6. A parallel plate capacitor is formed by stacking n equally spaced plates connected alternately. If capacitance between two adjacent plates is C then the resultant capacitance is

(a) $(n-1)C$ (b) $(n+1)C$

(c) C (d) nC

[AIEEE 2005]

Solution (a) $C_{eq} = (n-1)C$ (one less than the number of plates)

7. An air filled parallel plate capacitor has a capacity $2\ pF$. The separation between the plates is doubled and the inter space is filled with wax. If the capacity is increased to $6\ pF$, the dielectric constant of the wax is

(a) 2   (b) 4

(c) 3   (d) 6

[CET Karnatka 2005]

Solution (d) $K = \frac{C''}{C'} = \frac{6\ pF}{C/2} = \frac{6\ pF}{1\ pF} = 6$

8. If each capacitor has capacitance C in Fig. 20.32 (a) then find C_{AB}

(a) C (b) $\frac{C}{2}$

(c) $\frac{3C}{2}$ (d) none of these

Solution (c) Look into equivalent Fig. 20.32 (b) and (c). The dotted part is Wheatstone bridge with $C_{eq} = C$ then further equivalent circuit is shown in Fig. 20.32 (c).
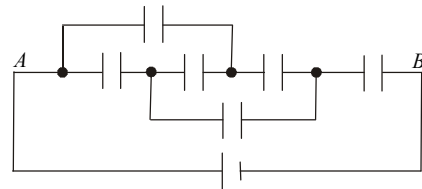
Fig. 20.32 (a)

Fig. 20.32 (b)

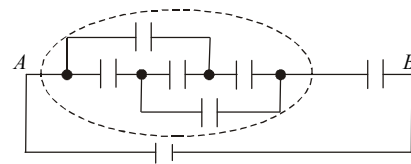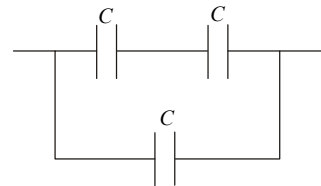**Fig. 20.32 (c)**

9. Square plates of area $2a^2$ are filled with dielectric of strength k_1, k_2 and k_3 as shown in Fig 20.33. Find C_{eq}.

(a) $\frac{\varepsilon_0 a^2 (k_1 k_3 + k_2 k_3)}{d (k_1 + k_2 + k_3)}$ (b) $\frac{2\varepsilon_0 a^2 (k_1 k_3 + k_2 k_3)}{d (k_1 + k_2 + 2k_3)}$

(c) $\frac{2\varepsilon_0 a^2 (k_1 k_3 + k_2 k_3)}{d (2k_1 + 2k_2 + k_3)}$ (d) none of these

Solution   (b) The equivalent capacitance circuit is where

$$C_1 = \frac{\varepsilon_0 k_1 a^2}{d}, \; C_2 = \frac{\varepsilon_0 k_2 a^2}{d} \text{ and}$$

$$C_3 = \frac{2\varepsilon_0 k_3 a^2}{d} \quad \frac{1}{C_{eq}} = \frac{1}{C_1 + C_2} + \frac{1}{C_3}$$

$$= \frac{d}{\varepsilon_0 a^2 (k_1 + k_2)} + \frac{d}{\varepsilon_0 a^2 2k_3}$$

$$C_{eq} = \frac{2\varepsilon_0 a^2 (k_1 + k_2) k_3}{(k_1 + k_2 + 2k_3) d}$$
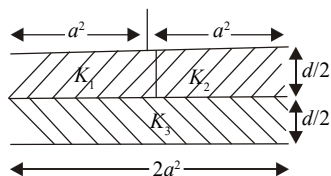
Fig. 20.33

10. Each capacitor has capacitance C in the Fig 20.34 (a). Find C_{AB}.

(a)
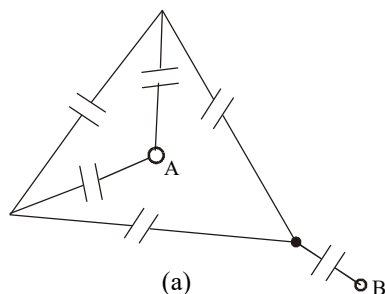
Fig. 20.34

(a) C (b) $2\,C$

(d) $C/2$ (d) $3C/2$

Solution   (c) Note that dotted part in the circuit is a Wheatstone bridge with $C_{eq} = C$ ∴ $C_{AB} = \dfrac{C}{2}$ from Fig 20.35 (c)
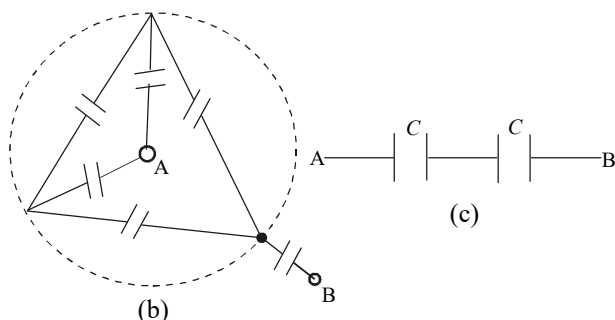
Fig. 20.35

11. Find C_{AB} in the infinite network shown in Fig 20.36 (a)
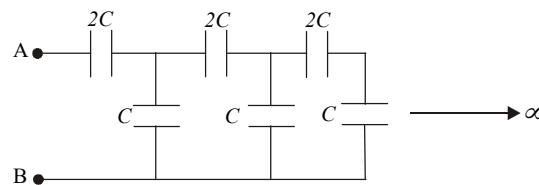
Fig. 20.36 (a)

(a) C (b) $-2C$

(c) $2C$ (d) $-C$

Solution   (a) Let X be the equivalent capacitance. If one more network is added capacitance remains unchanged. Thus from equivalent circuit of Fig 20.36 (b)

$$X = \frac{(C+X)2C}{X+3C} \text{ or } X^2 + 3CX = 2C^2 + 2CX \text{ or } X^2 + CX -$$

$2C^2 = 0$ or $(X + 2C)(X - C) = 0$ $X = C, X \neq -2C$

∵ capacitance is not negative.
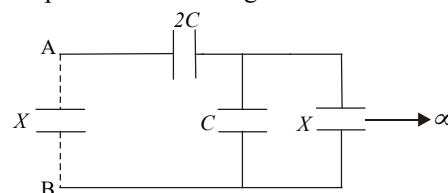
Fig. 20.36 (b)

12. Find C_{AB} if each capacitor is C in Fig 20.37 (a)
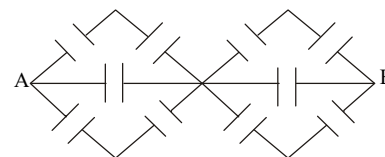
Fig. 20.37 (a)

(a) $3\,C$ (b) $2\,C$

(c) C (d) $C/2$

Solution   (c) The equivalent circuit is shown in Fig 20.37 (b) and (c). From Fig 20.37 (c) $C_{eq} = \dfrac{2C}{2} = C$

Fig. 20.37 (b)

Fig. 20.37 (c)

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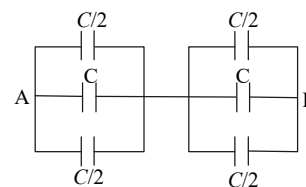
13. In the circuit shown in Fig 20.38 charge on 1 μF and $3\mu F$ capacitors respectively is

Fig. 20.38

(a) 7 μC, 3 μC　　　　　(b) 3 μC, 3 μC
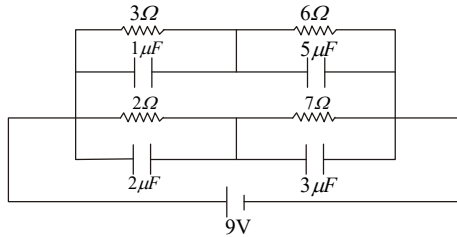
(c) 7 μC, 21 μC　　　　　(d) 3 μC, 21 μC

Solution　(d) $I = \dfrac{9V}{9\Omega} = 1\,A$

Potential drop across 1 μF capacitor $= 1 \times 3 = 3V$

Potential drop across 3 μF capacitor $= 1 \times 7 = 7V$

Charge on 1 μF capacitor $Q^1 = CV = 1 \times 3 = 3\,\mu C$

Similarly charge on $3\mu F$ capacitor $Q^2 = 7 \times 3 = 21\,\mu C$

14. Find the potential at D in Fig 20.39 taking potential at B to be zero.

(a) 24 V　　　　　(b) 8 V

(c) 12 V　　　　　(d) 16 V

Solution　(d) $V^2 = \dfrac{C_1 V_0}{C_1 + C_2} = \dfrac{12 \times 24}{18} = 16\ V$
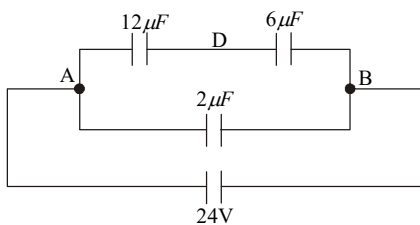
Fig. 20.39

15. Find potential drop across BD in the given Fig 20.40 (a).

(a) 5 V　　　　　　　　　(b) 7.5 V

(c) 10 V　　　　　　　　　(d) none of these

Solution　(b) See equivalent circuit Fig 20.40 (b)

$$V_{BD} = \frac{C_1}{C_1 + C_2}\,V_0 = \frac{10 \times 15}{20} = 7.5\ V$$
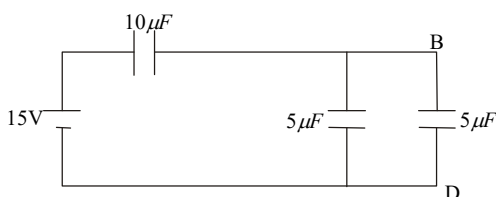
Fig. 20.40 (a)
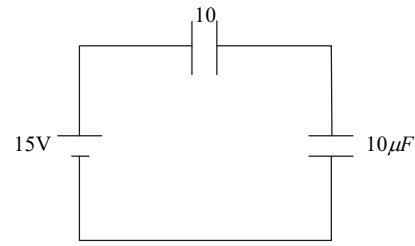
Fig. 20.40 (b)

16. If each capacitor is C find C_{AB} in the given circuit of Fig 20.41 (a)

(a) $\dfrac{3C}{4}$　　　　　　　　(b) 4 C

(c) $\dfrac{C}{4}$　　　　　　　　(d) $\dfrac{C}{2}$

Solution　(a) equivalent circuit of Fig 20.41 (a) is

$$C_{AB} = \frac{C \times 3C}{C + 3C} = \frac{3C}{4} \text{ shown as Fig 20.41 (b)}$$
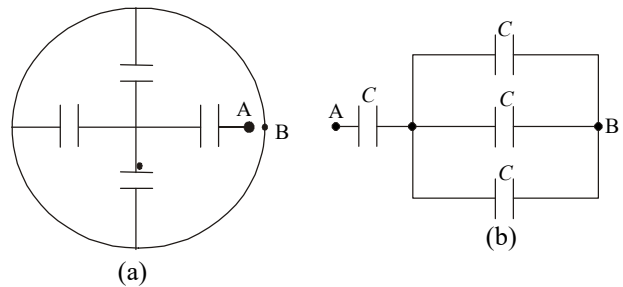
Fig. 20.41

17. Each capacitor in the circuit of Fig 20.42 is 4 μF. When the switch S is closed how much charge will flow through AB?

(a) 320 μC　　　　　　　(b) 213 μC

(c) 107 μC　　　　　　　(d) none of these

Solution　(b) Case (i) switch is open $Q = \dfrac{8 \times 4}{12} \times 40$

$= \dfrac{320}{3}\,\mu C$

Case (ii) switch is closed: $Q = 8 \times 40 = 320\ \mu C$

Charge flowing through $AB = 320 - \dfrac{320}{3} = \dfrac{640}{3}\ \mu C$
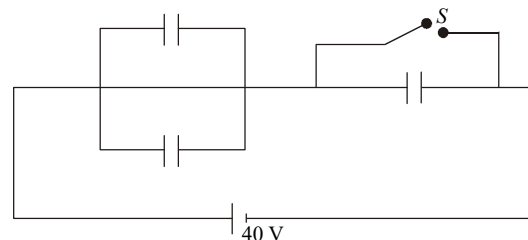
Fig. 20.42

18. A parallel plate capacitor has plate area 100cm² and separation between the plates is 1 cm. A glass plate (k_1 = 6) of thickness 6 mm and an ebonite plate (k_2 = 4) of thickness 4 mm are inserted. Find C_{eq}

(a) $4.085\,pF$ (b) $40.85\,pF$

(c) $.4085\,nF$ (d) $40.85\,nF$

Solution   (b) $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{\varepsilon_0 A\, k_1 k_2}{\left(k_1 d_2 + k_2 d_1\right)}$

$$= \dfrac{8.85 \times 10^{-12} \times 10^{-2} \times 6 \times 4}{\left(6 \times 6 \times 10^{-3} + 4 \times 4 \times 10^{-3}\right)}$$

$$= 4.085 \times 10^{-11}\,F$$
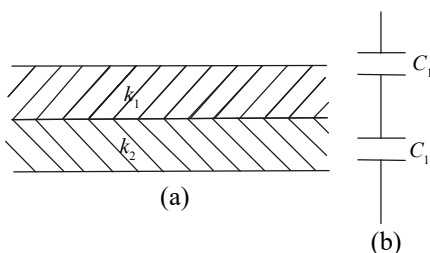
Fig. 20.43

19. A 5 μF capacitor is charged to 12 V. The positive plate of the capacitor is connected to the negative terminal of a 12V battery and vice versa. Find the heat developed in the connecting wires.

(a) $72\,\mu J$ (b) $720\,\mu J$

(c) $1.44\,mJ$ (d) $144\,mJ$

Solution   (c) $E = \dfrac{1}{2}\,CV^2 = \dfrac{1}{2} \times 5 \times 10^{-6} (24)^2 = 1.44\,mJ$
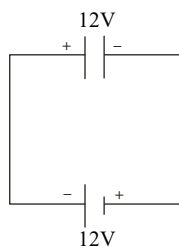
Fig. 20.44

20. Consider the assembly of 3 shells (conducting and concentric) of radii a, b and c as shown in Fig 20.45. Find the capacitance between A and B.

(a) $4\pi\varepsilon_0 \left[\dfrac{ba}{b-a} + \dfrac{bc}{c-b}\right]$ (b) $\dfrac{4\pi\varepsilon_0 (ba)(bc)}{b(b-a)+(c-b)c}$

(c) $\dfrac{4\pi\varepsilon_0 ca}{c-a}$ (d) none of these

Solution   (c) Presence of a thin sheet between parallel

plates does not affect the capacitance. Hence, $C = \dfrac{4\pi\varepsilon_0 ca}{c-a}$
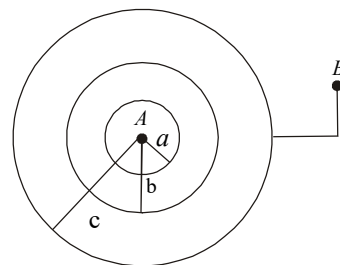
Fig. 20.45

21. A parallel plate capacitor with plate area 100 cm² and separation between the plate 5 mm is connected across a 24 V battery. The force of attraction between the plates is of the order of

(a) $10^{-6}\,N$ (b) $10^{-8}\,N$

(c) $10^{-4}\,N$ (d) $10^{-7}\,N$

Solution   (a) $F = \dfrac{Q^2}{2A\varepsilon_0} = \dfrac{(CV)^2}{2A\varepsilon_0} = \dfrac{\left(\dfrac{A\varepsilon_0}{d}\right)^2 V^2}{2A\varepsilon_0}$

$$= \dfrac{A\varepsilon_0 V^2}{2d^2} = \dfrac{10^{-2} \times 8.85 \times 10^{-12} \times 24^2}{2 \times 25 \times 10^{-6}}$$

$$= 1.08 \times 10^{-6}\,N.$$

22. A capacitor 10 μF charged to 50 V is joined to another uncharged 50 μC capacitor. Find the loss in energy.

(a) $1.04 \times 10^{-4}J$ (b) $4.01 \times 10^{-4}J$

(c) $6.25 \times 10^{-4}J$ (d) $1.64 \times 10^{-4}J$

Solution   (a) Energy loss $= \dfrac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

$$= \dfrac{10 \times 50 \times 10^{-12}}{2(10+50) \times 10^{-6}}(50-0)^2 = 1.04 \times 10^{-4}J$$

23. Two spheres of radius 5 cm and 10 cm, both charged to 120 μC, are joined by a metal wire and then metal wire is removed. What is the charge on each after removal of the wire?

(a) $120\,\mu C$, $120\,\mu C$ (b) $80\,\mu C$, $160\,\mu C$

(c) $100\,\mu C$, $140\,\mu C$ (d) None of these

Solution   (b) $Q_1' = \dfrac{(Q_1 + Q_2)r_1}{r_1 + r_2}$

$$= \dfrac{240 \times 5}{15} = 80\,\mu C$$

$$Q_2' = 240 - 80 = 160\,\mu C$$

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24. In the Fig 20.46 shown the potential drop across 3 μF capacitor when switch S is open and switch S is closed is

(a) 9 V, 8 V   (b) 9 V, 9 V

(c) 6 V, 8 V   (d) 12 V, 8 V

Solution   (d) when switch is open 18 V is applied across

6 μF and 3 μF capacitor $V_1 = \dfrac{18 \times 6}{6+3} = 12$ V

when the switch is closed potential drop across 8 Ω resistor is the potential drop across 3 μF capacitor i.e. 8 V.
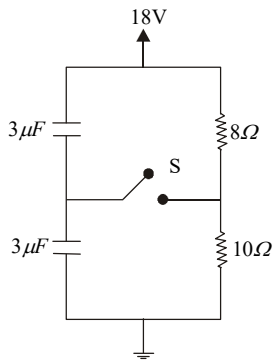
Fig. 20.46

25. Find the net capacitance between A and B in Fig 20.47.

Fig. 20.47

(a) $\dfrac{5}{2}$ μF (b) $\dfrac{2}{5}$ μF
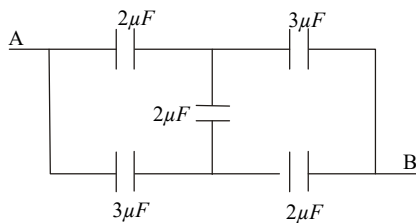
(c) $\dfrac{9}{22}$ μF (d) $\dfrac{22}{9}$ μF

Solution   (d) $C_{AB} = \dfrac{2C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2 + 2C_3}$

$$= \dfrac{2 \times 2 \times 3 + 2 \times 2 + 3 \times 2}{2 + 3 + 2 \times 2} = \dfrac{22}{9} \mu F$$

26. A 10 μF/400 V and a 4 μF/100 V capacitors are connected in series. Find the maximum potential which can be applied.

(a) 100 V   (b) 500 V

(c) 400 V   (d) 140 V

(e) None of these

Solution   (d) In series charge remains same.

The maximum charge which can be applied is 400 μC (maximum rating of 4μF/ 100 V) capacitor.

Then potential which can be applied is

$100 + 40 = 140$ V.
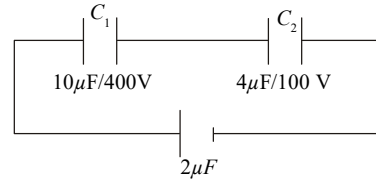
Fig. 20.48

27. Each side of a tetrahedral has a capacitor of capacitance C. Find the capacitance between a side.

(a) $\dfrac{C}{2}$ (b) 2 C

(c) C (d) $\dfrac{C}{3}$
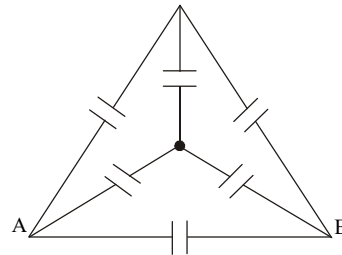
Fig. 20.49 (a)

Solution   (b) The equivalent circuit of Fig 20.49 (a) is shown in Fig 20.49 (b) and Fig 20.49 (c) respectively.

$\therefore C_{AB} = 2\ C$
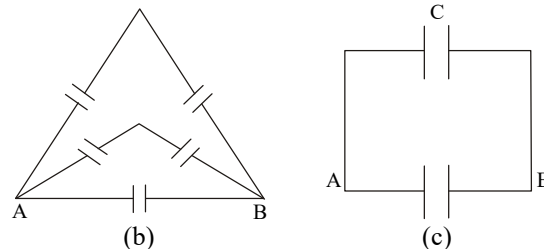
Fig. 20.49

28. The switch S is kept closed for a long time in Fig 20.50. It is opened at $t = 0$. Find the current in R_1 at $t = 1$ms.
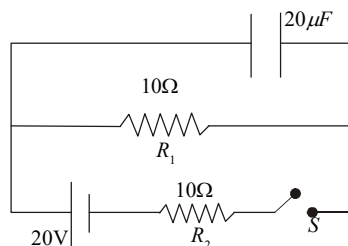
Fig. 20.50

(a) 11.2 mA (b) 12.4 mA

(c) 13.4 mA (d) 14.4 mA

Solution (c) $Q = Q_0\, e^{-t/RC}$, and $\dfrac{dQ}{dt} = i$

or $i = \dfrac{Q_0}{RC}\, e^{-t/RC}$

$= \dfrac{20 \times 20 \times 10^{-6}}{10 \times 20 \times 10^{-6}}\, e^{-5} = 2(.0067) = 13.4 \text{ mA}$

TYPICAL PROBLEMS

29. Find the potential drop between points x and y in the given Fig.

(a) 25 V (b) 50 V

(c) 75 V (d) 57 V
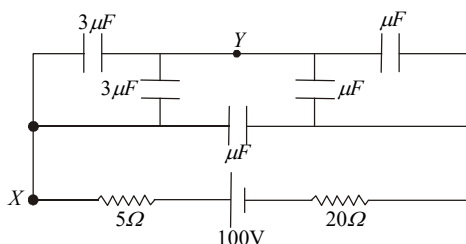
Fig. 20.51

Solution (a) Since no DC current passes through capacitor in steady state, no potential-drop occurs across resistors. Hence equivalent circuit is shown in Fig 20.52.
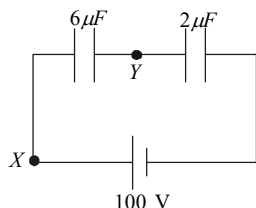
Fig. 20.52

$V_{xy} = \dfrac{100 \times 2}{6+2} = 25 \text{ V}$

30. Each capacitor in Fig 20.53 has capacitance C. Find capacitance across AB.

(a) $\dfrac{15C}{8}$ (b) $\dfrac{15C}{7}$

(c) $\dfrac{3C}{2}$ (d) $2C$
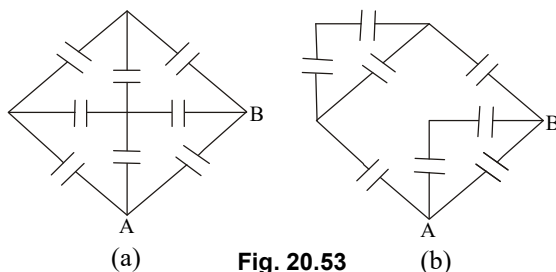
(a) **Fig. 20.53** (b)

Solution (a) Passing through equivalent circuits of Fig. 20.53 (b), (c) and (d) we find $C_{AB} = \dfrac{3C}{8} + \dfrac{3C}{2} = \dfrac{15C}{8}$
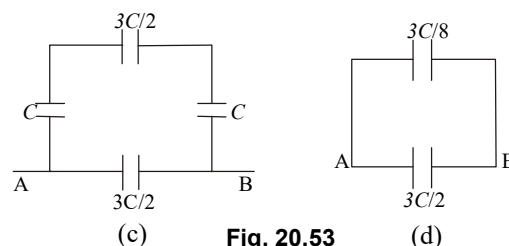
(c) **Fig. 20.53** (d)

31. Find C_{AB} in the given circuit of Fig 20.54 (a). Assume each capacitor is C.

(a) $\dfrac{15C}{8}$ (b) $\dfrac{15C}{7}$

(c) $\dfrac{3C}{2}$ (d) $2C$

Solution (b)

(a) (b)

(c) (d)
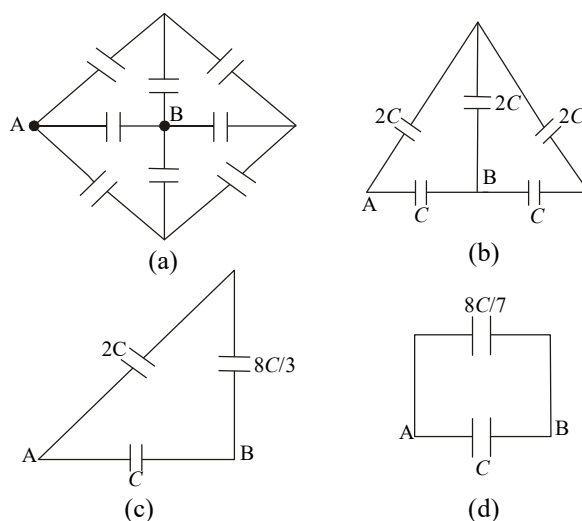
Fig. 20.54

(Olympiad 1999)

From equivalent circuit of Fig 20.54 (d)

$C_{AB} = \dfrac{8C}{7} + C = \dfrac{15C}{7}$

32. Each capacitor in the given circuit of Fig 20.55 has capacitance C. Find V_{AB}.
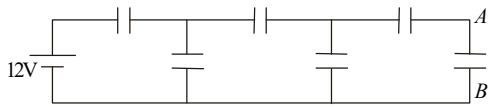
Fig. 20.55

(a) $\dfrac{65}{60}$ V

(b) $\dfrac{60}{65}$ V

(c) $\dfrac{30}{35}$ V

(d) none of these

Solution (b)
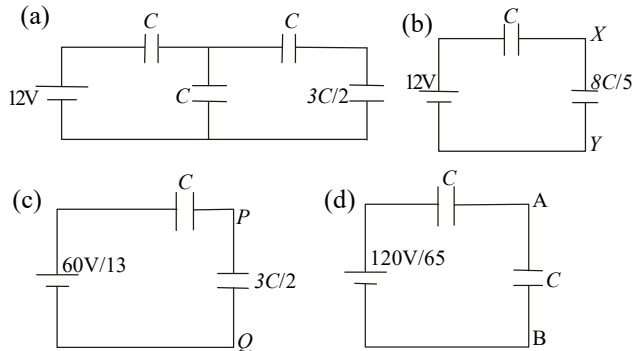
Fig. 20.56

From Fig 20.56 (b) $V_{XY} = \dfrac{12 \times C}{C + \dfrac{8C}{5}} = \dfrac{60}{13}$ V

From Fig 20.56 (c) $V_{PQ} = \dfrac{\dfrac{60}{13} \times C}{C + \dfrac{3C}{2}} = \dfrac{120}{65}$ V

From Fig 20.56 (d) $V_{AB} = \dfrac{\dfrac{120}{65} \times C}{C + C} = \dfrac{60}{65}$ V

33. If in the given Fig 20.57 switch S is closed, find the change in energy stored in the capacitors.
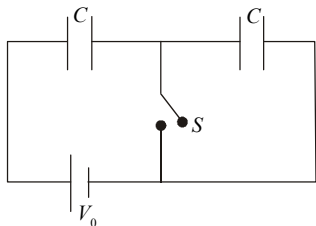
Fig. 20.57

(a) $\dfrac{CV_0^2}{2}$

(b) $\dfrac{CV_0^2}{4}$

(c) $\dfrac{CV_0^2}{8}$

(d) $\dfrac{3CV_0^2}{8}$

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Solution (b) When switch is open each capacitor has voltage $\dfrac{V_0}{2}$.

$\therefore$ energy stored $E_1 = \dfrac{1}{2}C\left(\dfrac{V_0}{2}\right)^2 + \dfrac{1}{2}C\left(\dfrac{V_0}{2}\right)^2 = \dfrac{CV_0^2}{4}$

After the switch is closed capacitor is not connected to battery discharges. Energy is stored in the capacitor connected to the battery of emf V_0. Therefore energy stored after switch is made ON.

$E_2 = \dfrac{1}{2}CV_0^2 \qquad \Delta E = E_2 - E_1 = \dfrac{CV_0^2}{4}$

34. What amount of heat will be generated in the circuit when switch S is shifted from position 1 to 2 in Fig 20.58.

(a) $\dfrac{1}{2}CV_1^2$

(b) $\dfrac{1}{2}CV_2^2$

(c) $\dfrac{1}{2}C\left(V_1^2 - V_2^2\right)$

(d) $\dfrac{1}{2}C\left(V_1 - V_2\right)^2$
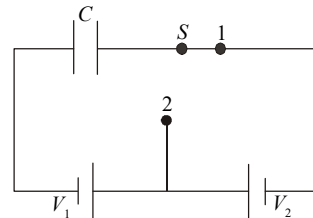
Fig. 20.58

Solution (b) Charge on capacitor C when switch is at position 1 $q_1 = C\left(V_1 - V_2\right)$

Charge on capacitor C when switch is at position 2 $q_2 = CV_1$ $\therefore \Delta q = CV_2$

Applying energy conservation,

$\dfrac{1}{2}CV_1^2 - \dfrac{1}{2}C(V_1 - V_2)^2 + $ heat energy $= \Delta q V_1$

Solving we get, heat energy $= \dfrac{1}{2}CV_2^2$

35. The capacitance between adjacent plates shown in Fig 20.59 (a) is 50 nF. A 1 μC charge is placed on the middle plate. Find the charge on the outer surface of upper plate and the potential difference between upper and middle plates.
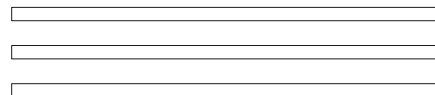
Fig. 20.59 (a)

(a) $0.5\,\mu C$, 10 V   (b) $1\,\mu C$, 20 V

(c) $0.5\,\mu C$, 20 V   (d) $1.0\,\mu C$, 40 V

Solution   (a) From the equivalent circuit of Fig 20.59 (b) we find charge on outer surface of upper plate is $0.5\,\mu C$.

$$V = \frac{Q}{C} = \frac{0.5\times10^{-6}}{50\times10^{-9}} = 10\ V$$
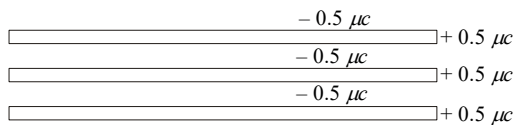
Fig. 20.59 (b)

36. The separation between the plates of C_1 of Fig 20.60 (a) is doubled and a dielectric of strength 6 is added in between the plates of C_2. Find the change in potential drop across C_1 and C_2.
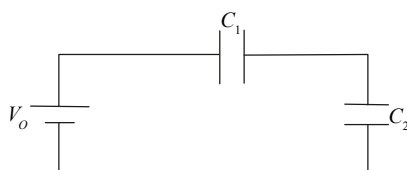
Fig. 20.60 (a)

Solution $C_1' = \dfrac{C_1}{2}$ and $C_2' = 6C_2$

From circuit of Fig 20.60 (a)
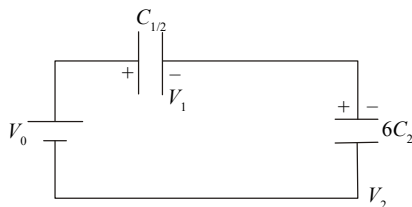
Fig. 20.60 (b)

$$V_1 = \frac{C_2 V_0}{C_1 + C_2} \qquad V_2 = \frac{C_1 V_0}{C_1 + C_2}$$

From circuit of Fig 20.60 (b)

$$V_1' = \frac{6C_2 V_0}{\dfrac{C_1}{2} + 6C_2} = \frac{12C_2 V_0}{C_1 + 12C_2};$$

$$V_2' = \frac{\dfrac{C_1}{2} V_0}{\dfrac{C_1}{2} + 6C_2} = \frac{C_1 V_0}{C_1 + 12C_2}$$

$$\Delta V_1 = \frac{12C_2 V_0}{C_1 + 12C_2} - \frac{C_2 V_0}{C_1 + C_2}$$

$$= C_2 V_0 \left[\frac{12(C_1 + C_2) - (C_1 + 12C_2)}{(C_1 + 12C_2)(C_1 + C_2)} \right]$$

$$= \frac{11 C_2 V_0 C_1}{(C_1 + 12C_2)(C_1 + C_2)} \text{ (increases)}$$

$$\Delta V_2 = \frac{C_1 V_0}{C_1 + C_2} - \frac{C_1 V_0}{C_1 + 12C_2}$$

$$= C_1 V_0 \left(\frac{C_1 + 12C_2 - (C_1 + C_2)}{(C_1 + C_2)(C_1 + 12C_2)} \right)$$

$$= \frac{11 C_1 C_2 V_0}{(C_1 + C_2)(C_1 + 12C_2)} \text{ (decreases)}$$

PASSAGE 1

Read the following passage and answer the questions given at the end.

A parallel plate capacitor has two parallel plates each of area A separated by a distance d. The space between plates is filled with a material having dielectric constant K. The material is not a perfect insulator but has a resistivity ρ. The capacitor is initially charged to Q_0 using a battery. The battery is removed. The capacitor discharges gradually by conduction through dielectric.

1. Calculate the current density in the dielectric.

(a) $\dfrac{Q_0}{\rho k \varepsilon_0 A} e^{-t/\rho k \varepsilon_0}$ (b) $\dfrac{Q_0}{\rho k \varepsilon_0} e^{-t/\rho k \varepsilon_0}$

(c) $\dfrac{Q_0}{\rho k \varepsilon_0 A} e^{-t/\rho k \varepsilon_0}$ (d) none of these

2. The displacement current density at any instant is

(a) $\dfrac{-Q_0}{\rho k \varepsilon_0 A} e^{-t/\rho k \varepsilon_0}$ (b) $\dfrac{Q_0}{\rho k \varepsilon_0 A} e^{-t/\rho k \varepsilon_0}$

(c) $\dfrac{-Q_0}{\rho k \varepsilon_0 A} e^{-t/\rho k \varepsilon_0 A}$ (d) $\dfrac{-Q_0}{\rho k \varepsilon_0 A} e^{-t/\rho k \varepsilon_0}$

Solution   1. (a) $Q = Q_0 e^{-t/RC}$

$$\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC}$$

$$\therefore \qquad i = \frac{Q_0}{\rho \dfrac{d}{A} \dfrac{K\varepsilon_0 A}{d}} e^{-t(\rho K \varepsilon_0)}$$

since $R = \rho \dfrac{d}{A}$.

or $\qquad J = \dfrac{i}{A} = \dfrac{Q_0}{\rho K \varepsilon_0 A} e^{-t(\rho K \varepsilon_0)}$

Solution 2. (d) Since $i_d + i_c = 0$ $\therefore$ $i_d = -i_c$

PASSAGE 2

Read the following passage and answer the questions given at the end.

To annoy your neighbour you develop a car alarm that produces sound at a particularly annoying frequency of 3500 Hz. To achieve it, the car alarm circuit must produce an alternating electric current of the same frequency. You choose an inductor and a capacitor in series. The maximum voltage across the capacitor could be 12 V (equal to the voltage of car battery). To produce sufficiently loud sound (to embrass your neighbaur) the capacitance must store 0.0160 J of energy.

1. What is the value of capacitance?

   (a) $22.2\ \mu F$ (b) $2.22\ \mu F$

   (c) $222\ \mu F$ (d) none of these

2. What is the inductance of the inductor?

   (a) $9.31\ \mu H$ (b) $.931\ \mu H$

   (c) $0.0931\ \mu H$ (d) $93.1\ \mu H$

Solution 1. (c) $\dfrac{1}{2} CV^2 = .016$

$$C = \frac{.032}{12^2} = \frac{.002}{9}\ F = 222\ \mu F$$

Solution 2. (a) $f = \dfrac{1}{2\pi\sqrt{LC}} \Rightarrow L = \dfrac{1}{4\pi^2 f^2 C}$

$$= \frac{9}{40 \times 3500 \times 3500 \times .002} = 9.31\ \mu H$$

QUESTIONS FOR PRACTICE

1. X and Y are large, parallel conducting plates close to each other. Each face has an area A, X is given a charge Q, Y is without any charge. Points A, B and C are as shown in the figure.
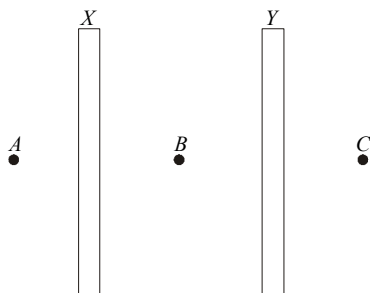
Fig. 20.61

   (a) The field at B is $\dfrac{Q}{2\varepsilon_0 A}$.

   (b) The field at B is $\dfrac{Q}{\varepsilon_0 A}$.

   (c) The fields at A, B and C are of the same magnitude.

   (d) The fields at A and C are of the same magnitude, but in opposite directions.

2. Three identical, parallel conducting plates A, B and C are placed as shown. Switches S_1 and S_2 are open, and can connect A and C to earth when closed. $+Q$ charge is given to B.
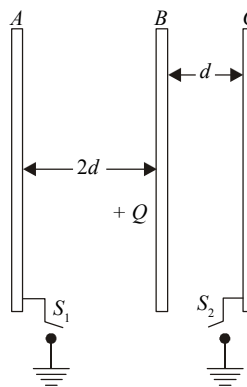
Fig. 20.62

   (a) If S_1 is closed with S_2 open, a charge of amount Q will pass through S_1.

   (b) If S_2 is closed with S_1 open, a charge of amount Q will pass through S_2.

   (c) If S_1 and S_2 are closed together, a charge of amount $Q/3$ will pass through S_1, and a charge of amount $2Q/3$ will pass through S_2.

   (d) All the above statements are incorrect.

3. A, B and C are three large, parallel conducting plates, placed horizontally. A and C are rigidly fixed and earthed. B is given some charge. Under electrostatic and gravitational forces, B may be
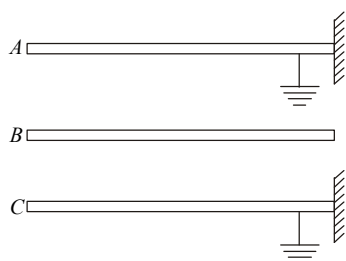
Fig. 20.63

(a) in equilibrium midway between A and C.

(b) in equilibrium if it is closer to A than to C.

(c) in equilibrium if it is closer to C than to A.

(d) B can never be in stable equilibrium.

4. A conductor A is given a charge of amount $+Q$ and then placed inside a deep metal can B, without touching it.
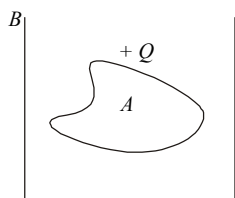
Fig. 20.64

(a) The potential of A does not change when it is placed inside B.

(b) If B is earthed, $+Q$ amount of charge flows from it into the earth.

(c) If B is earthed, the potential of A is reduced.

(d) Either (b) or (c) are true, or both are true only if the outer surface of B is connected to the earth and not its inner surface.

5. A parallel-plate capacitor is charged from a cell and then isolated from it. The seperation between the plates is now increased.

(a) The force of attraction between the plates will decrease.

(b) The field in the region between the plates will not change.

(c) The energy stored in the capacitor will increase.

(d) The potential difference between the plates will decrease.

6. A conducting sphere of radius R, carrying charge Q, lies inside an uncharged conducting shell of radius $2R$. If they are joined by a metal wire,

(a) $Q/3$ amount of charge will flow from the sphere to the shell

(b) $2Q/3$ amount of charge will flow from the sphere to the shell

(c) Q amount of charge will flow from the sphere to the shell

(d) $k\dfrac{Q^2}{4R}$ amount of heat will be produced

7. In an isolated parallel-plate capacitor of capacitance C, the four surfaces have charges Q_1, Q_2, Q_3 and Q_4 as shown. The potential difference between the plates is
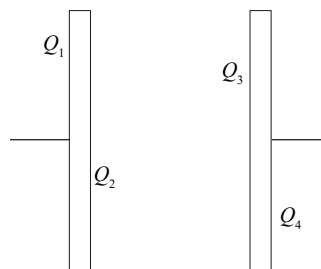
Fig. 20.65

(a) $\dfrac{Q_1 + Q_2}{C}$ (b) $\left|\dfrac{Q_2}{C}\right|$

(c) $\left|\dfrac{Q_3}{C}\right|$ (d) $\dfrac{1}{C}\left[(Q_1 + Q_2) - (Q_3 - Q_4)\right]$

8. When a charge of amount Q is given to an isolated metal plate X of surface area A, its surface charge density becomes σ_1. When an isolated identical plate Y is brought close to X, the surface charge density on X becomes σ_2. When Y is earthed, the surface charge density becomes σ_3.

(a) $\sigma_1 = \dfrac{Q}{A}$ (b) $\sigma_1 = \dfrac{Q}{2A}$

(c) $\sigma_1 = \sigma_2$ (d) $\sigma_3 = \dfrac{Q}{A}$

9. In the circuit shown, each capacitor has a capacitance C. The emf of the cell is ε. If the switch S is closed,
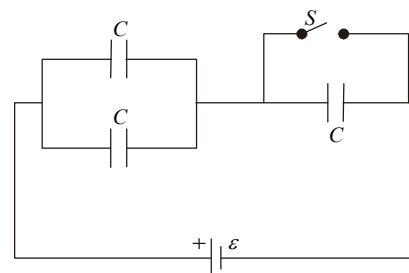
Fig. 20.66

(a) some charge will flow out of the positive terminal of the cell

(b) some charge will enter the positive terminal of the cell

(c) the amount of charge flowing through the cell will be $C\varepsilon$

(d) the amount of charge flowing through the cell will be $\dfrac{4}{3}C\varepsilon$

10. In the circuit shown, some potential difference is applied between A and B. If C is joined to D,
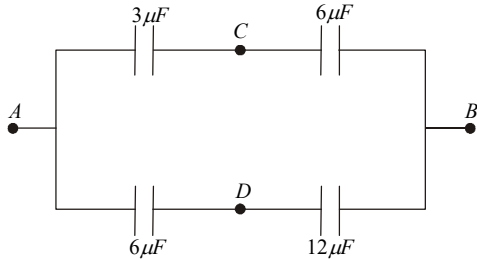
Fig. 20.67

(a) no charge will flow between C and D

(b) some charge will flow between C and D

(c) the equivalent capacitance between C and D will not change

(d) the equivalent capacitance between C and D will change

11. A parallel-plate capacitor is charged from a cell and then disconnected from the cell. The separation between the plates is now doubled.

(a) The potential difference between the plates will become double.

(b) The field between the plates will not change.

(c) The energy of the capacitor doubles.

(d) Some work will have to be done by an external agent on the plates.

12. In the circuit shown, the potential difference across the 3-μF capacitor is V, and the equivalent capacitance between A and B is C_{AB}.
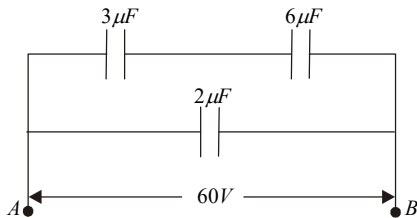
Fig. 20.68

(a) $C_{AB} = 4\mu F$ (b) $C_{AB} = \dfrac{18}{11}\mu F$

(c) $V = 20\ V$ (d) $V = 40\ V$

13. The separation between the plates of a parallel-plate capacitor is made double while it remains connected to a cell.

(a) The cell absorbs some energy.

(b) The electric field in the region between the plates becomes half.

(c) The charge on the capacitor becomes half.

(d) Some work has to be done by an external agent on the plates.

14. The two plates X and Y of a parallel-plates capacitor of capacitance C are given a charge of amount Q each. X is now joined to the positive terminal and Y to the negative terminal of a cell of emf $\varepsilon = Q/C$.

(a) Charge of amount Q will flow from the positive terminal to the negative terminal of the cell through the capacitor.

(b) The total cahrge on the plate X will be $2Q$.

(c) The total charge on the plate Y will be zero.

(d) The cell will supply C_{ε^2} amount of energy.

15. In a parallel-plate capacitor of plate area A, plate separation d and charge Q, the force of attraction between the plates is F.

(a) $F \propto Q^2$ (b) $F \propto \dfrac{1}{A}$

(c) $F \propto d$ (d) $F \propto \dfrac{1}{d}$

16. A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed. C is the common centre of A and B.
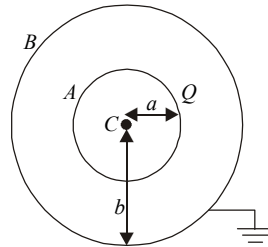
Fig. 20.69

(a) The field at a distance r from C, where $a \le r \le b$, is $k\dfrac{Q}{r^2}$.

(b) The potential at a distance r from C, where $a \le r \le b$, is $k\dfrac{Q}{r}$.

(c) The potenial difference between A and B is $kQ\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$.

(d) The potential at a difference r from C, where $a \le r \le b$, is $kQ\left(\dfrac{1}{r}-\dfrac{1}{b}\right)$.

17. The capacitance of a parallel-plate capacitor is C_0 when the region between the plates has air. This region is now filled with a dielectric slab of dielectric constant K. The capacitor is connected to a cell of emf ε_1 and the slab is taken out.

(a) Charge $\varepsilon C_0 (K-1)$ flows through the cell.

(b) Energy $\varepsilon^2 C_0 (K-1)$ is absorbed by the cell.

(c) The energy stored in the capacitor is reduced by $\varepsilon^2 C_0 (K-1)$.

(d) The external agent has to do $\dfrac{1}{2}\varepsilon^2 C_0 (K-1)$ amount of work to take the slab out.

18. A parallel-plate air capacitor of capacitance C_0 is connected to a cell of emf ε and then disconnected from it. A dielectric slab of dielectric constant K, which can just fill the air gap of the capacitor, is now inserted in it.

(a) The potential difference between the plates decreases K times.

(b) The energy stored in the capacitor decreases K times.

(c) The change in energy is $\dfrac{1}{2}C_0\varepsilon^2 (K-1)$.

(d) The change in energy is $\dfrac{1}{2}C_0\varepsilon^2 \left(1-\dfrac{1}{K}\right)$.

19. The gold leaf electroscope is charged so that its leaves somewhat diverge. If X-rays are incident on the electroscope then

(a) the divergence will decrease

(b) the divergence of leaves will remain unchanged

(c) the gold leaves will melt

(d) the divergence will increase

20. On removing the dielectric from a charged condenser, its energy

(a) increases
(b) remains unchanged
(c) decreases
(d) none of the above

21. A radioactive material is in the form of a sphere whose radius is 9×10^{-3} m. If 6.25×10^{12} β – particles are emitted per second by it then in how much time will a potential of 1volt be produced on the sphere if it is isolateed?

(a) 1.1 ms
(b) 10^{-8}
(c) 1.0 μs
(d) 11μs

22. 64 water drops combine to form a bigger drop. If the charge and potential of a small drop are q and V respectively, then the charge on bigger drop will be

(a) 2 q
(b) 4 q
(c) 16 q
(d) 64 q

23. A condenser of capcity $0.2\mu F$ is charged to a potential of 600 V. The battery is now disconnected and the condenser of capacity $1\mu F$ is connected across it. The potential of the condenser will reduce to

(a) 600 V
(b) 300 V
(c) 100 V
(d) 120 V

24. Two condensers each of capacitance $2\mu F$ are connected in parallel and this combination is connected in series with a $12\mu F$ capacitor. The resultant capctity of the system will be

(a) 16μF
(b) 13 μF
(c) 6 μF
(d) 3 μF

25. The distance between the plates of a parallel plate air condenser is d. If a copper plate of same area but thickness $d/2$ is placed between the plates then the new capacitance will become

(a) doubled
(b) half
(c) one fourth
(d) remain unchanged

26. The energy stored in a condenser is in the from of

(a) potential energy
(b) magnetic energy
(c) elastic energy
(d) kinetic energy

27. A condenser of capcity 500 μF is charged at the rate of 50 μF C/s. The time taken for charging the condenser 10 V will be

(a) 10 s
(b) 25 s
(c) 50 s
(d) 100 s

28. Two condensers of capacity 4 μF and 6 μF are connected in series. A potential difference of 500 V is applied between the outer plates of the compound capacitor. The numerical value of charge on each condenser will be

(a) 1200 μC
(b) 1200 C
(c) 6000 C
(d) 6000 C

29. Two parallel wires are suspended in vacuum. When the potential difference between the wires is 30 V then the charge on the wires is 104 μC. The capacitance of the system of wires will be

(a) 3.48 μF
(b) 5 μF
(c) 10.21μF
(d) 50 μF

30. Two capacitors C_1 and C_2 are connected in parallel. If a charge Q is given to the combination, the charge gets charged. Then the ratio of charge on C_1 to charge on C_2 is

(a) $\dfrac{1}{C_1 C_2}$
(b) $C_1 C_2$
(c) C_2 / C_1
(d) C_1 / C_2

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31. A parallel-plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved further apart by means of insulating handles

(a) the charge on capacitor increases

(b) the capacitance increases

(c) the voltage across the plates increases

(d) the electrostatic energy stored in the capacitor increases

32. A parallel-plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are give by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space betweenn the plates with battery still in connection. The corresponding quantities now given by Q, V, E and U are related with previous ones as

(a) $V > V_0$ (b) $U > U_0$

(c) $Q > Q_0$ (d) $E > E_0$

33. A parallel-plate condenser of plate area A and separation d is charged to potential V and then the battery is removed. Now a slab of dielectric constant k is introduced between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively the magnitude of charge on each plate, the electric field between the plates (after introduction of dielectric slab) and work done on the system in the process of introducing the slab, then

(a) $W = \dfrac{\varepsilon_0 A V h 2}{2d}(1 - 1/k)$ (b) $Q = \dfrac{\varepsilon_0 KAV}{d}$

(c) $Q = \dfrac{\varepsilon_0 AV}{d}$ (d) $E = \dfrac{V}{kd}$

34. An uncharged parallel-plate capacitor having a dielectric of constant k is connected to a similar air filled capacitor charged to a potential V. The two share the charge and the common potential is V'. The dielectric constant k is

(a) $\dfrac{V - V'}{V'}$ (b) $\dfrac{V' - V}{V'}$

(c) $\dfrac{V' - V}{V' + V}$ (d) $\dfrac{V' - V}{V}$

35. The plates of a capacitor are charged to a potential difference of 100 V and are then connected across a resistor. The potential difference across the capacitor decays exponentially with time. After 1 second the potential difference between the plates of the capacitor is 80 V. The fraction of the stored energy which has been dissipated is

(a) 1/5   (b) 1/25

(c) 9/25   (d) 15/25

36. Two capacitor $6\mu F/200\mu F$ and $1\mu F/60\ V$ are connected in series. The maximum emf which can be applied is

(a) 260 V (b) 200 V

(c) 70 V (d) none of these

37. The plates of a parallel-plate capacitor are not exactly parallel. The surface charge density, thererfore,

(a) is smallest where the plates are closest

(b) is higher at the closer end

(c) will not be uniform

(d) each plate will have the same potential at each point

38. A 1 mm thick paper of dielectric constant 4 lies between the plates of a parallel-plate capacitor. It is charged to 100 volt. The intensity of electric field between the plates of the condenser will be

(a) 100   (b) 100000

(c) 400000   (d) 25000

39. The plates of a parallel-plate condenser are being moved away with velocity v. If the plate separation at any instant of time is d then the rate of change of capacitance with time is proportional to

(a) d^2 (b) d

(c) d^{-2} (d) d^{-1}

40. A condenser of capacitance C_1 is charged to V_0 volt. The energy stored in it is U_0. It is connected in parallel to another uncharged condenser of capacitance C_2. The energy loss in the process is

(a) $\dfrac{C_1 C_2 U_0}{2(C_1 + C_2)}$ (b) $\left(\dfrac{C_1 - C_2}{C_1 + C_2}\right)^2 U_0$

(c) $\dfrac{C_1 U_0}{C_1 + C_2}$ (d) $\dfrac{C_2 U_0}{C_1 + C_2}$

41. A $4\mu F$ capacitor is charged to 50 V and another capacitor of 2 μF is charged to 100 V. The two condensers are connected such that their likely charged plates are connected together. The total energy of the system before and after joining will be in multiple of $10^{-2}\ J$.

(a) 3.0 and 2.67   (b) 2.67 and 3.0

(c) 1.5 and 1.33   (d) 1.33 and 1.5

42. A parallel-plate condenser of capacitance C is connected to a battery and is charged to potential V. Another condenser of capacity 2 C is connected to another battery and is charged to potential 2 V. The charging batteries are removed and now the condensers are connected in parallel in such a way that the positive plate of one is connected to negative plate of another. The final energy of this system is

(a) 25 CV^2/6   (b) 9 CV^2/2

(c) 3 CV^2/2   (d) zero

43. Two condensers, each of capacitance 1 μF, are connected in parallel. These are charged by a *DC* source of 200 volt. The total energy of their charges in Joule will be

(a) 0.06   (b) 0.04

(c) 0.02   (d) 0.01

44. On increasing the plate separation of a charged condenser its energy

(a) remains unchanged   (b) decreases

(c) increases   (d) none of these

45. The force of attraction between the plates of a charged condenser is

(a) q^2 ($2\varepsilon_0 A$)   (b) q^2 ($2\varepsilon_0 A^2$)

(c) $q/$ ($2\ \varepsilon_0 A^2$)   (d) none of these

46. A parallel plate condenser is connected to a battery of emf 4 volt. If a plate of dielectric constant 8 is inserted into it, then the potential difference on the condenser will be

(a) 32 V (b) 4 V

(c) 1/2 V (d) 2 V

47. A parallel plate condenser with plate separation d is charged with the help of a battery so that V_0 energy is stored in the system. The battery is now removed. A plate of dielectric constant k and thickness d is placed between the plates of condenser. The new energy of the system will be

(a) $V_0 k^{-2}$ (b) $k^2 V_0$

(c) $V_0 k^{-1}$ (d) kV_0

48. The area of each plate of a parallel plate capacitor is 2 m^2. The space between the plates is filled with materials of dielectric constants 2, 3 and 6 and their thickness are 0.4 mm and 1.2 mm respectively. The capacitance of the capacitor will be

(a) $8.94 \times 10^{-4} F$ (b) $6.94 \times 10^{-7} F$

(c) $2.94 \times 10^{-8} F$ (d) $10^{-8} F$

49. The intensity of an electric field between the plates of a charged condenser of plate area A will be

(a) $A / (q\varepsilon_0)$ (b) $qA/\varepsilon 0$

(c) $q/ (\varepsilon_0 A)$ (d) none of these

50. A battery of 100 V is connected to series combination of two identical parallel-plate condensers. If dielectric of constant 4 is slipped between the plates of second condenser, then the potential difference on the condensers will respectively become

(a) 80 V, 20 V (b) 75 V, 25 V

(c) 50 V, 80 V (d) 20 V, 80 V

51. The distance between the plates of a circular parallel plate condenser of diameter 40 mm, in order to make its capacitance equal to that of a metallic sphere of radius 1 m, will be

(a) 0.01 mm   (b) 0.1 mm

(c) 1 mm   (d) 10 mm

52. The minimum number of condensers each of capacitance of 2 μF, in order to obtain resultant capacitance of 5 μF will be

(a) 4   (b) 10

(c) 5   (d) 6

53. A 10 μF condenser is charged to a potential of 100 volt. It is now connected to another uncharged condenser. The common potential reached is 40 volt. The capacitance of second condenser is

(a) 2 μF (b) 10 μF

(c) 15 μF (d) 22 μF

54. When a thin mica sheet is placed between the plates of a condenser then the amount of charge, as compared to its previous value, on its plates will become

(a) unchanged   (b) zero

(c) less   (d) more

55. When dielectric medium of constant k is filled between the plates of a charged parallel-plate condenser, then the energy stored becomes, as compared to its previous value,

(a) k^{-2} times   (b) k^{-2} times

(c) k^{-1} times   (d) k times

56. A capacitor of capacitance C is connected to battery of emf V_0. Without removing the battery, a dielectric of strength ε_r is inserted between the parallel plates of the capacitor C, then the charge on the capacitor is

(a) CV_0 (b) $\varepsilon_r \ CV_0$

(c) $\dfrac{CV_0}{\varepsilon_r}$ (d) none of these

57. The capacitance of conducting metallic sphere will be 1μF if its radius is nearly

(a) 1.11 cm   (b) 10 cm

(c) 1.11 cm   (d) 9 km

58. The potential differene between the plates of a condenser of capacitance 0.5μF is 100 volt. It is connected to an uncharged condenser of capacity 0.2μF by a copper wire. The loss of energy in this process will be

(a) 0 J   (b) 0.5×10^{-3} J

(c) 0.7×10^{-3} J   (d) 10^{-3} J

59. The electric energy density between the plates of charged condenser is

(a) $q/2\varepsilon_0 A^2$ (b) $q/2\varepsilon_0 A$

(c) $q^2/(2\varepsilon_0 A^2)$ (d) none of these

60. Farad is not equivalent to

(a) CV^2 (b) J/V^2

(c) Q^2/J (d) Q/V

61. The energy stored between the plates of a condenser in *not* represented by

(a) $U = \dfrac{CV^2}{2}$ (b) $U = 2\,qV$

(c) $U = \dfrac{q^2}{2C}$ (d) $U = \dfrac{qV}{2}$

62. The capacitance of a spherical conductor of radius r is proportional to

(a) $1/r$ (b) r

(c) $1/r^2$ (d) r^2

63. The net charge on a condenser is

(a) infinity (b) $q/2$

(c) $2q$ (d) zero

64. Two charged conducting spheres are joined by a conducting wire then

(a) nothing will be conserved

(c) the total energy will be conserved

(c) the total charge will be conserved

(d) the total charge and energy will be conserved

65. The capacitance of a charged condenser is C and energy stored on account of charge on it is U, then the quantity of charge on the condenser will be

(a) $\sqrt{2UC}$ (b) $\sqrt{\dfrac{UC}{2}}$

(c) $2\,UC$ (d) zero

66. A 100 μF capacitor is charged to 200 volt. It is discharged through a 2 ohm resistance. The amount of heat generated will be

(a) 0.4 J (b) 0.2 J

(c) 2 J (d) 4 J

67. The capacitance of a condenser is 20 μF and it is charged to a potential of 2000 V. The energy stored in it will be

(a) zero (b) 40 J

(c) 80 J (d) 120 J

68. If the diameter of earth is 128×10^2 km then its capacitance will be

(a) $711\mu F$ (b) $331\mu F$

(c) $211\mu F$ (d) $111\mu F$

69. A condenser is charged to a potential difference of 200 volts as a result of which it gains charge of 0.1 coulomb. When it is dischargd then the energy released will be

(a) 1 J (b) 2 J

(c) 10 J (d) 20 J

70. The capacitance of a parallel plate capacitor in air is 2 μF. If dielectric medium is placed between the plates then the potential difference reduces to 1/6 of the original value. The dielectric constant of the medium is

(a) 6 (b) 3

(c) 2.2 (d) 4.4

71. When two condensers of capacitance $1\mu F$ and $2\,\mu F$ are connected is series then the effective capacitance will be

(a) $\dfrac{2}{3}\mu F$ (b) $\dfrac{3}{2}\,\mu F$

(c) $3\,\mu F$ (d) $4\,\mu F$

72. What will be area of pieces of paper in order to make a paper condenser of capacitance 0.04 μF, if the dielectric constant of paper is 2.5 and its thickness is 0.025 mm?

(a) 1m^2 (b) $2 \times 10^{-3}\,\text{m}^2$

(c) $4.51 \times 10^{-3}\,\text{m}^2$ (d) $10^{-3}\,\text{m}^2$

73. Three condensers each of capacitance 2 F, are connected in series. The resultant capacitance will be

(a) $6\,F$ (b) $5\,F$

(c) $2/3\,F$ (d) $3/2\,F$

74. Which material sheet should be placed between the plates of a parallel plate condenser in order to increase its capacitance?

(a) mica (b) copper

(c) tin (d) iron

75. Three condensers of capacity 2 μF, 4 μF and 8 μF respectively, are first connected in series and then connected in parallel. The ratio of equivalent capacitances in two cases will be

(a) $7:3$ (b) $49:4$

(c) $3:7$ (d) $4:49$

76. A conducting hollow sphere of radius 0.1 m is given a charge of 10 μC. The electric potential on the surface of sphere will be

(a) zero (b) $3 \times 10^5\,V$

(c) $9 \times 10^5\,V$ (d) $9 \times 10^9\,V$

77. The capacitance of parallel-plate capacitor is $4\mu F$. If a dielectric material of dielectric constant 16 is placed between the plates then the new capacitance will be

(a) 1/64 μF (b) 0.25 μF

(c) 64 μF (b) 40 μF

78. The energy acquired by a charged particle of 4 μC when it is accelerated through a potential difference of 8 volt will be

(a) 3.2×10^{-7} J   (b) 3.2×10^{-5} J

(c) 2×10^{-6} J   (d) 2×10^{-5} J

79. Two parallel-plate condensers of capacitance of 20 μF and 30 μF are charged to the potential of 30 V and 20 V respectively. If likely-charged plates are connected together then the common potential difference will be

(a) 10 V (b) 24 V

(c) 50 V (d) 100 V

80. 64 water drops having equal charges combine to form one bigger drop. The capacitance of bigger drop, as compared to that of smaller drop will be

(a) 4 times   (b) 8 times

(c) 16 times   (d) 64 times

PASSAGE 1

Read the following passage and answer the questions given at the end.

On your first day as an electrical technician you are asked to determine the leakage current in a capacitor. The capacitor consists of two parallel-plates. The plates have equal and opposite charge Q. The dielectric has dielectric constant K and a resistivity ρ. The plates have area A and separation between the two plates is d.

1. The leakage resistances is

(a) $R = \dfrac{\rho A}{d}$ (b) $R = \dfrac{\rho d}{A}$

(c) $R = \dfrac{\rho d}{KA}$ (d) $\dfrac{\rho d^2}{A} K$

2. The leakage current is

(a) $\dfrac{Q}{K\rho\varepsilon_0}$ (b) $\dfrac{QK}{\rho\varepsilon_0}$

(c) $\dfrac{Q}{\rho\varepsilon_0}$ (d) $\dfrac{Q\varepsilon_0}{\rho K}$

Solution   **1.** (b) $R = \dfrac{\rho d}{A}$

Solution   **2.** (a) $Q = CV$

or $V = \dfrac{Q}{C}$

$$= \frac{Q}{\dfrac{KA\varepsilon_0}{d}} = \frac{Qd}{KA\varepsilon_0}$$
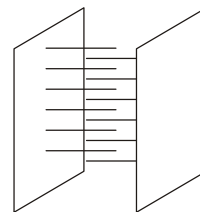

Fig. 20.70

$$I = \frac{V}{R}$$

$$= \frac{Qd}{KA\varepsilon_0 \dfrac{\rho d}{A}}$$

$$= \frac{Q}{K\rho\varepsilon_0}$$

PASSAGE 2

Read the following passage and answer the questions given at the end.

A fuel gauge uses a capacitor to determine the height of the fuel in the tank. The effective dielectric constant k_{eff} changes from a value of 1 when the tank is empty to a value of K, the dielectric constant of the fuel, when the tank is full. The appropriate electronic circuitry can determine the effective dielectric constant of the combined air and fuel between the capacitor plates. Each of the two rectangular plates has a width w and a length L (Fig 20.71). The height of the fuel between the plates is h. You can ignore any fringing effects.
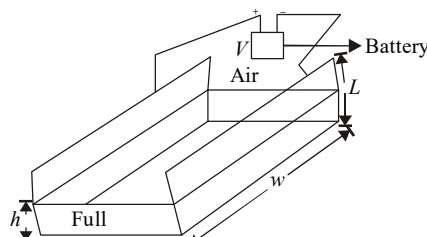

Fig. 20.71

1. The electronic gadget which senses the fuel height is called

(a) level tester   (b) strain gauge

(c) transducer   (d) capacitor

2. What is k_{eff} as a function of h?

(a) $\dfrac{(K-1)h}{L} + 1$ (b) $\dfrac{(K-1)h}{L}$

(c) $\dfrac{(K-1)h}{L} - 1$ (d) $\dfrac{(K+1)h}{L} - 1$

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3. When the tank is $\frac{3}{4}$ full $k_{eff} = $ .....

(a) $\frac{3K}{4}$

(b) $K - \frac{1}{4}$

(c) $(K-1)\frac{3}{4}$

(d) $\frac{K+1}{4}$

Solution **1.** (c)

Solution **2.** (a) $C = C_1 + C_2$

$$= \frac{k\varepsilon_0 wh}{d} + \frac{\varepsilon_0 w(L-h)}{d}$$

$$= \frac{\varepsilon_0 w}{d}[kh + L - h]$$

$$= \frac{\varepsilon_0 w}{d}[(k-1)h + L]$$

$$k_{eff} = \left[\frac{(k-1)h}{L} + 1\right]$$

Solution **3.** (d) If $h = \frac{3}{4}$, $L = \frac{k+1}{4}$

PASSAGE 3

Read the following passage and answer the questions given at the end.

The parallel-plate air capacitor shown in Fig 20.72 consists of two horizontal conducting plates of equal area A. The bottom plate rests on a fixed support and the top plate is suspended by four springs with spring constant k, positioned at each of the four corners of the top plate as shown in Fig. When uncharged, the plates are seprated by a distance z_0. A battery is connected to the plates and produces a potential diff erence V between them. This causes the plate separation to decrease to z. Neglect any fringing effects.
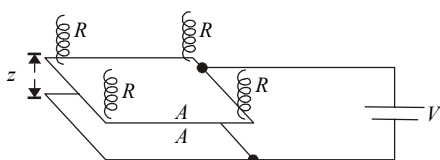

Fig. 20.72

1. Find the electrostatic force between the plates.

(a) $\frac{\varepsilon_0 A^2 V^2}{z^2}$

(b) $\frac{\varepsilon_0 A^2 V^2}{2z^2}$

(c) $\frac{\varepsilon_0^2 A^2 V^2}{2z^2}$

(d) $\frac{\varepsilon_0 A^2 V}{2z}$

2. Find the relation between potential difference V and separation between the plates z.

(a) $V^2 = \frac{Kz(z_0 - z)^2}{A\varepsilon_0}$

(b) $V^2 = \frac{2Kz(z_0 - z)^2}{A\varepsilon_0}$

(c) $V^2 = \frac{4Kz(z_0 - z)^2}{A\varepsilon_0}$

(d) $V^2 = \frac{Kz(z_0 - z)^2}{4A\varepsilon_0}$

3. Find the condition of stable equilibrium.

(a) z shall have positive root

(b) z shall have negative root

(c) no dependence on value of z

(d) none of these

Solution **1.** (b) $F = \frac{\sigma Q}{2\varepsilon_0}$

$$= \frac{Q^2}{2A\varepsilon_0} = \frac{C^2 V^2}{2A\varepsilon_0}$$

$$= \left(\frac{A\varepsilon_0}{z}\right)^2 \frac{V^2}{2A\varepsilon_0} = \frac{\varepsilon_0 A^2 V^2}{2z^2}$$

Solution **2.** (c) Using $\frac{1}{2} CV^2 = \frac{1}{2} kx^2$

or $$= \frac{1}{2}\frac{A\varepsilon_0 V^2}{z} = \frac{4}{2} k(z_0 - z)^2$$

or $$V^2 = \frac{4K}{A\varepsilon_0} z(z_0 - z)^2$$

Solution **3.** (b)

Answers to Questions for Practice

1.	(a, c, d)	**2.**	(a, b, c)	**3.**	(b, d)	**4.**	(a, b, c)	**5.**	(b, c)	**6.**	(c, d)	**7.**	(b, c)
8.	(b, c, d)	**9.**	(a, d)	**10.**	(a, c)	**11.**	(a, b, c, d)	**12.**	(a, d)	**13.**	(a, b, c, d)	**14.**	(a, b, c, d)
15.	(a, b)	**16.**	(a, c, d)	**17.**	(a, b, d)	**18.**	(a, b, d)	**19.**	(a)	**20.**	(a)	**21.**	(c)
22.	(d)	**23.**	(c)	**24.**	(d)	**25.**	(a)	**26.**	(a)	**27.**	(d)	**28.**	(a)
29.	(a)	**30.**	(d)	**31.**	(c, d)	**32.**	(b, c)	**33.**	(a, b, d)	**34.**	(a)	**35.**	(c)
36.	(c)	**37.**	(b, c, d)	**38.**	(d)	**39.**	(c)	**40.**	(d)	**41.**	(c)	**42.**	(c)
43.	(b)	**44.**	(c)	**45.**	(a)	**46.**	(b)	**47.**	(c)	**48.**	(c)	**49.**	(c)
50.	(a)	**51.**	(b)	**52.**	(a)	**53.**	(c)	**54.**	(a)	**55.**	(c)	**56.**	(b)
57.	(d)	**58.**	(c)	**59.**	(c)	**60.**	(a)	**61.**	(b)	**62.**	(b)	**63.**	(d)
64.	(c)	**65.**	(a)	**66.**	(c)	**67.**	(b)	**68.**	(a)	**69.**	(c)	**70.**	(a)
71.	(a)	**72.**	(c)	**73.**	(c)	**74.**	(a)	**75.**	(d)	**76.**	(c)	**77.**	(c)
78.	(b)	**79.**	(b)	**80.**	(a)									

EXPLANATION

2. (a,b,c) When either A or C is earthed (but not both together), a parallel-plate capacitor is formed with B, with $\pm Q$ charges on the inner surfaces. [The other plate, which is not earthed, plays no role]. Hence charge of amount $+Q$ flows to the earth.

When both are earthed together, A and C effectively become connected. The plates now form two capacitors in parallel, with capacitances in the ratio $1 : 2$, and hence share charge Q in the same ratio.

3. (b,d) As A and C are earthed, they are connected to each other. Hence '$A + B$' and '$B + C$' are two capacitors with the same potential difference. If B is closer to A than to C then the capacitance $C_{AB} > C_{BC}$. The upper surface of B will have greater charge than the lower surface. As the force of attraction between the plates of a capacitor is proportional to Q^2, there will be a net upward force on B. This can balance its weight.

6. (c,d) The capacitances of two are $C_1 = 4\pi\varepsilon_0 R$ and $C_2 = 4\pi\varepsilon_0(2R)$.

The initial energy, $E_i = \dfrac{Q^2}{2C_1}$.

The final energy, $E_f = \dfrac{Q^2}{2C_2}$.

The heat produced $= E_i - E_f$

$$= \frac{Q^2}{2}\left[\frac{1}{4\pi\varepsilon_0 R} - \frac{1}{2\times 4\pi\varepsilon_0 R}\right]$$

$$= k\frac{Q^2}{2R}\left[1 - \frac{1}{2}\right] = \frac{kQ^2}{4R}.$$